Investigation of the low-lying energy levels structure of the rich neutron isotopes $^{96-104}Mo$.

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Abstract

The low-lying levels structure, electric quadrupole transitions and the potential energy surfaces of the rich neutron $^{96-104}Mo$ nuclei have been studied using the Interacting Boson Model-1 (IBM-1).The agreement between theoretical prediction and experiment values were fairly good.

The results obtained and the values of parameters used in this calculations indicated that $^{96-100}Mo$ isotopes have a vibrational properties with the pairing effect which is starting from ^{100}Mo to ^{104}Mo isotopes while the quadrupole term appear in the last one as a acute competitor to the pairing force. The potential energy surface show smooth transition from vibration to gamma soft and finally to rotational like nuclei.

الخلاصة:

تمت دراسة مستويات الطاقة الواطئة والانتقالات رباعية القطب الكهربائية وسطوح تساوي الجهد لنوى $^{94-104}Mo$ الغنية بالنيترونات باستخدام نموذج البوزونات المتفاعلة الأول IBM-1.

التوقعات النظرية والقيم العملية متوافقة بوضوح .النتائج وقيم المتغيرات المستعملة في هذه الحسابات تشير إلى أن نظائر $^{96-100}Mo$ تمتلك صفات اهتزازية مع تأثير الازدواج الذي يبدأ من نظير ^{100}Mo إلى نظير ^{104}Mo في حين أن تأثير الحد رباعي القطب يبدو في النواة الاخيرة كمنافس شديد لقوة الازدواج .

ان سطوح تساوي الجهد تبين انتقال سلس من الاهتزازي إلى (Gamma soft) وأخيرا إلى النوى الشبيهة بالدورانية .

1. Introduction:

The IBM was introduced in 1974 by F. Iachello and A. Arima, it has been successfully applied to a wide range of nuclear collective phenomena[1-3]. A model of the atomic nucleus has to be able to describe nuclear properties such as spins and energies of the lowest levels, decay probabilities for the emission of gamma quanta's probabilities (spectroscopic factors) of transfer reactions, multipole moments and so forth.

The interacting boson model (IBM) is suitable for describing intermediate and heavy atomic nuclei. Adjusting a small number of parameters, it reproduces the majority of the low-lying states of such nuclei.

The IBM is based on the well-known shell model and on geometrical collective models of the atomic nucleus. Despite its relatively simple structure, it has proved to be a powerful tool. In addition, it is of considerable theoretical interest since it shows the dynamical symmetries of several nuclei, which are made visible using Lie algebras.

The essential idea is that the low energy collective degrees of freedom in nuclei can be described by proton and neutron bosons with spins of 0 and 2. These collective building blocks interact. Different choices of L=0 (s-boson) and L=2 (d-boson) energies and interaction strengths give rise to different types of collective spectra[4].

These bosons are interpreted as correlated pairs of protons and correlated pairs of neutrons in the valence shell. This interpretation places restriction on the boson number which is determined by counting the number of particle pairs (separately for protons and neutrons) if the shell is less than half filled. And by counting the number of hole pairs if the shell is more than half filled. If the bosons of neutrons and the bosons of protons were considered identical then the interacting boson model is in its simplest form which is called IBM-1[5]

The region of neutron –excess nuclei at mass near A=100 is an area of interest to many authors because of the observation of the phase transition (from spherical to well deformed nuclei and from well deformed to gamma soft). In the previous works [6] W. Luo and Y. Chen were studied the triaxial motion in Mo isotopes where the nuclear shapes of transitional Mo isotopes were calculated by means of a model based on the cranking approximation and the Strutinksy method.The extended level structure of $^{104-108}$Mo isotopes, with *g*-, γ- and the possible $K^{\pi} = 4^{+}$ 2γ phonon band structures have been studied in the interacting boson model-1by J . Gupta [7]. In 2004, S. Lalkovski and N. Minkov , describe the ground band in the even-even neutron-rich nuclei with 40 <Z< 50 using the interacting boson model [8]. M. Zielińska et al were studying the shape coexistence in even–even Mo isotopes by Coulomb excitation[9].

The Model Operators

The Hamiltonian of IBM-1[4] used is

$$H = \varepsilon\ \hat{n}_d + a_0\hat{P}^{\dagger}\hat{P} + a_1\hat{L}^{\dagger}\hat{L} + a_2\hat{Q}^{\dagger}\hat{Q} + a_3\hat{T}_3^{\dagger}\hat{T} + a_4\hat{T}_4^{\dagger}\ \hat{T}_4^{\dagger} \ldots.(1)$$

Where ε is the boson energy, the parameters a_i's designate the strengths of the, pairing, angular momentum, quadrupole, octupole, and hexadecapole interaction between bosons respectively. Where

$$\hat{n}_d = (d^{\dagger}d),\ \ \hat{P} = \frac{1}{2}\left[\left(d^{\dagger}.d\right)-\left(s^{\dagger}.s\right)\right],\ \ \hat{L} = \sqrt{10}\left(d^{\dagger}\times d\right)^{(1)}$$

$$\hat{Q} = \left(d^{\dagger}\times s + s^{\dagger}\times d\right)^{(2)} + \chi(d^{\dagger}\times d)^{(2)}\ \ and\ \hat{T}_l = \left(d^{\dagger}\times d\right)^{(l)}, l = 3,4$$

The $(s.d)$ and $(s^{\dagger}.d^{\dagger})$are the creation and annihilation operators of s and d. A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic transitions. The electric quadrupole transition B(E2) operator in the IBM-1has the form.

$$\hat{T}^{E2} = \alpha_2[d^{\dagger}\times \mathrm{s} + \mathrm{s}^{\dagger}\times \mathrm{d}]^{(2)} + \beta_2[d^{\dagger}\times d]^{(2)} ...(2)$$

The parameters α_2 and β_2 are adjusted to fit the experimental data. The classic limit of IBM-1 Hamiltonian can be obtained through the IBM coherent intrinsic state (boson condensate) introduced in references [10-12].

The geometric properties of interacting boson model are particularly important since they allow one to relate this model to the description of collective states in nuclei by shape variables . It is more convenient to use in the discussion of the geometric properties of the interacting boson model anther set of coherent states the projective states .These were introduced by Bore and Mottelson ,Gnocchio and Kirson and Dieperink ,Schollton and Iachello [10,11,13]

The corresponding variables ,α_{μ} , have a straightforward connection with the shape variable of Bohr and Mottelson .

The projective coherent states which also call intrinsic states ,are defined as

$$\left|N;\alpha_{\mu}\right\rangle = \left(S^{\dagger} + \sum \alpha_{\mu} d_{\mu}^{\dagger}\right)^{N}|0\rangle ......(3)$$

Where α_{μ} are five complex variables ,

Instead of using the five variables α_{μ} one can use the three euler angles (θ_1, θ_2 ,θ_3) defining the orientation in space of an intrinsic frame ,and two intrinsic variables β,γ, . The variables α_{μ} are related to the intrinsic variables by

$$\alpha_{\mu} = \sum a_{\nu} D_{\mu\nu}^{(2)}(\Omega)\ \ldots.\ (4)$$

Where Ω= (θ_1, θ_2 ,θ_3) and $\mathcal{D}_{\mu\nu}^{(2)}$ represents a Wigner $\mathcal{D}$ -function , and

$$a_0 = \beta\cos\gamma$$
$$a_{\pm 2} = 1/\sqrt{2}\beta\sin\gamma$$
$$a_{\pm 1} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The Euler angles θ_1, θ_2 ,θ_3 are shown in fig.1 in term of intrinsic variables β,γ the intrinsic state Fig. (1),can be written as $|N;\beta,\gamma\rangle = \{S^\dagger + \beta[\cos\gamma d_0^\dagger + 1/\sqrt{2}\sin\gamma(d_{+2}^\dagger + d_{-2}^\dagger)]\}^N|0\rangle$......(6)

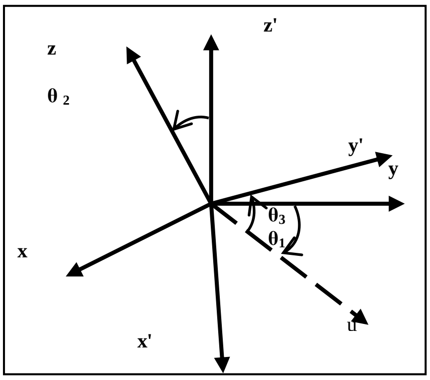

Fig.(1) the Euler angles θ_1,θ_2,θ_3.

The essential concepts for nuclear structure defines which the so called potential energy surface which represents potential energy of nucleon as a function for given factors β, γ for the relations[5].

$$E(N;\beta,\gamma) = \frac{\langle N;\beta,\gamma|H|N;\beta,\gamma\rangle}{\langle N;\beta,\gamma|N;\beta,\gamma\rangle} \text{................(7)}$$

where H is the interacting boson model Hamiltonian , and by minimizing $E(N;\beta,\gamma)$ with respect to β, γ ,

$$\frac{\partial E}{\partial \beta} = 0; \qquad \frac{\partial E}{\partial \gamma} = 0 \qquad \text{...............(8)}$$

this procedure gives the equilibrium 'shape ' corresponding to the boson Hamiltonian H [14]. found

$$E(N;\beta,\gamma) = E_0 + \frac{N}{(1+\beta^2)}(\varepsilon_S + \varepsilon_d\beta^2) + \frac{N(N-1)}{(1+\beta^2)^2}(f_1\beta^4 + f_2\beta^3\cos 3\gamma + f_3\beta^2 + \frac{1}{2}u_0)$$

with

$$f_1 = \frac{1}{10}c_0 + \frac{1}{7}c_2 + \frac{9}{35}c_4,$$

$$f_2 = -2(\frac{1}{35})^{\frac{1}{2}}v_2$$

$$f_3 = (\frac{1}{5})^{\frac{1}{2}}(v_0 + u_2) \text{..............................(9)}$$

N the boson number ,β is the magnitude of nuclear deformation γ is asymmetry angle and its value is between (0^0 and 60^0) f_1, f_2 ,f_3 and f_4 are coefficients conduct potential surface function .
Then the potential energy surface equation for the three symmetries can be given by the following equations [5]

$$E^{(I)}(N;\beta,\gamma) = E_0 + \varepsilon_d N\frac{\beta^2}{1+\beta^2} + f_1 N(N-1)\frac{\beta^4}{(1+\beta^2)^2}$$

$$E^{(II)}(N;\beta,\gamma) = E_0 - k^2\left[\frac{N}{(1+\beta^2)}(5+\frac{11}{4}\beta^2) + \frac{N(N-1)}{(1+\beta^2)^2}\times(\frac{\beta^4}{2} + 2\sqrt{2}\beta^3\cos 3\gamma + 4\beta^2)\right]$$

$$-k'\frac{6N\beta^2}{(1+\beta^2)}$$

$$E^{(III)}(N;\beta,\gamma) = E_0 + (2B+6C)\frac{A}{4}N(N-1)(\frac{1-\beta^2}{1+\beta^2})^2 \text{..........................(10)}$$

it is convenient to plot in each case the energy surface as a contour plot in the β-γ plane fig (2) , since E in (6) depends only on cos3 γ, it is sufficient to consider the portion of the β-γ plane with $0^o \le \gamma < 60^o$

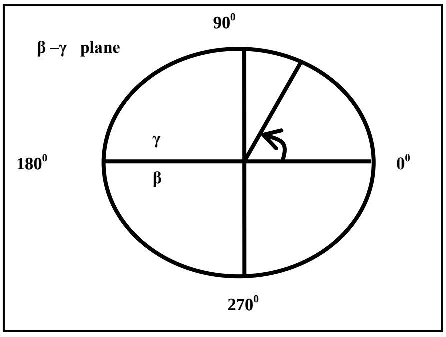

Fig.(2):The β-γ plane.

Calculations and results

Calculations of energy levels for even-even $^{96\text{-}104}$Mo isotopes were performed with the whole Hamiltonian (eq.1) using IBM-1 computer code .

For $^{96\text{-}104}$Mo nuclei (Z=42) have (6-10) bosons formed (4 proton hole bosons and (2-6) neutron particle bosons.

The parameters of equation (1) were calculated from the experimental schemes of these nuclei [15-19] and the analytical solutions for the three dynamical systems (see reference [4]). These parameters were tabulated in table (1) . The calculated and experimental energy levels and the parameters value are exhibit in figures (3-8).The calculations of B(E2) values were performed using computer code "FBEM". The parameters in E2 operator eq.(2) were determined by fitting the experimental B(E2;$2_1^+ \rightarrow 0_1^+$) data [15-21], and the parameters were listed in table(2),where $E2SD = \alpha_2, E2DD = \sqrt{5}\beta_2$ And $\beta_2 = \frac{-0.7}{5}\alpha_2, -\sqrt{7/2}\alpha_2 \; and \; = 0$ in SU(5), SU(3) and O(6) respectively[12]. and the converter coefficient between (e^2b^2) and (W.u) is $B(E2)w.u = \frac{B(E2)e^2b^2}{5.943\times10^{-6} A^{4/3} e^2 b^2}$

the values of the parameters which gave the best fit to experimental [15-19] are given in table (3) for potential energy surface which illustration in fig.(9).

Table (1) The parameters of the Hamiltonian equation and E2 operators used for the description of the $^{96\text{-}104}$Mo isotopes.

Isotope	N_b	ε	a_0	a_1	a_2	a_3	a_4	E2SD	E2DD
		In (MeV)						In unit (e^2b^2)	
^{96}Mo	6	0.5692	0.0	0.0342	0.0	0.0002	0.0009	0.1	-0.07
^{98}Mo	7	0.367	0.0	0.07	0.0	0.0001	0.0	0.1	-0.07
^{100}Mo	8	0.0056	0.0772	0.0	0.0	0.38	0.0	0.1	-0.07
^{102}Mo	9	0.0666	0.12	0.0337	0.0	0.055	0.p05	0.0877	0.0
^{104}Mo	10	0.0	0.0489	0.0198	0.0283	0.0	0.0	0.104	0.0

Table (2) Comparison between present values of B(E2)(in unit e^2b^2) for even-even $^{96-104}$Mo isotopes (Theo.) and experimental ones (Exp.) [15-21]. The quadrupole moment of 2_1^+ state listed in last line.

Transition	^{96}Mo		^{98}Mo		^{100}Mo		^{102}Mo		^{104}Mo	
	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.
$2_1^+ \rightarrow 0_1^+$	0.055	0.06	0.0536	0.07	0.102	0.192	0.2	0.168	0.266	0.269
$2_2^+ \rightarrow 0_1^+$	0.0028	0.0	0.0005	0.0	0.0017	0.0		0.0		0.026
$0_2^+ \rightarrow 2_1^+$	0.133	0.004	0.15	0.0048	0.253	0.0	0.198	0.0		0.0
$2_2^+ \rightarrow 2_1^+$	0.04	0.1	0.006	0.0		0.0009		0.23		0.069
$4_1^+ \rightarrow 2_1^+$	0.1	0.1	0.113	0.12	0.189	0.26	0.25	0.23	0.319	0.38
$0_3^+ \rightarrow 2_1^+$		0.0		0.0		0.0		0.0002		0.0
$2_3^+ \rightarrow 2_2^+$		0.0229		0.0286		0.0		0.0		0.012
$2_3^+ \rightarrow 0_1^+$		0.0		0.0		0.0074		0.0		0.0005
$Q2_1^+$	-0.2	-0.156	-0.26	-0.1565	-0.42 or -0.1	-0.26	-	0.0	-	1.31

Table(3):The parameters obtained from the programs IBMP code for $^{96-104}$Mo isotopes potential energy surface.

Isotope	□$_s$	□$_d$	f_1	f_2	f_3	f_4
	In (MeV)					
^{96}Mo	0.0	0.776	0.0	0.0	0.0	0.0
^{98}Mo	0.0	0.787	0.0	0.0	0.0	0.0
^{100}Mo	0.0	0.537	0.019	0.0	-0.039	0.0
^{102}Mo	0.0	0.437	0.056	0.0	-0.06	0.0
^{104}Mo	0.0	0.088	0.011	0.019	-0.138	0.0

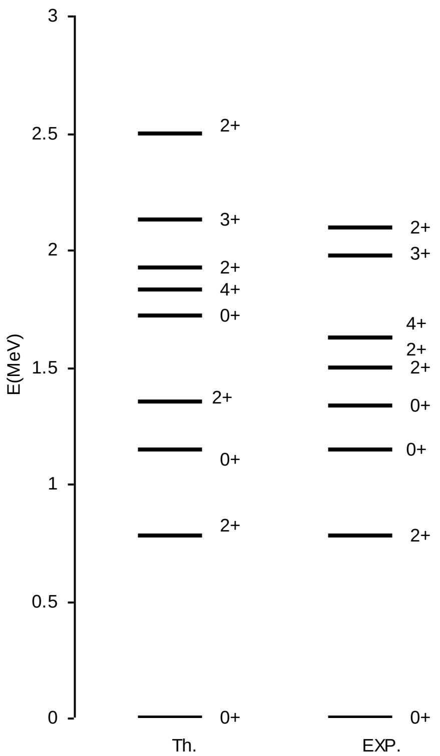

Fig. (3): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{96}Mo[15].

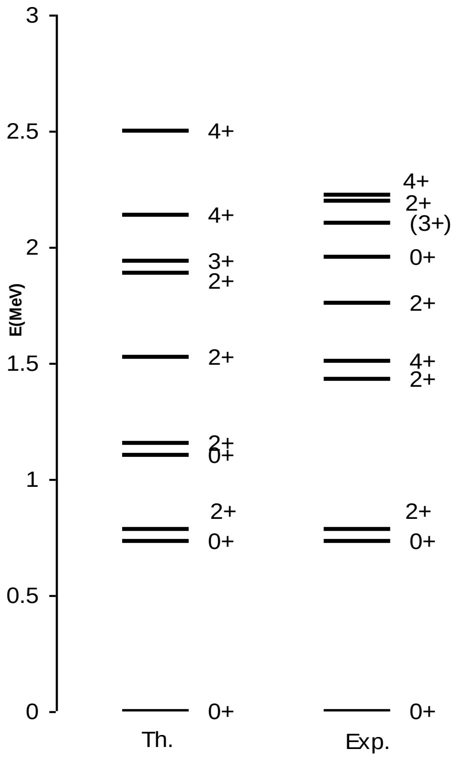

Fig. (4): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{98} Mo[16].

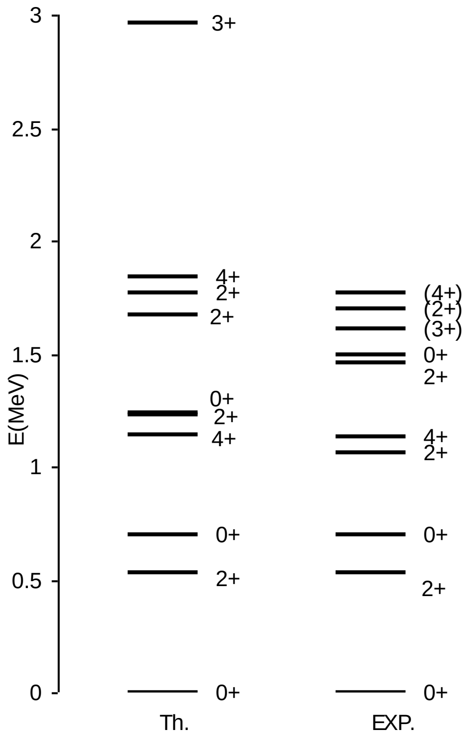

Fig. (5): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{100}Mo[17].

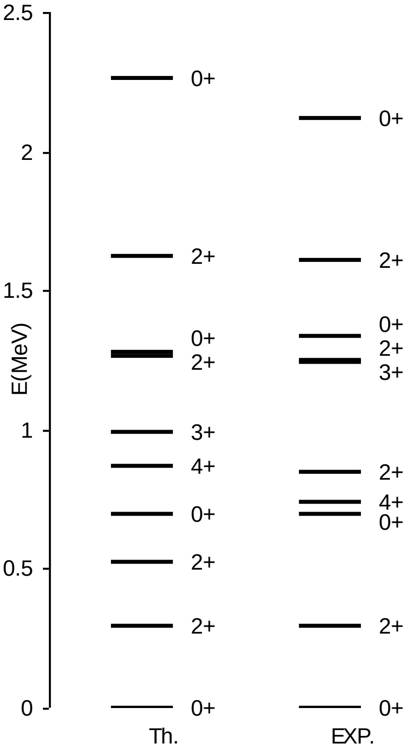

Fig. (6): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{102} Mo[18].

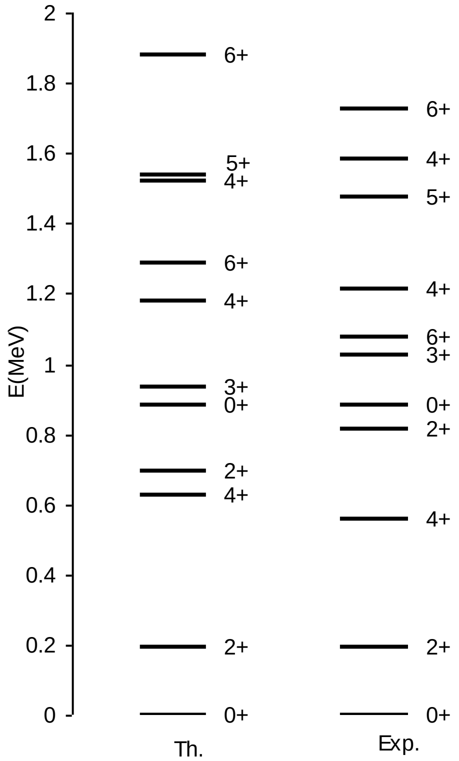

Fig. (7): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{104}Mo[19].

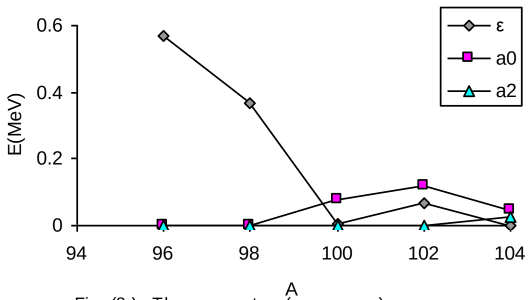

Fig.:(8) The parameters (eps. ,a_0 ,a_2) were calculated from the experimental schemes of $^{96\text{-}104}$Mo

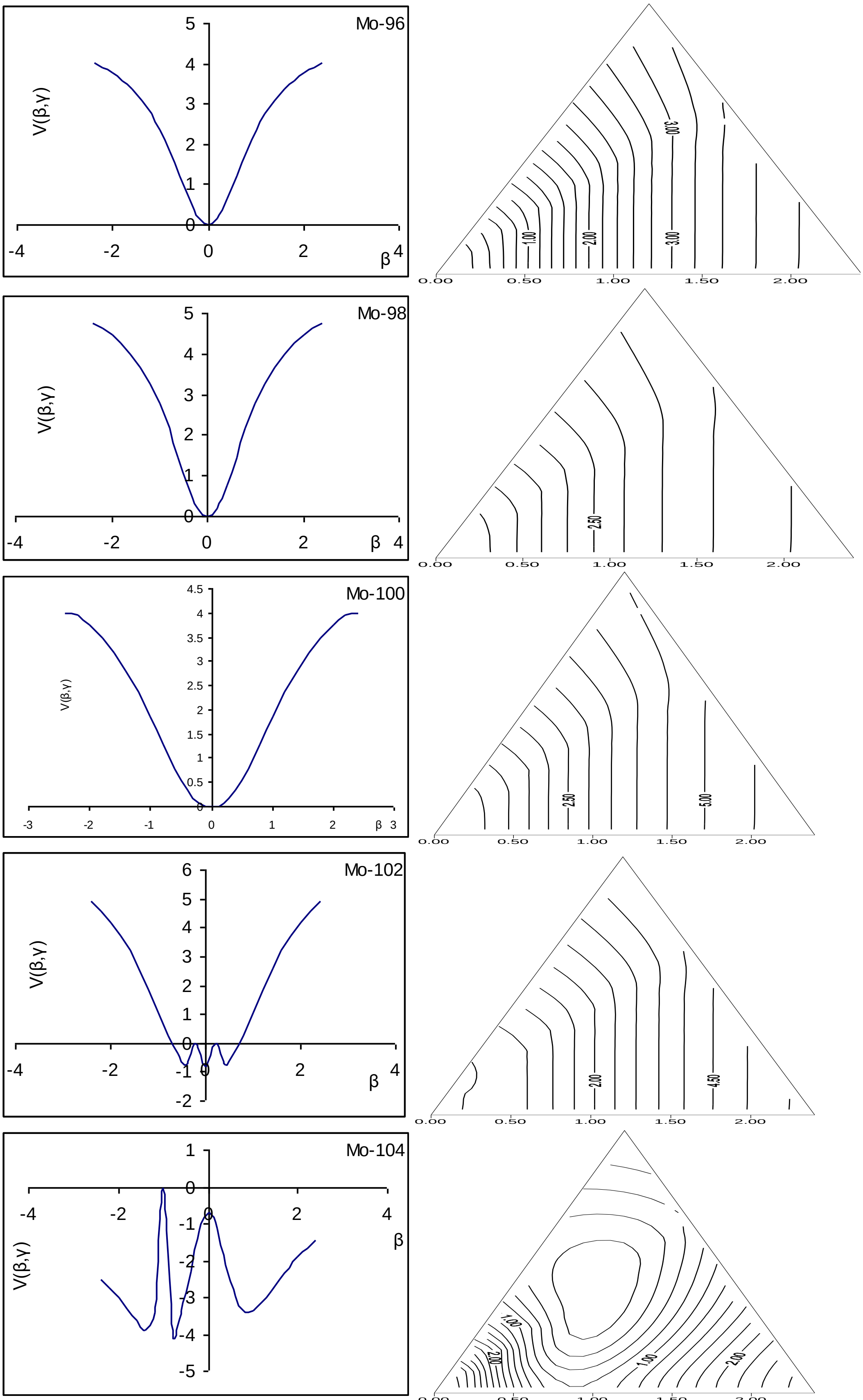

Fig .(9):The energy functional E(N; β,γ) as a function of β and the corresponding β-γ plot for $^{96-104}$Mo isotopes

4.Discussion and Conclusions

In the last decade the neutron rich nuclei in the 40≤ Z≤50 region have attracted both theoretical and experimental attention. Nuclei from this region of Segre chart exhibit vibrational, transitional ,and rotational types of collectivity.

In this paper the chain of $^{96\text{-}104}$Mo isotopes has been analyzed .In this chain nuclei evolve from spherical to deformed shape . We have performed an analysis of the corresponding shape transition to look for possible nuclei at or near deformed. The whole Hamiltonian has been used in the IBM-1 program and the values of parameters as shown in table (1).It seem clear that the value of □ in ^{96,98}Mo is larger than the values of pairing a_0 and quadrupole a_2 parameters ,the pairing effect is starting from ^{100}Mo to ^{104}Mo isotopes while the quadrupole term appear in the last one as a acute competitor to the pairing force, This is obvious in drawing the energy surfaces for these isotopes, where the form of contour lines in $^{96\text{-}100}$Mo (fig. 9) is similar to that of vibrational U(5) one, and the deformation becomes clear in contour lines of ^{102}Mo, while in ^{104}Mo the deformation is obvious, and this is due to the effect of rotation. The difference behavior in the last two can be interpreted if it looks at the neutron distribution in the shells the $^{96\text{-}100}$Mo all occupy the sub-level $1g_{7/2}$, while ^{102}Mo and ^{104}Mo occupy $2d_{5/2}$.

The potential energy surface show smooth transition from vibration to gamma soft and finally to rotational like nuclei.

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