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# Comparing Three Different Estimators of Fuzzy Reliability of Modified Weibull

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**Abstract:** *This paper deals with obtaining two parameters $(p, \theta)$, modified weibull, using entropy transformation, and also the cumulative distribution function, and also reliability function are obtained, and the rth moment formula about origin is derived. The two parameters $(p, \theta)$ estimated by Maximum Likelihood (MLE) and Moments Estimator and Proposed method and, then fuzzy reliability is compared by simulation methods.*

*Taking four size of sample (n= 20, 40, 60, and 80), and different sets of initial values of parameters $(p, \theta)$ as well as fuzzy parameter$(ki)$, and all the results are explained in tables.*

**Keywords: Entropy transformation (ET), Moments estimator (MOM), Maximum Likelihood Estimator (MLE), Fuzzy factor.**

# مقارنة ثلاث مقدرات للمعولية الضبابية لتوزيع ويبل المعدَل

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**المستخلص**

يتناول البحث تقدير المعلمات (p،θ) لتوزيع ويبل المعدل. وقد تم تقدير المعلمات (p،θ) باستخدام ثلاث طرائق وهي الإمكان الأعظم، والعزوم، وطريقة مقترحة وبعد ذلك طبقت تجارب المحاكاة باعتماد أربعة إحجام عينات (n= 20, 40, 60, 80) وكذلك قيمتان للمعلمة الضبابية (ki = 0.4, 0.7) وقيمتان للمعلمة (p= 2, 2.5) وكذلك قيمتان للمعلمة ($\theta$= 0.5, $\theta$ = 0.7) واتضح من الاستنتاجات ان اول مقدر للمعولية الضبابية هو مقدر العزوم ثم المقدر المقترح والمتمثل بالعزوم الموزونة الاحتمالية (PW) وأخيرا مقدر الإمكان الأعظم وقد عرضت نتائج المقارنة في جداول خاصة. وأن اهم النتائج التي تم التوصل لها كانت أن مقدر الامكان الاعظم كان الافضل.

**الكلمات المفتاحية: تحول الانتروبي، مقدر العزوم، مقدر الاحتمالية القصوى، عامل ضبابي.**

## Introduction

The lifetime data from exponential or Rayleigh or Weibull distribution have several desirable properties, and physical interpretations, which enable these distribution to be used frequently. One of these interpretation is to present modified weibull were chitany. In (2006) extenuated linear failure-rate distributions. And apply it in Engineering and information system. We know that weibull distribution is widely employed model for distributions of life spans, and reaction times, because its cumulative distribution function can be expressed explicitly as a simple function of random variables numerous writers, have been studied it extensively choen (1965,1973,1975) and Dubey (1966), Also hareter and Moore (1965,1967), also lemon (1974) and Zanakis and

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Mann(1981). While (Cohen and Betty) work on modified Maximum Likelihood and modified Moment estimators for three – parameters weibull distribution.

Weibull distribution received much interest in reliability theory , and it is used to describe real phenomena and modeling distribution of breaking strength of materials , and also it is used to fit tree diameters data , were this data play an important role in stand model . This distribution was introduced by BAILEY and DELL (1973) as a model for diameter distribution, and has been applied extensively in forestry, since it has ability to describe the wide range of Uniondale distributions. Also Cran in (1988) work on moment estimators for three parameter weibull distribution, while Tsionas . E .G .in (2000) introduced posterior analysis , prediction and reliability in three parameter weibull distribution .also weibull distribution used to describe real phenomena and modeling distribution of breaking strength of materials as indicated by Johnson et al (1994),and also murthy et al (2004) .in (2000) Tsionas , work on estimating Bayesian and also prediction and reliability in three parameters weibull. And Tian. Gives inference about coefficient of variation, in (2015), a group of researcher introduced the transmuted exponential weibull distribution with application were this distribution is flexible and useful for analyzing positive data that have bathtub shaped hazard rate function . the obtained new distribution can be used for modeling positive data in various fields like physical and biological sciences, Reliability theory hydrology , medicine and survival analysis and engineering . also in (2014) Felix Noyanim and et al introduce comparison between different methods for estimating parameters of weibull , using mean square error as a measure for comparison . in (2013) , Mahdi Teimouri and Arjunk. Gupta gives more attention for estimating reliability function for three parameters weibull, and variation and maximum likelihood estimate, and they indicate to Cran (1988) ,when he estimated three parameters by moments of weibull.

## Theoretical Parts

The p.d.f $f_T(p,\theta)$ of two parameter weibull, where $p$: is shape parameter, $\theta$: is scale parameter. Is given in equation (1), and its cumulative distribution function $f_T(t)$ , also is derived, as well as the reliability function, after we give these information we explained how modified weibull is obtained using entropy transformation.
Is defined in equation (1) below;

$$f_T(t,p,\theta) = \frac{p}{\theta^p} t^{p-1}\ e^{-\ (\frac{t}{\theta})^p} \qquad p > 0, \quad \theta > 0, \quad t > 0 \qquad \dots (1)$$

And the corresponding C.D.F. is:

$$F_T(t,p,\theta) = 1 - \ e^{-\ (\frac{t}{\theta})^p} \qquad \dots(2)$$

While the corresponding reliability function$R_T(t)$ is:

$$R_T(t) = \ e^{-\ (\frac{t}{\theta})^p} \qquad \dots(3)$$

While the mean and variance are:

$$E_t(T) = \theta^{\frac{1}{p}}\Gamma\left(1+\frac{1}{p}\right)$$

And the variance

$$V_t(T) = \theta^{\frac{2}{p}}\left[\Gamma\left(1+\frac{2}{p}\right) - \Gamma^2\left(1+\frac{1}{p}\right)\right]$$

Now we apply entropy transformation defined as:

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$$G_T(t) = F_T(t) + R_T(t)\ln R_T(t) \quad \dots (4)$$

To obtain new modified C D F of two parameters modified weibull which is

$$G_T(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^p} + e^{-\left(\frac{t}{\theta}\right)^p}\left(-\left(\frac{t}{\theta}\right)^p\right) \quad \dots(5)$$

$$G_T(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^p}\left[1 + \left(\frac{t}{\theta}\right)^p\right] \quad \dots(6)$$

Then new modified p.d.f two parameters weibull is

$$g_T(t) = \acute{G}_T(t)$$

$$g_T(t) = \frac{p}{\theta}\left(\frac{t}{\theta}\right)^{2p-1} e^{-\left(\frac{t}{\theta}\right)^p} \qquad p > 0, \quad \theta > 0, \quad t > 0 \quad \dots (7)$$

This p.d.f is new modified two parameters weibull , obtained from applying entropy transformation on equation (1) and corresponding new modified C.D.F

$$G_T(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^p}\left[1 + \left(\frac{t}{\theta}\right)^p\right] \quad \dots(8)$$

And the corresponding modified reliability is

$$R_T(ti) = e^{-\left(\frac{ti}{\theta}\right)^p}\left[1 + \left(\frac{t}{\theta}\right)^p\right] \quad \dots(9)$$

After the modified p.d.f of weibull obtained, and also modified C.D.F, and modified reliability function we work on estimating parameters $(p, \theta)$ by different method

## Estimation Parameters $(\mathbf{p}, \boldsymbol{\theta})$:

First all we must to estimate the two parameters of modified weibull by method of Maximum likelihood and second by moments and third, by proposed method as follows:

### 1. Maximum likelihood Estimators (MLE)

Let $t_{1,} t_{2,} t_{3,}, \dots\dots, t_n$ be a random sample from p.d.f in equation (7) then for modified weibull.

$$L = \prod_{i=1}^{n} g_T(ti)$$

$$= p^n \theta^{-2np} \prod_{i=1}^{n} ti^{2p-1} e^{-\Sigma\left(\frac{ti}{\theta}\right)^p} \quad \dots(10)$$

$$\log L = \text{n} \log p - 2np\theta + (2p-1)\sum_{i=1}^{n} \log ti - \sum_{i=1}^{n}\left(\frac{ti}{\theta}\right)^p$$

$$\frac{\partial \log L}{\partial p} = \frac{n}{p} - 2\,n\log\theta + 2\sum_{i=1}^{n}\log ti - \sum_{i=1}^{n}\left(\frac{ti}{\theta}\right)^p (1)\log\left(\frac{ti}{\theta}\right)$$

From $\frac{\partial \log L}{\partial p} = 0$

Then

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$$\frac{n}{\hat{p}} = 2nlog\,\theta - 2\sum_{i=1}^{n}\log ti + \sum_{i=1}^{n}(\frac{ti}{\theta})^{p}\log(\frac{ti}{\theta}) \qquad \ldots(11)$$

From equation (11), we can find the Maximum Likelihood estimator of parameter ($p$) as:

$$\hat{p}_{MLE} = n\Big/\left[2nlog\,\theta - 2\sum_{i=1}^{n}\log ti + \sum_{i=1}^{n}(\frac{ti}{\theta})^{p}\log(\frac{ti}{\theta})\right]$$

And then

$$\frac{\partial \log L}{\partial\theta} = \frac{-\,2\text{np}}{\theta} + p\sum_{i=1}^{n} ti^{p}\,\theta^{-\,p-1}$$

$$\frac{\partial \log L}{\partial\theta} = 0$$

$$\frac{-\,2\text{np}}{\theta} = \frac{p}{\theta^{p+1}}\sum_{i=1}^{n} ti^{p}$$

$$2\text{n} = \frac{\sum_{i=1}^{n} ti^{p}}{\theta^{p}}$$

$$\hat{\theta}_{MLE} = \left(\frac{\sum_{i=1}^{n} ti^{p}}{2n}\right)^{\frac{1}{p}}$$

## 2. Moment Estimators (MOM)

To find moments estimators of new modified weibull , first of all we derive the (r th) moments formula of this modified weibull as :

$$\mu\acute{r} = E(x^{r}) = \int_{0}^{\infty} t^{r} \quad f(t,p,\theta)dt$$

$$\mu\acute{r} = \int_{0}^{\infty} t^{r}\frac{p}{\theta^{\,2\,p}} \; t^{2p-1}e^{-\,(\frac{t}{\theta})^{p}}\,dt$$

$$\mu\acute{r} = \frac{p}{\theta^{\,2\,p}}\int_{0}^{\infty} t^{r} \;\; ti^{r+2p-1}e^{-\,\frac{ti}{\theta}^{\,p}}\,dt$$

Let $Z = (\frac{t}{\theta})^{p}$

$$\theta Z^{\frac{1}{p}} = t$$

$$dt = \frac{\theta}{p}Z^{\frac{1}{p}\,-\,1}dz$$

$$\acute{\mu}_{r=}\frac{p}{\theta^{\,2\,p}}\int_{0}^{\infty} (\theta\, Z^{\frac{1}{p}}\;)^{r+2p-1}\,e^{-\,z}\frac{\theta}{p}Z^{\frac{1}{p}\,-\,1}dz$$

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$$\acute{\mu}_{r=}\theta^r \int_0^\infty Z^{\frac{r}{p}+1}\, e^{-\,z}\ dz$$

$$\acute{\mu}_{r=}\theta^r\ \Gamma\left(\frac{r}{p}+2\right)$$

$$\mu_{1=}\ E(z) =\ \theta\ \Gamma\left(\frac{1}{p}+2\right)$$

While

$$\mu_{2=}\ E(z^2) =\ \theta^2\ \Gamma\left(\frac{2}{p}+2\right)$$

So Variance (T)

$$v\ (T) =\ \theta^2\ \Gamma\left(\frac{2}{p}+2\right) - \theta^2\Gamma^2\left(\frac{1}{p}+2\right)$$

$$= \theta^2\left[\Gamma\left(\frac{2}{p}+2\right) - \Gamma^2\left(\frac{1}{p}+2\right)\right]$$

From $E(T)$ and $E(T^2)$ we can find moments estimators of $(p,\theta)$

From $\frac{\sum_{i=1}^{n} ti}{n} =\ \hat{\theta}\ \Gamma\left(\frac{1}{\hat{p}}+2\right)$

And $\frac{\sum_{i=1}^{n} ti^2}{n} =\ \hat{\theta}^2\ \Gamma\left(\frac{2}{\hat{p}}+2\right)$

$$\hat{\theta}_{Mom} = \frac{\bar{t}}{\Gamma\left(\frac{1}{\hat{p}}+2\right)} \qquad \ldots(12)$$

And $\hat{p}_{Mom}$ obtained using $\hat{\theta}_{Mom}$

Solving equation (12) numerically, to find ( $\hat{\theta}_{Mom}$) , depending on observations( $t_{1,}t_{2,}t_{3,\ldots\ldots\ldots,}t_n$) (for computing $\bar{t}$ ) , (and also on given value of $\hat{p}$ ).

## 3. Proposed Estimators Method (Prop)

Were $\theta$: is scale parameter, and $p$: is shape parameter.

This method depend on using have one scale parameter $(\theta)$ and one shape parameter $(p)$ ,so according to the generated observations we can estimate shape parameter $(p)$ , depending on non parametric estimator of scale parameter $(\theta)$,

Were $\hat{\theta} = Y_{(1)}$ which is min. (observation).

(i. e ) the smallest observation and the shape parameter $(p)$,which is either known or can be calculated from formula (13) ,which is found in Reference (10).

$$\hat{p} = \frac{Ln\ 2}{Ln\ (\ \frac{Median}{\hat{\theta}})} \qquad \ldots(13)$$

Also the two parameters $(p,\theta)$ can be estimated by probability weighted Moments (PWM) as:

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$$\hat{p} = \frac{\hat{\mu}_{1,0\ 1} - \hat{\mu}_{1,0,0}}{(\ 2\hat{\mu}_{1,0,1} - \hat{\mu}_{1,0,0}\ )}$$

Were: $\hat{\mu}_{1,0,0} = \bar{t}\ \ or\ \ \bar{Y}$

And

$$\hat{\mu}_{1,0,1} = \frac{1}{n(n-1)} \sum_{i=1}^{n} (\mathrm{i}-1) Y_{(i)}$$

Were $Y_{(1)} < Y_{(2)} < Y_{(3)}\ \ \ldots\ldots < Y_{(n)}$

And the scale parameter

$$\hat{\theta} = \frac{\bar{t}\ (\ \hat{p} - 1)}{\hat{p}}$$

## Simulation Procedure

The comparison of fuzzy reliability estimators of new modified two parameters weibull is done through simulation, and the smallest $\hat{h}\ (ti\ , p, \theta)$ is considered the best , now we explain the results of simulation by tables .

The C.D.F of new modified two parameters weibull under entropy transformation is

$$G_T(ti) = 1 - \ e^{-\left(\frac{ti}{\theta}\right)^p} - \left(\frac{ti}{\theta}\right)^p \ e^{-\left(\frac{ti}{\theta}\right)^p}$$

$$= 1 - \left(1 + \left(\frac{ti}{\theta}\right)^p\right) \ e^{-\left(\frac{ti}{\theta}\right)^p}$$

Let:

$$U_i = G_T(ti)$$

$$U_i = \ 1 - \left(1 + \left(\frac{ti}{\theta}\right)^p\right) \ e^{-\left(\frac{ti}{\theta}\right)^p}$$

$$\left(1 + \left(\frac{ti}{\theta}\right)^p\right) \ e^{-\left(\frac{ti}{\theta}\right)^p} = (1 - Ui)\ \ , \qquad Zi = (1 - Ui), when\ \ Ui \in (0,1)$$

$$\left(1 + \left(\frac{ti}{\theta}\right)^p\right) \ e^{-\left(\frac{ti}{\theta}\right)^p} = Zi$$

$$1 + \left(\frac{ti}{\theta}\right)^p = Zi\ \ e^{-\left(\frac{ti}{\theta}\right)^p}$$

$$\left(\frac{ti}{\theta}\right)^p = Zi\ \ e^{\left(\frac{ti}{\theta}\right)^p} - 1$$

$$\frac{ti}{\theta} = \left[Zi\ \ e^{\left(\frac{ti}{\theta}\right)^p} - 1\right]$$

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And Generation $(ti)$ by using inverse transformation of C.D.F

$$ti = \theta\left[Zi \quad e^{\left(\frac{ti}{\theta}\right)^{p}} - 1\right] \qquad ti > 0\ ,\ p > 0\ ,\ \theta > 0$$

This is the formula for simulation and

$$Zi = 1 - Ui \qquad 0 < Ui \leq 1$$

We give the values of initial values, and the results of simulation

$$\mathrm{h}(ti, \theta, p) = \frac{g_T(ti)}{R_T(ti)}$$

$$= \frac{p\ ti^{2p-1}}{\theta^{2\,p}}$$

Using generated values $(t_i)$ for (n = 20, 40, 60), were $(p, \theta)$ are estimated by methods of Moments, and maximum likelihood and proposed once (L-Moments).

These observations are generated according to different sample size n = (20, 40, 60, 80), and different values for the summary of initial value are given as:

| Experiment | θ | $p$ | $\alpha$ | $Ki$ |
|---|---|---|---|---|
| 1 | 0.7 | 2 | 0.3 | 0.4 |
| 2 | 0.5 | 2.5 | 0.5 | 0.7 |

The following tables give fuzzy reliability estimators of modified weibull using different values of $(t_i)$ and initial values $(p, \theta)\hat{k}i$. Were the comparison is done due to the largest values of fuzzy Reliability estimators , so for each method of estimation , we count ,the Maximum values of Reliability estimators, and then the results of comparison is in tables , and we depend on higher percentage of preference for each estimator.

**Table 1: Fuzzy Reliability Estimator When: $p = 2\ , \alpha = 0.3,\ \theta = 0.3,\ Ki = 0.4$**

| n | $t_i$ | Real | $R_{mle}$ | $R_{mom}$ | $R_{prop}$ | Best |
|---|---|---|---|---|---|---|
| 20 | 1.5 | 0.6012 | 0.5831 | 0.6064 | 0.5726 | Mle |
| | 2.5 | 0.6333 | 0.6335 | 0.6526 | 0.6236 | Mom |
| | 3.5 | 0.6572 | 0.6927 | 0.6826 | 0.6542 | Mle |
| | 4.5 | 0.6899 | 0.7076 | 0.7031 | 0.6774 | Mle |
| | 5.5 | 0.6977 | 0.7236 | 0.7191 | 0.6936 | Mle |
| | 6.5 | 0.7001 | 0.7364 | 0.7318 | 0.7082 | Mle |
| | 7.5 | 0.7092 | 0.7456 | 0.7421 | 0.7183 | Mle |
| | 8.5 | 0.7167 | 0.7525 | 0.7504 | 0.7415 | Mle |
| | 9.5 | 0.7232 | 0.7606 | 0.7573 | 0.7426 | Mle |
| | 10.5 | 0.7342 | 0.7657 | 0.7634 | 0.7832 | Prop |

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| | | | | | | |
|---|---|---|---|---|---|---|
| 40 | 1.5 | 0.6012 | 0.5624 | 0.5344 | 0.5594 | Prop |
| | 2.5 | 0.6333 | 0.6133 | 0.6021 | 0.6683 | Prop |
| | 3.5 | 0.6572 | 0.6354 | 0.6443 | 0.6420 | Mom |
| | 4.5 | 0.6899 | 0.6662 | 0.6734 | 0.6644 | Mom |
| | 5.5 | 0.6977 | 0.6855 | 0.6932 | 0.6623 | Mom |
| | 6.5 | 0.7001 | 0.7004 | 0.7084 | 0.6954 | Mom |
| | 7.5 | 0.7092 | 0.7214 | 0.7233 | 0.6820 | Mom |
| | 8.5 | 0.7167 | 0.7239 | 0.7401 | 0.7066 | Mom |
| | 9.5 | 0.7232 | 0.7365 | 0.7386 | 0.7231 | Mom |
| | 10.5 | 0.7342 | 0.7421 | 0.7452 | 0.7294 | Mom |
| 60 | 1.5 | 0.6012 | 0.5721 | 0.6064 | 0.5526 | Mom |
| | 2.5 | 0.6333 | 0.6226 | 0.6516 | 0.6016 | Mom |
| | 3.5 | 0.6572 | 0.6567 | 0.6826 | 0.6213 | Mom |
| | 4.5 | 0.6899 | 0.6827 | 0.7143 | 0.6672 | Mom |
| | 5.5 | 0.6977 | 0.7034 | 0.7219 | 0.7082 | Mom |
| | 6.5 | 0.7001 | 0.7236 | 0.7329 | 0.7188 | Mom |
| | 7.5 | 0.7092 | 0.7438 | 0.7412 | 0.7274 | Mle |
| | 8.5 | 0.7167 | 0.7464 | 0.7603 | 0.7362 | Mle |
| | 9.5 | 0.7232 | 0.7578 | 0.7574 | 0.7415 | Mle |
| | 10.5 | 0.7342 | 0.7624 | 0.7572 | 0.7416 | Mle |
| 80 | 1.5 | 0.6012 | 0.5726 | 0.5677 | 0.5534 | Mle |
| | 2.5 | 0.6333 | 0.6223 | 0.6283 | 0.6026 | Mle |
| | 3.5 | 0.6572 | 0.6568 | 0.6523 | 0.6352 | Mle |
| | 4.5 | 0.6899 | 0.6822 | 0.6736 | 0.6476 | Mle |
| | 5.5 | 0.6977 | 0.6994 | 0.7024 | 0.6887 | Mom |
| | 6.5 | 0.7001 | 0.7132 | 0.7231 | 0.7006 | Mom |
| | 7.5 | 0.7092 | 0.7167 | 0.7306 | 0.7174 | Mom |
| | 8.5 | 0.7167 | 0.7241 | 0.7372 | 0.7236 | Mom |
| | 9.5 | 0.7232 | 0.7332 | 0.7361 | 0.7245 | Mom |
| | 10.5 | 0.7342 | 0.7441 | 0.7452 | 0.7342 | Mom |

**Table 2: Fuzzy Reliability Estimator When: $p = 2.5$ , $\alpha = 0.3, \theta = 0.7, Ki = 0.4$**

| n | $t_i$ | Real | $R_{mle}$ | $R_{mom}$ | $R_{prop}$ | Best |
|---|---|---|---|---|---|---|
| 20 | 1.5 | 0.5604 | 0.5823 | 0.6062 | 0.0.726 | Mom |
| | 2.5 | 0.6016 | 0.6226 | 0.6506 | 0.6016 | Mom |
| | 3.5 | 0.6333 | 0.6576 | 0.6816 | 0.6662 | Mom |
| | 4.5 | 0.6572 | 0.6827 | 0.7134 | 0.6846 | Mom |
| | 5.5 | 0.6572 | 0.7086 | 0.7235 | 0.6924 | Mom |
| | 6.5 | 0.6889 | 0.7234 | 0.7422 | 0.7081 | Mom |
| | 7.5 | 0.7204 | 0.7336 | 0.7346 | 0.7276 | Mom |
| | 8.5 | 0.7050 | 0.7426 | 0.7574 | 0.7352 | Mom |
| | 9.5 | 0.7167 | 0.7525 | 0.7634 | 0.7414 | Mom |
| | 10.5 | 0.7234 | 0.7621 | 0.7663 | 0.7524 | Mom |

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|---|---|---|---|---|---|---|
| 40 | 1.5 | 0.5604 | 0.6122 | 0.6014 | 0.6083 | Mle |
| | 2.5 | 0.6016 | 0.6463 | 0.6388 | 0.6421 | Mle |
| | 3.5 | 0.6333 | 0.6883 | 0.6646 | 0.6646 | Mle |
| | 4.5 | 0.6572 | 0.7026 | 0.6662 | 0.6820 | Mle |
| | 5.5 | 0.6572 | 0.7232 | 0.7006 | 0.6956 | Mle |
| | 6.5 | 0.6889 | 0.7243 | 0.7124 | 0.7066 | Mle |
| | 7.5 | 0.7204 | 0.7325 | 0.7218 | 0.7158 | Mle |
| | 8.5 | 0.7050 | 0.7338 | 0.7327 | 0.7232 | Mle |
| | 9.5 | 0.7167 | 0.7371 | 0.7374 | 0.7284 | Mle |
| | 10.5 | 0.7234 | 0.7624 | 0.7399 | 0.7366 | Mle |
| 60 | 1.5 | 0.5604 | 0.5612 | 0.5824 | 0.5726 | Mom |
| | 2.5 | 0.6016 | 0.6226 | 0.6342 | 0.6352 | Prop |
| | 3.5 | 0.6333 | 0.6676 | 0.6686 | 0.6497 | Mom |
| | 4.5 | 0.6572 | 0.6917 | 0.6844 | 0.6944 | Prop |
| | 5.5 | 0.6572 | 0.7087 | 0.7120 | 0.7128 | Prop |
| | 6.5 | 0.6889 | 0.7236 | 0.7273 | 0.7271 | Prop |
| | 7.5 | 0.7204 | 0.7348 | 0.7384 | 0.7389 | Prop |
| | 8.5 | 0.7050 | 0.7438 | 0.7477 | 0.7422 | Mom |
| | 9.5 | 0.7167 | 0.7515 | 0.7556 | 0.7509 | Mom |
| | 10.5 | 0.7234 | 0.7576 | 0.7622 | 0.7624 | Prop |
| 80 | 1.5 | 0.5604 | 0.5709 | 0.5716 | 0.5624 | Prop |
| | 2.5 | 0.6016 | 0.6231 | 0.6869 | 0.6132 | Mom |
| | 3.5 | 0.6333 | 0.6338 | 0.7103 | 0.6468 | Mom |
| | 4.5 | 0.6572 | 0.6644 | 0.7151 | 0.6718 | Mom |
| | 5.5 | 0.6572 | 0.6771 | 0.7334 | 0.6892 | Mom |
| | 6.5 | 0.6889 | 0.7038 | 0.7422 | 0.7092 | Mom |
| | 7.5 | 0.7204 | 0.7181 | 0.7486 | 0.7131 | Mom |
| | 8.5 | 0.7050 | 0.7287 | 0.7492 | 0.7233 | Mom |
| | 9.5 | 0.7167 | 0.7267 | 0.7372 | 0.7305 | Mom |
| | 10.5 | 0.7234 | 0.7446 | 0.7309 | 0.7369 | Mom |

**Table 3: Fuzzy Reliability Estimator When: $p = 2$ , $\alpha = 0.3, \theta = 0.5$ , $Ki = 0.4$**

| n | $t_i$ | Real | $R_{mle}$ | $R_{mom}$ | $R_{prop}$ | Best |
|---|---|---|---|---|---|---|
| 20 | 1.5 | 0.3106 | 0.3068 | 0.3386 | 0.3186 | Mom |
| | 2.5 | 0.3653 | 0.3698 | 0.3974 | 0.3832 | Mom |
| | 3.5 | 0.4165 | 0.4096 | 0.4511 | 0.4236 | Mom |
| | 4.5 | 0.4452 | 0.4728 | 0.4803 | 0.4513 | Mom |
| | 5.5 | 0.4657 | 0.4836 | 0.4951 | 0.4716 | Mom |
| | 6.5 | 0.4816 | 0.4842 | 0.5068 | 0.4872 | Mom |
| | 7.5 | 0.4942 | 0.4942 | 0.5163 | 0.4994 | Mom |
| | 8.5 | 0.5127 | 0.5043 | 0.5242 | 0..5093 | Mom |
| | 9.5 | 0.5206 | 0.5127 | 0.5303 | 0.5179 | Mom |
| | 10.5 | 0.5326 | 0.5197 | 0.5342 | 0.5244 | Mom |

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|---|---|---|---|---|---|---|
| 40 | 1.5 | 0.3106 | 0.3206 | 0.3262 | 0.3242 | Mom |
| | 2.5 | 0.3653 | 0.3742 | 0.3875 | 0.3787 | Mom |
| | 3.5 | 0.4165 | 0.4147 | 0.4286 | 0.4196 | Mom |
| | 4.5 | 0.4452 | 0.4427 | 0.4539 | 0.4478 | Mom |
| | 5.5 | 0.4657 | 0.4632 | 0.4736 | 0.4684 | Mom |
| | 6.5 | 0.4816 | 0.4788 | 0.4892 | 0.4965 | Prop |
| | 7.5 | 0.4942 | 0.4902 | 0.5011 | 0.5085 | Prop |
| | 8.5 | 0.5127 | 0.5013 | 0.5109 | 0.5148 | Prop |
| | 9.5 | 0.5206 | 0.5054 | 0.5129 | 0.5217 | Prop |
| | 10.5 | 0.5326 | 0.5136 | 0.5256 | 0.5321 | Prop |
| 60 | 1.5 | 0.3106 | 0.3226 | 0.3332 | 0.3224 | Mom |
| | 2.5 | 0.3653 | 0.3847 | 0.3842 | 0.3862 | Prop |
| | 3.5 | 0.4165 | 0.4246 | 0.4237 | 0.4227 | Mom |
| | 4.5 | 0.4452 | 0.4525 | 0.4511 | 0.4511 | Mom |
| | 5.5 | 0.4657 | 0.4886 | 0.4703 | 0.4713 | Prop |
| | 6.5 | 0.4816 | 0.5005 | 0.4867 | 0.4867 | Prop |
| | 7.5 | 0.4942 | 0.5196 | 0.4983 | 0.4989 | Prop |
| | 8.5 | 0.5127 | 0.5256 | 0.5068 | 0.5086 | Prop |
| | 9.5 | 0.5206 | 0.6165 | 0.5172 | 0.5172 | Mle |
| | 10.5 | 0.5326 | 0.6268 | 0.5236 | 0.5238 | Mle |
| 80 | 1.5 | 0.3106 | 0.3862 | 0.4006 | 0.3933 | Mom |
| | 2.5 | 0.3653 | 0.4077 | 0.4724 | 0.4667 | Mom |
| | 3.5 | 0.4165 | 0.4928 | 0.5166 | 0.5117 | Mom |
| | 4.5 | 0.4452 | 0.4447 | 0.5469 | 0.5423 | Mom |
| | 5.5 | 0.4657 | 0.5019 | 0.5688 | 0.5644 | Mom |
| | 6.5 | 0.4816 | 0.5328 | 0.5811 | 0.5917 | Prop |
| | 7.5 | 0.4942 | 0.5822 | 0.5942 | 0.6047 | Prop |
| | 8.5 | 0.5127 | 0.6016 | 0.6046 | 0.6133 | Prop |
| | 9.5 | 0.5206 | 0.6167 | 0.6133 | 0.6206 | Prop |
| | 10.5 | 0.5326 | 0.6123 | 0.6206 | 0.6324 | Prop |

**Table 4: Fuzzy Reliability Estimator When: $p = 2.5$ , $\alpha = 0.3$, $\theta = 0.5$ , $Ki = 0.4$**

| n | $t_i$ | Real | $R_{mle}$ | $R_{mom}$ | $R_{prop}$ | Best |
|---|---|---|---|---|---|---|
| 20 | 1.5 | 0.3102 | 0.3066 | 0.3369 | 0.3186 | Mom |
| | 2.5 | 0.3752 | 0.3688 | 0.3772 | 0.3832 | Prop |
| | 3.5 | 0.4154 | 0.4069 | 0.4352 | 0.4204 | Mom |
| | 4.5 | 0.4452 | 0.4372 | 0.4611 | 0.4615 | Prop |
| | 5.5 | 0.4816 | 0.4574 | 0.4802 | 0.4725 | Mom |
| | 6.5 | 0.4842 | 0.4683 | 0.4952 | 0.4872 | Mom |
| | 7.5 | 0.4943 | 0.4807 | 0.5008 | 0.4993 | Mom |
| | 8.5 | 0.5127 | 0.5032 | 0.5163 | 0.5093 | Mom |
| | 9.5 | 0.5206 | 0.5242 | 0.5243 | 0.5175 | Mom |
| | 10.5 | 0.597 | 0.5306 | 0.5308 | 0.5244 | Mom |

| | | | | | | |
|---|---|---|---|---|---|---|
| 40 | 1.5 | 0.3102 | 0.3114 | 0.3226 | 0.3115 | Mle |
| | 2.5 | 0.3752 | 0.3762 | 0.3845 | 0.3672 | Mle |
| | 3.5 | 0.4154 | 0.4268 | 0.4247 | 0.3892 | Mle |
| | 4.5 | 0.4452 | 0.4452 | 0.4108 | 0.4162 | Mom |
| | 5.5 | 0.4816 | 0.4656 | 0.4525 | 0.4354 | Mle |
| | 6.5 | 0.4842 | 0.4812 | 0.4628 | 0.4722 | Mom |
| | 7.5 | 0.4943 | 0.4936 | 0.4883 | 0.4663 | Mle |
| | 8.5 | 0.5127 | 0.5036 | 0.5005 | 0.4825 | Mle |
| | 9.5 | 0.5206 | 0.5122 | 0.5104 | 0.4885 | Mle |
| | 10.5 | 0.597 | 0.5189 | 0.5256 | 0.4946 | Mle |
| 60 | 1.5 | 0.3102 | 0.3223 | 0.3116 | 0.3084 | Prop |
| | 2.5 | 0.3752 | 0.3842 | 0.3672 | 0.3026 | Prop |
| | 3.5 | 0.4154 | 0.3937 | 0.4272 | 0.3614 | Prop |
| | 4.5 | 0.4452 | 0.4207 | 0.4466 | 0.4426 | Mle |
| | 5.5 | 0.4816 | 0.4422 | 0.4543 | 0.4632 | Mle |
| | 6.5 | 0.4842 | 0.4765 | 0.4606 | 0.4789 | Mom |
| | 7.5 | 0.4943 | 0.4867 | 0.4721 | 0.4915 | Mom |
| | 8.5 | 0.5127 | 0.4867 | 0.4922 | 0.5016 | Mle |
| | 9.5 | 0.5206 | 0.4889 | 0.5062 | 0.5099 | Mle |
| | 10.5 | 0.597 | 0.4182 | 0.5188 | 0.5172 | Mle |
| 80 | 1.5 | 0.3102 | 0.3068 | 0.3386 | 0.3178 | Mom |
| | 2.5 | 0.3752 | 0.3699 | 0.3974 | 0.3415 | Mom |
| | 3.5 | 0.4154 | 0.4096 | 0.4349 | 0.3442 | Prop |
| | 4.5 | 0.4452 | 0.4372 | 0.4611 | 0.4016 | Mom |
| | 5.5 | 0.4816 | 0.4574 | 0.4803 | 0.4232 | Mom |
| | 6.5 | 0.4842 | 0.4949 | 0.4952 | 0.4526 | Mom |
| | 7.5 | 0.4943 | 0.6032 | 0.5166 | 0.4807 | Mle |
| | 8.5 | 0.5127 | 0.6421 | 0.5242 | 0.4962 | Mle |
| | 9.5 | 0.5206 | 0.6242 | 0.5308 | 0.4973 | Mle |
| | 10.5 | 0.597 | 0.6308 | 0.5233 | 0.5201 | Mle |

**Table 5: Fuzzy Reliability Estimator When: $p = 2.5$ , $\alpha = 0.3, \theta = 0.7, Ki = 0.7$**

| n | $t_i$ | Real | $R_{mle}$ | $R_{mom}$ | $R_{prop}$ | Best |
|---|---|---|---|---|---|---|
| 20 | 1.5 | 0.3162 | 0.3068 | 0.3387 | 0.3188 | Mom |
| | 2.5 | 0.3753 | 0.3686 | 0.3974 | 0.3832 | Mom |
| | 3.5 | 0.4165 | 0.4096 | 0.4348 | 0.4234 | Mom |
| | 4.5 | 0.4452 | 0.4574 | 0.4622 | 0.4513 | Mom |
| | 5.5 | 0.4817 | 0.4728 | 0.4805 | 0.4716 | Mom |
| | 6.5 | 0.4842 | 0.4949 | 0.4887 | 0.4872 | Mle |
| | 7.5 | 0.5543 | 0.5032 | 0.4906 | 0.4994 | Mle |
| | 8.5 | 0.5624 | 0.5223 | 0.5062 | 0.5093 | Mle |
| | 9.5 | 0.5636 | 0.5306 | 0.5243 | 0.5176 | Mle |
| | 10.5 | 0.5826 | 0.5455 | 0.5307 | 0.5244 | Mle |

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|---|---|---|---|---|---|---|
| **40** | 1.5 | 0.3162 | 0.3116 | 0.3366 | 0.3068 | Mom |
| | 2.5 | 0.3753 | 0.3762 | 0.3876 | 0.3704 | Mom |
| | 3.5 | 0.4165 | 0.4246 | 0.4351 | 0.4112 | Mom |
| | 4.5 | 0.4452 | 0.4525 | 0.4506 | 0.4393 | Mle |
| | 5.5 | 0.4817 | 0.4626 | 0.4803 | 0.4600 | Mom |
| | 6.5 | 0.4842 | 0.4728 | 0.4962 | 0.4752 | Mom |
| | 7.5 | 0.5543 | 0.4884 | 0.5068 | 0.4983 | Mom |
| | 8.5 | 0.5624 | 0.5006 | 0.5242 | 0.5066 | Mom |
| | 9.5 | 0.5636 | 0.5103 | 0.5308 | 0.5136 | Mom |
| | 10.5 | 0.5826 | 0.5168 | 0.5422 | 0.5217 | Mom |
| **60** | 1.5 | 0.3162 | 0.3163 | 0.3314 | 0.3068 | Mom |
| | 2.5 | 0.3753 | 0.3805 | 0.3758 | 0.3704 | Mle |
| | 3.5 | 0.4165 | 0.4212 | 0.4169 | 0.4112 | Mle |
| | 4.5 | 0.4452 | 0.4493 | 0.4451 | 0.4383 | Mle |
| | 5.5 | 0.4817 | 0.4688 | 0.4657 | 0.4602 | Mle |
| | 6.5 | 0.4842 | 0.4844 | 0.4815 | 0.4756 | Mle |
| | 7.5 | 0.5543 | 0.5067 | 0.4836 | 0.4882 | Mle |
| | 8.5 | 0.5624 | 0.5163 | 0.5042 | 0.5066 | Mle |
| | 9.5 | 0.5636 | 0.5242 | 0.5162 | 0.5366 | Prop |
| | 10.5 | 0.5826 | 0.5342 | 0.5602 | 0.5314 | Mom |
| **80** | 1.5 | 0.3162 | 0.112 | 0.3262 | 0.3655 | Prop |
| | 2.5 | 0.3753 | 0.3806 | 0.3876 | 0.3944 | Prop |
| | 3.5 | 0.4165 | 0.4212 | 0.4266 | 0.3996 | Prop |
| | 4.5 | 0.4452 | 0.4483 | 0.4416 | 0.4102 | Prop |
| | 5.5 | 0.4817 | 0.4699 | 0.4526 | 0.4226 | Prop |
| | 6.5 | 0.4842 | 0.4212 | 0.5040 | 0.5042 | Mle |
| | 7.5 | 0.5543 | 0.4688 | 0.5217 | 0.5317 | Prop |
| | 8.5 | 0.5624 | 0.4803 | 0.5148 | 0.5321 | Mle |
| | 9.5 | 0.5636 | 0.4989 | 0.5316 | 0.5967 | Prop |
| | 10.5 | 0.5826 | 0.5163 | 0.5418 | 0.5466 | Prop |

**Table 6: Fuzzy Reliability Estimator When: $p = 2.5$ , $\alpha = 0.5, \theta = 0.7$ , $Ki = 0.7$**

| n | $t_i$ | Real | $R_{mle}$ | $R_{mom}$ | $R_{prop}$ | Best |
|---|---|---|---|---|---|---|
| **20** | 1.5 | 0.3101 | 0.3068 | 0.3287 | 0.3186 | Mom |
| | 2.5 | 0.3753 | 0.3697 | 0.3874 | 0.3832 | Mom |
| | 3.5 | 0.4165 | 0.4096 | 0.4501 | 0.4204 | Mom |
| | 4.5 | 0.4450 | 0.4372 | 0.4956 | 0.4413 | Mom |
| | 5.5 | 0.4517 | 0.4572 | 0.5068 | 0.4626 | Mom |
| | 6.5 | 0.4658 | 0.4716 | 0.5163 | 0.4772 | Mom |
| | 7.5 | 0.4871 | 0.4853 | 0.5242 | 0.4852 | Mom |
| | 8.5 | 0.4942 | 0.4946 | 0.5308 | 0.5106 | Mom |
| | 9.5 | 0.5094 | 0.5036 | 0.6309 | 0.5242 | Mom |
| | 10.5 | 0.5163 | 0.5097 | 0.5361 | 0.5524 | Prop |

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|---|---|---|---|---|---|---|
| 40 | 1.5 | 0.3101 | 0.3116 | 0.3216 | 0.3164 | Mom |
| | 2.5 | 0.3753 | 0.3662 | 0.3845 | 0.3832 | Mom |
| | 3.5 | 0.4165 | 0.4066 | 0.4248 | 0.4241 | Mom |
| | 4.5 | 0.4450 | 0.4352 | 0.4525 | 0.4523 | Mom |
| | 5.5 | 0.4517 | 0.4450 | 0.4736 | 0.4724 | Mom |
| | 6.5 | 0.4658 | 0.4656 | 0.4776 | 0.4885 | Prop |
| | 7.5 | 0.4871 | 0.4812 | 0.5005 | 0.5010 | Prop |
| | 8.5 | 0.4942 | 0.4932 | 0.5104 | 0.5112 | Prop |
| | 9.5 | 0.5094 | 0.5104 | 0.5166 | 0.5193 | Prop |
| | 10.5 | 0.5163 | 0.5166 | 0.5242 | 0.5263 | Prop |
| 60 | 1.5 | 0.3101 | 0.3206 | 0.3314 | 0.3251 | Mom |
| | 2.5 | 0.3753 | 0.3320 | 0.3162 | 0.3142 | Mle |
| | 3.5 | 0.4165 | 0.5568 | 0.3705 | 0.3786 | Prop |
| | 4.5 | 0.4450 | 0.5571 | 0.4212 | 0.4196 | Mle |
| | 5.5 | 0.4517 | 0.5538 | 0.4492 | 0.4468 | Mle |
| | 6.5 | 0.4658 | 0.6120 | 0.4483 | 0.4586 | Mle |
| | 7.5 | 0.4871 | 0.6132 | 0.4652 | 0.4682 | Mle |
| | 8.5 | 0.4942 | 0.6422 | 0.4782 | 0.5065 | Mle |
| | 9.5 | 0.5094 | 0.6510 | 0.4806 | 0.5148 | Mle |
| | 10.5 | 0.5163 | 0.6721 | 0.5632 | 0.5248 | Mle |
| 80 | 1.5 | 0.3101 | 0.3115 | 0.3387 | 0.3186 | Mom |
| | 2.5 | 0.3753 | 0.3742 | 0.3864 | 0.3832 | Mom |
| | 3.5 | 0.4165 | 0.4172 | 0.4346 | 0.4234 | Mom |
| | 4.5 | 0.4450 | 0.4454 | 0.4511 | 0.4513 | Prop |
| | 5.5 | 0.4517 | 0.4462 | 0.4702 | 0.4616 | Mom |
| | 6.5 | 0.4658 | 0.4820 | 0.4802 | 0.4773 | Mle |
| | 7.5 | 0.4871 | 0.4945 | 0.5062 | 0.4984 | Mom |
| | 8.5 | 0.4942 | 0.5046 | 0.5163 | 0.5082 | Mom |
| | 9.5 | 0.5094 | 0.5132 | 0.5232 | 5155 | Mom |
| | 10.5 | 0.5163 | 0.5200 | 0.5246 | 0.6236 | Prop |

**Table 7: Fuzzy Reliability Estimator When: $p = 2$ , $\alpha = 0.5$, $\theta = 0.5$, $Ki = 0.7$**

| n | $t_i$ | Real | $R_{mle}$ | $R_{mom}$ | $R_{prop}$ | Best |
|---|---|---|---|---|---|---|
| 20 | 1.5 | 0.3116 | 0.3216 | 0.3338 | 0.3184 | Mom |
| | 2.5 | 0.3763 | 0.3846 | 0.3827 | 0.3832 | Mle |
| | 3.5 | 0.4165 | 0.4248 | 0.4523 | 0.4242 | Mom |
| | 4.5 | 0.4452 | 0.4525 | 0.4886 | 0.4523 | Mom |
| | 5.5 | 0.4658 | 0.4626 | 0.5012 | 0.4628 | Mom |
| | 6.5 | 0.4813 | 0.5006 | 0.5193 | 0.4939 | Mom |
| | 7.5 | 0.4927 | 0.5134 | 0.5246 | 0.5056 | Mom |
| | 8.5 | 0.5036 | 0.5286 | 0.5303 | 0.5166 | Mom |
| | 9.5 | 0.5122 | 0.5258 | 0.5412 | 0.5226 | Mom |
| | 10.5 | 0.5189 | 0.5316 | 0.5523 | 0.5238 | Mom |

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|---|---|---|---|---|---|---|
| 40 | 1.5 | 0.3116 | 0.3110 | 0.2814 | 0.3768 | Prop |
| | 2.5 | 0.3763 | 0.3752 | 0.3216 | 0.3199 | Mle |
| | 3.5 | 0.4165 | 0.4172 | 0.3429 | 0.3478 | Mle |
| | 4.5 | 0.4452 | 0.4493 | 0.4072 | 0.3862 | Mle |
| | 5.5 | 0.4658 | 0.4699 | 0.4233 | 0.4434 | Mle |
| | 6.5 | 0.4813 | 0.5032 | 0.4282 | 0.4065 | Mle |
| | 7.5 | 0.4927 | 0.5233 | 0.4526 | 0.4212 | Mle |
| | 8.5 | 0.5036 | 0.5431 | 0.4217 | 0.4272 | Mle |
| | 9.5 | 0.5122 | 0.5052 | 0.4276 | 0.4313 | Mle |
| | 10.5 | 0.5189 | 0.5523 | 0.4362 | 0.4411 | Mle |
| 60 | 1.5 | 0.3116 | 0.3068 | 0.3396 | 0.31142 | Mom |
| | 2.5 | 0.3763 | 0.3688 | 0.3878 | 0.3787 | Mom |
| | 3.5 | 0.4165 | 0.4096 | 0.4352 | 0.4196 | Mom |
| | 4.5 | 0.4452 | 0.4372 | 0.4611 | 0.4472 | Mom |
| | 5.5 | 0.4658 | 0.4728 | 0.4803 | 0.4684 | Mom |
| | 6.5 | 0.4813 | 0.4949 | 0.4952 | 0.4852 | Mom |
| | 7.5 | 0.4927 | 0.5032 | 0.5123 | 0.5085 | Mom |
| | 8.5 | 0.5036 | 0.5088 | 0.5192 | 0.5148 | Mom |
| | 9.5 | 0.5122 | 0.6412 | 0.6241 | 0.7232 | Mom |
| | 10.5 | 0.5189 | 0.6566 | 0.7011 | 0.7142 | Mom |
| 80 | 1.5 | 0.3116 | 0.3226 | 0.3262 | 0.3142 | Mom |
| | 2.5 | 0.3763 | 0.3768 | 0.3876 | 0.3787 | Mom |
| | 3.5 | 0.4165 | 0.4172 | 0.4266 | 0.4196 | Mom |
| | 4.5 | 0.4452 | 0.4354 | 0.4285 | 0.4737 | Mom |
| | 5.5 | 0.4658 | 0.4427 | 0.4566 | 0.4896 | Prop |
| | 6.5 | 0.4813 | 0.4663 | 0.4893 | 0.4844 | Prop |
| | 7.5 | 0.4927 | 0.4832 | 0.4737 | 0.4806 | Prop |
| | 8.5 | 0.5036 | 0.4645 | 0.4902 | 0.4821 | Prop |
| | 9.5 | 0.5122 | 0.5064 | 0.4915 | 0.4832 | Mle |
| | 10.5 | 0.5189 | 0.6132 | 0.4922 | 0.4665 | Mle |

**Table 8: Fuzzy Reliability Estimator When: $p = 2$ , $\alpha = 0.5$, $\theta = 0.5$, $Ki = 0.7$**

| n | $t_i$ | Real | $R_{mle}$ | $R_{mom}$ | $R_{prop}$ | Best |
|---|---|---|---|---|---|---|
| 20 | 1.5 | 0.2956 | 0.2956 | 0.4226 | 0.3885 | Mom |
| | 2.5 | 0.3376 | 0.3376 | 0.4902 | 0.4722 | Mom |
| | 3.5 | 0.3872 | 0.3872 | 0.5316 | 0.4562 | Mom |
| | 4.5 | 0.4032 | 0.4032 | 0.5404 | 0.5236 | Mom |
| | 5.5 | 0.4155 | 0.4155 | 0.5512 | 0.5304 | Mom |
| | 6.5 | 0.4256 | 0.4256 | 0.5672 | 0.5442 | Mom |
| | 7.5 | 0.4396 | 0.4396 | 0.6069 | 0.5807 | Mom |
| | 8.5 | 0.4462 | 0.4462 | 0.6192 | 0.5697 | Mom |
| | 9.5 | 0.4520 | 0.4520 | 0.6275 | 0.6175 | Mom |
| | 10.5 | 0.4466 | 0.4466 | 0.6345 | 0.6244 | Mom |

| | | | | | | |
|---|---|---|---|---|---|---|
| 40 | 1.5 | 0.2956 | 0.2981 | 0.4026 | 0.4613 | Prop |
| | 2.5 | 0.3376 | 0.3366 | 0.4738 | 0.5063 | Prop |
| | 3.5 | 0.3872 | 0.3634 | 0.5117 | 0.5244 | Prop |
| | 4.5 | 0.4032 | 0.3827 | 0.5324 | 0.5368 | Prop |
| | 5.5 | 0.4155 | 0.3978 | 0.5644 | 0.5589 | Prop |
| | 6.5 | 0.4256 | 0.4097 | 0.5801 | 0.5756 | Prop |
| | 7.5 | 0.4396 | 0.4193 | 0.5849 | 0.5887 | Prop |
| | 8.5 | 0.4462 | 0.4272 | 0.5932 | 0.5982 | Prop |
| | 9.5 | 0.4520 | 0.4339 | 0.6037 | 0.6078 | Prop |
| | 10.5 | 0.4466 | 0.4371 | 0.6206 | 0.6161 | Mom |
| 60 | 1.5 | 0.2956 | 0.3816 | 0.3917 | 0.3869 | Mom |
| | 2.5 | 0.3376 | 0.4647 | 0.4702 | 0.4538 | Mom |
| | 3.5 | 0.3872 | 0.5088 | 0.5166 | 0.5122 | Mom |
| | 4.5 | 0.4032 | 0.5405 | 0.5688 | 0.5407 | Mom |
| | 5.5 | 0.4155 | 0.5565 | 0.5857 | 0.5628 | Mom |
| | 6.5 | 0.4256 | 0.5732 | 0.6086 | 0.6114 | Prop |
| | 7.5 | 0.4396 | 0.5852 | 0.6133 | 0.6192 | Prop |
| | 8.5 | 0.4462 | 0.5862 | 0.6246 | 0.6299 | Prop |
| | 9.5 | 0.4520 | 0.3622 | 0.5241 | 0.5536 | Prop |
| | 10.5 | 0.4466 | 0.3785 | 0.3864 | 0.6252 | Prop |
| 80 | 1.5 | 0.2956 | 0.3917 | 0.3774 | 0.3852 | Mle |
| | 2.5 | 0.3376 | 0.4647 | 0.4623 | 0.4583 | Mle |
| | 3.5 | 0.3872 | 0.5177 | 0.5063 | 0.5033 | Mle |
| | 4.5 | 0.4032 | 0.5422 | 0.6064 | 0.5339 | Mom |
| | 5.5 | 0.4155 | 0.5644 | 0.6114 | 0.5682 | Mom |
| | 6.5 | 0.4256 | 0.5821 | 0.6192 | 0.6054 | Mom |
| | 7.5 | 0.4396 | 0.5942 | 0.6233 | 0.6127 | Mom |
| | 8.5 | 0.4462 | 0.6208 | 0.6425 | 0.6315 | Mom |
| | 9.5 | 0.4520 | 0.6328 | 0.6324 | 0.6306 | Mle |
| | 10.5 | 0.4466 | 0.6425 | 0.6441 | 0.6252 | Mom |

## 4. Conclusion

From result of simulation for comparing different fuzzy Reliability function we find from table (1) that $\hat{R}_{mom}$ is best with percentage $=\frac{21}{40}=$ 52% and second one is $\hat{R}_{MLE}$ with percentage $\frac{16}{40}$ = 40% and $\hat{R}_{prop}$ with percentage $\frac{3}{40}$ = 7.5%.

While from table (2) $\hat{R}_{MLE} = \frac{27}{40} = 67.5\%$ , $\hat{R}_{Mom} = \frac{12}{40} = 30\%$ *and* $\hat{R}_{prop} \frac{1}{40} = 2.5\%$.

From table (3), we find that the first best estimator of fuzzy reliability function is $\hat{R}_{mom} = \frac{17}{40} = 42.5\%$ , $\hat{R}_{prop} = \frac{19}{40} = 47.5\%$ *and* $\hat{R}_{Mle} = \frac{4}{40} = 10\%$

From table (4) the result of comparison indicates that $\hat{R}_{Mle} = \frac{17}{40} = 42.5\%$, $\hat{R}_{mom} = \frac{17}{40} =$ 42.5% and $\hat{R}_{prop} = \frac{6}{40}$ =15%.

The result table (5) indicates that $\hat{R}_{Mle}$ is best with percentage $= \frac{15}{40}$ = 37.5%, $\hat{R}_{mom} = \frac{15}{40} =$ 37.5% and $\hat{R}_{prop} = \frac{10}{40}$=25%.

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The result table (6) indicates that $\hat{R}_{mom}$ is best with percentage $= \frac{22}{40} =$ 55%, $\hat{R}_{Mle} = \frac{9}{40} =$ 22.5% and $\hat{R}_{prop} = \frac{9}{40}$=22.5%.

The result of fuzzy reliability estimator from table (7) indicates that $\hat{R}_{mom} = \frac{19}{40} =$ 47.5%, $\hat{R}_{Mle} = \frac{14}{40}$= 35% and $\hat{R}_{prop} = \frac{7}{40}$ =17.5%.

And the result of fuzzy reliability estimator from table (8) indicates that $\hat{R}_{mom} = \frac{22}{40} =$ 55%, $\hat{R}_{prop} = \frac{14}{40}$= 35% and $\hat{R}_{Mle} = \frac{4}{40}$ =10%.

We notice that the best first estimators are equally likely between Moments estimators and Maximum likelihood one, while the proposed become the third best as the results of comparison from different tables. Here are the results of preference of Reliability estimators for all tables percentage of preference of three estimators:

| Tables | $\hat{R}_{MLE}$ | $\hat{R}_{Mom}$ | $\hat{R}_{prop}$ | Best |
|---|---|---|---|---|
| Table (1) | $\frac{16}{40} = 40$ | $\frac{21}{40}$ =52 | $\frac{3}{40}$ =7.5 | Mom |
| Table(2) | $\frac{27}{40}$=67.5 | $\frac{12}{40} = 30$ | $\frac{1}{40} = 2.5$ | Mle |
| Table(3) | $\frac{4}{40} = 10$ | $\frac{17}{40} = 42.5$ | $\frac{19}{40} = 47.5$ | Prop |
| Table(4) | $\frac{17}{40} = 42.5$ | $\frac{17}{40}$=42.5 | $\frac{6}{40}$=15 | Mle , Mom |
| Table(5) | $\frac{15}{40}$=37.5 | $\frac{15}{40} = 37.5$ | $\frac{10}{40}$=25 | Mle , Mom |
| Table(6) | $\frac{9}{40} = 22.5$ | $\frac{22}{40}$=55 | $\frac{9}{40}$=22.5 | Mom |
| Table(7) | $\frac{14}{40}$=35 | $\frac{19}{40} = 47.5$ | $\frac{7}{40}$=17 | Mom |
| Table(8) | $\frac{4}{40}$=10 | $\frac{22}{40} = 55$ | $\frac{14}{40}$=35 | Mom |

The result of fuzzy Reliability estimators indicates that the first best one is Mom with percentage for all tables (50%), then MLE and Prop.

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