مجلة الإدارة والاقتصاد
المجلد الرابع
العدد الثالث عشر

Comparing Different Estimators for Scale Parameters of Maxwell Distribution

Mohammed Kadhim Hawash, Haider Adnan Ameer

Dijlah University College

Abstract

This paper deals with comparing four estimators of scale parameter (θ), for Maxwell distribution. The first is Pitman estimator which is denoted by $(\widehat{\theta}_1)$, the second is moment estimator $(\widehat{\theta}_2)$, the third is maximum likelihood estimator $(\widehat{\theta}_3)$, and fourth is proposed Bayes estimator, where (θ) considered random variable having prior distribution [g(θ)], this Bayes estimator denoted by$(\widehat{\theta}_4)$, these estimators compared using the statistical measure mean square error (MSE), taking initial values of $(\theta = 3, \theta = 5, c = 1,2)$.

Keywords: Maxwell Distribution, Scale Parameter, Pitman Estimator, Moment Estimator, Maximum Likelihood Estimator, Bayes Estimator.

1. Introduction

The Maxwell distribution, named for James Clerk Maxwell1869, is the distribution of the magnitude of a three-dimensional random vector whose coordinates are independent, identically distributed, mean 0 normal variables. The distribution has a number of applications in settings where magnitudes of normal variables are important, particularly in physics. It is also called the Maxwell-Boltzmann distribution in honor also of Ludwig Boltzmann. The Maxwell distribution is closely related to the Rayleigh distribution, which governs the magnitude of a two-dimensional random vector whose coordinates are independent, identically distributed, mean 0 normal variables. Maxwell–Boltzmann statistics describes the average distribution of non-interacting material particles over various energy states in thermal equilibrium, and is applicable when the temperature is high enough or the particle density is low enough to render quantum effects negligible.

In (2012), Al – Baldawi, Tasnim H.K. compare tree different Bayes estimator of the scale parameter of Maxweel distribution. Also Al – Obedy, Nadia, J. (2012), introduce a comparison of the scale parameter for Maxweel using different prior estimators, (i.e) she compare Bayes estimator with maximum likelihood using simulation procedure. Because this distribution play an important role in physics, chemistry, engineering, genetics, hydrology, and medicine, so we work on estimating the scale parameter of this distributing using maximum likelihood method and proposed Bayes estimator, also proposed estimator depending on minimum variance unbiased estimator.

2. The model

Maxwell failure model is;

$$f(t;\theta)=\frac{4}{\sqrt{\pi}\,\theta^3}t^2e^{-\left(\frac{t}{\theta}\right)^2} \qquad t>0 \tag{1}$$

$$E(t)=\frac{4}{\sqrt{\pi}\,\theta^3}\int_0^\infty (t)t^2e^{-\left(\frac{t}{\theta}\right)^2}dt \tag{2}$$

Let $y=\frac{t}{\theta} \;\rightarrow\; t=y\theta \qquad dt=\theta dy$

$$E(t)=\frac{4}{\sqrt{\pi}\,\theta^3}\int_0^\infty (y\theta)^3\; e^{-y^2}\theta dy$$

$$E(t)=\frac{4}{\sqrt{\pi}\,\theta^3}\theta^4\int_0^\infty y^3\; e^{-y^2}dy$$

Let $Z=y^2 \;\rightarrow y=\sqrt{Z} \qquad dy=\frac{1}{2\sqrt{Z}}\;dz$

$$E(t)=\frac{4}{\sqrt{\pi}}\theta\int_0^\infty (z)^{\frac{3}{2}}\; e^{-Z}\frac{1}{2\sqrt{Z}}\;dz$$

$$E(t)=\frac{4\theta}{\sqrt{\pi}}\frac{1}{2}\int_0^\infty Z\; e^{-Z}\;dz$$

$$E(t)=\frac{2\theta}{\sqrt{\pi}}\Gamma(2)=\frac{2\theta}{\sqrt{\pi}} \tag{3}$$

Also we can find $E(t^2)$;

$$E(t^2)=\int_0^\infty t^2 f(t;\theta)dt \tag{4}$$

$$E(t^2)=\frac{4}{\sqrt{\pi}\,\theta^3}\int_0^\infty t^4e^{-\left(\frac{t}{\theta}\right)^2}dt$$

Let $y=\frac{t}{\theta} \;\rightarrow\; t=y\theta \quad dt=\theta dy$

$$E(t^2)=\frac{4}{\sqrt{\pi}\,\theta^3}\int_0^\infty (y\theta)^4e^{-y^2}\theta dy$$

Therefore;

$$E(t^2)=\frac{4\theta^2}{\sqrt{\pi}}\int_0^\infty y^4e^{-y^2}\;dy$$

Let $Z=y^2 \;\rightarrow Z^{\frac{1}{2}}=y \qquad dy=\frac{1}{2\sqrt{Z}}\;dz$

$$E(t^2)=\frac{4\theta^2}{\sqrt{\pi}}\int_0^\infty (Z^{\frac{1}{2}})^4e^{-Z}\frac{1}{2\sqrt{Z}}\;dz$$

$$E(t^2)=\frac{2\theta^2}{\sqrt{\pi}}\int_0^\infty Z^{\frac{3}{2}}e^{-Z}\;dz$$

$$E(t^2)=\frac{2\theta^2}{\sqrt{\pi}}\Gamma\left(\frac{5}{2}\right)=\frac{2\theta^2}{\sqrt{\pi}}\frac{3}{2}\frac{1}{2}\sqrt{\pi}=\frac{3\theta^2}{2} \tag{5}$$

$$v(t)=\sigma_t^2=E(t^2)-[E(t)]^2$$

$$=\frac{3\theta^2}{2}-\left(\frac{2\theta}{\sqrt{\pi}}\right)^2$$

$$= \frac{3\theta^2}{2} - \frac{4\theta^2}{\frac{22}{7}}$$

$$v(t) = \frac{5\theta^2}{22} \qquad (6)$$

3. Estimation Method for Scale Parameter (θ)

3.1 Pitman Estimator for (θ)

Let $(t_1, t_2, \ldots, t_n)$ be a random sample from density $f(t;\theta)$, where $\theta > 0$ is a scale parameter, the estimator;

$$t(x_1, x_2, \ldots, x_n) = \frac{\int_0^\infty \left(\frac{1}{\theta^2}\right) \prod_{i=1}^n f(x_i, \theta) d\theta}{\int_0^\infty \left(\frac{1}{\theta^3}\right) \prod_{i=1}^n f(x_i, \theta) d\theta} \qquad (7)$$

Is called Pitman estimator of scale parameter.

Now we find $E\left(\frac{1}{\theta^2}\right)$;

$$\prod_{i=1}^n f(t_i, \theta) = \left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{\theta^{3n}} \prod_{i=1}^n t_i^2\, e^{-\sum_{i=1}^n \left(\frac{t_i}{\theta}\right)^2}$$

$$E\left(\frac{1}{\theta^2}\right) = \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n t_i^2 \int_0^\infty \frac{1}{\theta^2} \frac{1}{\theta^{3n}} e^{-\sum_{i=1}^n \left(\frac{t_i}{\theta}\right)^2} d\theta$$

$$= \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n t_i^2 \int_0^\infty \left(\frac{1}{\theta}\right)^{2+3n} e^{-\frac{1}{\theta^2} \sum_{i=1}^n t_i^2} d\theta$$

Let $y = \frac{1}{\theta} \quad \theta = \frac{1}{y} \quad d\theta = -\frac{1}{y^2}\, dy$

$$E\left(\frac{1}{\theta^2}\right) = \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n t_i^2 \int_0^\infty (y)^{2+3n} e^{-y^2 \sum_{i=1}^n t_i^2} \frac{1}{y^2} dy$$

$$E\left(\frac{1}{\theta^2}\right) = \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n t_i^2 \int_0^\infty y^{3n} e^{-y^2 \sum_{i=1}^n t_i^2} dy$$

Let $Z = y^2 \quad \rightarrow \sqrt{Z} = y \quad dy = \frac{1}{2\sqrt{Z}}\, dz$

$$E\left(\frac{1}{\theta^2}\right) = \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n t_i^2 \int_0^\infty Z^{\frac{3n}{2}} e^{-Z \sum_{i=1}^n t_i^2} \frac{1}{2\sqrt{Z}} dZ$$

$$E\left(\frac{1}{\theta^2}\right) = \left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{2} \prod_{i=1}^n t_i^2 \int_0^\infty Z^{\frac{3n}{2} - \frac{1}{2}} e^{-Z \sum_{i=1}^n t_i^2} dZ$$

$$E\left(\frac{1}{\theta^2}\right) = \frac{\left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{2} \prod_{i=1}^n t_i^2}{\left(\sum_{i=1}^n t_i^2\right)^{\frac{3n}{2}+\frac{1}{2}}} \Gamma\left(\frac{3n+1}{2}\right) \qquad (8)$$

Similarly;

$$\int_0^\infty \frac{1}{\theta^3} \prod_{i=1}^n f(x_i, \theta)\, d\theta \qquad (9)$$

$$\int_0^{\infty} \frac{1}{\theta^3}\left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{\theta^{3n}} \prod_{i=1}^{n} t_i^2\, e^{-\sum_{i=1}^{n}\left(\frac{t_i}{\theta}\right)^2} d\theta$$

$$\left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^{n} t_i^2 \int_0^{\infty} \left(\frac{1}{\theta}\right)^{3+3n} e^{-\frac{1}{\theta^2}\sum_{i=1}^{n} t_i^2} d\theta$$

Let $y = \frac{1}{\theta}$ $\quad \theta = \frac{1}{y}$ $\quad d\theta = -\frac{1}{y^2}\, dy$

$$\left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^{n} t_i^2 \int_0^{\infty} y^{3+3n}\, e^{-y^2 \sum_{i=1}^{n} t_i^2} \frac{1}{y^2}\, dy$$

$$\left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^{n} t_i^2 \int_0^{\infty} y^{1+3n}\, e^{-y^2 \sum_{i=1}^{n} t_i^2}\, dy$$

Let $Z = y^2 \;\rightarrow \sqrt{Z} = y$ $\quad dy = \frac{1}{2\sqrt{Z}}\, dz$

$$\left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^{n} t_i^2 \int_0^{\infty} Z^{\frac{1+3n}{2}}\, e^{-Z \sum_{i=1}^{n} t_i^2}\, \frac{1}{2\sqrt{Z}} dZ$$

$$\left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{2} \prod_{i=1}^{n} t_i^2 \int_0^{\infty} Z^{\frac{3n}{2}}\, e^{-Z \sum_{i=1}^{n} t_i^2}\, dZ$$

$$= \frac{\left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{2} \prod_{i=1}^{n} t_i^2}{\left(\sum_{i=1}^{n} t_i^2\right)^{\frac{3n}{2}+1}}\, \Gamma\left(\frac{3n}{2}+1\right) \qquad (10)$$

∴ The formula of Pitman estimator for the parameter (θ) is;

$$t(x_1, x_2, \dots, x_n) = \frac{\left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{2} \prod_{i=1}^{n} t_i^2 \Gamma\left(\frac{3n+1}{2}\right)}{\left(\sum_{i=1}^{n} t_i^2\right)^{\frac{3n}{2}+\frac{1}{2}}} \times \frac{\left(\sum_{i=1}^{n} t_i^2\right)^{\frac{3n}{2}+1}}{\left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{2} \prod_{i=1}^{n} t_i^2 \Gamma\left(\frac{3n}{2}+1\right)}$$

$$\hat{\theta}_1 = \frac{\left(\sum_{i=1}^{n} t_i^2\right)^{\frac{1}{2}} \Gamma\left(\frac{3n+1}{2}\right)}{\Gamma\left(\frac{3n}{2}+1\right)} \qquad (11)$$

Which is a function of $(t_1, t_2, \dots, t_n)$

3.2 Moment Estimator

Since the $p.d.f$ in equation (1) has only one parameter (θ), this can be estimated by moment method from solving;

$$m_1 = E(t) \tag{12}$$

$m_1 = \frac{\sum_{i=1}^{n} t_i}{n}$ sample mean, then;

$$\bar{t} = \frac{2\,\hat{\theta}_{MOM}}{\sqrt{\pi}}$$

$$\hat{\theta}_{MOM} = \frac{\bar{t}\,\sqrt{\pi}}{2} \tag{13}$$

3.3 Maximum Likelihood Estimator

The third estimator of (θ) is obtained now by using likelihood function $[L(\theta)]$, and finding $(\hat{\theta})$ which maximize $[logL(\theta)]$, as follows;

$$f(t;\theta) = \frac{4}{\sqrt{\pi}}\frac{1}{\theta^3} t_i^2\, e^{-\frac{t_i^2}{\theta^2}}$$

$$L(\theta) = \prod_{i=1}^{n} f(t_i;\theta)$$

$$L(\theta) = \left(\frac{4}{\sqrt{\pi}}\right)^n \theta^{-3n} \prod_{i=1}^{n} t_i^2\, e^{-\frac{\sum_{i=1}^{n} t_i^2}{\theta^2}}$$

$$\log L(\theta) = n\log\left(\frac{4}{\sqrt{\pi}}\right) - 3n\log\theta + \sum_{i=1}^{n} log t_i^2 - \frac{\sum_{i=1}^{n} t_i^2}{\theta^2} \tag{14}$$

$$\frac{\partial \log L(\theta)}{\partial\theta} = -\frac{3n}{\theta} + 2\frac{\sum_{i=1}^{n} t_i^2}{\theta^3}$$

$putting\ \frac{\partial \log L(\theta)}{\partial\theta} = 0$

$$-3\,n\,\theta^2 + 2\sum_{i=1}^{n} t_i^2 = 0$$

$$3\,n\,\theta^2 = 2\sum_{i=1}^{n} t_i^2$$

$$\theta^2 = \frac{2\sum_{i=1}^{n} t_i^2}{3n}$$

$$\hat{\theta}_3 = \hat{\theta}_{MLE} = \sqrt{\frac{2\sum_{i=1}^{n} t_i^2}{3n}} \tag{15}$$

Which is the maximum likelihood estimator of (θ).

3.4 Bayes Estimator

In this method the parameter (θ) is considered as random variable having prior distribution as;

$$g(\theta) = e^{-\frac{c}{\theta^2}} \qquad \theta > 0$$

Then the Bayes estimator is the $(\hat{\theta}_{Bayes})$ minimizing the risk function $[R(\hat{\theta},\theta)]$ which is the expected loss. First of all we must find $[h(\theta|t)]$.

The Bayes estimator of $[\tau(\theta)]$ is denoting by;

$$\tau_i^* = t_i^*(x_1, x_2, \dots, x_n)$$

With respect to the loss function $[L(\hat{\theta}, \theta)]$ and prior distribution $[g(\theta)]$, it is defined as the estimator with smallest Bayes risk (i.e) it is the estimator that minimizes;

$$\int_{\forall\theta} L[t(x_1, x_2, \dots, x_n; \theta)]\, f(\theta|t) d\theta$$

$$\prod_{i=1}^{n} f(t_i, \theta) g(\theta) = \left(\frac{4}{\sqrt{\pi}}\right)^n \left(\frac{1}{\theta^3}\right)^n \prod_{i=1}^{n} t_i^2\, e^{-\frac{\sum_{i=1}^n t_i^2}{\theta^2}} e^{-\frac{c}{\theta^2}}$$

$$f(t) = \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^{n} t_i^2 \int_0^\infty \left(\frac{1}{\theta}\right)^{3n} e^{-\frac{(c+\sum_{i=1}^n t_i^2)}{\theta^2}} d\theta$$

Let $y = \frac{1}{\theta^2}$ $\quad \theta^2 = \frac{1}{y}$ $\quad \theta = \frac{1}{\sqrt{y}} = y^{-\frac{1}{2}}$ $\quad d\theta = -\frac{1}{2}\, y^{-\frac{3}{2}} dy$

$$f(t) = \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^{n} t_i^2 \int_0^\infty (Y)^{\frac{3n}{2}} e^{-y\,(c+\sum_{i=1}^n t_i^2)} \frac{1}{2}\, y^{-\frac{3}{2}} dy$$

$$f(t) = \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^{n} \frac{t_i^2}{2} \int_0^\infty y^{\frac{3n-3}{2}} e^{-y\,(c+\sum_{i=1}^n t_i^2)} dy$$

According to Gamma function;

$$f(t) = \frac{\left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n \frac{t_i^2}{2}}{(c+\sum_{i=1}^n t_i^2)^{\left(\frac{3n-1}{2}\right)}} \Gamma\left(\frac{3n-1}{2}\right) \qquad (16)$$

Therefore, $h(\theta|t)$ has been obtained.

$$f(t) = \left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^{n} \frac{t_i^2}{2} \int_0^\infty y^{\frac{3n}{2}-\frac{3}{2}} e^{-y\,(c+\sum_{i=1}^n t_i^2)} dy$$

$$f(t) = \frac{\left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n \frac{t_i^2}{2}}{(c+\sum_{i=1}^n t_i^2)^{\left(\frac{3n-1}{2}\right)}} \Gamma\left(\frac{3n-1}{2}\right) \qquad (17)$$

$$h(\theta|t) = \frac{\prod_{i=1}^n f(t_i, \theta) g(\theta)}{f(t)}$$

$$h(\theta|t) = \frac{\left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n t_i^2 \left(\frac{1}{\theta}\right)^{3n} e^{-\frac{(c+\sum_{i=1}^n t_i^2)}{\theta^2}}}{\frac{\left(\frac{4}{\sqrt{\pi}}\right)^n \prod_{i=1}^n \frac{t_i^2}{2}}{(c+\sum_{i=1}^n t_i^2)^{\left(\frac{3n-1}{2}\right)}} \Gamma\left(\frac{3n-1}{2}\right)}$$

$$h(\theta|t) = \frac{2(c+\sum_{i=1}^n t_i^2)^{\left(\frac{3n-1}{2}\right)} \left(\frac{1}{\theta^2}\right)^{\frac{3n}{2}} e^{-\frac{(c+\sum_{i=1}^n t_i^2)}{\theta^2}}}{\Gamma\left(\frac{3n-1}{2}\right)} \qquad (18)$$

we can show that;

$$\int_0^\infty h(\theta|t)\,d\theta = 1$$

This indicates that $h(\theta|t)$ is $p.d.f$. To find the Bayes estimator of (θ), under square error loss function, the risk is;

$$R(\hat{\theta},\theta) = E(\hat{\theta}-\theta)^2$$

$$= \int_{\forall\theta} (\hat{\theta}-\theta)^2\, h(\theta|t)d\theta$$

Then the Bayes estimator of (θ) is $(\hat{\theta}_{Bayes})$ which represent the conditional mean $[E(\theta|t)]$, or which so called posterior mean $E(\theta|t)$:

$$E(\theta|t) = \int_{\forall\theta} \theta\, h(\theta|t)d\theta$$

$$= \frac{2(c+\sum_{i=1}^n t_i^2)^{\left(\frac{3n-1}{2}\right)}}{\Gamma\left(\frac{3n-1}{2}\right)} \int_0^\infty \theta \left(\frac{1}{\theta^2}\right)^{\frac{3n}{2}} e^{-\frac{(c+\sum_{i=1}^n t_i^2)}{\theta^2}} d\theta$$

Let $y = \frac{1}{\theta^2}$ $\quad \theta^2 = \frac{1}{y}$ $\quad \theta = \frac{1}{\sqrt{y}} = y^{-\frac{1}{2}}$ $\quad d\theta = -\frac{1}{2}\, y^{-\frac{3}{2}}\, dy$

$$E(\theta|t) = k \int_0^\infty y^{-\frac{1}{2}} (y)^{\frac{3n}{2}} e^{-y(c+\sum_{i=1}^n t_i^2)} \frac{1}{2}\, y^{-\frac{3}{2}}\, dy$$

$$E(\theta|t) = \frac{k}{2} \int_0^\infty y^{\frac{3n-4}{2}}\, e^{-y(c+\sum_{i=1}^n t_i^2)}\, dy$$

$$E(\theta|t) = \frac{k}{2} \frac{1}{(c+\sum_{i=1}^n t_i^2)^{\frac{3n-2}{2}}} \int_0^\infty [y(c+\sum_{i=1}^n t_i^2)]^{\frac{3n-4}{2}}\, e^{-y(c+\sum_{i=1}^n t_i^2)} (c+\sum_{i=1}^n t_i^2)\, dy$$

$$E(\theta|t) = \frac{k}{2} \frac{1}{(c+\sum_{i=1}^n t_i^2)^{\frac{3n-2}{2}}}\, \Gamma\left(\frac{3n-2}{2}\right)$$

$$E(\theta|t) = \frac{2(c+\sum_{i=1}^n t_i^2)^{\frac{3n-1}{2}}}{\Gamma\left(\frac{3n-1}{2}\right)} \; \frac{\Gamma\left(\frac{3n-2}{2}\right)}{2(c+\sum_{i=1}^n t_i^2)^{\frac{3n-2}{2}}}$$

$$E(\theta|t) = (c+\sum_{i=1}^n t_i^2)^{\frac{3n-1-3n+2}{2}} \; \frac{\Gamma\left(\frac{3n-2}{2}\right)}{\Gamma\left(\frac{3n-1}{2}\right)}$$

$$\hat{\theta}_4 = E(\theta|t) = (c+\sum_{i=1}^n t_i^2)^{\frac{1}{2}} \; \frac{\Gamma\left(\frac{3n-2}{2}\right)}{\Gamma\left(\frac{3n-1}{2}\right)} \qquad (19)$$

4. Simulation Procedure

To compare the four estimator of the scale parameter of Maxwell distribution, we apply simulation procedure which depend, first on the cumulative distribution function, which is;

$$F_X(x) = pr(X \le x) = \int_0^x f(t)dt$$

$$F_T(t) = \int_0^x \frac{4}{\sqrt{\pi}} \frac{1}{\theta^3} t^2 e^{-\left(\frac{t}{\theta}\right)^2} dt \qquad (20)$$

According to integral of incomplete Gamma;

$$\gamma(S,X) = \int_0^x t^{S-1} \quad e^{-t} dt \qquad (21)$$

Where;

$$\gamma(S,X) = \sum_{k=0}^{\infty} \frac{X^S e^{-X} X^k}{S(S+1)\dots\dots(S+k)}$$

$$= X^S\,\Gamma(S)e^{-X}\sum_{k=0}^{\infty}\frac{X^k}{\Gamma(S+k+1)} \qquad (22)$$

After generating values of Maxweel distribution, Monte – Carlo simulation is performed to compare the four estimators of (θ), by using mean square error (MSE), where;

$$MSE(\theta) = \sum_{i=1}^{R} \frac{(\theta_i - \widehat{\theta}_i)^2}{R} \qquad (23)$$

Taking initial values for (θ), $(\theta = 3,5)$, and sample size $(n = 25, 50, 75, 100)$, taking values of constant $(c = 1,2)$, as follows.

Table (1): Average values of different estimators of scale parameter with mean square error.

n	θ	c	$\widehat{\theta}_1$	$\widehat{\theta}_2$	$\widehat{\theta}_3$	$\widehat{\theta}_4$
25	3	1	2.3165	2.3148	2.3416	2.1864
			0.3712	0.3051	0.3026	0.1801
		2	2.4267	2.4669	2.5449	2.1855
			0.3326	0.3021	0.2044	0.1730
	5	1	4.4678	2.4593	2.4396	2.1544
			0.33201	0.3005	0.2330	0.1730
		2	4.2517	2.546	2.5171	2.1774
			0.3206	0.2972	0.2259	0.1696
50	3	1	2.3619	2.4396	2.4950	2.1337
			0.3093	0.2362	0.2002	0.1675
		2	2.1456	2.5463	2.6028	2.752
			0.639	0.3004	0.1833	0.1380

	5	1	2.2886	2.6038	2.3255	2.752
			0.2304	0.4030	0.1534	0.1380
		2	2.2246	2.4299	2.8236	2.1957
			0.2044	0.1534	0.1997	0.1335
75	3	1	2.3177	2.0125	1.7668	2.0993
			0.1908	0.1104	0.2404	0.1038
		2	2.0769	2.0076	1.8693	1.8796
			0.1658	0.1102	0.2403	0.1102
	5	1	2.3158	2.999	2.464	2.1364
			0.3051	0.1140	0.2018	0.1029
		2	2.4467	2.996	2.030	2.4806
			0.4102	0.1014	0.2240	0.2011
100	3	1	2.1479	2.6028	2.6052	2.1252
			0.2396	0.1833	0.1585	0.1144
		2	2.0209	2.4299	2.1044	2.4201
			0.1946	0.1534	0.1454	0.1433
	5	1	2.0786	2.4299	2.1957	2.1500
			0.1804	0.1534	0.1335	0.1343
		2	2.0076	2.0125	2.0993	2.0994
			0.1846	0.1104	0.1035	0.1025

Note: the first figure represents the average estimates, second figure is MSE.

Conclusions

1. The estimation of scale parameter (θ) is necessary to compute the mean time to failure for this distribution, also to find the efficiency of estimator from comparing variance of different estimators.

2. For large sample size $(n = 100)$, the best estimators are, maximum likelihood, and proposed Bayes estimators since it gives smallest MSE, as indicated by table (1) for $(n = 100)$, $\{MSE(\hat{\theta}_3) = 0.1035, MSE(\hat{\theta}_4) = 0.1025\}$ $for\ \theta = 5, c = 2$.

References

[1] Ali Kasmi, S.M., Aslam, M. and Ali, S. (2012): on the Baysian estimation for two component mixture of Maxwell distribution, assuming type I censored data. Int. J. of Applied Science and Technology: 2(1): 197- 218.

[2] Bekker , A. and Roux , J.J. (2005)," Reliability characteristics of the Maxwell distribution": a Bayes estimation study", Comm. Stat (Theory & Meth.), Vol. 34, No. 11, pp. 2169-2178.

[3] Fisher, Len. (2009). The Perfect Swarm: the Science of Complexity in Everyday Life (Maxwell-Boltzmann distribution of students, pg. 24). Basic Books.

[4] Gabriela V. Cohen Freue, (2007), " The Pitman estimator of the Cauchy location parameter ", Journal of statistical planning & inference.

[5] Galit Shmueli, Sharad Borle, Peter Boatwright, (2005), " A useful distribution for fitting discrete data: revival of the Conway–Maxwell–Poisson distribution ", Appl. Statist. (2005)54, Part 1, pp. 127–142.

[6] Kadane, J. B., Shmueli, G., Minka, T. P., Borle, S. and Boatwright, P. (2003) Conjugate analysis of the Conway-Maxwell-Poisson distribution. To be published.

[7] Kasmi, A. S. M., Aslam, M. and Ali. S. (2011) A note of Maximum likelihood estimator for the mixture of Maxwell distributions using type-I censored scheme, The open stat. and prob. Journal, (3): 31 – 35.

[8] Krishna, H and Malik, M. (2009): Reliability estimation in Maxwell distribution with Type-II censored data, Int. Journal of Quality and Reliability management, 26 (2): 184 – 195.

[9] Maxwell – Boltzmann (2008), available at: http://wikipedia.org/wiki/ Maxwell – Boltzmann distribution (accessed June 20 , 2008).

[10] Maxwell Distribution (2008), available at: http://math world. Wolfram.com/ Maxwell distribution html (accessed June 20 , 2008).

[11] M. Shakil, B. M. Golam Kibria, J. N. Singh, (2006), " DISTRIBUTION OF THE RATIO OF MAXWELL AND RICE RANDOM VARIABLES , Int. J. Contemp. Math. Sci., Vol. 1, 2006, no. 13, 623 – 637.

[12] Rinne, H. (2009): The Weibull Distribution: A Handbook, Chapman & Hall/CRC.

[13] Sanku Dey(2011), "Bayesian estimation and prediction for Maxwell distribution", st. Anthony's College, Shillong, Meghalaya. India.

[14] Tasnim H.K. Al-Baldawi, (2013), "Comparison of Maximum Likelihood and some Bayes Estimators for Maxwell Distribution based on Non-informative Priors", J. Baghdad for Sci. Vol.10(2).