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Efficient Method for Solving System of Nonlinear PDEs

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Abstract

This paper presents an analysis solution for systems of nonlinear partial differential equations using decomposition method. Two illustrated examples has been introduced, and the method has shown a high-precision, fast approach to solve nonlinear system of PDEs with initial conditions, there is no need to convert the nonlinear terms into the linear ones due to the Adomian polynomials, not requiring any discretization or assumption for a small parameter to be present in the problem. The steps of the method are easy implemented and high accuracy.

Keywords: System of PDEs, Decomposition Method, Convergence Analysis.

1. Introduction

Systems of partial differential equations (PDEs) have been use to described many important models in real life, such as contamination, distribution of shallow water, heat, waves contamination and the chemical reaction – distribution model [1-4]. The general ideas and key characteristics of these systems are generally applicable [5]. In recent years, many authors have focused on solving non-linear systems of PDEs using various methods such that HAM [6], VIM [7], DTM [8], HPM [9,10], ADM [11,12], coupled Laplace decomposition method [13], and semi analytic technique [14]. Recently, decomposition method and its modifications have been used in wider scope to solve different types of PDEs. In 2001 Wazwaz and Al-sayed [15] presented a modification of the ADM for non-linear operator, that is replaced the process of dividing f into two parts by infinite series of components. Another modification is the restarted ADM [16]. In 2005, Wazwaz [17] found another modification to the ADM to overcome the difficulties that arise when the equation consist singular points. This modification represent useful for similar models with singularities. Luo [18] was proposed another modification based on separates the ADM into two steps and so is termed the two steps ADM (TSAMD) the purpose behind the proposed scheme is to identify the exact solution more readily and eliminate some calculations. Here we used ADM for solving systems of nonlinear PDEs with initial conditions.

2. Solving System of Nonlinear PDEs by ADM

This section consist the procedure of the ADM to solve system of nonlinear PDEs. Firstly writes the system of nonlinear PDEs as follows:

$$L_t u + R\big(u(x,y,t)\big) + N_1(u,v) = h_1(x,y,t) \quad (1)$$
$$L_t v + R(v(x,y,t)) + N_2(u,v) = h_2(x,y,t)$$

with ICs:

$$u(x,y,0) = f(x,y)$$
$$v(x,y,0) = g(x,y) \quad (2)$$

Where $x, y \in R, L$ is a linear differential operator $\left(L_t = \frac{\partial}{\partial t}\right)$, R is a remained of the linear operator, N_1 *and* N_2 are nonlinear operators and $h_1(x,y,t), h_2(x,y,t)$ are the nonhomogeneous part. Take $L_t^{-1} = \int_0^t (.)dt$ on the system (1), we have:

$$u(x,y,t) = f(x,y) + L_t^{-1}(h_1) - L_t^{-1}R(u(x,y,t)) - L_t^{-1}[N_1(u,v)]$$
$$v(x,y,t) = g(x,y) + L_t^{-1}(h_2) - L_t^{-1}R(v(x,y,t)) - L_t^{-1}[N_2(u,v)] \quad (3)$$

$u(x,y,t), v(x,y,t)$ can be represented by the decomposition series:

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n(x,y,t)$$
$$v(x,y,t) = \sum_{n=0}^{\infty} v_n(x,y,t) \quad (4)$$

$N_1(u,v), N_2(u,v)$ are nonlinear terms can be represented by Adomain polynomials

$$N_1(u,v) = \sum_{n=0}^{\infty} A_n(x,y,t)$$
$$N_2(u,v) = \sum_{n=0}^{\infty} B_n(x,y,t) \quad (5)$$

$$A_n or\ B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}(F\sum_{i=0}^{\infty}\big(\lambda^i u_i\big))_{\lambda=0}\ ,$$
$$n = 0,1,2,\ldots (6)$$

Now substituting equation (4), (5) into equation (3), to obtain

$$\sum_{n=0}^{\infty} u_n(x,y,t) = f(x,y) + L_t^{-1}(h_1) - L_t^{-1}R(u_n(x,y,t)) - L_t^{-1}(\sum_{n=0}^{\infty} A_n)$$
$$\sum_{n=0}^{\infty} v_n(x,y,t) = g(x,y) + L_t^{-1}(h_2) - L_t^{-1}R\big(v_n(x,y,t)\big) - L_t^{-1}(\sum_{n=0}^{\infty} B_n) \ (7)$$

We get recursive relation:

$$u_0(x,y,t) = f(x,y) + L_t^{-1}(h_1)$$
$$u_{k+1}(x,y,t) = L_t^{-1}R(u_k(x,y,t)) - L_t^{-1}(A_k), \quad k \geq 0 \quad (8)$$
$$v_0(x,y,t) = g(x,y) + L_t^{-1}(h_2)$$
$$v_{k+1}(x,y,t) = L_t^{-1}R(v_k(x,y,t)) - L_t^{-1}(B_k), k \geq 0 \quad (9)$$

In the next section we give an illustrative example

3. Illustrative Examples

In this section ADM has been used to solve system of nonlinear PDEs

Example 1

Consider the following system of 2D, nonlinear system of Burgers equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$
$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}$$

Subject to IC: $u(x,y,0) = x + y$,

$$w(x,y,0) = \mathrm{x} - \mathrm{y}\ , (\mathrm{x},\mathrm{y},\mathrm{t}) \in R^2 \times [0, \frac{1}{\sqrt{2}})$$

Solution Take $L_x^{-1} = \int_0^t (.)dt$; for the system, to obtain

$u(x,y,t) = u(x,y,0) + L_x^{-1}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - u\frac{\partial u}{\partial x} - w\frac{\partial u}{\partial y}\right]$

$w(x,y,t) = w(x,y,0) + L_x^{-1}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - u\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial y}\right]$

$u(x,y,t) = x + y + L_x^{-1}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - u\frac{\partial u}{\partial x} - w\frac{\partial u}{\partial y}\right]$

$w(x,y,t) = x - y + L_x^{-1}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - u\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial y}\right]$

$u(x,y,t), w(x,y,t)$ can be represented by the decomposition series

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n(x,y,t)$$

$$w(x,y,t) = \sum_{n=0}^{\infty} w_n(x,y,t)$$

$u\frac{\partial u}{\partial x}, w\frac{\partial u}{\partial y}$ *and* $u\frac{\partial w}{\partial x}, w\frac{\partial w}{\partial y}$ are nonlinear terms can be represented by Adomian polynomials as:

$$u\frac{\partial u}{\partial x} = \sum_{n=0}^{\infty} A_n \ , \ w\frac{\partial u}{\partial y} = \sum_{n=0}^{\infty} B_n$$

and

$$u\frac{\partial w}{\partial x} = \sum_{n=0}^{\infty} \boldsymbol{C}_n \ , w\frac{\partial w}{\partial y} = \sum_{n=0}^{\infty} D_n$$

$\sum_{n=0}^{\infty} u_n(x,y,t) = x + y + L_t^{-1}\left[\frac{\partial^2 u_n}{\partial x^2} + \frac{\partial^2 u_n}{\partial y^2} - \sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n\right]$

$\sum_{n=0}^{\infty} w_n(x,y,t) = x - y + L_t^{-1}\left[\frac{\partial^2 w_n}{\partial x^2} + \frac{\partial^2 w_n}{\partial y^2} - \sum_{n=0}^{\infty} C_n - \sum_{n=0}^{\infty} \boldsymbol{D}_n\right]$

$u_0 = x + y$

$$u_1 = L_t^{-1}\left[\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} - A_0 - B_0\right]$$

$$u_{k+1} = L_t^{-1}\left[\frac{\partial^2 u_k}{\partial x^2} + \frac{\partial^2 u_k}{\partial y^2} - A_k - B_k\right], k \geq 1$$

$w_0 = x - y$

$$w_1 = L_t^{-1}\left[\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} - C_0 - D_0\right]$$

$$w_{k+1} = L_t^{-1}\left[\frac{\partial^2 w_k}{\partial x^2} + \frac{\partial^2 w_k}{\partial y^2} - C_k - D_k\right], k \geq 1$$

The Adomian polynomials for the nonlinear term $u\frac{\partial u}{\partial x}$, $w\frac{\partial u}{\partial y}$ are computed by:

$A_0 = u_0\frac{\partial u_0}{\partial x}$, $A_1 = u_1\frac{\partial u_0}{\partial x} + u_0\frac{\partial u_1}{\partial x}$

$B_0 = w_0\frac{\partial u_0}{\partial y}$, $B_1 = w_1\frac{\partial u_0}{\partial y} + w_0\frac{\partial u_1}{\partial y}$

and $u\frac{\partial w}{\partial x}$, $w\frac{\partial w}{\partial y}$ are computed by:

$C_0 = u_0\frac{\partial w_0}{\partial x}$, $C_1 = u_1\frac{\partial w_0}{\partial x} + u_0\frac{\partial w_1}{\partial x}$

$D_0 = w_0\frac{\partial w_0}{\partial x}$, $D_1 = w_1\frac{\partial w_0}{\partial x} + w_0\frac{\partial w_1}{\partial x}$

$$u_1 = L_t^{-1}\left[\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} - A_0 - B_0\right]$$

$A_0 = (x+y)(1) = x + y$

$B_0 = (x-y)(1) = x - y$

$$u_1 = \int_0^t [-(x+y) - (x-y)]ds$$

$$u_1 = \int_0^t -2x ds = -2xt$$

$$w_1 = L_t^{-1}\left[\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} - C_0 - D_0\right]$$

$C_0 = (x+y)(1) = x + y$

$D_0 = (x-y)(-1) = -x + y$

$$w_1 = \int_0^t [-(x+y) - (-x+y)]ds$$

$$w_1 = \int_0^t -2y ds = -2yt$$

$$u_2 = L_t^{-1}\left[\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} - A_1 - B_1\right]$$

$$A_1 = (-2xt)(1) + (x+y)(-2t)$$
$$= -2xt - 2xt - 2yt$$
$$= -4xt - 2yt$$
$$B_1 = (-2yt)(1) + (x-y)(0) = -2yt$$

$$u_2 = \int_0^t [-(-4xs - 2ys) - (-2ys)]ds$$

$$u_2 = \int_0^t [(4xs + 2ys + 2ys)]ds$$

$$u_2 = \int_0^t [(4xs + 4ys)]ds = 2xt^2 + 2yt^2$$

$$w_2 = L_t^{-1}\left[\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} - C_1 - D_1\right]$$

$$C_1 = (-2xt)(1) + (x+y)(0) = -2xt$$
$$D_1 = (-2yt)(-1) + (x-y)(-2t)$$
$$D_1 = 2yt - 2xt + 2yt = 4yt - 2xt$$

$$w_2 = \int_0^t [(2xs - 4ys + 2xs)]ds$$

$$w_2 = \int_0^t [(4xs - 4ys)]ds = 2xt^2 - 2yt^2$$

$$u_3 = L_t^{-1}\left[\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} - A_2 - B_2\right]$$

$$A_2 = u_2\frac{\partial u_0}{\partial x} + u_1\frac{\partial u_1}{\partial x} + u_0\frac{\partial u_2}{\partial x}$$
$$A_2 = (2xt^2 + 2yt^2)(1) + (-2xt)(-2t) + (x+y)(2t^2)$$
$$A_2 = 2xt^2 + 2yt^2 + 4xt^2 + 2xt^2 + 2yt^2$$
$$A_2 = 8xt^2 + 4yt^2$$

$$B_2 = w_2\frac{\partial u_0}{\partial x} + w_1\frac{\partial u_1}{\partial x} + w_0\frac{\partial u_2}{\partial x}$$
$$B_2 = (2xt^2 - 2yt^2)(1) + (-2yt)(0) + (x-y)(2t^2)$$
$$= 4xt^2 - 4yt^2$$

$$u_3 = \int_0^t [-(8xs^2 + 4ys^2) - (4xs^2 - 4ys^2)]\,ds$$

$$u_3 = \int_0^t [-8xs^2 - 4ys^2 - 4xs^2 + 4ys^2]ds$$

$$u_3 = \int_0^t (-12xs^2)ds = -4xt^3$$

$$w_3 = L_t^{-1}\left[\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} - C_2 - D_2\right]$$

$$C_2 = u_2\frac{\partial w_0}{\partial x} + u_1\frac{\partial w_1}{\partial x} + u_0\frac{\partial w_2}{\partial x}$$
$$C_2 = (2xt^2 + 2yt^2)(1) + (-2xt)(0) + (x+y)(2t^2)$$
$$C_2 = 2xt^2 + 2yt^2 + 2xt^2 + 2yt^2$$
$$= 4xt^2 + 4yt^2$$

$$D_2 = w_2\frac{\partial w_0}{\partial x} + w_1\frac{\partial w_1}{\partial x} + w_0\frac{\partial w_2}{\partial x}$$

$$D_2 = (2xt^2 - 2yt^2)(-1) + (-2yt)(-2t) + (x-y)(-2t^2)$$

$$D_2 = -4xt^2 + 8yt^2$$

$$w_3 = \int_0^t [-(4xs^2 + 4ys^2) - (-4xs^2 + 8ys^2)]ds$$

$$w_3 = \int_0^t [-4xs^2 - 4ys^2 + 4xs^2 - 8ys^2]ds$$

$$w_3 = -4yt^3$$

$$u_4 = 4xt^4 + 4yt^4$$

$$w_4 = 4xt^4 - 4yt^4$$

$$u(x,y,t) = u_0 + u_1 + u_2 + u_3 + u_4 + \cdots$$

$$u(x,y,t) = x + y - 2xt + 2xt^2 + 2yt^2 - 4xt^3 + 4xt^4 + 4yt^4 + \cdots$$

$$u(x,y,t) = x + y + 2xt^2 + 2yt^2 + 4xt^4 + 4yt^4 + \cdots - 2xt - 4xt^3 - 8xt^5 - \cdots$$

$$u(x,y,t) = (x+y)(1 + 2t^2 + 4t^4 + \cdots) - 2xt(1 + 2t^2 + 4t^4 + \cdots)$$

That is closed to the exact solution:

$$u(x,y,t) = (x+y)\left(\frac{1}{1-2t^2}\right) - 2xt\left(\frac{1}{1-2t^2}\right) = \frac{x+y-2xt}{1-2t^2}$$

$$w(x,y,t) = w_0 + w_1 + w_2 + w_3 + w_4 + \cdots$$

$$w(x,y,t) = x - y - 2yt + 2xt^2 - 2yt^2 - 4yt^3 + 4xt^4 - 4yt^4 + \cdots$$

$u(x,t), v(x,t)$ can be represented by the decomposition series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$$

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t)$$

vv_x, uv_x and $u_x v$ are nonlinear terms can be

$$w(x,y,t) = (x - y + 2xt^2 - 2yt^2 + 4xt^4 - 4yt^4 + 8xt^6 - 8yt^6 + \cdots) + (-2yt - 4yt^3 - 8yt^5 - \cdots)$$

$$w(x,y,t) = (x-y)(1 + 2t^2 + 4t^4 + 8t^6 + \cdots) - 2yt(1 + 2t^2 + 4t^4 + 8t^6 + \cdots)$$

That is closed to the exact solution:

$$w(x,y,t) = (x-y)\left(\frac{1}{1-2t^2}\right) - (2yt)\left(\frac{1}{1-2t^2}\right) = \frac{x-y-2yt}{1-2t^2}$$

Example 2

Consider a system of 3^{rd} order nonlinear PDE

$$u_t + vv_x = 0;\ v_t + v_{xxx} + uv_x + u_x v = 0$$

Subject to ICs: $u(x,0) = 2\,sech^2(x)$, $v(x,0) = 2\,sech(x)$

Solution

Take $L_t^{-1} = \int_0^t (.)dt$ for the system, we obtain

$$u(x,t) = u(x,0) + L_x^{-1}[-vv_x]$$

$$v(x,t) = v(x,0) + L_x^{-1}[-v_{xxx} - uv_x - u_x v]$$

$$u(x,t) = 2\,sech^2(x) + L_x^{-1}[-vv_x]$$

$$v(x,t) = 2\,sech(x) + L_x^{-1}[-v_{xxx} - uv_x - u_x v]$$

represented by Adomian polynomials:

$vv_x = \sum_{n=0}^{\infty} A_n,\ uv_x = \sum_{n=0}^{\infty} B_n$

and $u_x v = \sum_{n=0}^{\infty} C_n$

$$\sum_{n=0}^{\infty} u_n(x,t) = 2\ sech^2(x) + L_x^{-1}\left[-\sum_{n=0}^{\infty} A_n\right]$$

$$\sum_{n=0}^{\infty} v_n(x,t) = 2\ sech(x) + L_x^{-1}\left[-\frac{\partial^3 v_n}{\partial x^3} - \sum_{n=0}^{\infty} B_n - \sum_{n=0}^{\infty} C_n\right]$$

$$u_0 = 2\ sech^2(x)$$

$$u_1 = L_x^{-1}[-(-4\ tanh(x)sech^2(x))]$$

$$u_1 = \int_0^t (\,4\ tanh(x)sech^2(x))\,ds$$

$$v_1 = L_x^{-1}\left[-\frac{\partial^3 v_0}{\partial x^3} - B_0 - \boldsymbol{C}_0\right]$$

The Adomian polynomials for the nonlinear term vv_x are computed by:

$A_0 = v_0 v_{0x}\ , A_1 = v_1 v_{0x} + v_0 v_{1x}$

And $uv_x, u_x v$ are computed by:

$B_0 = u_0 v_{0x}\ ,\quad B_1 = u_1 v_{0x} + u_0 v_{1x}$

$c_0 = u_{0x} v_0\ ,\quad c_1 = u_{1x} v_0 + u_{0x} v_1$

$$v_1 = L_x^{-1}\begin{bmatrix} -10tanh(x)sech^3(x) + 2\ tanh^3(x)sech(x) \\ +4\ tanh(x)sech^3(x) + 8\ tanh(x)sech^3(x) \end{bmatrix}$$

$$v_1 = L_x^{-1}[2\ tanh(x)sech^3(x) + 2\ tanh^3(x)sech(x)]$$

$$v_1 = L_x^{-1}\big[2\ tanh(x)sech(x)\big(sech^2(x) + tanh^2(x)\big)\big]$$

$$v_1 = \int_0^t (\,2\ tanh(x)sech(x))ds$$

$$v_1 = \big(2\ tanh(x)sech(x)\big)t$$

$$u_2 = L_x^{-1}[-A_1]$$

$$A_1 = v_1 v_{0x} + v_0 v_{1x}$$

$$\mathrm{A_1} = (2\ \mathrm{tanh(x)sech(x)t})\big(-2\ \mathrm{tanh(x)sech(x)}\big)$$

$$A_0 = v_0 v_{0x} = \big(2\ sech(x)\big)\big(-2\ tanh(x)\ sech(x)\big)$$
$$= -4\ tanh(x)sech^2(x)$$

$$u_1 = L_x^{-1}[-A_0]$$

$$u_{k+1} = L_x^{-1}[-A_k],\qquad k \geq 1$$

$$v_0 = 2\ sech(x)$$

$$u_1 = (4\ tanh(x)sech^2(x))t$$

$$v_1 = L_x^{-1}\left[-\frac{\partial^3 v_0}{\partial x^3} - B_0 - \boldsymbol{C}_0\right]$$

$$v_{k+1} = L_x^{-1}\left[-\frac{\partial^3 v_k}{\partial x^3} - B_k - \boldsymbol{C}_k\right],\ k \geq 1$$

$$\frac{\partial^3}{\partial x^3}(2\ sech(x)) = -10tanh(x)sech^3(x)$$
$$-\ 2\ tanh^3(x)sech(x)$$

$$B_0 = u_0 v_{0x} = \big(2\ sech^2(x)\big)\big(-\ 2\ tanh(x)\ sech(x)\big)$$

$$= -4\ tanh(x)sech^3(x)\ c_0 = u_{0x} v_0$$
$$= (-4\ tanh(x)sech^2(x))\big(2\ sech(x)\big)$$

$$= -8\ tanh(x)sech^3(x)$$

$$u_1 = L_x^{-1}[-A_0]$$

$+\left(2\mathrm{sech}(\mathrm{x})\right)(2\ \mathrm{sech}^3(\mathrm{x})\mathrm{t}-2\ \tanh^2(\mathrm{x})\mathrm{sech}(\mathrm{x})\mathrm{t})$

$$A_1=-4\ tanh^2(x)sech^2(x)t+4\ sech^4(x)t$$

$$-4\ tanh^2(x)sech^2(x)t$$

$$A_1=-8\ tanh^2(x)sech^2(x)t + 4\ sech^4(x)t$$

$$u_2=L_x^{-1}[-(-8\ tanh^2(x)sech^2(x)t + 4\ sech^4(x)t)]$$

$$u_2=L_x^{-1}[8\ tanh^2(x)sech^2(x)t-4sech^4(x)t]$$

$$u_2=L_x^{-1}\left[4sech^2(x)t\left(2\ tanh^2(x)-sech^2(x)\right)\right]$$

$$u_2=L_x^{-1}\left[4sech^2(x)t\left(\frac{cosh(2x)-1}{cosh^2(x)}-\frac{1}{cosh^2(x)}\right)\right]$$

$$u_2=L_x^{-1}[4sech^4(x)t(\cosh(2x)-2)]$$

$$u_2=\int_0^t(4sech^4(x)(\cosh(2x)-2)s)ds$$

$$u_2=4sech^4(x)(\cosh(2x)-2)\frac{t^2}{2}$$

$$u_2=-2\ sech^4(x)(2-\cosh(2x))t^2$$

$$v_2=L_x^{-1}\left[-\frac{\partial^3 v_1}{\partial x^3}-B_1-C_1\right]$$

$$\frac{\partial^3}{\partial x^3}(2\ tanh(x)sech(x)t)=-10sech^5(x)t$$

$$+36tanh^2(x)sech^3(x)t-2tanh^4(x)\mathrm{sech}(x)t$$

$$v_2=L_x^{-1}\left[-\left(\begin{matrix}-10sech^5(x)t+36tanh^2(x)sech^3(x)t-\\2tanh^4(x)\mathrm{sech}(x)t\end{matrix}\right)\right.$$

$$-(-12\ tanh^2(x)sech^3(x)t+4\ sech^5(x)t)-$$

$$\left.(8\ sech^5(x)t-24\ tanh^2(x)sech^3(x)t)\right]$$

$$v_2=L_x^{-1}\left[\begin{matrix}10sech^5(x)t-36tanh^2(x)sech^3(x)t+\\2tanh^4(x)sech(x)t+12\ tanh^2(x)sech^3(x)t\\-4\ sech^5(x)t-8\ sech^5(x)t+24\ tanh^2(x)sech^3(x)t\end{matrix}\right]$$

$$v_2=L_x^{-1}[-2\ sech^5(x)t+2\ tanh^4(x)sech(x)t]$$

$$B_1=u_1v_{0x}+u_0v_{1x}$$

$$B_1=(4\ tanh(x)sech^2(x)t)\left(-2tanh(x)sech(x)\right)$$

$$+\left(2sech^2(x)\right)(2\ sech^3(x)t-2tanh^2(x)sech(x)t)$$

$$B_1=-8\ tanh^2(x)sech^3(x)t + 4\ sech^5(x)t$$

$$-4\ \ tanh^2(x)sech^3(x)t$$

$$B_1=-12\ tanh^2(x)sech^3(x)t + 4\ sech^5(x)t$$

$$C_1=u_{1x}v_0+u_{0x}v_1$$

$$C_1=(4\ sech^4(x)t-8\ tanh^2(x)sech^2(x)t)\left(2sech(x)\right)+(-4\ \ tanh(x)sech^2(x))(2\ tanh(x)sech(x)t)$$

$$C_1=8\ sech^5(x)t-16\ tanh^2(x)sech^3(x)t - 8tanh^2(x)sech^3(x)t$$

$$C_1=8\ sech^5(x)t-24\ tanh^2(x)sech^3(x)t$$

$$v_2 = L_x^{-1}\left[2\,sech(x)t\left(tanh^4(x) - sech^4(x)\right)\right]$$

$$v_2 = L_x^{-1}\left[2\,sech(x)t\left(tanh^2(x) - sech^2(x)\right)\right]$$

$$v_2 = L_x^{-1}\left[2\,sech(x)t\left(\frac{\frac{1}{2}(cosh(2x)-1)}{cosh^2(x)} - \frac{1}{cosh^2(x)}\right)\right]$$

$$v_2 = L_x^{-1}\left[2\,sech^3(x)t\left(\frac{1}{2}cosh(2x) - \frac{3}{2}\right)\right]$$

$$v_2 = \int_0^t (\,sech^3(x)(cosh(2x)-3)s)\,ds$$

$$v_2 = -\frac{1}{2}sech^3(x)(3 - cosh(2x))t^2$$

and so on

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$$
$$= u_0 + u_1 + u_2 + \cdots$$

$$u(x,t) = 2sech^2(x) + \left(4tanh(x)sech^2(x)\right)t$$
$$-\left(2sech^4(x)(2 - cosh(2x))\right)t^2 + \cdots$$

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) \quad = v_0 + v_1 + v_2 + \cdots$$

$$v(x,t) = 2sech(x) + \left(2tanh(x)sech(x)\right)t$$
$$-\frac{1}{2}(3 - cosh(2x))sech^3(x)t^2 + \cdots$$

That is closed to
$u(x,t) = 2sech^2(x-t)$,
$v(x,t) = 2sech(x-t)$
This is exact solution.

4. Conclusions

In this article, decomposition method is used to solve system of nonlinear PDEs to get exact analytical solution, where numerical method are used to solved the same examples but cannot be getting exact analytical solution. Moreover, the convergence concept of the decomposition series was thoroughly investigated to confirm the rapid convergence of the resulting series. So, this approach is very efficient, easy implementation and rapid convergence to the exact solutions.

References

[1] Batiha B, Noorani MSM, Hashim I. Numerical simulations of systems of PDEs by Variational iteration method. Phys. Lett. A. 2008; 372: 822–829.
[2] Wazwaz AM. The Variational iteration method for solVing linear and nonlinear systems of PDEs, Comput. Math. Appl. 2007; 54: 895–902.

[3] Tawfiq LNM and Altaie H. 2020. Recent Modification of Homotopy Perturbation Method for Solving System of Third Order PDEs. Journal of Physics: Conference Series. 1530 (012073): 1-8. IOP Publishing.
[4] Somjate Duangpithak. Variational iteration method for special nonlinear partial differential eqUations. Int. JoUrnal of Math. Analysis. 2012; 6(22): 1071-1077.

[5] Enadi, M. O., and Tawfiq, L.N.M., 2019, New Technique for Solving Autonomous Equations, Ibn Al-Haitham Journal for Pure and Applied Science, 32(2), p: 123-130, Doi: 10.30526/32.2.2150.
[6] Adomian G., 1995, The diffusion-brusselator eqUation, Comput. Math. Appl. 29: 1 - 3.

[7] Tawfiq, L.N.M. and Naoum, R.S., 2007, Density and approximation by using feed forward Artificial neural networks. Ibn Al-Haitham Journal for Pure & Applied Sciences., 20(1): 67-81.

[8] Abdul-Majid Wazwaz. The Variational iteration method for solving linear and nonlinear systems of PDEs. Computers and Mathematics with Applications. 2007; 54: 895–902.

[9] Tawfiq LNM, Rasheed HW. 2013, On Solution of Non Linear Singular Boundary Value Problem. IHJPAS. 26(3): 320- 8.

[10] Tawfiq LNM, 2005, On Training of Artificial Neural Networks, Al-Fatih journal, 1(23), 130-139

[11] Salih, H., Tawfiq, L.N.M, Yahya, Z.R.I, and Zin, S.M., 2018, Solving Modified Regularized Long Wave Equation Using Collocation Method. Journal of Physics: Conference Series. 1003(012062): 1-10. doi :10.1088/1742-6596/1003/1/012062.

[12] Tawfiq, L.N.M. and Salih, O. M., 2019, Design neural network based upon decomposition approach for solving reaction diffusion equation, Journal of Physics: Conference Series, **1234** (012104):1-8.

[13] Tawfiq, L.N.M, Jasim, K.A., and Abdulhmeed, E.O., 2015, Mathematical Model for Estimation the Concentration of Heavy Metals in Soil for Any Depth and Time and its Application in Iraq, International Journal of Advanced Scientific and Technical Research, 4(5), 718-726.

[14] Wazwaz AM. The modified decomposition method for analytic treatment of differential eqUations. Appl. Math. CompUt.(2006). 173(1):165-176.

[15] Tawfiq, L.N.M, Al-Noor, N.H., and Al-Noor, T. H., 2019, Estimate the Rate of Contamination in Baghdad Soils By Using Numerical Method, Journal of Physics: Conference Series, **1294** (032020),1-10, doi:10.1088/1742-6596/1294/3/032020.

[16] Tawfiq, L.N.M, Jasim KA, and Abdulhmeed, E.O., 2015, Pollution of soils by heavy metals in East Baghdad in Iraq. International Journal of Innovative Science, Engineering & Technology, 2(6): 181-187.

[17] Kareem ZH and Tawfiq LNM, 2020, Recent Modification of Decomposition Method for Solving Nonlinear Partial Differential Equations. Journal of Advances in mathematics. 18: 154-161.

[18] Debnath L. Nonlinear Water Waves. Boston: Academic Press.(1994).

[19] Tawfiq, L.N.M, Jasim K.A., and Abdulhmeed, E.O., 2016, Numerical Model for Estimation the Concentration of Heavy Metals in Soil and its Application in Iraq. Global Journal of Engineering science and Researches. 3(3): 75- 81.

[20] Adomian G. Nonlinear Stochastic Operator Equations, Academic Press, San Diego.(1986).

[21] Abassy TA, El-Tawil MA, Saleh HK . The solution of KdV and mKdV equations Using Adomian padé approximation. Int. J. Nonlinear Sci. Numer. Simul. (2004). 5(4): 327-339.

[22] Tawfiq, L.N.M., and Jaber, A.K., 2016, Mathematical Modeling of Groundwater Flow, Global Journal of Engineering Science and Researches, 3(10), 15-22. doi: 10.5281/zenodo.160914.

[23] Tawfiq, L.N.M, and Hassan, M. A., 2018, Estimate the Effect of Rainwaters in Contaminated Soil by Using Simulink Technique, Journal of Physics, 1003(012057): 1-6.

[24] Lesnic D. The decomposition method for initial Value problems. Appl. Math. Comput.(2006). 181(1):206-213.

[25] Lesnic D. A nonlinear reaction-diffusion process Using the Adomian decomposition method. Int. Commun. Heat Mass Transfer.(2007).34 (2): 129-135.

[26] Tawfiq, L.N.M. and Jabber, A.K., 2018, Steady State Radial Flow in Anisotropic and Homogenous in Confined Aquifers, Journal of Physics, 1003(012056) : 1-11.

[27] Tawfiq, L.N.M., and Jaber, A. K., 2017, Solve The Ground Water Model Equation Using Fourier Transforms Method, International Journal of Advances in Applied Mathematics and Mechanics, 5(1), 75-80.

[28] Enadi, M.O., Tawfiq, L.N.M., 2019, New Approach for Solving Three Dimensional Space Partial Differential Equation, Baghdad Science Journal, 16(3): 786-792.