# Using Markov Chains to Establish the System of Reliability and Preventive Maintenance with Application Part

**Mohanad T. Yaseen[1] , Dr. Faris Mahdi Alwan[2]**

[1] University of Baghdad, Faculty of Administration and Economics, Department of Statistics,

moh597@yahoo.com, 009647901290797

[2] University of Baghdad, Faculty of Administration and Economics, Department of Statistics.

## Abstract

In this research, the problem of calculating reliability by Using Markov chains to establish the system of reliability and preventive maintenance and the periods during which the decision maker must perform preventive maintenance using Maximum Likelihood Estimation method and White method and the comparison between them will be addressed, as these two methods will be applied to data collected for one of the institutions and find the optimal solution and the comparison between the two methods by comparing them to where List the advantages and disadvantages of each of the two methods.

**Keywords**: Markov Chains, Reliability Systems, Maximum Likelihood Estimation method, White method

**Corresponding Author**
Mohanad T. Yaseen
Department of Statistics
University of Baghdad, Faculty of Administration and Economics
E-mail: moh597@yahoo.com , the1eng83@gmail.com

# استعمال سلاسل ماركوف في تصميم نظام معولية وصيانة وقائية مع تطبيق عملي

**مهند طه ياسين[1] فارس مهدي علوان[2]**
**[1،2]جامعة بغداد - كلية الادارة والاقتصاد - قسم الاحصاء**

**المستخلص**
سيتم في هذا البحث معالجة مشكلة احتساب المعولية باستعمال سلاسل ماركوف في تصميم نظام معولية وصيانة وقائية والفترات التي يجب على متخذ القرار اجراء صيانة وقائية فيها باستعمال طريقة الامكان الاعظم وطريقة وايت والمقارنة بينهما، حيث سيتم تطبيق هذين الطريقتين على بيانات تم جمعها لاحد المؤسسات وايجاد الحل الامثل والمفاضلة بين الطريقتين عن طريق المقارنة بينهما حيث سيتم ذكر مميزات ومساويء كل اسلوب من الاسلوبين
**الكلمات المفتاحية : سلاسل ماركوف، أنظمة المعقولية، طريقة تقدير الإمكان الأعظم، طريقة White**

 **ISSN (1681- 6870)**

## 1. Introduction

The concept of reliability is one of the important concepts that accompanied the technological development of engineering, electronic and electrical systems, devices and complex machines in many fields. As it is considered one of the important methods for estimating the efficiency of the system's work and its ability to continue to work, by building and analyzing the error tree with **both** static and dynamic types, which is a way to design a system that can be analyzed by drawing graphs and determining the factors that cause the system's failure. And there are many methods for analyzing the dynamic error tree and the method of Markov chains (with continuous time) is one of the important common methods, because of the multiplicity of applications that are used in many administrative and engineering fields and in the field of communication, and what distinguishes[1].

Markov chains are represented by a contract that indicates the different cases of the system and the parentheses of the link that indicate failure and repair rates between these cases, and there is a second method for calculating reliability is dynamic Biz networks that can be used in complex systems in particular, including engineering, mechanical and electrical and have a great importance in calculating reliability. It can be expressed as follows (they are probabilistic networks based on the graph theory guided in the form of arcs and nodes and each node represents to a discontinuous random variable and the arcs represent to the direct conditional probability relationships between the connected nodes).

## 2. Mathematical Model

### 2.1 Weibull distribution

It is one of the most important continues distributions and failure distributions, as the Swedish scientist (Wallodii Weibull) in 1939 derived it. He outlined some of his applications and was used to describe diversity in failure cases and to describe the failure of some electrical devices such as a vacuum valve [6] .

This distribution emerged in the Second World War because of its wide applications in the field of reliability and life tests were the focus of attention of researchers, they mentioned its properties and its parameters were estimated and many researches were done in the field of its use and a lot of research was published about it because of its importance in theory and practice.

The probability density function for the two-parameter Weibull distribution is:

$$f(t) = \begin{cases} \frac{\rho}{\theta} t^{\rho-1} e^{\frac{t^{\rho}}{\theta}} & , I_{(0,\infty)} \ ; \ \rho, \theta > 0 \\ 0 & o.w \end{cases} \qquad (1)$$

Where it represents:

$\rho$: *Shape parameter*

$\theta$: *Scale parameter*

Where the aggregate failure distribution function, reliability function, failure rate function, and aggregate failure rate function for the Weibull model of failure are respectively as follows:

$$F(t) = 1 - e^{-\frac{t^{\rho}}{\theta}} \qquad (2)$$

$$R(t) = e^{-\frac{t^{\rho}}{\theta}} \qquad (3)$$

 **ISSN (1681- 6870)**

Where the failure rate $h(t)$ growing when $(\rho > 1)$ and they are diminishing when $(\rho < 1)$
Its value is constant when $(\rho = 1)$.
There are several methods for estimating the Weibull model parameters and the approximate reliability function, as follows:

### 2.1.1. Maximum Likelihood Estimation method (MLE)

It is one of the most important estimation methods which aims to make the possibility function of random variables at its greatest end. That's where the function of the Maximum Likelihood Estimation of distribution Weibull is as follows[6]:

$$L = \frac{\rho^n}{\theta^n} e^{\frac{\sum_{i=1}^n t_i^p}{\theta}} \prod_{i=1}^n t_i^{p-1} \quad (4)$$

$$\ln L = n\ln\rho - n\ln\theta \frac{\sum_{i=1}^n t_i^p}{\theta} + (\rho - 1)\sum_{i=1}^n \ln t_i \quad (5)$$

By finding the partial derivative of the function relative to the parameters ρ, θ we get the following:

$$\frac{\partial \ln L}{\partial \rho} = \frac{n}{\rho^{\wedge}} - \frac{\sum_{i=1}^n t_i^p \ln t_i}{\theta} + \sum_{i=1}^n \ln t_i = 0 \quad (6)$$

$$\frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta} + \frac{\sum_{i=1}^n t_i^p}{\theta^2} = 0 \quad (7)$$

$$\hat{\theta} = \frac{\sum_{i=1}^n t_i^p}{n} \quad (8)$$

We use the Newton-Raphson method to find a numerical solution and we have used the R program that is used to find the estimation of feature values using the MLE method.

### 2.1.2. White method

The idea of this method is to use the c.d.f aggregate density function to formulate a simple linear regression model as follows [3]:

$$F(t) = 1 - e^{-\frac{t^\rho}{\theta}} \quad (9)$$

$$1 - F(t) = e^{-\frac{t^\rho}{\theta}} \quad (10)$$

$$\ln[1 - F(t)] = -\frac{t^\rho}{\theta} \quad (11)$$

$$\ln[1 - F(t)]^{-1} = \frac{t_i^\rho}{\theta} \quad (12)$$

$$\ln\ln[1 - F(t)]^{-1} = \rho \ln t_i - \ln\theta \quad (13)$$

The following linear regression model is obtained:

$$y_i = a + bx_i + r_i \quad (14)$$

Where it represents:
$r_i$ : Random error variable
i: 1,…,n
And by applying the least squares method, we get:

The (15th & the 2nd International) Conference of Statistical Applications ISSN (1681- 6870)

$$\hat{y} = \hat{a} + \hat{b}x_i \quad (15)$$

$$\hat{b}_{LS} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} \quad (16)$$

as:

$$\hat{a}_{LS} = \bar{Y} - \hat{b}\bar{X} \quad (17)$$

It can be obtained $\hat{\theta}$ and $\hat{\rho}$ as follows:

$\hat{\rho} = b_{LS}$

$\hat{\theta} = e^{\hat{a}_{LS}}$

## 2.2. Reliability Function

The reliability function of a particular system is defined as the probability that a system running successfully during a specified period of time will remain and mathematically illustrates this if R (t) symbolizes the reliability of a particular system in time (t) then [4]**:**

$$R(t) = \Pr(T>t) \quad (18)$$

(T): Represents the continuous random variable and denotes the time period required for a failure to occur during the period [0, t].

$$\begin{aligned} R(t) &= \Pr(T>t) \\ &= 1-\Pr(T\le t) \\ &= 1-F(t) \end{aligned}$$

And if T distributed as Weibull distribution, it can be written as:

$$R(t) = e^{-\frac{t^{\rho}}{\theta}} \quad (19)$$

Among the most important characteristics of the reliability function is that it is positive, continuous, and decreasing for all t values within the period [0, t] i.e.

$$\left.\begin{aligned} 0 \le R(t) \le 1 \\ \lim_{t\to 0} R(t) = 1 \\ \lim_{t\to\infty} R(t) = 0 \end{aligned}\right\}$$

This means that the ability of the component to operate in time t = 0 with a probability equal to one and at the passage of a certain time period as a result of its operation, this probability decreases due to aging.

## 2.3. Markov Chain in general

It is one of the commonly used models that have an important place in many important applications in several fields, including industrial and engineering. It is a special case of Markov operations, which is a sequence of random variables, which achieves the Markov property, since the systems that have this property are known as Markov chains and may be of an intermittent time or a continuous time. The evolution of the Markov chain is treated with a series of transfers between certain process values and is known as chain states. The probability law of a chain in a particular case depends on the condition itself and not on how the chain reached that particular state [2], Cases may be specific or not specific.

The application of Markov chains is in the reliability study because the state space for each component contains only two values mostly, namely the working state (operation) and we will symbolize it with the symbol 0 and the failure status (malfunction) and symbolize it with code 1 as there are cases in which the status space may contain more than two values but This type of Markov chain will not be included in the interest.

### 2.3.1. Markov model for one component

Markov chains can be applied to find the reliability of a specific system co4nsisting of one component and that has a failure rate λ as in Figure (1) that shows system states [5], where the system operating status of the system denoted by code $S_1$ (0) is symbolized and the failure status of the system is denoted by code $S_2$ (1).

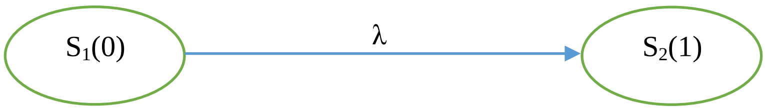



**Figure (1) depicts transitions in a single component system with a failure rate of λ**

In the case of repairing this component after the failure that affected it with a repair rate μ, the reliability model is represented in the figure (2):

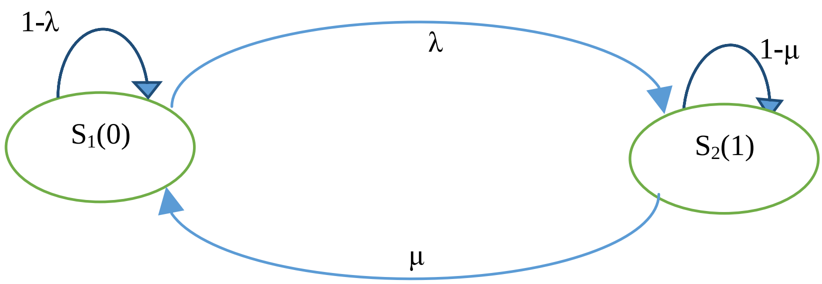



**Figure (2) shows the transmission cases in one component system with failure rate λ and repair rate μ**

The transition probability matrix between the operation and failure stages is defined as follows:

$$M = \begin{bmatrix} e^{\frac{-t_1^{p_1}}{\theta_1}} & 1 - e^{\frac{-t_1^{p_1}}{\theta_1}} \\ 1 - e^{\frac{-t_2^{p_2}}{\theta_2}} & e^{\frac{-t_2^{p_2}}{\theta_2}} \end{bmatrix} \qquad (20)$$

$p_1, \theta_1$, estimated weibul dist. for working time
$p_2, \theta_2$, estimated weibul dist. for stopping time
$t_1$ , the time in days for working
$t_2$ , the time in days for stop

 ISSN (1681- 6870)

## 3. Application to Real Data

To understand the nature of the work, the data that was the number of faults for each stage and the repair times for it were collected over a period of three years for the operational period at the station above.
The data was used for the obligatory stops of the station .The data collected for failure times and working times were examined and were taking two-parameter Weibull distribution , Also, the random variable of failure time is independent of working time.

**Table (1) Parameters estimation of working and stopping time by the two methods**

| *Method* | | $\hat{\rho}$ | $\hat{\theta}$ | *R(t)* |
|---|---|---|---|---|
| mle | Stop time | 0.6341801 | 1.2290110 | $e^{\frac{-t_2^{0.6341801}}{1.2290110}}$ |
| | Work time | 1.407537 | 10.966080 | $e^{\frac{-t_1^{1.407537}}{10.966080}}$ |
| white | Stop time | 0.7871528 | 1.3115305 | $e^{\frac{-t_2^{0.7871528}}{1.3115305}}$ |
| | Work time | 1.015913 | 6.122806 | $e^{\frac{-t_1^{1.015913}}{6.122806}}$ |

### 3.1. The Transition Probability Matrix

The transition matrix for working $t_1$ days and stopping for $t_2$ days, given the MLE estimators is:

$$M = \begin{bmatrix} e^{\frac{-t_1^{1.407537}}{10.966080}} & 1 - e^{\frac{-t_1^{1.407537}}{10.966080}} \\ 1 - e^{\frac{-t_2^{0.6341801}}{1.2290110}} & e^{\frac{-t_2^{0.6341801}}{1.2290110}} \end{bmatrix} \quad (21)$$

The transition matrix for working $t_1$ days and stopping for $t_2$ days, given the White estimators is:

$$M = \begin{bmatrix} e^{\frac{-t_1^{1.015913}}{6.122806}} & 1 - e^{\frac{-t_1^{1.015913}}{6.122806}} \\ 1 - e^{\frac{-t_2^{0.7871528}}{1.3115305}} & e^{\frac{-t_2^{0.7871528}}{1.3115305}} \end{bmatrix} \quad (22)$$

So the transition probability matrix for working 3 days and stopping 0.5 day by MLE estimators become

$$M = \begin{bmatrix} e^{\frac{-3^{1.407537}}{10.966080}} & 1 - e^{\frac{-3^{1.407537}}{10.966080}} \\ 1 - e^{\frac{-0.5^{0.6341801}}{1.2290110}} & e^{\frac{-0.5^{0.6341801}}{1.2290110}} \end{bmatrix} = \begin{bmatrix} 0.6517659 & 0.3482341 \\ 0.4079998 & 0.5920002 \end{bmatrix}$$

and transition probability matrix for working 3 days and stopping 0.5 day by White estimators become

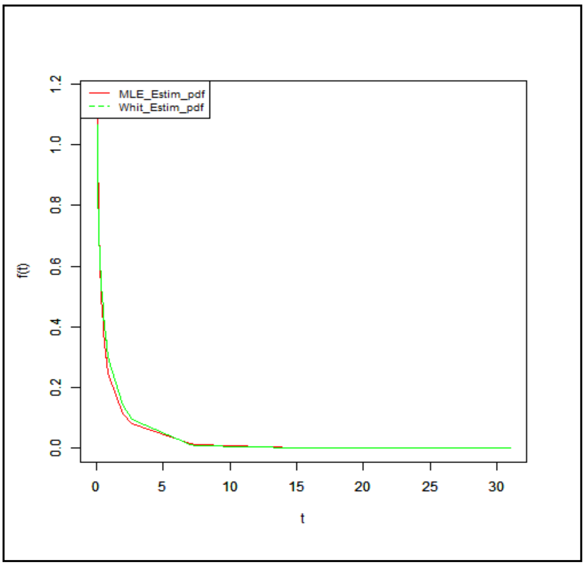
$$M = \begin{bmatrix} e^{\frac{-3^{1.015913}}{6.122806}} & 1 - e^{\frac{-3^{1.015913}}{6.122806}} \\ 1 - e^{\frac{-0.5^{0.7871528}}{1.3115305}} & e^{\frac{-0.5^{0.7871528}}{1.3115305}} \end{bmatrix} = \begin{bmatrix} 0.6073728 & 0.3926272 \\ 0.3571475 & 0.6428525 \end{bmatrix}$$

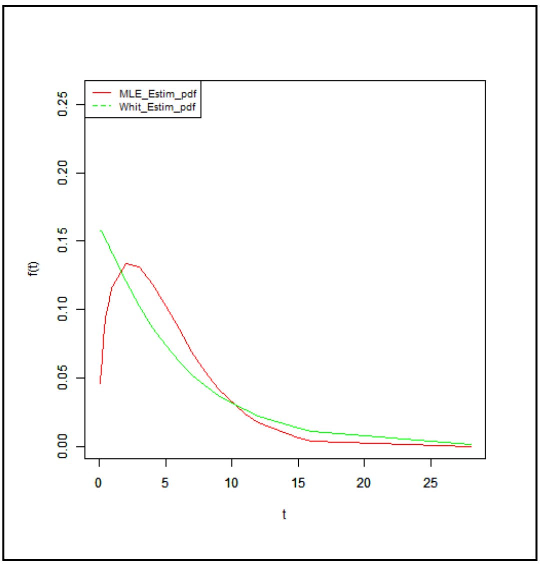
## 4. Simulation

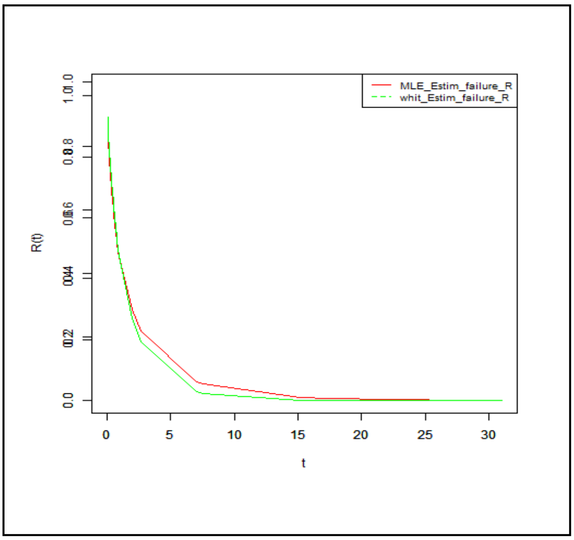
Random numbers were generated following the Weibull distribution with the same values for the parameters whose values were found by the mle method. The sample size of 70,130,160 were generated for working time and stop time. We applied both methods to generated data and replicated the steps for 1000 times and calculated the mse of parameters and the average mse of the model dependent variable. The parameters we used in the simulation are $\hat{p}_1$=1.407537, $\hat{\theta}_1$ =10.966080, $\hat{p}_2$=0.6341801, $\hat{\theta}_2$ =1.2290110.

**Table (2) Parameters mse and model mse of working and stopping time by the two methods and three sample sizes.**

| Sample size | Method | | MSE $\hat{p}$ | MSE $\hat{\theta}$ | Average MSE Y |
|---|---|---|---|---|---|
| n=70 | mle | Stop time | 0.006039885 | 0.034117779 | 0.37549023 |
| | | Work time | 0.01224406 | 9.08049098 | 0.27447136 |
| | white | Stop time | 0.007815646 | 0.082036080 | 0.38479123 |
| | | Work time | 0.09438147 | 9.67358408 | 0.28487236 |
| n=130 | mle | Stop time | 0.002039885 | 0.014117779 | 0.15549023 |
| | | Work time | 0.01224406 | 8.08049098 | 0.05332136 |
| | white | Stop time | 0.003815646 | 0.012036080 | 0.18479123 |
| | | Work time | 0.02438147 | 7.67358408 | 0.08466356 |
| n=160 | mle | Stop time | 0.001039885 | 0.004117779 | 0.05549023 |
| | | Work time | 0.00224406 | 3.08049098 | 0.0025466 |
| | white | Stop time | 0.000815646 | 0.002036080 | 0.0127683 |
| | | Work time | 0.00038147 | 5.67358408 | 0.0216576 |





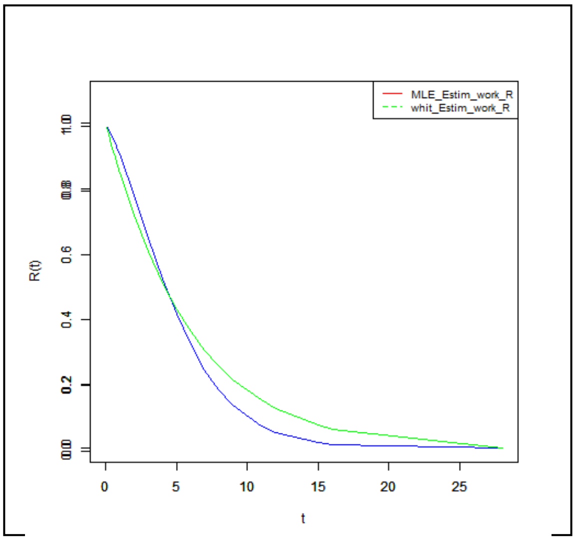




**Figure (3) The graph of pdf and reliability functions by the estimated parameters of two methods. The top two graphs from left to right represent the pdf of stop, work time, respectively. Whereas, the two in the bottom represent the reliability functions of stop, work time, respectively.**

## 5. Discussion and Conclusions

After completing the practical side, the following conclusions were reached:

1. The proposed mle method achieved the advantage of exceeding the mle method, with the sample size n = 160,
2. By comparing the results of the mean square error (MSE) standard between the proposed methods, the MLE method is best in this paper.
3. A comparison of the capabilities used in the research was found to be superior to the MLE estimates by comparison with the other estimation.

## 6. References

1. عبد علي، سوسن صبيح- فندي، صالح جعفر-عبدمطلك، ستار– 2009- قياس معولية الفرن الدوار في معمل سمنت كبيسة – هندسة الانتاج والمعادن – الجامعة التكنلوجيا – مجلة الهندسة والتكنلوجيا – المجلد 27– العدد 11.
2. العكيلي، ماجد جاسب عبدالله – 2012 – تقدير دالة المعولية لتوزيع ويبل باستخدام التوزيع اللامعلمي من خلال الربط بين التحليل العنقودي والعمليات التصادفية – اطروحه دكتوراه في الاحصاء – كلية الادارة والاقتصاد – الجامعة المستنصرية.
3. Shin S.K. and Kate, B.K. (1980),"Life Testing and Reliability Estimation", Willey Eastern Limited, New Delhi.
4. Verma, Ajit Kumar & Ajit, Srividya & Karanki, Durga. (2010). Reliability and Safety Engineering. Springer Series in Reliability Engineering.
5. Weber P., Jouffe L. – 2003- Reliability modelling with dynamic Bayesian networks - IFAC- pp.57-62. Hal-00128475 – Washington- D.C., USA.
6. Wayne, C. E. and Rootunmala, V. M. (1986). "Minimum Expected loss Estimators of the shape and scale parameter of the Weibull Distribution" IEEE Transaction on Reliability.