2024, 21(12): 3831-3835
https://doi.org/10.21123/bsj.2024.9237
P-ISSN: 2078-8665 - E-ISSN: 2411-7986

Baghdad Science Journal

On pre- Open Regular Spaces

Afraa R. Sadek

Department of Mathematics, College of Science, University of Baghdad, Baghdad, Iraq.

Received 13/06/2023, Revised 18/03/2024, Accepted 20/03/2024, Published Online First 20/05/2024, Published 01/12/2024

Abstract

In this paper, certain types of regularity of topological spaces have been highlighted, which fall within the study of generalizations of separation axioms. One of the important axioms of separation is what is called regularity, and the spaces that have this property are not few, and the most important of these spaces are Euclidean spaces. Therefore, limiting this important concept to topology is within a narrow framework, which necessitates the use of generalized open sets to obtain more good characteristics and preserve the properties achieved in general topology. Perhaps the reader will realize through the research that our generalization preserved most of the characteristics, the most important of which is the hereditary property. Two types of regular spaces have been presented, namely the topological space Rp and the topological space S-Rp. The properties of these two spaces and their relationship with each other, as well as the effect of functions on them, have been studied. In addition several theorems have been proved regarding the sufficient and necessary conditions to make the topological spaces Rp-regular or S-Rp-regular. The above concepts have been linked with a new type of Hausdorff space and the concepts under study are reinforced with examples.

Keywords: Pre-open set, Pre- closed set, Pre- open- function, Pre- continuous- function, Pre-irresolute map, Regular space.

Introduction

Throughout this paper Z means a topological space (Z,τ) without separation axioms, unless it is explicitly referenced. The nature of this work is to explore and extract a specific new genre, the generalized separation axioms have been constructed by several authors[1-4]. A pre-open was studied in 1982 by Mashhour, Abd El – Monsef and El- Deeb[5], a subset M is a pre-open set if $M \subseteq \mathrm{int}(\mathrm{Cl}(M))$, where the $\mathrm{Cl}(M)$ and the $\mathrm{int}(M)$ are the closure and the interior operators of a set M respectively[6]. $(Z - M)$ is labeled a pre-closed, where M is a pre-open. If M is a subset of a space Z, then the pre-closure of M means the intersection of all pre-closed sets in Z that contain M which is denoted by $Cl_p(M)$ for instance[7,8].

In this paper, our goal is to generalize the concept of regularity of spaces by using the pre-open sets[9,10]. Many results were proven and illustrated by examples. Further many properties of such spaces have been investigated. The assortment of pre-open (resp. pre-closed) of Z will be denoted by P.O(Z) (resp. P.C(Z)) and say that a set H in a space Z is pre-closed neighborhood [11-13] of a point c if H is pre-closed and contains a pre-open set to which c belongs. For each pair of topological spaces Z and Y a function f: Z→Y is called pre-irresolute[14,15] if f^{-1}(H)∈P.O(Z) for each H∈P.O(Y) and f is termed M-

2024, 21(12): 3831-3835
https://doi.org/10.21123/bsj.2024.9237
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

pre-closed (resp. M-pre-open) if f(K)∈P.C(Y) (resp. f(K)∈P.O(Y)) for all K∈P.C(Z) (resp. K∈P.O(Z))[16,17].

The pre - regular spaces

In the beginning of this section, a definition of pre-regular spaces is provided by:

Definition 1: A space Z is called pre-regular space and shortly R_p- space if for each $c \in Z$ and for every closed set W in Z such that $c \notin W$, there exist $G, H \in P.O(Z)$, such that $c \in G$ and $W \subseteq H$ and $G \cap H = \emptyset$.

Note: It is easy to see that every regular space is pre-regular. The following example shows that the convers in general is not true.

Example1: Let $Z = \{c_1, c_2, c_3\}$, $\tau = \{Z, \emptyset, \{c_1, c_2\}\}$ Then $PO(Z) = \{Z, \emptyset, \{c_1\}, \{c_2\}, \{c_1, c_2\}, \{c_1, c_3\}, \{c_2, c_3\}\}$. It is clear that (Z, τ) is R_p- space while it is not regular. The following theorem gives a characterization of a space to be R_p- space.

Theorem 1: A space Z is R_p- space ⇔ for every $c \in Z$ and for every open subset G, there is a pre-open subset L,whereas $c \in L \subseteq Cl_p(L) \subseteq G$.

Proof: Necessity; Since $c \notin G^c$ and G^c is closed, so by pre-regularity of Z there are two disjoint $L, W \in P.O(Z)$ where $c \in L$ and $G^c \subseteq W$, but W^c is pre-closed and $L \subseteq W^c$, this mean that $Cl_p(W^c) = W^c$ and since $Cl_p(L) \subseteq Cl_p(W^c)$, hence $L \subseteq Cl_p(L) \subseteq W^c \subseteq G$, therefore $c \in L \subseteq Cl_p(L) \subseteq G$.

Sufficient; Let E be a closed and $c \notin E$, so $c \in E^c$ and E^c is open implies that there exist pre-open set K whereas $c \in K \subseteq Cl_p(K) \subseteq E^c$. Now $E \subseteq (Cl_p(K))^c$, and since $Cl_p(K)$ is pre-closed and $K \cap (Cl_p(K))^c = \emptyset$, therefore $c \in K$ and $E \subseteq (Cl_p(K))^c$ implies Z is R_p- space.

Corollary 1: A space Z is R_p- space if for every $c \in Z$ and for every pre-open subset H includes c there exists a pre-open subset W containing c and $Cl_p(W) \subseteq H$.

Proof: Obvious since every closed set is pre-closed set[18].

Theorem 2: A space Z is $Rp\text{-}space$ ⇔ For each $c \in Z$, the set of pre-closed contained in each neighbourhood of c form a base for that neighbourhood.

Proof: Necessity; Give $c \in Z$ and neighborhood O of c, so there is an open subset $L \subseteq Z$ whereas $c \in L \subseteq O$ implies $c \notin Z - L$ and $(Z - L)$ is closed, thus by the pre regularity there are $G, H \in P.O(Z)$ such that $c \in G$, $(Z - L) \subseteq H$ and $H \cap G = \emptyset$. Thus $c \in G \subseteq Z - H \subseteq L \subseteq O$, so $Z - H$ is pre-closed neighborhood of c contain in the given neighborhood O.

Sufficient; Let $c \in Z$ and the closed set $E \subseteq Z - \{c\}$. Since $Z - E$ is open and contained c , there is a pre-closed neighborhood O of c such that $O \subseteq Z - E$. Now let $H = Z - O$, then H is pre-open and $\subseteq H$. But O is neighborhood of c , so there is an open set G such that $c \in G \subseteq O$. Thus $H \cap G \subseteq O \cap (Z - O) = \emptyset$ and since G is pre-open[18] implies Z is R_p-space.

Proposition 1: If a space Z is $Rp\text{-}space$ then for every $c \in Z$ and every open set H containing c, there exists a $N \in P.O(Z)$, whereas $c \in N$ and $Cl(int(N)) \subseteq H$.
Proof: Since $c \notin H^c$ and H^c is closed, so there exist disjoint pre-open sets G_1 and G_2 such that $c \in G_1$ and $H^c \subseteq G_2$. Now $G_1 \subseteq (G_2)^c \subseteq H$ also $(G_2)^c$ is pre-closed, hence $Cl(int(G_1)) \subseteq Cl(int((G_2)^c)) \subseteq (G_2)^c \subseteq H$. thus G_1 is the required pre-open set.

The following theorem shows that the pre regularity is hereditary property.

Theorem 3: Let (Y, τ_Y) be a clopen subspace of R_p-space (Z, τ_Z), then (Y, τ_Y) is pre-$\mathcal{R}$- subspace of (Z, τ_Z).

Proof: Let $c \in Y$ and K be a closed set in Y such that $c \notin K$. Since K is closed in Y, so there is a closed set E in Z such that $K = E \cap Y$[19]. Now $c \notin E$ and (Z, τ_Z) is R_p- space implies there exist $G, H \in P.O(Z)$ such that $c \in H$, $E \subseteq G$ and $H \cap G = \emptyset$. Y is open, so it is pre-open[18] and G is pre-open, thus $Y \cap G \subseteq int_{\tau_Z}(Cl_{\tau_Z}(Y)) \cap int_{\tau_Z}(Cl_{\tau_Z}(G)) = int_{\tau_Z}[(Cl_{\tau_Z}(Y)) \cap (Cl_{\tau_Z}(G))] = int_{\tau_Z}[Y \cap (Cl_{\tau_Z}(G))] \subseteq int_{\tau_Y}[Y \cap (Cl_{\tau_Z}(G))]$[20]. Now it is sufficient to show that $Y \cap (Cl_{\tau_Z}(G)) = Cl_{\tau_Y}(Y \cap G)$, clear that $Cl_{\tau_Y}(Y \cap G) \subseteq Y \cap (Cl_{\tau_Z}(G))$. To show $Y \cap (Cl_{\tau_Z}(G)) \subseteq Cl_{\tau_Y}(Y \cap G)$, let $t \in Y \cap$

2024, 21(12): 3831-3835
https://doi.org/10.21123/bsj.2024.9237
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

$(Cl_{\tau_Z}(G))$, if $t \notin Cl_{\tau_Y}(Y \cap G)$, then there is an open neighborhood O of t in Y whereas $O \cap (Y \cap G) = \emptyset$. But Y is open, hence O is open in Z [6] which contains t and $O \cap G = \emptyset$ which is imposible since $t \in Cl_{\tau_Z}(G)$. Thus $Y \cap G$ is pre-open containing K.Similarly $Y \cap H$ is pre-open which contain t, and since $(Y \cap G) \cap (Y \cap H) = Y \cap (H \cap G) = \emptyset$ which complete the proof.

Definition 2: A space Z is called pre - Hausdorff space ⇔ for any elements $c_1 \neq c_2$ in Z, there are pre-open sets H and G satisfy $c_1 \in G$, $c_2 \in H$ and $H \cap G = \emptyset$.

The following example shows that the quotient topology of R_p- space is pre-Hausdorff.

Example 2: Let Z be a R_p- space and N be any clopen subset of Z. Define a relation R on Z as follows: c R v ⇔ c,v $\in N$ or c, v $\notin N$. It can be seen that R is an equivalence relation on Z. To prove Z/R with the quotient topology is pre-Hausdorff space take [c], [v] ∈ Z/R such that [c]≠[v] hence either c or v belong to N. Now by the pre regularity of Z there are two pre-open subsets K_1 and K_2 in Z whereas $[c] \in N \subseteq K_1$ and $[v] \in K_2$, implies that the quotient space is pre-Hausdorff.

Theorem 4: Let $f: Z \rightarrow Y$ be a closed, pre-irresolute, injective function and Y is R_p- space, then Z is R_p- space.

Proof: let $c \in Z$ and L be any closed subset of Z such that $c \notin L$, then $f(L)$ is closed subset of Y whereas $f(c) \notin f(L)$ and since Y is R_p- space, so there are two disjoint pre-open subsets H, W whereas $f(L) \subseteq H$ and $f(c) \in W$. Clear that $L \subseteq f^{-1}(f(L)) \subseteq f^{-1}(H)$ and $c \in f^{-1}(W)$ also $f^{-1}(H) \cap f^{-1}(W) = \emptyset$ see[21] and since f is pre-irresolute, hence $f^{-1}(H)$, $f^{-1}(W)$ are pre-open subsets of Z, which complete the proof.

Theorem 5: Let $f: Z \rightarrow Y$ be a bijective, M-pre-open, continuous function and Z is R_p- space, then Y is R_p- space.

Proof: Let L be a closed set in Y and v $\notin$ L, then $f^{-1}(L) \subseteq Z$ and $f^{-1}(v) \notin f^{-1}(L)$. Since f is continuous then $f^{-1}(L)$ is closed in Z and by the pre-regularity of Z there are $N_1, N_2 \in P.O(Z)$ whereas $N_1 \cap N_2 = \phi$ such that $f^{-1}(L) \subseteq N_1$ and $f^{-1}(v) \in N_2$. Now f is M-pre-open, hence $f(N_1)$ and $f(N_2)$ are disjoin pre-open subsets in Y include L and v respectively.

Strongly pre- regular spaces

In the beginning of this section a definition of strongly-pre-regular spaces is presented.

Definition 3: A space Z is termed strongly pre-regular space and shortly S-R_p- space if for each $c \in Z$ and for every pre-closed set K in Z such that $c \notin K$, there are two disjoint open sets G and H where $c \in G$ and $K \subseteq H$.

It is easy to see that every S-R_p- space is R_p- space but the convers is not necessarily true, as shown below

Example 3: Take $Z = \{c_1, c_2, c_3\}$, $\tau = \{Z, \emptyset, \{c_1, c_2\}\}$, then $P.O(Z) = \{Z, \emptyset, \{c_1\}, \{c_2\}, \{c_1, c_2\}, \{c_1, c_3\}, \{c_2, c_3\}\}$ and $P.C(Z) = \{Z, \emptyset, \{c_1\}, \{c_2\}, \{c_3\}, \{c_1, c_3\}, \{c_2, c_3\}\}$. Clear that (Z, τ) is R_p- space but not S-R_p- space.

The following two theorems gives characterizations for S-R_p- spaces.

Theorem 6: A space Z is S-R_p- space ⇔ for every $c \in Z$ and for every pre-open set G includes c, there exist an open set N whereas $c \in N \subseteq Cl(N) \subseteq G$.

Proof: Necessity; Since $\mathbf{c} \notin \boldsymbol{G}^{\mathbf{c}}$ and $\boldsymbol{G}^{\mathbf{c}}$ is pre-closed, so by the S-$\boldsymbol{R_p}$- regularity of **Z** there are two disjoint open subsets **N** and **H** where $\mathbf{c} \in \boldsymbol{N}$ and $\boldsymbol{G}^{\mathbf{c}} \subseteq \boldsymbol{H}$. Now $\boldsymbol{H}^{\mathbf{c}}$ is closed and $\boldsymbol{N} \subseteq \boldsymbol{H}^{\mathbf{c}}$, this mean that $\boldsymbol{Cl}(\boldsymbol{H}^{\mathbf{c}}) = \boldsymbol{H}^{\mathbf{c}}$ and $\boldsymbol{Cl}(\boldsymbol{N}) \subseteq \boldsymbol{Cl}(\boldsymbol{H}^{\mathbf{c}})$, hence $\boldsymbol{N} \subseteq \boldsymbol{Cl}(\boldsymbol{N}) \subseteq \boldsymbol{H}^{\mathbf{c}} \subseteq \boldsymbol{G}$, therefore $\mathbf{c} \in \boldsymbol{N} \subseteq \boldsymbol{Cl}(\boldsymbol{N}) \subseteq \boldsymbol{G}$.
Sufficiency; Let L be a pre-closed and $c \notin L$, so $c \in L^c$ and L^c is pre-open implies that there exist open set N whereas $c \in N \subseteq Cl(N) \subseteq L^c$ implies $L \subseteq (Cl(N))^c$. But $Cl(N)$ is closed and $N \cap (Cl(N))^c = \emptyset$, therefore $c \in N$ and $L \subseteq (Cl(N))^c$, hence Z is S-R_p- space.

The following theorem gives us an advantage of strongly pre-regular spaces.

Theorem 7: If Z is a S-R_p- space, then for every $c \in Z$ and for every pre-open set K includes c, there exists closed set L whereas $Cl_p(\text{int}(L)) \subseteq K$.

Proof: Consider K is a pre-open and c $\in K$, then c $\notin K^c$ and K^c is pre-closed. Since Z is S-R_p- space, so there are disjoint open sets L_1 and L_2 such that c $\in L_1$ and $K^c \subseteq L_2$ implies $L_1 \subseteq (L_2)^c \subseteq K$. But $int((L_2)^c) \subseteq (L_2)^c$ [20]. Further $(L_2)^c$ is pre-closed, hence $Cl_p(int((L_2)^c)) \subseteq (L_2)^c \subseteq K$.

Theorem 8: Let $f\colon Z \to W$ be a continuous, M-pre-closed, injective function and W is S-R_p- space, then Z is S-R_p- space.

Proof: let c $\in$Z and $H \in P.C(Z)$ whereas $c \notin H$, then f (H) is pre-closed subset of Wwhereas f(c) $\notin f(H)$ and since W is S-R_p- space, so there are two disjoint open sets K, L whereas $f(H) \subseteq K$ and f (c) $\in L$. Now $H \subseteq f^{-1}(f\ (H)) \subseteq f^{-1}(K)$ and c $\in f^{-1}(L)$[21] also $f^{-1}(K) \cap f^{-1}(L) = \emptyset$. But f is continuous, thus $f^{-1}(K), f^{-1}$ (L) are open subsets of Z, which complete the proof.

Lastly, a definition of the weak form of pre-Hausdorff spaces will be introduced.

Definition 4: A space Z is called weakly pre-Hausdorff space if for each distinct points c, v in Z such that c $\notin Cl_p(H_v)$, where H_v is any pre-open set containing v, there exist disjoint pre-open sets H,G containing c and v respectively

Proposition 2: Every S-R_p- space is weakly pre-Hausdorff space.

Proof: Let Z be a S-R_p- space and c, v $\in$ Z such that c $\neq$ v and let c $\notin Cl_p(H_v)$, where H_v be any pre-open set containing v. Since $Cl_p(H_v)$ is pre-closed[5,17], so by the S-R_p -regularity of Z there are two disjoint open subsets G_c and G_v whereas c $\in G_c$ and v $\in Cl_p(H_v) \subseteq G_v$ implies Z is weakly pre-Hausdorff space.

Conclusion

The separation axioms are considered one of the most important topics in topology, and in particular in general topology, because of their role in classifying topological spaces. The research focused on two types of topological spaces, namely, Rp-regular and S-Rp-regular as a generalization of the regular spaces within the generalizations of the separation axioms. Many results have been presented about the characteristics of these types of spaces and their relationship to each other and the rest of the spaces such as the Hausdorff spaces. It is worth noting that the two most important conclusions obtained are the first that the Rp-regularity is hereditary and the second states that every S-Rp-space is weakly pre-Hausdorff space after the introduction of this new type of Hausdorff space, despite the invalidity of these two conclusions before generalization.

Author's Declaration

- Conflicts of Interest: None.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Baghdad.

References

1. Levine N. Semi-open sets and semi-continuity in topological spaces. Amer Math Mon. 1963; 70(1): 36-41. https://doi.org/10.1080/00029890.1963.11990039.
2. Kamel GA, Dib KA. Generalized topology and the family of monotonic maps $\Gamma(X)$. J Egypt Math Soc. 2023 Dec; 31(1): 1-27. https://doi.org/10.1186/s42787-023-00162-5.
3. Aziz NI, Jasim TH. On Generalized N*-Closed Set in NANO-N* Topological Spaces With Some Properties. 2nd International Conference on Physics and Applied Sciences (ICPAS 2021), College of Education, Mustansiriyah University, Baghdad, Iraq, 26-27 May. J Phys. Conf Ser. 2021; 1963(1). https://doi.org/10.1088/1742-6596/1963/1/012157.
4. Al- Shami TM, El-Shafi ME. On soft compact and soft Lindelöf spaces via soft pre-open sets. Ann Fuzzy Math. Inform. 2019 Feb; 17(1): 79-100. https://doi.org/10.30948/afmi.2019.17.1.79.
5. Mashhour AS, Abd El – Monsef ME, Hasanein IA. On Pretopological Spaces. Bull Math Dela Soc Roum. 1984; 28(76): 39-45.
6. Sierpinski W. General Topology. USA: Dover Publications; 2020 Apr 15. Chap 2, Topological spaces; p. 40-43. .

7. Hwaidi AH, Hussein JH. λ –Semi – Open Sets in Generalized Open Spaces. Proceedings of the 1st International Conference on Advance Research in Pure and Applied Science (ICARPAS 2021): Third Annual Conference of Al-Muthanna University/College of Science, 24–25 March 2021, Al-Samawah, Iraq. AIP Conf Proc. 2022; 2398(1): 060078. https://doi.org/10.1063/5.0094717.
8. Al- Omeri WF, Jafari S. Neutrosophic Pre-Continuous Multifunctions and Almost PreContinuous Multifunctions. Neutrosophic Sets Syst. 2019 Oct; 27(1): 53-69.
9. Ali HJ, Hassan RF. On Light Mapping and Certain Concepts by Using $m_X N$-Open Sets. Baghdad Sci J. 2020 Jan 2; 17(1(Suppl.)): 371-377. https://dx.doi.org/10.21123/bsj.2020.17.1(Suppl.).0371.
10. Al-Omeri W, Jafari S. On Generalized Closed Sets and Generalized Pre-Closed Sets in Neutrosophic Topological Spaces. Math. 2018; 7(1): 1-12. https://doi.org/10.3390/math7010001.
11. Ali AA, Sadek AR. On regular δ-semi-open space. J Interdiscip Math. 2021; 24(4): 953-960. https://doi.org/10.1080/09720502.2021.1885813.
12. Jasim AH. Results on a Pre-$\mathbf{T}_2$ Space and Pre-Stability. Baghdad Sci J. 2019 Mar; 16(1): 111-115. https://doi.org/10.21123/bsj.2019.16.1.0111.
13. Kadzam AR, Yousif YY. Fibrewise totally separation axioms. J Interdiscip Math. 2022; 25(2): 511-520. https://doi.org/10.1080/09720502.2021.2003012.
14. Darwesh HM, Namiq SF. Pre-irresolute Functions in Closure Spaces. Eur J Pure Appl Math. 2019; 12(3): 1082-1095. https://doi.org/10.29020/nybg.ejpam.v12i3.3317.
15. Kannan K, Nagaveni N, Saranya S. On β ̂g-Continuous and β ̂g- Irresolute maps in topological spaces. Procedia Comput Sci. 2015; 47: 368-373. https://doi.org/10.1016/j.procs.2015.03.218.
16. Sathiyaraj J, Valivel A, Maheshwari OV. On Double Fuzzy M-open Mappings and Double Fuzzy M-closed Mappings. Appl Appl Math Int J (AAM). 2020 Dec; 15(2): 928-941.
17. Esmaeel RB. On semi- p- open sets. M.Sc. [thesis]. College of Education/ Ibn Al-Haitham: Baghdad University; 2004.
18. Rao VV, Rao YS. Neutrosophic Pre-open Sets and Pre-closed Sets in Neutrosophic Topology. Int J Chemtech Res. Infinite study. 2017; 10(10): 449-458.
19. Morris SA. Topology Without Tears. USA: University of New England; 2023.
20. Kelley JL. General topology. Graduate Texts in Mathematics, 27. USA: Springer Science & Business Media; 1955. Chap 1, Topological Spaces; p. 50-53.
21. Sadek AR P. P-L. Compact Topological Ring. Iraqi J Sci. 2016; 57(4B): 2754-2759.

عن الفضاءات المنتظمة pre-open

عفراء راضي صادق

قسم علوم الرياضيات، كلية العلوم، جامعة بغداد، بغداد، العراق

الخلاصة

في هذا البحث تم تسليط الضوء على انواع معينة من الانتظام للفضاءات التبولوجية والذي يقع ضمن دراسة تعميمات بديهيات الفصل. ومن البديهيات المهمة للفصل ما تسمى الانتظام والفضاءات التي تمتلك هذه الخاصية ليست قليلة، وأهم هذه الفضاءات هي الفضاءات الإقليدية. ولذلك فإن حصر هذا المفهوم المهم في الطوبولوجيا يكون ضمن إطار ضيق مما يستلزم استخدام المجموعات المفتوحة المعممة للحصول على المزيد من الخصائص الجيدة والحفاظ على الخصائص المتحققة في الطوبولوجيا العامة. ولعل القارئ سيدرك من خلال البحث أن تعميمنا حافظ على أغلب الصفات وأهمها الصفة الوراثية. تم تقديم نوعين من الفضاءات المنتظمة وهما الفضاء التبولوجي R_p والفضاء التبولوجي S-R_p. تم دراسة خصائص هذين الفضاءين وعلاقتهم مع بعضهم وكذلك تاثير الدوال عليهم اضافة الى ذلك تم برهان العديد من المبرهنات الخاصة بالشروط الكافية والضرورية لجعل الفضاء التبولوجي R_p-regular او S-R_p-regular. تم ربط المفاهيم اعلاه مع نوع جديد من الفضاءات الهاوزدورفية وتعزيز المفاهيم قيد الدراسة بالامثلة

الكلمات المفتاحية: المجموعة المفتوحة الابتدائية، المجموعة المغلقة الابتدائية، الدالة المفتوحة الابتدائية، الدالة المستمرة الابتدائية، المجموعة المترددة الابتدائية، فضاء منتظم.