S-Coprime Submodules

Inaam M.A. Hadi
Rasha I. Khalaf
Dept. of Mathematics/ College of Education for Pure Science (Ibn-Al-Haitham)/ University of Baghdad

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Abstract

In this paper, we introduce and study the concept of S-coprime submodules, where a proper submodule N of an R-module M is called S-coprime submodule if $\frac{M}{N}$ is S-coprime R-module. Many properties about this concept are investigated.

Key word: s-coprime submodules, coprime submodules, s-coprime modules and coprime modules.

Introduction

Let R be a commutative ring with unity and let M be an R-module. Annine in [1] introduce the concept coprime module, where an R-module M is called coprime if $\underset{R}{\text{ann}}\,M = \underset{R}{\text{ann}}\,\frac{M}{N}$ for all $N \subsetneqq M$. Wijayanti in [2], Khalaf in [3] studied this concept. Also Ali and Khalaf in [4] introduced and studied the concept of S-coprime R-module where an R-module M is called S-coprime if $\underset{R}{\text{ann}}\,M = \underset{R}{\text{ann}}\,\frac{M}{N}$, for all small submodules N of M ($N \ll M$); that $N \ll M$ if $N + W \neq M$ for all $W \subsetneqq M$. Hence it is clear that every coprime module is S-coprime module.

Ali in [5], studied the concept of coprime submodule, where a proper submodule N of M is called coprime submodule if $\frac{M}{N}$ is coprime R-module. In this paper, we introduce the concept of S-coprime submodule, a proper submodule N of an R-module M is called S-coprime submodule if $\frac{M}{N}$ is S-coprime module. Moreover, we study and give the basic properties related with these concepts. Also we give many relationships between this concept and other related concepts.

S.1 Basic Properties of S-Coprime Submodules

First we give the following definition.

Definition 1.1:

Let N be a proper submodule of an R-module M. N is called S-coprime submodule if $\frac{M}{N}$ is S-coprime R-module.

A proper ideal of a ring R is S-coprime ideal if $\frac{R}{I}$ is S-coprime R-module.

Remarks and Examples 1.2:

(1) It is clear that every coprime submodule is S-coprime submodule. However the converse is true as the following example shows:

The submodule <0> in the Z-module Z is an S-coprime submodule since Z/<0> ≃ Z is S-coprime module [4, Rem. and Ex.2.1], but <0> is not coprime submodule, since $\frac{Z}{<0>} \cong Z$ is not a coprime Z-module.

(2) Every proper submodule N of a coprime R-module M is S-coprime submodule.

Proof: Since M is coprime R-module, then by [5, Rem and Ex.1.1(4)], every submodule N of M is an coprime submodule. Thus the result is followed (1).

(3) Any small submodule of S-coprime module is an S-coprime submodule.

Proof: Let N be a small submodule of S-coprime R-module M. Then by [4, Th.12], $\frac{M}{N}$ is S-coprime module and hence N is S-coprime submodule

Note that the converse of (3) is not true in general, for example the submodule $<\bar{2}>$ of the Z-module Z_6 (which is S-coprime) is S-coprime submodule, but it is not small.

(4) Any submodule of hollow (or chained) S-coprime R-module is a S-coprime submodule, where an R-module M is called hollow if every proper submodule of M is small [6].
An R-module M is called chained if the lattice of submodules of M is linearly ordered by inclusion [7].
Proof: It follows directly by (3).

(5) If n is a square free integer then <n> is an S-coprime submodule of the Z-module Z.
Proof: If n is a square free, then by [8], $\frac{Z}{<n>}$ is semisimple, hence by [4, Rem. and Ex. (2)], $\frac{Z}{<n>}$ is S-coprime Z-module. Thus <n> is an S-coprime submodule.

(6) If $W \leq N < M$ such that N is an S-coprime submodule, then it is not necessarily that W is an S-coprime submodule, for example:
N = <2> in the Z-module is an S-coprime submodule but W = <4> ⊂ N is not an S-coprime submodule, since $\frac{Z}{W} \cong Z_4$ which is not an S-coprime Z-module.

(7) If A is an S-coprime submodule of an R-module M and B is an S-coprime submodule in A, then it is not necessary that B is an S-coprime submodule of M as the following example shows:
Consider the Z-module Z_{24}. A = $<\bar{2}>$ is an S- coprime submodule in Z_{24}. B = $<\bar{4}> \leq$ A, B is an S-coprime submodule in A, since $\frac{A}{B} \cong Z_2$ which is an S-coprime module. However B is not an S-coprime submodule of Z_{24} because $\frac{Z_{24}}{B} \cong Z_4$ which is not an S-coprime Z-module.

Recall that an R-module M is S-coprime module iff for each $r \in R - \{0\}$, $rM \ll M$ implies rM = (0), [4, Th.3].
The following results are characterizations of an S-coprime module.

Proposition 1.3:
Let N < M. Then the following statements are equivalent:
(1) N is an S-coprime submodule.
(2) For each $r \in R - \{0\}$, $\frac{rM+N}{N} \ll \frac{M}{N}$ implies $r \in [N:M]$.
(3) [N:M] = [W:M] for all $\frac{W}{N} \ll \frac{M}{N}$.

Proof: (1) ⇔ (2) N is an S-coprime submodule equivalent to $\frac{M}{N}$ is an S-coprime R-module, which is equivalent to $\forall\, r \in R - \{0\}$, $r\frac{M}{N} \ll \frac{M}{N} \Rightarrow r\frac{M}{N} = 0_{\frac{M}{N}}$ by [4, Th.3], that is $\frac{rM+N}{N} \ll \frac{M}{N}$ implies rM + N = N and hence $r \in [N:M]$.

(1) ⇔ (3) N is an S-coprime submodule means that $\frac{M}{N}$ is an S-coprime module, which means $\operatorname{ann}\frac{M}{N} = \operatorname{ann}\frac{M/N}{W/N}$ for all $\frac{W}{N} \square \frac{M}{N}$; that [N:M] = [W:M] for all $\frac{W}{N} \square \frac{M}{N}$.

Corollary 1.4:

Let I be an ideal of a ring R. Then I is an S-coprime ideal iff I = J for all ideal J of R s.t. $\frac{J}{I} \square \frac{R}{I}$.

Recall that an R-module M is called antihopfian if $M \cong \frac{M}{N}$ for all N ⪇ M, [9].

Proposition 1.5:

Every submodule N of an antihopfian R-module is an Scoprime submodule.

Proof: Since M is antihopfian module, then by [3], M is an coprime module. Hence the results follows by Rem. and Ex. 1.2(2).

Remark 1.6:

If A < B ≤ M such that A is an S-coprime submodule in M, then it is not necessarily that A is an S-coprime submodule in B; for example:

Consider the Z-module Z_{p^∞}, if $A = <\frac{1}{p^2} + Z>$ and $B = <\frac{1}{p^4} + Z>$. Since Z_{p^∞} is an antihopfian, A is an S-coprime submodule of Z_{p^∞} by prop.1.3. But $\frac{B}{A} \cong Z_{p^2}$ which is not an S-coprime module. Thus A is not an S-coprime submodule in B.

The following result shows that the concepts coprime submodule and S-coprime submodule are equivalent under the class of hollow (chained) modules.

Proposition 1.7:

If M is a hollow (or chained) R-module, N < M. Then N is an S-coprime submodule iff N is an coprime submodule.

Proof: (⇒) If M is hollow R-module, then it is clear that $\frac{M}{N}$ is hollow. Since N is an S-coprime submodule, then $\frac{M}{N}$ is an S-coprime R-module. By [4,Prop.7], $\frac{M}{N}$ is an coprime R-module and hence N is an coprime submodule of M.

(⇐) It is clear (see Rem. and Ex. 1.2(1)). If M is a chained, then the result follows obviously, since every chained module is hollow.

Recall that a proper submodule N of an R-module M is called semimaximal if $\frac{M}{N}$ is a semisimple R-module [10].

Remark 1.8:

Every semimaximal submodule N of an R-module M is an S-coprime, but not conversely.

Proof: Since N is semimaximal, then $\frac{M}{N}$ is a semisimple R-module, then by [4, Rem. and Ex.2(2)], $\frac{M}{N}$ is S-coprime module. Thus N is an S-coprime submodule.

Note that <0> in the Z-module is an S-coprime submodule (see Rem. and Ex. 1.2(11)) but <0> is not a semimaximal submodule, since $\frac{Z}{<0>} \cong Z$ is not semisimple.

Proposition 1.9:

Let M be an R-module, let I be an ideal of R such that $I \subseteq \underset{R}{\operatorname{ann}} M$ and let N be a submodule of M. Then N is an S-coprime R-submodule of M iff N is an S-coprime $\bar{R}$-submodule, where $\bar{R} = R/I$.

Proof: ($\Rightarrow$) Let N be an S-coprime R-submodule. Then $\frac{M}{N}$ is an S-coprime R-module. Hence by [4 , Prop.5], $\frac{M}{N}$ is an S-coprime $\bar{R}$-module. Thus N is S-coprime $\bar{R}$-module.

($\Leftarrow$) The proof is similarly, so it is omitted.

Proposition 1.10:

Let f: M $\longrightarrow$ M′ be an R-epimorphism and let N < M such that N is an S-coprime submodule of M and ker $f \subseteq$ N. Then f (N) is an S-coprime submodule of M′.

Proof: Since N is an S-coprime submodule, then $\frac{M}{N}$ is an S-coprime R-module. To prove f (N) is an S-coprime submodule of M′, we must prove $\frac{M'}{f(N)}$ is an S-coprime R-module. Define g: $\frac{M}{N} \longrightarrow \frac{M'}{f(N)}$ by g(m + N) = f (m) + N for all m + N $\in \frac{M}{N}$. It is easy to check that g is an isomorphism; that is $\frac{M}{N} \cong \frac{M'}{f(N)}$. Thus $\frac{M'}{f(N)}$ is an S-coprime R-module and hence f (N) is an S-coprime submodule of M′.

Corollary 1.11:

Let N, K be submodules of an R-module M such that N $\supseteq$ K. Then N is an S-coprime submodule of M iff $\frac{N}{K}$ is an S-coprime submodule of $\frac{M}{K}$.

Proof: ($\Rightarrow$) Since N is an S-coprime submodule, $\frac{M}{N}$ is an S-coprime R-module. But $\frac{M}{N} \cong \frac{M/K}{N/K}$, thus $\frac{M/K}{N/K}$ is an S-coprime R-module; that is $\frac{N}{K}$ is an S-coprime submodule of $\frac{M}{K}$.

($\Leftarrow$) The proof of the converse is similarly, so it is omitted.

Proposition 1.12:

Let N, W be submodules of an R-module M such that W ⊇ N. If N is an S-coprime submodule of M and W ≪ M, then W is a S-coprime submodule of M.

Proof: Since N is an S-coprime submodule, then $\frac{M}{N}$ is an S-coprime R-module. But W ≪ M implies $\frac{W}{N} \ll \frac{M}{N}$.Hence by [4,Th.12], $\frac{M/N}{W/N}$ is an S-coprime module. But $\frac{M/N}{W/N} \cong \frac{M}{W}$, thus $\frac{M}{N}$ is an S-coprime module and hence W is an S-coprime submodule.

Corollary 1.13:

Let N ≪ M, W ≪ M. If N is an S-coprime (or W is an S-coprime) submodule of M, then N + W is an S-coprime submodule of M.

Proof: By [8], N + W ≪ M. Hence the result is followed prop.1.12, directly.

Next, we consider the direct sum of S-coprime submodules.

Proposition 1.14:

Let N_1 and N_2 be S-coprime submodules of R-modules M_1, M_2 respectively. Then $N_1 \oplus N_2$ is an S-coprime submodule in $M_1 \oplus M_2$.

Proof: Since N_1 and N_2 are S-coprime submodules of M_1, M_2 respectively, then $\frac{M_1}{N_1}$ and $\frac{M_2}{N_2}$ are S-coprime R-modules. Hence by [4,Prop.18], $\frac{M_1}{N_1} \oplus \frac{M_2}{N_2}$ is an S-coprime R-module. But $\frac{M_1 \oplus M_2}{N_1 \oplus N_2} \cong \frac{M_1}{N_1} \oplus \frac{M_2}{N_2}$, it follows that $\frac{M_1 \oplus M_2}{N_1 \oplus N_2}$ is a S-coprime R-module. Therefore $N_1 \oplus N_2$ is an S-coprime submodule of $M_1 \oplus M_2$.

S.2 S-Coprime Submodules and Multiplication Modules:

First we have the following result:

Proposition 2.1:

Let M be a multiplication R-module and let N ≪ M. Then N is an S-coprime submodule if and only if N is a maximal small submodule of M.

Proof: (⇒) Assume there exists a small submodule W such that W ⊇ N. Hence $\frac{W}{N} \ll \frac{M}{N}$. But N is an S-coprime submodule, [N:M] = [W:M] by prop.1.3(1⇔3). On the other hand, M is a multiplication R-module, so N = [N:M]M = [W:M]M = W. Thus N is a maximal small submodule of M.

(⇐) To prove N is an S-coprime submodule. Let $\frac{W}{N} \ll \frac{M}{N}$. Since N ≪ M by hypothesis, so that W ≪ M. Thus W = N because N is a maximal small submodule of M. Then it is clear that [W:M] = [N:M] and hence by prop.1.3(1⇔3), N is an S-coprime submodule.

Corollary 2.2:

Let I ≪ R. Then I is an S-coprime ideal of R if and only if I is a maximal ideal of R.

Theorem 2.3:

Let M be a faithful finitely generalted multiplication R-module and let $N \subsetneqq M$. Then N is an S-coprime submodule if and only if [N:M] is an S-coprime ideal.

Proof: ($\Rightarrow$) Since N is an S-coprime submodule of M, then $\frac{M}{N}$ is an S-coprime R-module. But M is a multiplication module implies $\frac{M}{N}$ is a multiplication R-module. Hence (0) is the only small submodule in $\frac{M}{N}$ by [4, Rem. and Ex. 2(7)]. But M is finitely generated R-module, so $\frac{M}{N}$ is finitely generated. Also $\frac{M}{N}$ is a faithful $\bar{R} = R/\underset{R}{\text{ann}}\, M \cong R$. It follows that $[\bar{0} \underset{R}{:} \frac{M}{N}] \ll R$ [11, Prop.1.1.8]; that is $\text{ann}\frac{M}{N} \ll R$; i.e. $[N \underset{R}{:} M] \ll R$. Again by [11, Prop.1.1.8], $N \ll M$ and hence by Prop.2.1, N is a maximal small submodule of M. It follows that $[N \underset{R}{:} M]$ is a maximal small ideal of R. To see this: suppose there exists a small ideal I of R such that $I \underset{\neq}{\supset} [N \underset{R}{:} M]$. Then by [12, Th.3.1], IM $\underset{\neq}{\supset}$ [N:M]M = N and by [12], $IM \ll M$. Thus we get a contradiction, since N is a maximal small submodule of M. Therefore $[N \underset{R}{:} M]$ is a maximal small ideal of R and so $[N \underset{R}{:} M]$ is an S-coprime ideal of R.

($\Leftarrow$) To prove N is an S-coprime submodule, we shall prove $[N \underset{R}{:} M]$ = $[W \underset{R}{:} M]$ for all $\frac{W}{N} \ll \frac{M}{N}$. Since M is multiplication W = [W:M]M, N = [N:M]M, we claim that $\frac{[W:M]}{[N:M]} \ll \frac{R}{[N:M]}$. To see this: assume that $\frac{[W:M]}{[N:M]} + \frac{K}{[N:M]} = \frac{R}{[N:M]}$, hence [W:M] +K = R. It follows that W + KM = M and so $\frac{W}{N} + \frac{KM}{N} = \frac{M}{N}$.Hence $\frac{KM}{N} = \frac{M}{N}$, which implies that KM = M and hence K = R, since M is a faithful finitely generated multiplication. Thus $\frac{K}{[N:M]} = \frac{R}{[N:M]}$ and so $\frac{[W:M]}{[N:M]} \ll \frac{R}{I}$. But [N:M] is an S-coprime ideal of R, so by Cor.1.4, [N:M] = [W:M]. Then by prop.1.3(1$\Leftrightarrow$3), N is an S-coprime submodule.

Corollary 2.4:

Let M be a finitely generated faithful multiplication R-module and let N < M. Then the following statements are equivalent:

(1) N is an S-coprime submodule in M.
(2) [N:M] is an S-coprime ideal in R.
(3) N = IM for some S-coprime ideal I of R.

Proof: (1) $\Leftrightarrow$ (2) It follows by Th.2.3.

(2) $\Rightarrow$ (3) It is clear, since N = [N;M]M and [N:M] is an S-coprime ideal of R.

(3) $\Rightarrow$ (2) Since M is a finitely generated faithful multiplication module, then M = ± N and we can take I = [N:M] by [12]. Thus [N:M] is an S-coprime ideal of R.

Ali in [13], introduced the concept S*-coprime module, where an R-module is an S*-coprime if for each $f \in End(M)$, Im $f \ll M$ implies $f = 0$.

Corollary 2.5:

Let M be a finitely generated faithful multiplication R-module and let N < M. Then the following statements are equivalent:

(1) N is an S-coprime submodule of M.

(2) $\frac{M}{N}$ is an S-coprime R-module.

(3) R is an S-coprime ring.

(4) M is an S-coprime R-module.

(5) $\frac{M}{N}$ is an S*-coprime R-module.

Proof: (1) $\Leftrightarrow$ (2) It is clear.

(3) $\Leftrightarrow$ (4) by [4, Prop.11].

(2) $\Leftrightarrow$ (3) by [4, Prop.11].

(2) $\Leftrightarrow$ (5) by [13, Prop.1.1].

Proposition 2.6:

Let M be an R-module such that $J(M) \ll M$. Then J(M) is an S-coprime submodule of M, where J(M) is the intersection of all maximal submodules of M, if M has maximal submodules and J(M) = M if M has no maximal submodule, [8].

Proof: It is easy to check that $\frac{M}{J(M)}$ has no nonzero small submodule, since $J(M) \ll M$. It follows that $\frac{M}{J(M)}$ is an S-coprime module and hence J(M) is an S-coprime submodule.

Corollary 2.7:

Let M be a multiplication R-module. Then J(M) is an S-coprime submodule of M.

Proof: By [12, Cor.2.6], $J(M) \ll M$. Hence the result is followed Prop.2.6.

Corollary 2.8:

For any ring R, J(R) is an S-coprime ideal of R.

S.3 S-Coprime and other Related Concepts:

Proposition 3.1:

If M has a finite number of maximal submodules, then J(M) is an S-coprime submodule.

Proof: Let $L_1, L_2, \ldots, L_n$ be the maximal submodules of M. Then $J(M) = \bigcap_{i=1}^{n} L_i$ and so that $\frac{M}{J(M)} \cong$ submodule of $\bigoplus_{i=1}^{n} \frac{M}{L_i}$ [14]. It follows that $\frac{M}{J(M)}$ is a semisimple R-module and so S-coprime module. Thus J(M) is an S-coprime submodule.

Recall that an R-module M is called local if M has a unique maximal submodule, [9].

Corollary 3.2:

If M is a local ring then J(M) is an S-coprime submodule.

Recall that an R-module M is called weakly supplemented if for each $A \leq M$, there exists $B \leq M$ such that $A + B = M$ and $A \cap B \ll M$, [15].

Proposition 3.3:

If M is a weakly supplemented, then J(M) is an S-coprime submodule.

Proof: Since M is a weakly supplemented R-module, then $\frac{M}{J(M)}$ is a semisimple R-module by [4,Rem. And Ex. 1.2(2)]. Hence $\frac{M}{J(M)}$ is an S-coprime module. Thus J(M) is an S-coprime submodule.

Proposition 3.4:

Let M be a weakly supplemented R-module and let $A < M$. If A is an S-coprime submodule, then there exists $B \leq M$ such that $A + B = M$, $A \cap B$ is an S-coprime in B.

Proof: Since M is weakly supplemented and $A < M$, then there exists $B \leq M$ such that $A + B = N$ and $A \cap B \ll M$. But A is an S-coprime submodule, so $\frac{M}{A}$ is an S-coprime R-module. On the other hand, $\frac{M}{A} = \frac{A+B}{A} \cong \frac{B}{A \cap B}$. Thus $\frac{B}{A \cap B}$ is an S-coprime module and hence $A \cap B$ is an S-coprime submodule in B.

Proposition 3.5:

Let M be an S*-coprime R-module. Then every small submodule of M is an S-coprime submodule.

Proof: Let $N \ll M$. Since M is an S*-coprime module, then M is an S-coprime module [13,Rem. and Ex. 1.2(1)], and hence N is an S-coprime submodule by Rem. and Ex. 1.2(3).

Recall that an R-module M is called scalar module if for each $f \in End(M)$, there exists $r \in R - \{0\}$, such that $f(x) = rx$ for all $x \in M$ [16].

A ring R is called regular ring (in the sense of Von Neumann) if for each $x \in R$, there exists $y \in R$ such that $x = xyx$ [9].

To prove our next result, we prove the following lemma.

Lemma 3.6:

Let M be a scalar module over a regular ring R. Then M is an S-coprime module.

Proof: To prove M is an S-coprime R-module. Let $r \in R - \{0\}$ and suppose that $rM \ll M$. So that to show $rM = (0)$. Since M is a scalar R-module, then $\underset{R}{End}(M) \cong R/\underset{R}{ann}\, M$ [17]. But R is a regular ring implies that $R/\underset{R}{ann}\, M$ is a regular ring. Thus $\underset{R}{End}(M)$ is a regular ring. Let $f \in End(M)$ such that $f(m) = rm$ for each $m \in M$. Then $f(M) = rM$. But by [9, Exc 17(a), p.272] $\underset{R}{End}(M) = rM + \ker f$, hence $\ker f = M$. Since $rM \ll M$. Thus $f = 0$ and then $rM = 0$. Therefore M is an S-coprime R-module.

Theorem 3.7:

Let M be a scalar R-module over a regular ring R. Then every small submodule of M is an S-coprime submodule.

Proof: It follows by Lemma 3.6 and Rem. and Ex. 1.2(3).

To prove the next result we need the following Lemma which is proved by Ali and Khalaf in [4]

Lemma 3.8:
Let M be a chained module over a regular ring R, then the following statements are equivalent:
(1) M is an S-coprime R-module.
(2) M is coprime R-module.
(3) M is a prime R-module.
(4) M is a quasi-Dedekind R-module.

Theorem 3.9:
Let M be a chained module over a regular ring and let N < M. Then the following statements are equivalent:
(1) N is an S-coprime submodule in M.
(2) N is a coprime submodule in M.
(3) N is a prime submodule in M.

We end our paper by this result:

Proposition 3.10:
Let N < M. If N is an S-coprime E-submodule in M, then N is an S-coprime R-submodule in M, where E = $\underset{R}{\text{End}}(M)$.

Proof: Since N is an S-coprime E-submodule, then $\frac{M}{N}$ is an S-coprime E-module. Hence by [4, Prop.2.5], $\frac{M}{N}$ is an S-coprime R-module. Thus N is an S-coprime R-submodule in M.

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رشا إبراهيم خلف
قسم علوم الرياضيات / كلية التربية للعلوم الصرفة (ابن الهيثم) /جامعة بغداد

استلم البحث في : 23 ايلول 2012 ، قبل البحث في : 3 شباط 2013

الخلاصة

في هذا البحث قدمنا ودرسنا مفهوم المقاسات الجزئية الاولية المضادة من النمط – S، حيث يكون الموديول الجزئي الفعلي N من مقاس M على R، مقاسا جزئيا اوليا ًمضادا من النمط S اذا كان $\frac{M}{N}$ مقاسا ًعلى R من النمط –S. وقد اعطيت العديد من الخواص المتعلقة بهذا المفهوم.

الكلمات المفتاحية: المقاسات الجزئية المضادة من النمط S، المقاسات الجزئية المضادة، المقاسات المضادة من النمط S، المقاسات المضادة.