Addressing Multi-Choice Problem in the Linear Programming Model by Integer Programming and Fuzzy Methods with Practical Application

Taha A.Nadhim[1] , Abdul Jabbar K.Bakheet[2]
[1] Faculty of Administration and Economics, Department of Statistics, University of Baghdad
[2] Faculty of Administration and Economics, Department of Statistics, University of Baghdad

Abstract
In this paper, the problem of multi-choice in the linear programming model will be addressed by using integer programming method with a new technique and fuzzy programming method and comparing them. Where the trade-off between the two methods will be achieved by the final profit and the optimal production quantities and the amount of surplus in each of the two methods where the advantages and disadvantages of each method will be mentioned.

Keywords: Multi-Choice Mathematical Programming, Optimization, Fuzzy

Corresponding Author
Taha A. Nadhim
Department of Statistics,
Faculty of Administration and Economics, , University of Baghdad
E-mail: Tahaalhayali2@gmail.com

معالجة مشكلة تعدد الخيارات باستعمال تقنية البرمجة العددية متعددة الخيارات وطريقة البرمجة الخطية الضبابية مع تطبيق عملي

طه ازهر محمد ناظم [1]، عبد الجبار خضر بخيت [2]
[1] جامعة بغداد ، كلية الادارة والاقتصاد ، قسم الاحصاء
[2] جامعة بغداد ، كلية الادارة والاقتصاد ، قسم الاحصاء

المستخلص: سيتم في هذا البحث معالجة مشكلة تعدد الخيارات في انموذج البرمجة الخطية باستعمال اسلوبي البرمجة العددية بتقنية جديدة واسلوب البرمجة الخطية الضبابية والمقارنة بينهما ، حيث سيتم تطبيق هذين الاسلوبين على انموذج رياضي عددي لاحد المؤسسات وايجاد الحل الامثل والمفاضلة بين الطريقتين عن طريق المقارنة بين الربح المتحقق وكميات الانتاج المثلى وكمية الفائض في كل من الاسلوبين حيث سيتم ذكر مميزات ومساويء كل اسلوب من الاسلوبين .

الكلمات المفتاحية: البرمجة العددية متعددة الخيارات، الأمثلية، المنطق الضبابي

ISSN (1681- 6870)

Introduction

In the reality of the production process, there is a process called optimization and this means finding the minimum or maximum limit of some quantities of production and that will be by building a mathematical model called the objective function subject to a linear system of equality or inequality called restrictions as there is a lot of research deals with these problems in many ways and all These methods have yielded useful results and the application of the linear programming model in industrial problems requires the introduction of new tools commensurate with these problems, that the decision-making approach under uncertainty has followed a variety of methods of building models as the common approach to decision-making However, in certain cases, it is believed that the parameters in the decision-making model are multiple-choice in nature. In the recent period, many wonderful works and researches have emerged to solve these problems, where the appropriate constraint is chosen using binary variables. The binary variables required for the constraints are the same as the total number of options for this limitation in this paper.

Mathematical Models

1- Multi-choice integer programming

The mathematical model of a multi-choice linear programming problem is presented as:

Find$(X_1, X_2, \dots, X_n)$ so as to::

$$Max\ Z = \sum_{j=1}^{n} C_j X_j$$

Subject to: $\sum_{j=1}^{n} a_{ij} X_{ij} \le (b_{i1}, b_{i2}, \dots, b_{ik}) \qquad , i = 1,2,\dots,m$

$X_j \ge 0 \qquad , j = 1,2,\dots,n$

The right-hand side of each constraint has a set of i k number of choice where is one choice of them should be selected. For solving the multi-choice linear programming problem, we should transform the problem to a standard mathematical programming problem.

Linear Transformation

Let $(b_{i1}, b_{i2}, \dots, b_{ik})$ be choices values (right hand side) for i^{th} of constraints L_i and its k_i

k_i : is The least common multiple of numbers $(k_1, k_2, \dots, k_n)$

To get to the main goal of choosing only one value from thek_i options, the idea is the number of k_i variables called $(Z_1, Z_2, \dots, Z_n)$ to create a set of $r_i = \frac{k}{k_i}$ of linear combinations

$$L_{i1} = b_{i1}Z_1 + b_{i2}Z_2 + \dots + b_{iki}Z_{ki} = \sum_{t=1}^{ki} b_{it}Z_t$$

$$L_{i2} = b_{i1}Z_{ki+1} + b_{i2}Z_{ki+2} + \dots + b_{iki}Z_{2ki} = \sum_{t=1}^{ki} b_{it}Z_{k+t}$$

$$L_{i3} = b_{i1}Z_{ki+1} + b_{i2}Z_{2ki+2} + \dots + b_{iki}Z_{3ki} = \sum_{t=1}^{ki} b_{it}Z_{2k+t}$$

ISSN (1681- 6870)

$$L_{iri} = b_{i1} Z_{(ri-1)ki+1} + b_{i2} Z_{(ri-1)ki+2} + \cdots + b_{iki} Z_{riki} = \sum_{t=1}^{ki} b_{it} Z_{(ri-1)k+t}$$

The right side i^{th} th of the constraints will be replaced by the following formula:

$$L_i = \sum_{j=1}^{ri} L_{ij} = \sum_{j=1}^{ri} \left(\sum_{t=1}^{ki} b_{it} Z_{(j-1)k+t} \right)$$

Now, we can write the mathematical model as follows:
To find $X = (X_1, X_2, \dots, X_n)$

$$Max\ Z = \sum_{j=1}^{n} C_j X_j$$

Subject To

$$\sum_{j=1}^{n} a_{ij} X_{ij} \le L_i \qquad , i = 1,2, \dots, m$$

$$Z_p = 0/1 \qquad p = 1,2, \dots, k$$

$$\sum_{p=1}^{k} Z_p = 1$$

$$X_j \ge 0 \qquad , j = 1,2, \dots, n$$

2- Fuzzy linear programming Method

2.1. Robusta's Ranking Technique for triangular fuzzy number

Robust ranking technique which satisfies compensation, linearity, and additively properties and provides results which consist human intuition. If ã is a fuzzy number, then the Robust Ranking is defined by
Let: $\tilde{A} = (\alpha_L, \alpha_m, \alpha_u)$
If ã is a fuzzy number then the Robust Ranking is defined by

$$R(\tilde{A}) = \int_0^1 (0.5)(\alpha_L + \alpha_u) d\alpha$$

And $\alpha - cut$ is :

$$A(X) = [X/M(X) \ge X \qquad \forall X \in (0,1)]$$

Now (α_L, α_u) is $X - cut$ for fuzzy number $(\tilde{A})$

2.2. Ranking Functions for Trapezoidal Fuzzy Number:

Let :
$\tilde{A} = (\alpha_L, \alpha_m, \alpha_u, \alpha_r)$
The rank function to handle this number will be as follows:

$$R(\tilde{A}) = \frac{1}{2} \int_0^1 (\inf a_\alpha + \operatorname{Sup} a_\alpha) d\alpha$$

ISSN (1681- 6870)

And

$\tilde{A} > a_\alpha = \alpha - cut$

This leads us to

$$R(\tilde{A}) = \frac{1}{4}(\alpha_L + \alpha_m + \alpha_u + a_r)$$

2.3. Ranking Functions for Hexagonal Fuzzy Number:

Let $\tilde{A} = (\alpha_L, \alpha_m, \alpha_u, a_r, a_n, a_s)$ is a fuzzy number , Ranking Functions will be as follow :

$$R(\tilde{A}) = \frac{2\alpha_L + 3\alpha_m + 4\alpha_u + 4a_r + 3a_n + 2a_s}{18}$$

Numerical Example

A factory produce Ten types of medicines($X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}$). The following model consists of a target function, which is a function of maximizing profits and constraints restricting the model. These constraints consist of constraints (raw materials, demand, available working time, production capacity):

$MAX\ (P) = (172.75)X_1 + (215.68)X_2 + (121.75)X_3 + (128.66)X_4 + (1318)X_5 + (53.835)X_6 + (52.09)X_7 + (74.495)X_8 + (222.04)X_9 + (175.58)X_{10}$

Subject to

A- Restrictions of production requirements (gram)

1- $(0.04)X_1 + (0.04)X_4 + (0.04)X_7 \le (150000, 160000, 250000)$

2- $(2.4)X_1 \le (7250000, 8000000, 8350000, 8700000)$

3- $(0.09)X_1 + (0.09)X_3 + (0.09)X_4 + (0.09)X_5 + (0.09)X_6 + (0.09)X_7 + (0.09)X_8 + (0.09)X_9 + (0.09)X_{10} \le (300000, 365000, 395000)$

4- $(0.03)X_1 + (0.03)X_2 + (0.03)X_3 + (0.03)X_4 + (0.03)X_5 + (0.03)X_6 + (0.03)X_7 + (0.03)X_8 + (0.03)X_9 + (0.03)X_{10} \le (150000, 175000, 195000)$

5- $(0.1)X_1 + (0.2)X_2 + (0.1)X_3 + (0.1)X_4 + (0.15)X_6 + \ + (0.1)X_8 + (0.05)X_{10} \le (450000, 566000, 725000)$

6- $(15)X_1 \le (45000000, 46000000, 47000000)$

7- $(0.02)X_1 + (0.1)X_2 \le (70000, 80000, 88000)$

8- $(0.25)X_1 + (0.7)X_2 + (0.27)X_3 + (0.27)X_4 + (0.09)X_5 + (0.5)X_6 + (0.17)X_8 + (0.07)X_9 + (0.11)X_{10} \le (846000, 858000, 865000)$

9- $(0.75)X_1 + (0.3)X_2 + (0.6)X_3 + (0.6)X_4 + (0.24)X_5 + (0.7)X_6 + (0.78)X_8 + (0.7)X_9 \le (3000000, 3500000, 4200000)$

10- $(0.0005)X_1 \le (1750000, 1800000, 1850000)$

11- $(10)X_1 + (5)X_2 + (1.83)X_3 + (1)X_4 + (0.97)X_5 + (15)X_6 + (1)X_7 + (0.97)X_8 + (0.97)X_9 \le (37000000, 38000000, 39000000, 39500000)$

12- $(20)X_1 + (5)X_2 + (5)X_3 + (5)X_4 + (9.72)X_5 + (20)X_6 + (5)X_7 + (9.72)X_8 + (9.72)X_9 \le (75000000, 80000000, 82000000, 85000000)$

13- $(0.97)X_8 \le (68000, 70000, 72000)$

14- $(0.01)X_1 \le (30000, 35000, 40000)$

15- $(0.11)X_2 \le (22000, 23000, 25000)$

16- $(0.0015)X_2 \le (6730, 6850, 7000)$

17- $(0.05)X_3 \le (4800, 5000, 5200)$

18- $(0.001)X_3 + (0.0019)X_4 + (0.02)X_5 + (0.07)X_8 + (0.03)X_9 + (0.001)X_{10} \le (7000, 8000, 9500)$

19- $(0.00033)X_3 + (0.0002)X_4 + (0.002)X_6 \le (950000, 1000000, 1050000)$

20- $(5.82)X_5 \le (80000, 85000, 95000, 113000)$

21- $(38.89)X_5 + (25)X_6 + (30)X_7 + (38.89)X_8 + (38.89)X_9 \leq$ $(9500000,10000000,10500000,11000000)$
22- $(0.15)X_5 + (0.05)X_8 + (0.15)X_9 \leq (64000,65000,70000)$
23- $(0.19)X_5 + (0.19)X_8 + (0.29)X_9 \leq (69000,75000,85000)$
24- $(0.62)X_6 + (1)X_7 \leq (235000,280000,310000)$
25- $(0.05)X_7 \leq (38000,39000,40000)$
26- $(0.1)X_6 + (0.1)X_7 \leq (80000,81000,82000)$
27- $(0.02)X_7 \leq (8000)$
28- $(0.0005)X_7 \leq (1100)$
29- $(1)X_6 \leq (496000)$
30- $(0.2)X_6 \leq (40000,41500,43000)$
31- $(0.08)X_{10} \leq (23560,28000,34000)$

B- Restriction of working time (minute):

32- $(0.087)X_1 + (0.0817)X_2 + (0.0817)X_3 + (0.0817)X_4 + (0.0917)X_5 + (0.085)X_6 + (0.085)X_7 + (0.0917)X_8 + (0.0917)X_9 + (0.085)X_{10} \leq (7831440,9118800,10441920)$

C- On-demand (number of units):

33- $(1)X_1 \geq (2000000,2200000,2300000,2400000,2600000,2800000)$
34- $(1)X_2 \geq (30000,36000,40000,42000)$
35- $(1)X_3 \geq (28000,36000)$
36- $(1)X_4 \geq (300000,342000,383000)$
37- $(1)X_5 \geq (5000,6000,8000,12000)$
38- $(1)X_6 \geq (25000,36000)$
39- $(1)X_7 \geq (88000,120000)$
40- $(1)X_8 \geq (17500,28000)$
41- $(1)X_9 \geq (64500,68000,80000,90000,100000,120000)$
42- $(1)X_{10} \geq (176000,250000,400000,500000)$

D- Capacity constraints (number of units):

43- $(1)X_1 \leq (2716377,4154232)$
44- $(1)X_2 \leq (32000,51408)$
45- $(1)X_3 \leq (31824,40000)$
46- $(1)X_4 \leq (348649,563203)$
47- $(1)X_5 \leq (6000,13000)$
48- $(1)X_6 \leq (30371,45754)$
49- $(1)X_7 \leq (90875,130798)$
50- $(1)X_8 \leq (18175,29360)$
51- $(1)X_9 \leq (70627,122113)$
52- $(1)X_{10} \leq (183765,550000)$

E- Constraint of Negativity: j

$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10} \geq 0$ and integer

Optimal Solution

In this paper we will solve this model to get the optimal solution by two methods as follow:

1- Multi-choice integer programming

Through the use of multi-choice linear programming technology will transform the mathematical model into the following form:

ISSN (1681- 6870)

$MAX\ (P) = (172.75)X_1 + (215.68)X_2 + (121.75)X_3 + (128.66)X_4 + (1318)X_5$
$+(53.835)X_6 + (52.09)X_7 + (74.495)X_8 + (222.04)X_9 + (175.58)X_{10}$

Subject to

A- Restrictions of production requirements (gram)

1- $(0.04)X_1 + (0.04)X_4 + (0.04)X_7 \leq (150000Z_1 + 160000Z_2 + 250000Z_3 + 150000Z_4 + 160000Z_5 + 250000Z_6 + 150000Z_7 + 160000Z_8 + 250000Z_9 + 150000Z_{10} + 160000Z_{11} + 250000Z_{12})$

2- $(2.4)X_1 \leq (7250000Z_1 + 8000000Z_2 + 8350000Z_3 + 8700000Z_4 + 7250000Z_5 + 8000000Z_6 + 8350000Z_7 + 8700000Z_8 + 7250000Z_9 + 8000000Z_{10} + 8350000Z_{11} + 8700000Z_{12})$

3- $(0.09)X_1 + (0.09)X_3 + (0.09)X_4 + (0.09)X_5 + (0.09)X_6 + (0.09)X_7 + (0.09)X_8 + (0.09)X_9 + (0.09)X_{10} \leq (300000Z_1 + 365000Z_2 + 395000Z_3 + 300000Z_4 + 365000Z_5 + 395000Z_6 + 300000Z_7 + 365000Z_8 + 395000Z_9 + 300000Z_{10} + 365000Z_{11} + 395000Z_{12})$

4- $(0.03)X_1 + (0.03)X_2 + (0.03)X_3 + (0.03)X_4 + (0.03)X_5 + (0.03)X_6 + (0.03)X_7 + (0.03)X_8 + (0.03)X_9 + (0.03)X_{10} \leq (150000Z_1 + 175000Z_2 + 195000Z_3 + 150000Z_4 + 175000Z_5 + 195000Z_6 + 150000Z_7 + 175000Z_8 + 195000Z_9 + 150000Z_{10} + 175000Z_{11} + 195000Z_{12})$

5- $(0.1)X_1 + (0.2)X_2 + (0.1)X_3 + (0.1)X_4 + (0.15)X_6 + \ + (0.1)X_8 + (0.05)X_{10} \leq (450000Z_1 + 566000Z_2 + 725000Z_3 + 450000Z_4 + 566000Z_5 + 725000Z_6 + 450000Z_7 + 566000Z_8 + 725000Z_9 + 450000Z_{10} + 566000Z_{11} + 725000Z_{12})$

6- $(15)X_1 \leq (45000000Z_1 + 46000000Z_2 + 47000000Z_3 + 45000000Z_4 + 46000000Z_5 + 47000000Z_6 + 45000000Z_7 + 46000000Z_8 + 47000000Z_9 + 45000000Z_{10} + 46000000Z_{11} + 47000000Z_{12})$

7- $(0.02)X_1 + (0.1)X_2 \leq (70000Z_1 + 80000Z_2 + 88000Z_3 + 70000Z_4 + 80000Z_5 + 88000Z_6 + 70000Z_7 + 80000Z_8 + 88000Z_9 + 70000Z_{10} + 80000Z_{11} + 88000Z_{12})$

8- $(0.25)X_1 + (0.7)X_2 + (0.27)X_3 + (0.27)X_4 + (0.09)X_5 + (0.5)X_6 + (0.17)X_8 + (0.07)X_9 + (0.11)X_{10} \leq (846000Z_1 + 858000Z_2 + 865000Z_3 + 846000Z_4 + 858000Z_5 + 865000Z_6 + 846000Z_7 + 858000Z_8 + 865000Z_9 + 846000Z_{10} + 858000Z_{11} + 865000Z_{12})$

9- $(0.75)X_1 + (0.3)X_2 + (0.6)X_3 + (0.6)X_4 + (0.24)X_5 + (0.7)X_6 + (0.78)X_8 + (0.7)X_9 \leq (3000000Z_1 + 3500000Z_2 + 4200000Z_3 + 3000000Z_4 + 3500000Z_5 + 4200000Z_6 + 3000000Z_7 + 3500000Z_8 + 4200000Z_9 + 3000000Z_{10} + 3500000Z_{11} + 4200000Z_{12})$

10- $(0.0005)X_1 \leq (1750000Z_1 + 1800000Z_2 + 1850000Z_3 + 1750000Z_4 + 1800000Z_5 + 1850000Z_6 + 1750000Z_7 + 1800000Z_8 + 1850000Z_9 + 1750000Z_{10} + 1800000Z_{11} + 1850000Z_{12})$

11- $(10)X_1 + (5)X_2 + (1.83)X_3 + (1)X_4 + (0.97)X_5 + (15)X_6 + (1)X_7 + (0.97)X_8 + (0.97)X_9 \leq (37000000Z_1 + 38000000Z_2 + 39000000Z_3 + 39500000Z_4 + 37000000Z_5 + 38000000Z_6 + 39000000Z_7 + 39500000Z_8 + 37000000Z_9 + 38000000Z_{10} + 39000000Z_{11} + 39500000Z_{12})$

12- $(20)X_1 + (5)X_2 + (5)X_3 + (5)X_4 + (9.72)X_5 + (20)X_6 + (5)X_7 + (9.72)X_8 + (9.72)X_9 \leq (75000000Z_1 + 80000000Z_2 + 82000000Z_3 + 85000000Z_4 + 75000000Z_5 + 80000000Z_6 + 82000000Z_7 + 85000000Z_8 + 75000000Z_9 + 80000000Z_{10} + 82000000Z_{11} + 85000000Z_{12})$

13- $(0.97)X_8 \leq (68000Z_1 + 70000Z_2 + 72000Z_3 + 68000Z_4 + 70000Z_5 + 72000Z_6 + 68000Z_7 + 70000Z_8 + 72000Z_9 + 68000Z_{10} + 70000Z_{11} + 72000Z_{12})$

14- $(0.01)X_1 \leq (30000Z_1 + 35000Z_2 + 40000Z_3 + 30000Z_4 + 35000Z_5 + 40000Z_6 + 30000Z_7 + 35000Z_8 + 40000Z_9 + 30000Z_{10} + 35000Z_{11} + 40000Z_{12})$

ISSN (1681- 6870)

15- $(0.11)X_2 \leq$
$(22000Z_1+23000Z_2+25000Z_3+22000Z_4+23000Z_5+25000Z_6+22000Z_7+23000Z_8+25000Z_9+22000Z_{10}+23000Z_{11}+25000Z_{12})$

16- $(0.0015)X_2 \leq$
$(6730Z_1+6850Z_2+7000Z_3+6730Z_4+6850Z_5+7000Z_6+6730Z_7+6850Z_8+7000Z_9+6730Z_{10}+6850Z_{11}+7000Z_{12})$

17- $(0.05)X_3 \leq$
$(4800Z_1+5000Z_2+5200Z_3+4800Z_4+5000Z_5+5200Z_6+4800Z_7+5000Z_8+5200Z_9+4800Z_{10}+5000Z_{11}+5200Z_{12})$

18- $(0.001)X_3 + (0.0019)X_4 + (0.02)X_5 + (0.07)X_8 + (0.03)X_9 + (0.001)X_{10} \leq$
$(7000Z_1+8000Z_2+9500Z_3+7000Z_4+8000Z_5+9500Z_6+7000Z_7+8000Z_8+9500Z_9+7000Z_{10}+8000Z_{11}+95000Z_{12})$

19- $(0.00033)X_3 + (0.0002)X_4 + (0.002)X_6 \leq$
$(950000Z_1+1000000Z_2+1050000Z_3+950000Z_4+1000000Z_5+1050000Z_6+950000Z_7+1000000Z_8+1050000Z_9+950000Z_{10}+1000000Z_{11}+1050000Z_{12})$

20- $(5.82)X_5 \leq$
$(80000Z_1+85000Z_2+95000Z_3+113000Z_4+80000Z_5+85000Z_6+95000Z_7+113000Z_8+80000Z_9+85000Z_{10}+95000Z_{11}+113000Z_{12})$

21- $(38.89)X_5 + (25)X_6 + (30)X_7 + (38.89)X_8 + (38.89)X_9 \leq$
$(10000000Z_1+10500000Z_2+11000000Z_3+10000000Z_4+10500000Z_5+11000000Z_6+10000000Z_7+10500000Z_8+11000000Z_9+10000000Z_{10}+10500000Z_{11}+11000000Z_{12})$

22- $(0.15)X_5 + (0.05)X_8 + (0.15)X_9 \leq$
$(64000Z_1+65000Z_2+70000Z_3+64000Z_4+65000Z_5+70000Z_6+64000Z_7+65000Z_8+70000Z_9+64000Z_{10}+65000Z_{11}+70000Z_{12})$

23- $(0.19)X_5 + (0.19)X_8 + (0.29)X_9 \leq$
$(69000Z_1+75000Z_2+85000Z_3+69000Z_4+75000Z_5+85000Z_6+69000Z_7+75000Z_8+85000Z_9+69000Z_{10}+75000Z_{11}+85000Z_{12})$

24- $(0.62)X_6 + (1)X_7 \leq$
$(235000Z_1+280000Z_2+310000Z_3+235000Z_4+280000Z_5+310000Z_6+235000Z_7+280000Z_8+310000Z_9+235000Z_{10}+280000Z_{11}+310000Z_{12})$

25- $(0.05)X_7 \leq$
$(38000Z_1+39000Z_2+40000Z_3+38000Z_4+39000Z_5+40000Z_6+38000Z_7+39000Z_8+40000Z_9+38000Z_{10}+39000Z_{11}+40000Z_{12})$

26- $(0.1)X_6 + (0.1)X_7 \leq$
$(80000Z_1+81000Z_2+82000Z_3+80000Z_4+81000Z_5+82000Z_6+80000Z_7+81000Z_8+82000Z_9+80000Z_{10}+81000Z_{11}+82000Z_{12})$

27- $(0.02)X_7 \leq (8000)$

28- $(0.0005)X_7 \leq (1100)$

29- $(1)X_6 + \leq (496000)$

30- $(0.2)X_6 \leq$
$(40000Z_1+41500Z_2+43000Z_3+40000Z_4+41500Z_5+43000Z_6+40000Z_7+41500Z_8+43000Z_9+40000Z_{10}+41500Z_{11}+43000Z_{12})$

31- $(0.08)X_{10} \leq$
$(23560Z_1+28000Z_2+34000Z_3+23560Z_4+28000Z_5+34000Z_6+23560Z_7+28000Z_8+34000Z_9+23560Z_{10}+28000Z_{11}+34000Z_{12})$

32- *Restriction of working time (minute):*

33- $(0.087)X_1 + (0.0817)X_2 + (0.0817)X_3 + (0.0817)X_4 + (0.0917)X_5 + (0.085)X_6 + (0.085)X_7 + (0.0917)X_8 + (0.0917)X_9 + (0.085)X_{10} \leq$

ISSN (1681- 6870)

$(7831440Z_1+9118800Z_2+10441920Z_3+7831440Z_4+9118800Z_5+10441920Z_6+7831440Z_7+9118800Z_8+10441920Z_9+7831440Z_{10}+9118800Z_{11}+10441920Z_{12})$

34- *On-demand (number of units)*

35- $(1)X_1 \geq (2000000Z_1+2200000Z_2+2300000Z_3+2400000Z_4+2600000Z_5+2800000Z_6+2000000Z_7+2200000Z_8+2300000Z_9+2400000Z_{10}+2600000Z_{11}+2800000Z_{12})$

36- $(1)X_2 \geq (30000Z_1+36000Z_2+40000Z_3+42000Z_4+30000Z_5+36000Z_6+40000Z_7+42000Z_8+30000Z_9+36000Z_{10}+40000Z_{11}+42000Z_{12})$

37- $(1)X_3 \geq (28000Z_1+36000Z_2+28000Z_3+36000Z_4+28000Z_5+36000Z_6+28000Z_7+36000Z_8+28000Z_9+36000Z_{10}+28000Z_{11}+36000Z_{12})$

38- $(1)X_4 \geq (300000Z_1+342000Z_2+383000Z_3+300000Z_4+342000Z_5+383000Z_6+300000Z_7+342000Z_8+383000Z_9+300000Z_{10}+342000Z_{11}+383000Z_{12})$

39- $(1)X_5 \geq (5000Z_1+6000Z_2+8000Z_3+12000Z_4+5000Z_5+6000Z_6+8000Z_7+12000Z_8+5000Z_9+6000Z_{10}+8000Z_{11}+12000Z_{12})$

40- $(1)X_6 \geq (25000Z_1+36000Z_2+25000Z_3+36000Z_4+25000Z_5+36000Z_6+25000Z_7+36000Z_8+25000Z_9+36000Z_{10}+25000Z_{11}+36000Z_{12})$

41- $(1)X_7 \geq (88000Z_1+120000Z_2+88000Z_3+120000Z_4+88000Z_5+120000Z_6+88000Z_7+120000Z_8+88000Z_9+120000Z_{10}+88000Z_{11}+120000Z_{12})$

42- $(1)X_8 \geq (17500Z_1+28000Z_2+17500Z_3+28000Z_4+17500Z_5+28000Z_6+17500Z_7+28000Z_8+17500Z_9+28000Z_{10}+17500Z_{11}+28000Z_{12})$

43- $(1)X_9 \geq (64500Z_1+68000Z_2+80000Z_3+90000Z_4+100000Z_5+120000Z_6+64500Z_7+68000Z_8+80000Z_9+90000Z_{10}+100000Z_{11}+120000Z_{12})$

44- $(1)X_{10} \geq (176000Z_1+250000Z_2+400000Z_3+500000Z_4+176000Z_5+250000Z_6+400000Z_7+500000Z_8+176000Z_9+250000Z_{10}+400000Z_{11}+500000Z_{12})$

45- *Capacity constraints (number of units):*

46- $(1)X_1 \leq (2716377Z_1+4154232Z_2+2716377Z_3+4154232Z_4+2716377Z_5+4154232Z_6+2716377Z_7+4154232Z_8+2716377Z_9+4154232Z_{10}+2716377Z_{11}+4154232Z_{12})$

47- $(1)X_2 \leq (32000Z_1+51408Z_2+32000Z_3+51408Z_4+32000Z_5+51408Z_6+32000Z_7+51408Z_8+32000Z_9+51408Z_{10}+32000Z_{11}+51408Z_{12})$

48- $(1)X_3 \leq (31824Z_1+40000Z_2+31824Z_3+40000Z_4+31824Z_5+40000Z_6+31824Z_7+40000Z_8+31824Z_9+40000Z_{10}+31824Z_{11}+40000Z_{12})$

49- $(1)X_4 \leq (348649Z_1+563203Z_2+348649Z_3+563203Z_4+348649Z_5+563203Z_6+348649Z_7+563203Z_8+348649Z_9+563203Z_{10}+348649Z_{11}+563203Z_{12})$

50- $(1)X_5 \leq (6000Z_1+13000Z_2+6000Z_3+13000Z_4+6000Z_5+13000Z_6+6000Z_7+13000Z_8+6000Z_9+13000Z_{10}+6000Z_{11}+13000Z_{12})$

51- $(1)X_6 \leq$
$(30371Z_1+45754Z_2+30371Z_3+45754Z_4+30371Z_5+45754Z_6+30371Z_7+45754Z_8+30371Z_9+45754Z_{10}+30371Z_{11}+45754Z_{12})$
52- $(1)X_7 \leq$
$(90875Z_1+130798Z_2+90875Z_3+130798Z_4+90875Z_5+130798Z_6+90875Z_7+130798Z_8+90875Z_9+130798Z_{10}+90875Z_{11}+130798Z_{12})$
53- $(1)X_8 \leq$
$(18175Z_1+29360Z_2+18175Z_3+29360Z_4+18175Z_5+29360Z_6+18175Z_7+29360Z_8+18175Z_9+29360Z_{10}+18175Z_{11}+29360Z_{12})$
54- $(1)X_9 \leq$
$(70627Z_1+122113Z_2+70627Z_3+122113Z_4+70627Z_5+122113Z_6+70627Z_7+122113Z_8+70627Z_9+122113Z_{10}+70627Z_{11}+122113Z_{12})$
55- $(1)X_{10} \leq$
$(183765Z_1+550000Z_2+183765Z_3+550000Z_4+183765Z_5+550000Z_6+183765Z_7+550000Z_8+183765Z_9+550000Z_{10}+183765Z_{11}+550000Z_{12})$
56- $Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 + Z_7 + Z_1 + Z_9 + Z_{10} = 1$
57- *Constraint of Negativity:*
58- $Z_p = 0/1 \quad , \quad p=1,2,\ldots\ldots, k$

$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10} \geq 0$ *and Integer*

2- Fuzzy Linear Programming

Through the use of mathematical methods of fuzzy numbers, we have converted multiple options on the right side to one number and thus the model has been transformed into a normal programming model as follow:

$$MAX\ (P) = (172.75)X_1 + (215.68)X_2 + (121.75)X_3 + (128.66)X_4 + (1318)X_5 + (53.835)X_6 + (52.09)X_7 + (74.495)X_8 + (222.04)X_9 + (175.58)X_{10}$$

Subject to

A- Restrictions of production requirements (gram)

1- $(0.04)X_1 + (0.04)X_4 + (0.04)X_7 \leq (200000)$
2- $(2.4)X_1 \leq (8075000)$
3- $(0.09)X_1 + (0.09)X_3 + (0.09)X_4 + (0.09)X_5 + (0.09)X_6 + (0.09)X_7 + (0.09)X_8 + (0.09)X_9 + (0.09)X_{10} \leq (347500)$
4- $(0.03)X_1 + (0.03)X_2 + (0.03)X_3 + (0.03)X_4 + (0.03)X_5 + (0.03)X_6 + (0.03)X_7 + (0.03)X_8 + (0.03)X_9 + (0.03)X_{10} \leq (172500)$
5- $(0.1)X_1 + (0.2)X_2 + (0.1)X_3 + (0.1)X_4 + (0.15)X_6 + \; + (0.1)X_8 + (0.05)X_{10} \leq (587500)$
6- $(15)X_1 \leq (46000000)$
7- $(0.02)X_1 + (0.1)X_2 \leq (79000)$
8- $(0.25)X_1 + (0.7)X_2 + (0.27)X_3 + (0.27)X_4 + (0.09)X_5 + (0.5)X_6 + (0.17)X_8 + (0.07)X_9 + (0.11)X_{10} \leq (855500)$
9- $(0.75)X_1 + (0.3)X_2 + (0.6)X_3 + (0.6)X_4 + (0.24)X_5 + (0.7)X_6 + (0.78)X_8 + (0.7)X_9 \leq (3600000)$
10- $(0.0005)X_1 \leq (1800000)$
11- $(10)X_1 + (5)X_2 + (1.83)X_3 + (1)X_4 + (0.97)X_5 + (15)X_6 + (1)X_7 + (0.97)X_8 + (0.97)X_9 \leq (38375000)$
12- $(20)X_1 + (5)X_2 + (5)X_3 + (5)X_4 + (9.72)X_5 + (20)X_6 + (5)X_7 + (9.72)X_8 + (9.72)X_9 \leq (80500000)$
13- $(0.97)X_8 \leq (70000)$

ISSN (1681- 6870)

14- $(0.01)X_1 \leq (35000)$
15- $(0.11)X_2 \leq (23500)$
16- $(0.0015)X_2 \leq (6865)$
17- $(0.05)X_3 \leq (5000)$
18- $(0.001)X_3 + (0.0019)X_4 + (0.02)X_5 + (0.07)X_8 + (0.03)X_9 + (0.001)X_{10} \leq (8250)$
19- $(0.00033)X_3 + (0.0002)X_4 + (0.002)X_6 \leq (1000000)$
20- $(5.82)X_5 \leq (93250)$
21- $(38.89)X_5 + (25)X_6 + (30)X_7 + (38.89)X_8 + (38.89)X_9 \leq (10250000)$
22- $(0.15)X_5 + (0.05)X_8 + (0.15)X_9 \leq (67000)$
23- $(0.19)X_5 + (0.19)X_8 + (0.29)X_9 \leq (77000)$
24- $(0.62)X_6 + (1)X_7 \leq (272500)$
25- $(0.05)X_7 \leq (39000)$
26- $(0.1)X_6 + (0.1)X_7 \leq (81000)$
27- $(0.02)X_7 \leq (8000)$
28- $(0.0005)X_7 \leq (1100)$
29- $(1)X_6 \leq (496000)$
30- $(0.2)X_6 \leq (41500)$
31- $(0.08)X_{10} \leq (28780)$

B- Restriction of working time (minute):

32- $(0.087)X_1 + (0.0817)X_2 + (0.0817)X_3 + (0.0817)X_4 + (0.0917)X_5 + (0.085)X_6 + (0.085)X_7 + (0.0917)X_8 + (0.0917)X_9 + (0.085)X_{10} \leq (9136680)$

C- On-demand (number of units):

33- $(1)X_1 \geq (2377777.778)$
34- $(1)X_2 \geq (37000)$
35- $(1)X_3 \geq (32000)$
36- $(1)X_4 \geq (341500)$
37- $(1)X_5 \geq (7750)$
38- $(1)X_6 \geq (30500)$
39- $(1)X_7 \geq (104000)$
40- $(1)X_8 \geq (22750)$
41- $(1)X_9 \geq (86277.78)$
42- $(1)X_{10} \geq (291000)$

D- Capacity constraints (number of units):

43- $(1)X_1 \leq (3435304.5)$
44- $(1)X_2 \leq (41704)$
45- $(1)X_3 \leq (35912)$
46- $(1)X_4 \leq (455926)$
47- $(1)X_5 \leq (9500)$
48- $(1)X_6 \leq (38062.5)$
49- $(1)X_7 \leq (110836.5)$
50- $(1)X_8 \leq (23767.5)$
51- $(1)X_9 \leq (96370)$
52- $(1)X_{10} \leq (366882.5)$

E- Constraint of Negativity:

$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10} \geq 0$ and integer
The above mathematical models are treated by LINGO software.
The solution is obtained as follow:

ISSN (1681- 6870)

Optimal production quantities and net profit by fuzzy linear programming	Optimal production quantities and net profit with multi-choice integer programming technology	Product
2649856	2645121	Cold-M
37000	36000	Dexam
32000	36000	Ventomaxin
341500	342000	Histamaxin
9500	13000	Naliman susp
30500	36000	Babylet
110836	120000	Eximan
22750	28000	Mefaman susp
96370	100424	Metroman susp
359750	350000	Solvoman
619770200	624253700	Net profit (IQD)

Discussion and Conclusions

We can see through the application of the mathematical model and find the optimal solution in two methods (multi-choice integer programming technique and fuzzy linear programming technology) that the first method achieved a higher profit than it is the method of fuzzy linear programming and that the integer programming technique was more exploitation of raw materials available than the other method Also, the integer programming method saves more time for the lab than it is by the fuzzy solution. At the same time, the first method deals with the resources available to the lab in a realistic and actual way, i.e. one of the options (resources) that are physically available within It is not necessary to request additional raw materials or waste large quantities of these materials, contrary to the existing options available through the use of fuzzy linear programming method which imposes on the owner of the amounts of new resources resulting from the integration of these options and come up with new options due to the use of fuzzy mathematical methods Certain is different from the actual reality available from those resources, which makes the task difficult for the decision maker, which may not be commensurate with the possibilities of that material and human institution, especially if the demand is fluctuating or if the institution is experiencing a specific financial crisis, they must exploit all that is available Keep on the balance of the institution, and one of the most important advantages of the method of multi-choice integer programming that it can deal with an unlimited number of options without adherence to new mathematical and complex methods, as this technology has a fixed mechanism dealing with all options whatever the number in addition to ease of use and design of a program ready to be applied Unlike Fuzzy Linear Programming, which deals with a limited number of options (**triangular**, **trapezoidal**, **Hexagonal**, and **Octet**), To date, no more than eight foggy options

ISSN (1681- 6870)

have been dealt with, in addition to the difficulty of implementing them , also its several mathematical processes should be made that It is difficult for people who are not academic or who have no statistical or mathematical competencies to do it.

References

[1] Srinivasan, A. and Geetharamani, G., 2013. Method for solving fuzzy assignment problem. Applied Mathematical Sciences, 7(113), pp.5607-5619.

[2] Hiller, F. and Lieberman, G. (1990) "Introduction to Operations Research". Mc Graw-Hill, New York.

[3] Rajendran, Dr. M. Ananthanarayanan (2018), " Fuzzy Criticalpath Method with Hexagonal and Generalised Hexagonal Fuzzy Numbers Using Ranking Method ", International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 15 (2018) pp. 11877-11882.

[4] Taha. H. A. ,2007," Operations Research: An Introduction ‘“ Pearson Education, Inc., Eighth Edition.

[5] Tarek A. Khalil1, Yashpal Singh Raghav2*, N. Badra3 (2016) "Optimal Solution of Multi-Choice Mathematical Programming Problem Using a New Technique". American Journal of Operations Research, 2016, 6, 167-172.

[6] Wakas S. Khalaf, "Meeting the Desired Fuzzy Shortest Possible Duration to Complete the Project at Fuzzy Least Cost”, Australian Journal of Basic and Applied Sciences, 9(23) July 2015, Pages: 136-143.