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On some Separation Axioms in Soft Lattice Topological Spaces

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Abstract

In the realm of topology, various constraints are frequently imposed on the types of topological spaces under examination and these constraints are defined by what are known as separation axioms. Also, the separation axioms can be seen as additional conditions that may be incorporated into the definition of topological spaces. Separation axioms serve various purposes in lattice theory. They provide tools for classifying and comparing different lattices, revealing their structural and topological properties. While traditional separation axioms like T_0, T_1, T_2, etc., still play a role, their interpretation and implications differ in the context of soft sets and lattice structures. This paper introduces the separation axioms, soft lattice T_i-space (for i = 4, 5, 6), within the context of a soft lattice topological space and investigates several of their associated properties. Studying lattices through these axioms can reveal connections between their order-theoretic properties and their topological features. This work goes beyond simply applying separation axioms to soft lattice topological spaces. It ventures into unveiling soft lattice invariant properties. Additionally, the study explores soft lattice invariant properties that are derived from these soft lattice T_i-space concepts, specifically, soft lattice hereditary and soft lattice topological properties. In conclusion, the study's exploration of soft lattice invariant properties pushes the boundaries of understanding soft lattice topological spaces. By delving into the essence of these structures and their local and global characteristics, the study opens doors to exciting theoretical possibilities and potential applications in diverse fields.

Keywords: Invariant properties, Soft set, Soft lattice, Soft lattice topology, Soft lattice T_i-space $(i = 4, 5, 6)$, Soft lattice normal space.

Introduction

Molodtsov [1] is the first to describe soft set theory in the year 1999. This approach to modeling, vagueness and uncertainties is entirely new. Several applications in different directions of soft set theory have been shown by Molodtsov[1]. Also, Maji et al.[2] conducted a study on Molodtsov's soft sets, wherein they established definitions that revolved around the equivalence of two soft sets, soft set inclusion and containment, the soft set complement, the null soft set, and the absolute soft set. They complemented these definitions with illustrative examples and explanation of fundamental properties. Additionally, various researchers[3-6] have explored the algebraic aspects of set theory in managing uncertain situations. The concept of soft set has been extended to soft lattices and soft fuzzy sets by Li F.[7] in 2010. Shabir and Naz[8] introduced the concept of soft topological spaces in the year 2011 and studied some basic properties. Also, the concept of soft continuous mapping and soft continuous mapping between two soft topological spaces are studied by several authors. On a soft topological space, Tantawy et al.[9] established the separation axioms $T_i(i = 0,1,2,3,4,5)$ in 2015 and investigated some of its characteristics. They clarified that, in some soft mappings, these axioms constitute soft topological features. Also, Soft Urysohn space was studied by Ramkumar et al.[10]. A considerable amount of research has been dedicated to the investigation of soft separation axioms [11-14].

Sandhya and Baiju [15] proposed soft lattice topological spaces over an initial universe U with a predefined set of parameters N using the notion of soft set initiated by Molodtsov [1]. They elucidated key attributes of soft lattice topological spaces and defined what constitutes soft lattice open and soft lattice closed sets are. Additionally, as an extension of set closure, they introduced a generalized concept known as the soft lattice closure of a soft lattice. In the context of parameterized topologies within an initial universe, the role of parameters was highlighted, and each parameter was assigned its individual topological space, emphasising its significance in the overall framework. The authors [15] established that a soft lattice topological space generates a parameterized family of topologies in the initial universe, although the reverse may not hold true. This suggests that constructing a soft lattice topological space is not feasible if specific topologies are given for each parameter. The properties related to the continuity of soft lattice continuous mappings, encompassing aspects such as injectivity, surjectivity, bijectivity, and the composition of soft lattice mappings are also investigated by them. Moreover, the concept of soft lattice continuous mapping between two soft lattice topological spaces and the Cartesian product in a soft lattice topological space is also studied. These mappings maintained a stable set of parameters across the initial universe and the intriguing findings included the exploration of soft open and closed lattice mappings, soft lattice homeomorphism etc.

The objective of this study is to obtain the soft lattice separation axioms, soft lattice T_i -spaces (i=4,5,6), in a soft lattice topological space. Also, investigated the invariant properties such as soft lattice hereditary property and soft lattice topological property.

The motivation for our study stems from the desire to understand and characterize the properties that dictate how points and sets can be distinguished or separated within a given space. These axioms provide a framework for exploring the level of 'closeness' of points in a soft lattice topological setting, offering insights into the structure and behavior of various mathematical spaces. The study of separation axioms leads to a better understanding of the underlying features that underpin topological spaces and is important in a variety of mathematical applications, including analysis, geometry, and functional analysis.

The previous work by the same authors[16] introduced soft lattice separation axioms, denoted as soft lattice T_i-spaces, for soft lattice topological spaces when i takes values $0, 1, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}$. In the current paper, authors extend this concept by defining and providing foundational results for soft lattice T_i-spaces with i equal to 4, 5, and 6, within the context of soft lattice topological spaces. The notions of soft lattice normal and soft lattice regular spaces are also explored and establish a necessary condition for a soft lattice topological space to qualify as a soft lattice T_i-space with i values of 4, 5, or 6. The study also delves into soft lattice invariant properties, specifically the soft lattice hereditary property and soft lattice topological properties

Materials and Methods

By assuming the consistency of C, take C as a complete lattice in the context of this investigation. Define a unary operation denoted as $': C \longrightarrow C$ as a quasi-complementation, provided it exhibits two key properties: first, it is an involution (i.e., $c'' = c$ for all $c \in C$) and second, inverts the ordering (i.e., $c \leq d \Longrightarrow d' \leq c'$).

Definition 1:[7] Consider the triplet $M = (h, U, C)$, where (i) C is a complete lattice, (ii) $h: U \longrightarrow \wp(C)$ is a mapping, (iii) U is a universe set, then M is called the soft lattice denoted by h_N^C. i.e., for every $u \in U, h_N^C$ is a soft lattice over C, if the image of $h(u)$ under h is a sub lattice of C.

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Definition 2:[15] Consider an initial universe set U and a non-empty set of parameters N. Suppose that C_T is the collection of all complete and uniquely complemented soft lattices over C satisfying the following conditions:
(i) φ and C belong to C_T.
(ii) the arbitrary union of soft lattices from C_T belongs to C_T.
(iii) the finite intersection of soft lattices from C_T is a member of C_T.
then, C_T is a soft lattice topology on C and the 3-tuple (C, C_T, N) is termed as a soft lattice topological space or a soft lattice space defined over C.

Definition 3:[15] If (C, C_T, N) is a soft lattice topological space over C, then the soft lattice open sets in C are defined as the members of C_T.

Definition 4:[15] Let (C, C_T, N) be a soft lattice topological space over C and D be a subset of C which is non-empty. Then $C_{T_D} = \{D h_N^C \mid h_N^C \in C_T\}$ is said to be the soft relative lattice topology on D and (D, C_{T_D}, N) is called a soft lattice subspace of h_N^C.

Definition 5: [16] Suppose (C, C_T, N) is a soft lattice topological space over C and $c_1, c_2 \in C$ such that $c_1 \neq c_2$. If there exist soft lattice open sets h_N^C and k_N^C such that $c_1 \in h_N^C$ and $c_2 \notin h_N^C$ or $c_2 \in k_N^C$ and $c_1 \notin k_N^C$, then (C, C_T, N) is called a soft lattice T_0-space.

Definition 6: [16] Let (C, C_T, N) represents a soft lattice topological space over C and $c_1, c_2 \in C$ such that $c_1 \neq c_2$. If there exist soft lattice open sets h_N^C and k_N^C such that $c_1 \in h_N^C$ and $c_2 \notin h_N^C$ and $c_2 \in k_N^C$ and $c_1 \notin k_N^C$, then (C, C_T, N) is called a soft lattice T_1-space.

Definition 7: [16] For any $c \in C$, c_N^C denotes the soft lattice set over C for which $c(n) = \{c\}$ for every $n \in N$.

Definition 8: [16] Suppose (C, C_T, N) is a soft lattice topological space over C and $c_1, c_2 \in C$ s.t. $c_1 \neq c_2$. If ∃ soft lattice open sets h_N^C and k_N^C s.t. $c_1 \in h_N^C$, $c_2 \in k_N^C$ and $h_N^C \cap k_N^C = \phi$, then (C, C_T, N) is said to be a soft lattice T_2-space or soft lattice Hausdorff space.

Definition 9: [16] Consider (C, C_T, N) as a soft lattice topological space over C. Then (C, C_T, N) is a soft lattice $T_{2\frac{1}{2}}$-space or soft lattice Urysohn space if for $c_1, c_2 \in C$ such that $c_1 \neq c_2$, there exist two soft lattice open sets h_N^C and k_N^C such that $c_1 \in h_N^C$ and $c_2 \in k_N^C$ and $\overline{h_N^C} \cap \overline{k_N^C} = \phi$.

Definition 10: [16] Let (C, C_T, N) is a soft lattice topological spaces over C, k_N^C be a soft lattice closed set in C and $c_1 \in C$ such that $c_2 \notin k_N^C$. If there exist soft lattice open sets h_{1N}^C and h_{2N}^C such that $c_1 \in h_{1N}^C$, $k_N^C \subset h_{2N}^C$ and $h_{1N}^C \cap h_{2N}^C = \phi$, then (C, C_T, N) is called a soft lattice regular space.

Definition 11: [16] Let (C, C_T, N) be a soft lattice topological space over C, then it is said to be a soft lattice T_3-space if it is soft lattice regular and soft lattice T_1- space.

Theorem 1: [16] A soft lattice T_3-space is a soft lattice T_2-space.

Theorem 2: [16] If (C, C_T, N) is a soft lattice T_3-space, then (C, C_{T_n}, N) is a soft lattice T_3-space for some parameter $n \in N$.

Theorem 3: [16] Let's consider (C, C_T, N) as a soft lattice topological spaces over C. If (C, C_T, N) is a soft lattice regular space and $l_1 \in C$ is a soft lattice closed

set, then (C, C_T, N) is a soft lattice T_3-space.

Definition 12: [16] (C, C_T, N) represents a soft lattice topological spaces over C, then (C, C_T, N) is characterized as a soft lattice completely regular space if for every soft lattice closed subset h_N^C and any soft lattice point $c_N^C \notin h_N^C$, there is a soft lattice continuous function $h_k: (C, C_T, N) \longrightarrow (C, C_T, N)$ such that $h(c) = \phi$ and $h(h_N^C) = C$. Otherwise, say c and h_N^C can be separated by a soft lattice continuous function.

Definition 13: [16] A soft lattice topological space (C, C_T, N) is characterized as a soft lattice T_3-space

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if it is a soft lattice completely regular space and a soft lattice T_1-space.

Proposition 1: [16] Consider a soft lattice topological spaces (C, C_T, N) over C and let D be a non-empty subset of C. If (C, C_T, N) is a soft lattice T_1-space, then $\left(D, C_{T_D}, N\right)$ is a soft lattice T_1-space.

Results and Discussion

Soft lattice separation axioms:

Definition 14: Let (C, C_T, N) be a soft lattice topological space over C, h_N^C and k_N^C be soft lattice closed sets s.t. $h_N^C \cap k_N^C = \phi$. If $\exists$ soft lattice open sets h_{1N}^C and h_{2N}^C s.t. $h_N^C \subset h_{1N}^C, k_N^C \subset h_{2N}^C$ and $h_{1N}^C \cap h_{2N}^C = \phi$, then (C, C_T, N) is called a soft lattice normal space.

Definition 15: Let (C, C_T, N) be a soft lattice topological space over C. Then (C, C_T, N) is said to be a soft lattice T_4-space if it is soft lattice normal and soft lattice T_1-space.

Theorem 5: Every soft lattice T_4-space is a soft lattice T_3-space.

Proof: Let (C, C_T, N) be a soft lattice T_4-space. Then (C, C_T, N) is soft lattice normal and soft lattice T_1-space. Let h_N^C and k_N^C be soft lattice closed sets s.t. $h_N^C \cap k_N^C = \phi$. Then by theorem 3, (C, C_T, N) is a soft lattice T_3-space.

Remark 1: The example given below demonstrates that the converse of the previous theorem may not be true in general.

Example 1: Let $C = \{c_1, c_2\}$, $N = \{n_1, n_2\}$ and $C_T = \{\emptyset, C, f_{1N}^C, f_{2N}^C, f_{3N}^C\}$ is a soft lattice topological space over C, where $f_{1N}^C, f_{2N}^C, f_{3N}^C$ are soft lattices over C, given as follows:
$f_1(n_1) = C$, $f_1(n_2) = \{c_2\}$,
$f_2(n_1) = \{c_1\}$, $f_2(n_2) = C$,
$f_3(n_1) = \{c_1\}$, $f_3(n_2) = \{c_2\}$.
Thus (C, C_T, N) is a soft lattice topological spaces over C.
Here (C, C_T, N) is a soft lattice T_3-space over C which is also a soft lattice T_1-space over C but not a soft lattice T_4-space because let h_N^C and k_N^C be soft lattice closed sets such that $h_N^C \cap k_N^C = \phi$. If there do not exist soft lattice open sets f_{1N}^C and f_{2N}^C such that $h_N^C \subset f_{1N}^C, k_N^C \subset f_{2N}^C$ and $f_{1N}^C \cap f_{2N}^C = \phi$, then (C, C_T, N) is not a soft lattice normal space.
Hence every soft lattice T_3-space is not necessarily soft lattice T_4-space.

Theorem 4: [16] The property of being soft lattice T_i-space $\left(i = 0, 1, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}\right)$ is a soft lattice topological property or it is preserved under a soft lattice homeomorphism.

Theorem 6: If (C, C_T, N) is a soft lattice T_4-space over C, then $\left(C, C_{T_n}, N\right)$ is a soft lattice T_4-space for each parameter $n \in N$.

Proof: Proof follows from Theorem 1 and Theorem 2.

Theorem 7: Let (C, C_T, N) be a soft lattice topological space over C. If (C, C_T, N) is soft lattice regular and c_{1N}^C is a soft lattice closed set for each $c_1 \in C$, then (C, C_T, N) is a soft lattice T_4-space.

Proof: As every soft lattice T_4-space is a soft lattice T_3-space by theorem 5. Also, by theorem 3, if (C, C_T, N) is a soft lattice normal space and if c_{1N}^C is a soft lattice closed set for each $c_1 \in C$, then (C, C_T, N) is a soft lattice T_3-space. Hence (C, C_T, N) is a soft lattice T_4-space.

Definition 16: Let (C, C_T, N) be a soft lattice topological space over C and A_N^C, B_N^C be two non-empty soft lattice subsets over C. Then A_N^C and B_N^C are separated soft lattice sets if $A_N^C \cap \overline{B_N^C} = \phi$ and $\overline{A_N^C} \cap B_N^C = \phi$.

Definition 17: Let (C, C_T, N) be a soft lattice topological space over C. A soft lattice topological space (C, C_T, N) is said to be soft lattice completely normal space if for any two non-empty separated soft lattice sets $A_N^C, B_N^C, \exists F_N^C, G_N^C \in C_T$ s.t. $A_N^C \subset F_N^C, B_N^C \subset G_N^C$ and $F_N^C \cap G_N^C = \phi$.

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Definition 18: A soft lattice topological space (C, C_T, N) is said to be a soft lattice T_5-space if it is soft lattice completely normal and soft lattice T_1-space.

Theorem 8: Every soft lattice completely normal space is a soft lattice normal space and hence every soft lattice T_5-space is a soft lattice T_4-space.

Proof: Let (C, C_T, N) be a soft lattice completely normal space and A_N^C , B_N^C are two soft lattice sets. Then $\overline{A_N^C} = A_N^C$ and $\overline{B_N^C} = B_N^C$ and follows that A_N^C, B_N^C are seperated sets. Also, since (C, C_T, N) is soft lattice completely normal space, $\exists\ F_N^C, G_N^C \in C_T$ s.t. $A_N^C \subset F_N^C, B_N^C \subset G_N^C$ and $F_N^C \cap\ G_N^C = \phi$. Then (C, C_T, N) is soft lattice normal. Hence, every soft lattice completely normal space is a soft lattice normal space. Also, by Definition 18, every soft lattice T_5-space is a soft lattice T_4-space.

Remark 2: The example given below demonstrates that the converse of the previous theorem may not be true in general.

Example 2: Let $C = \{c_1, c_2, c_3, c_4\}$, $N = \{n_1, n_2\}$ and
$C_T = \{\emptyset, C, f_{1N}^C, f_{2N}^C, f_{3N}^C, f_{4N}^C, f_{5N}^C, f_{6N}^C, f_{7N}^C, f_{8N}^C\}$ is a soft lattice topological space over C, where
$f_{1N}^C, f_{2N}^C, f_{3N}^C, f_{4N}^C, f_{5N}^C, f_{6N}^C, f_{7N}^C, f_{8N}^C$ are soft lattices over C, given as follows:
$f_1\ (n_1) = \{c_1, c_2, c_4\}$, , $f_1\ (n_2) = \{c_1, c_2, c_3\}$,
$f_2\ (n_1) = \{c_1, c_3, c_4\}$, $f_2\ (n_2) = \{c_1, c_2, c_3\}$,
$f_3\ (n_1) = \{c_1, c_4\}$, $f_3\ (n_2) = \{c_1, c_2, c_3\}$,
$f_4\ (n_1) = \{c_2, c_3\}$, $f_4\ (n_2) = \{c_1, c_2, c_3\}$,
$f_5\ (n_1) = \{c_2\}$, $f_5\ (n_2) = \{c_1, c_2, c_3\}$,
$f_6\ (n_1) = \{c_3\}$, $f_6\ (n_2) = \{c_1, c_2, c_3\}$,
$f_7\ (n_1) = \emptyset$, $f_7\ (n_2) = \{c_1, c_2, c_3\}$,
$f_8\ (n_1) = C$, $f_8\ (n_2) = \{c_1, c_2, c_3\}$.
Thus (C, C_T, N) is a soft lattice topological spaces over C.
Here (C, C_T, N) is a soft lattice T_4-space over C which is also a soft lattice T_1-space and soft lattice normal space over C but not a soft lattice T_5-space because if for any two non-empty separated soft lattice sets A_N^C, B_N^C, there does not exist $F_N^C, G_N^C \in C_T$ such that $A_N^C \subset F_N^C, B_N^C \subset\ G_N^C$ and $F_N^C \cap G_N^C = \phi$, then (C, C_T, N) is not a soft lattice completely normal space.

Hence every soft lattice T_4-space is not necessarily soft lattice T_5-space. Also, every soft lattice normal space need not be a soft lattice completely normal space.

Theorem 9: If (C, C_T, N) is a soft lattice topological space over C and $\left(F_N^C, C_{T_{F_N^C}}, N\right)$ is soft lattice normal subspace of (C, C_T, N) for all $F_N^C \in\ C_T$, then (C, C_T, N) is soft lattice completely normal space.

Proof: Let A_N^C, B_N^C be two non-empty separated sets in C, then

$$A_N^C \cap \overline{B_N^C} = \phi \ \square\square\square\ \overline{A_N^C} \cap B_N^C = \phi. \quad (1)$$

Since $\overline{A_N^C}, \overline{B_N^C} \in C_T{}'$, $\left(\overline{A_N^C} \cap \overline{B_N^C}\right)' \in C_T$.

Assume that

$$F_N^C = \left(\overline{A_N^C} \cap \overline{B_N^C}\right)' \in C_T. \quad (2)$$

Let $C_{T_{F_N^C}}$ denotes the C_T-relative topology for F_N^C.

Moreover, $F_N^C \cap \overline{A_N^C}$ and $F_N^C \cap \overline{B_N^C}$ are $C_{T_{F_N^C}}$ - closed subsets of F_N^C s.t.

$\left(F_N^C \cap \overline{A_N^C}\right) \cap \left(F_N^C \cap \overline{B_N^C}\right) = F_N^C \cap \left(\overline{A_N^C} \cap\ \overline{B_N^C}\right) = \left(\overline{A_N^C} \cap \overline{B_N^C}\right)' \cap \left(\overline{A_N^C} \cap \overline{B_N^C}\right) = \phi$. [From Eq. 2]

Now $F_N^C \cap \overline{A_N^C}$ and $F_N^C \cap \overline{B_N^C} \in C_{T}{}'_{F_N^C}$ s.t. $\left(F_N^C \cap \overline{A_N^C}\right) \cap \left(F_N^C \cap\ \overline{B_N^C}\right) = \phi$.

But $\left(F_N^C, C_{T_{F_N^C}}, N\right)$ is soft lattice normal subspace of (C, C_T, N), so $\exists\ C_N^C, D_N^C \in C_{T_{F_N^C}}$ s.t. $F_N^C \cap \overline{A_N^C} \subset C_N^C$,

$F_N^C \cap \overline{B_N^C} \subset D_N^C$ and $C_N^C \cap D_N^C = \phi$.

Since $C_N^C, D_N^C \in C_{T_{F_N^C}}, F_N^C \in C_T$, then $C_N^C, D_N^C \in C_T$.

From Eq.1, $A_N^C \cap \overline{B_N^C} = \phi$, and hence

$$A_N^C \subseteq \left(\overline{B_N^C}\right)' \subseteq \left(\overline{B_N^C}\right)' \cup \left(\overline{A_N^C}\right)' = \left(\overline{B_N^C} \cap \overline{A_N^C}\right)' = F_N^C.$$

Also, $B_N^C \cap \overline{A_N^C} = \phi$, implies $B_N^C \subseteq \left(\overline{A_N^C}\right)' \subseteq \left(\overline{A_N^C}\right)' \cup \left(\overline{B_N^C}\right)' = \left(\overline{A_N^C} \cap \overline{B_N^C}\right)' = F_N^C$.

Now $A_N^C \subseteq F_N^C, A_N^C \subseteq \overline{F_N^C}$ which implies that $A_N^C = A_N^C \cap \overline{F_N^C} \subseteq F_N^C \cap \overline{A_N^C} \subseteq C_N^C$.

Also $B_N^C \subseteq F_N^C, B_N^C \subseteq \overline{B_N^C}$, implies that $B_N^C = B_N^C \cap \overline{B_N^C} \subseteq F_N^C \cap \overline{B_N^C} \subseteq D_N^C$.

Consequently, A_N^C and B_N^C are two seperated subsets of C and $\exists\, C_N^C, D_N^C \in C_T$ s.t. $A_N^C \subseteq C_N^C, B_N^C \subseteq D_N^C$

and $C_N^C \cap D_N^C = \phi$.

Hence (C, C_T, N) is soft lattice completely normal space.

Definition 19: Let (C, C_T, N) be a soft lattice topological space over C. It is said to be a soft lattice perfectly normal space if it is soft lattice normal and every soft lattice closed subset contains countable intersection of soft lattice open subsets.

Note 1: A soft lattice topological space (C, C_T, N) is a soft lattice perfectly normal space if and only if every soft lattice closed set is a zero set.

Theorem 10: Every soft lattice perfectly normal space is soft lattice normal.

Proof: Let (C, C_T, N) be a soft lattice perfectly normal space. Then by Definition 19, it follows that (C, C_T, N) is soft lattice normal.

Definition 20: Let (C, C_T, N) be a soft lattice topological space over C. A soft lattice topological space (C, C_T, N) is a called soft lattice T_6-space if it is a soft lattice perfectly T_4-space.

Theorem 11: Every soft lattice T_6-space is a soft lattice T_4-space.

Proof: Proof follows directly from Definition 20, i.e., a soft lattice topological space (C, C_T, N) is said to be soft lattice T_6-space if it is a soft lattice perfectly T_4-space.

Remark 3: The example given below demonstrates that the converse of the previous theorem may not be true in general.

Example 3: Let $C = \{c_1, c_2, c_3, c_4\}$, $N = \{n_1, n_2\}$ and
$C_T = \{\emptyset, C, f_{1N}^C, f_{2N}^C, f_{3N}^C, f_{4N}^C, f_{5N}^C, f_{6N}^C, f_{7N}^C, f_{8N}^C\}$ is a soft lattice topological space over C, where
$f_{1N}^C, f_{2N}^C, f_{3N}^C, f_{4N}^C, f_{5N}^C, f_{6N}^C, f_{7N}^C, f_{8N}^C$ are soft lattices over C, given as follows:

$f_1(n_1) = \{c_1, c_2, c_4\}$, , $f_1(n_2) = \{c_1, c_2, c_3\}$,
$f_2(n_1) = \{c_1, c_3, c_4\}$, $f_2(n_2) = \{c_1, c_2, c_3\}$,
$f_3(n_1) = \{c_1, c_4\}$, $f_3(n_2) = \{c_1, c_2, c_3\}$,
$f_4(n_1) = \{c_2, c_3\}$, $f_4(n_2) = \{c_1, c_2, c_3\}$,
$f_5(n_1) = \{c_2\}$, $f_5(n_2) = \{c_1, c_2, c_3\}$,
$f_6(n_1) = \{c_3\}$, $f_6(n_2) = \{c_1, c_2, c_3\}$,
$f_7(n_1) = \emptyset$, $f_7(n_2) = \{c_1, c_2, c_3\}$,
$f_8(n_1) = C$, $f_8(n_2) = \{c_1, c_2, c_3\}$.

Thus (C, C_T, N) is a soft lattice topological spaces over C.
Here (C, C_T, N) is a soft lattice T_4-space over C which is also a soft lattice T_1-space and soft lattice normal space over C but not a soft lattice T_6-space because every soft lattice closed subset does not contains countable intersection of soft lattice open subsets, then (C, C_T, N) is not a soft lattice perfectly normal space. Hence every soft lattice T_4-space is not necessarily soft lattice T_6-space.

Figure 1 represents the diagrammatic implications between soft lattice T_i -spaces for (i = 0, 1, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, 5, 6).

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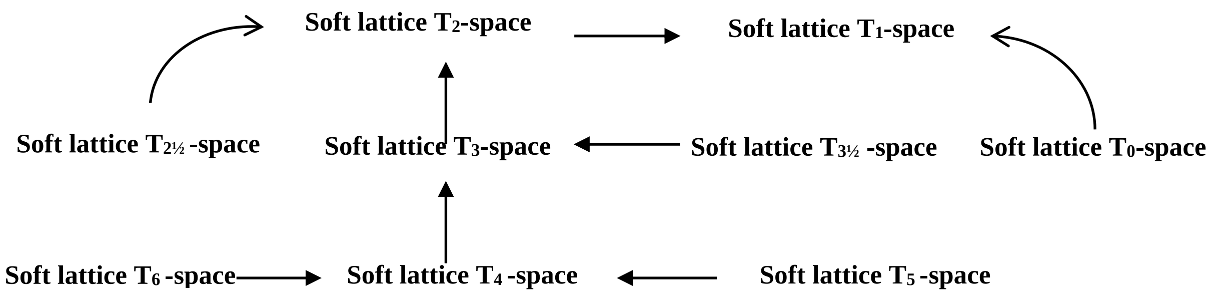

Figure 1. Diagram representing the implications between soft lattice T_i -spaces

Soft lattice hereditary property:

Theorem 12: Every closed soft lattice subspace of a soft lattice normal space is a soft lattice normal space.

Proof: Let (C, C_T, N) be a soft lattice normal space and (Y, C_{T_Y}, N) be a closed soft lattice subspace of (C, C_T, N).

To prove: (Y, C_{T_Y}, N) is a soft lattice normal space. Let h_N^C, k_N^C be two non-empty distinct soft lattice closed sets of C.

Since $h_N^C, k_N^C \in C_{T_Y}'$, by Definition 4, $\exists\, h_{1N}^C$ and $h_{2N}^C \in C_T{}'$ s.t. $h_N^C = Y_N^C \cap h_{1N}^C$ and $k_N^C = Y_N^C \cap h_{2N}^C$.

Also, since $Y_N^C \in C_T{}'$ and h_{1N}^C , $h_{2N}^C \in C_T{}'$, then $Y_N^C \cap h_{1N}^C, Y_N^C \cap h_{2N}^C \in C_T{}'$ and therefore $h_N^C, k_N^C \in C_{T_Y}'$.

Now h_N^C, k_N^C are two non-empty distinct soft lattice closed sets of C and (C, C_T, N) be a soft lattice normal space, then $\exists F_N^C, G_N^C \in C_T$ s.t. $h_N^C \subset F_N^C, k_N^C \subset G_N^C$ and $F_N^C \cap G_N^C = \phi$. This implies that $Y_N^C \cap F_N^C, Y_N^C \cap G_N^C \in C_{T_Y}$ s.t. $h_N^C \subset Y_N^C \cap F_N^C, k_N^C \subset Y_N^C \cap G_N^C$ and $(Y_N^C \cap F_N^C) \cap (Y_N^C \cap G_N^C) = Y_N^C \cap (F_N^C \cap G_N^C) = Y_N^C \cap \phi = \phi$.

So (Y, C_{T_Y}, N) is a soft lattice normal space.

Theorem 13: Let (C, C_T, N) be a soft lattice topological space over C and Y be a subset of C which is not empty. If (C, C_T, N) is a soft lattice T_4-space, then (Y, C_{T_Y}, N) is a soft lattice T_4-space.

Proof: Given that (C, C_T, N) is a soft lattice T_4-space. Then by Proposition 1 and Theorem 12, (Y, C_{T_Y}, N) is a soft lattice T_1-space and soft lattice normal space respectively.

Hence (Y, C_{T_Y}, N) is a soft lattice T_4-space.

Theorem 14: Let (C, C_T, N) be a soft lattice T_5-space. Then every soft lattice subspace (Y, C_{T_Y}, N) of the soft lattice topological space (C, C_T, N) is a soft lattice T_5-space.

Proof: Since (C, C_T, N) is a soft lattice T_5-space, by Definition 18, it is soft lattice completely normal and a soft lattice T_1-space. As every soft lattice subspace of a soft lattice T_1-space is a soft lattice T_1-space, it follows that (Y, C_{T_Y}, N) is a soft lattice T_1-space by Proposition 1.

To prove: (Y, C_{T_Y}, N) is a soft lattice completely normal space.

Let A_N^C, B_N^C be two seperated sets of Y, then

$A_N^C \cap \overline{B_N^C} = \phi$ □□□ $\overline{A_N^C} \cap B_N^C = \phi$.
3

Also $\overline{A_N^C} = Y_N^C \cap \overline{A_N^C}$ □□□ $\overline{B_N^C} = Y_N^C \cap \overline{B_N^C}$.
4

Substituting Eq 4 in Eq 3, $(Y_N^C \cap \overline{A_N^C}) \cap B_N^C = \phi$ and $(Y_N^C \cap \overline{B_N^C}) \cap A_N^C = \phi$.

Since $A_N^C, B_N^C \subset Y_N^C$, $A_N^C \cap \overline{B_N^C} = \phi$ and $\overline{A_N^C} \cap B_N^C = \phi$ and it implies that A_N^C, B_N^C are two seperated subsets of C.

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As (C, C_T, N) is a soft lattice completely normal space, $\exists F_N^C, G_N^C \in C_T$ s.t. $A_N^C \subset F_N^C, B_N^C \subset G_N^C$

and $F_N^C \cap G_N^C = \phi$.

Since $A_N^C \subset Y_N^C$ and $A_N^C \subset F_N^C$, implies that $A_N^C \subset Y_N^C \cap F_N^C$.

Also $B_N^C \subset Y_N^C$ and $A_N^C \subset G_N^C$, then $B_N^C \subset Y_N^C \cap G_N^C$.

Since $F_N^C, G_N^C \in C_T$, $Y_N^C \cap F_N^C, Y_N^C \cap G_N^C \in C_{T_Y}$ s.t. $A_N^C \subset Y_N^C \cap F_N^C, B_N^C \subset Y_N^C \cap G_N^C$ and $(Y_N^C \cap F_N^C) \cap (Y_N^C \cap G_N^C) = Y_N^C \cap (F_N^C \cap G_N^C) = Y_N^C \cap \phi = \phi$.

Thus (Y, C_{T_Y}, N) is a soft lattice completely normal space and hence every soft lattice subspace (Y, C_{T_Y}, N) of the soft lattice T_5-space (C, C_T, N) is a soft lattice T_5-space.

The proof of the following Lemma 1 follows from Theorem 14.

Lemma 1: Let (C, C_T, N) be a soft lattice T_5-space. Then every soft lattice subspace (Y, C_{T_Y}, N) of the soft lattice topological space is a soft lattice completely normal space.

Theorem 15: Let (C, C_T, N) be a soft lattice T_6-space. Then every soft lattice subspace (Y, C_{T_Y}, N) of the soft lattice topological spaces (C, C_T, N) is a soft lattice T_6-space.

Proof: Let (C, C_T, N) be a soft lattice T_6-space. Then it is a perfectly soft lattice T_4-space. Also, $Y \subset C$ s.t. (Y, C_{T_Y}, N) be the soft lattice subspace of (C, C_T, N).

To prove: (Y, C_{T_Y}, N) is a perfectly soft lattice T_4-space.

Since (C, C_T, N) is a soft lattice T_6-space, then it is soft lattice perfectly normal space and soft lattice T_1-space. Now by theorem 10, every soft lattice perfectly normal space is a soft lattice normal space. Also, by Proposition 1 and Theorem 14, it follows that (Y, C_{T_Y}, N) is a soft lattice perfectly normal space and soft lattice T_1-space. Therefore, (Y, C_{T_Y}, N) is a perfectly soft lattice T_4-space.

The proof of the following Lemma 2 follows from Theorem 15.

Lemma 2: Let (C, C_T, N) be a soft lattice T_6-space. Then every soft lattice subspace (Y, C_{T_Y}, N) of the soft lattice topological space is a soft lattice perfectly normal space.

Soft lattice topological property:

Theorem 16: The property of being soft lattice T_i-space $(i = 4, 5, 6)$ is a soft lattice topological property or it is preserved under soft lattice homeomorphism.

Proof: Proof follows from Theorem 4.

Conclusion

Soft lattice separation axioms and soft lattice T_i -spaces (i=4,5,6) are obtained within a soft lattice topological space in this study. Also investigated the invariant properties such as soft lattice hereditary property and soft lattice topological property. This work is an initiative to a new framework of soft lattice separation axioms. As a future work, one can study these concepts with respect to other generalizations of soft lattice open sets such as soft lattice preopen and soft lattice semi-open sets.

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Authors' Declaration

- Conflicts of Interest: None.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Manipal Academy of Higher Education, India.

Authors' Contribution Statement

This work was carried out in collaboration between all authors. S. S. P. developed the idea of separation axioms (T_i, i= 4, 5, 6) on soft lattice topological spaces. S. S. P. and B. T derived some of the observations and methods of introducing these axioms. S. S. P. wrote the manuscript. B. T. edited the manuscript with the revised idea. All authors read and approved the final manuscript.

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في بعض بديهيات الانفصال في الفضاءات الطوبولوجية ذات الشبكة الرخوة

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الخلاصة

في عالم الطوبولوجيا، يتم فرض قيود مختلفة في كثير من الأحيان على أنواع الفضاءات الطوبولوجية قيد الدراسة ويتم تعريف هذه القيود من خلال ما يعرف ببديهيات الفصل. كما يمكن اعتبار بديهيات الفصل بمثابة شروط إضافية يمكن دمجها في تعريف الفضاءات الطوبولوجية. تخدم بديهيات الفصل أغراضًا مختلفة في نظرية الشبكة. أنها توفر أدوات لتصنيف ومقارنة الشبكات المختلفة، والكشف عن خصائصها الهيكلية والطوبولوجية. في حين أن بديهيات الفصل التقليدية مثل T0، T1، T2، وما إلى ذلك، لا تزال تلعب دورًا، فإن تفسيرها وآثارها تختلف في سياق المجموعات الناعمة والهياكل الشبكية. يقدم هذا البحث بديهيات الفصل، الفضاء T_i للشبكة الناعمة (لـ i = 4، 5، 6)، في سياق الفضاء الطوبولوجي للشبكة الناعمة ويبحث في العديد من الخصائص المرتبطة بها. يمكن لدراسة الشبكات من خلال هذه البديهيات أن تكشف عن الروابط بين خصائصها النظرية وخصائصها الطوبولوجية. يتجاوز هذا العمل مجرد تطبيق بديهيات الفصل على المساحات الطوبولوجية ذات الشبكة الناعمة. إنها تغامر بالكشف عن الخصائص الثابتة للشبكة الناعمة. بالإضافة إلى ذلك، تستكشف الدراسة الخصائص الثابتة للشبكة الناعمة المستمدة من مفاهيم T_i-space للشبكة الناعمة، وعلى وجه التحديد، الخصائص الطوبولوجية للشبكة الناعمة الوراثية والشبكة الناعمة. في الختام، فإن استكشاف الدراسة للخصائص الثابتة للشبكة الناعمة يدفع حدود فهم الفضاءات الطوبولوجية للشبكة الناعمة. ومن خلال الخوض في جوهر هذه الهياكل وخصائصها المحلية والعالمية، تفتح الدراسة الأبواب أمام إمكانيات نظرية مثيرة وتطبيقات محتملة في مجالات متنوعة.

الكلمات المفتاحية: لخصائص الثابتة، المجموعة الناعمة، الشبكة الناعمة، طوبولوجيا الشبكة الناعمة، الشبكة الناعمة T_i-space (i=4,5,6)، المساحة العادية للشبكة الناعمة.