An Efficient Algorithm for Solving Variational Problems Using Hermite Polynomials

Halah Rahman Jaber
halah.jaber@yahoo.com
Al-Mustansiriyah University
College of Science - Department of Mathematics

Abstract: *An algorithm for solving variational problems with fixed and free boundary conditions using Hermite polynomials is proposed. The properties of Hermite polynomials with the operational matrix of integration are used to reduce a variational problem to the solution of algebraic equations. The method verifies an accurate approximate solution with using small numbers of polynomials comparing to other methods. Several examples have been applied to the proposed method.*

Keywords: variational problem, Hermite polynomials, operational matrix of integration.

1. Introduction

Orthogonal functions and polynomial series have received consideration attention in dealing with various problems of dynamic system. The main characteristic of this technique is to reduce these problems to those of solving a system of algebraic equations thus greatly simplifying the problem and making it computationally plausible.

The available sets of orthogonal functions can be divided into three classes. The first class is the set of piecewise constant basis function, the second class consists of the set of orthogonal polynomials and the third class is the widely used set of sine-cosine functions in Fourier series. In these methods, a truncated orthogonal series is used for solving variational problem with the goal of obtaining efficient computational solutions. Typical examples are the rationalized Haar functions [1], Walsh-hybrid[2], generalized Laguerre polynomials[3], block pulse function [4], Chebyshev polynomials[5], Fourier series [6] and Bernstein[7].

In this paper we use Hermite polynomials for solving variational problems. The method consists of reducing the variational problem into a set of algebraic equations by first expanding the candidate function as a Hermite function with unknown coefficients. The operational matrix of integration is given. The matrix used to evaluate the coefficients of Hermite functions in such a way that the necessary conditions for extremization are imposed. The proposed method is computationally attractive and applications are demonstrated through illustrative examples.

2. Hermite Polynomials with Some New Properties

Hermite polynomials are a classical orthogonal polynomial sequence that arises in probability. They are named after Charles Hermite (1864).

The explicit expression of Hermite polynomials of degree N is defined by [8]:

$$H_n(t) = n!\sum_{l=0}^{n/2} \frac{(-1)^{n/2-l}}{(2l)!(n/2-l)!}(2t)^{2l}$$

The recurrence relation is given by the formula[8]:

$$\mathrm{H}_{n+1}(t) = 2t\mathrm{H}_n(t) - 2n\mathrm{H}_{n-1}(t)$$

The function approximation by Hermite polynomials is defined as fellow:

A function f(t) defined over $[0, t_f)$ may be expanded as

$$f(t) = \sum_{i=0}^{\infty} c_i H_i(t) \tag{1}$$

Where

$c_i = < f(t), H_n(t) >$

in which $<,>$ denotes the inner product.

If the inner infinite series in eq.(1) is truncated, then eq.(1) can be written

$$f(t) = \sum_{i=0}^{N} c_i H_i(t) = c^T H(t) \tag{2}$$

where

$c = [c_0, c_1, \dots, c_N]^T$

$$H(t) = [H_0(t), H_1(t), \dots, H_N(t)]^T \tag{3}$$

The integration of $H_n(t)$ of order n can be obtained using the following formula:

$$\int H_n(t)dt = \begin{cases} \frac{1}{2(n+1)} H_{n+1} & \text{if n even} \\ \frac{1}{2(n+1)} [H_{n+1}(t) + \frac{2n!}{\left(\frac{n-1}{2}\right)!} H_0 & \text{if n odd} \end{cases} \tag{4}$$

The first few integration of Hermite polynomials are:

$$\int H_0(t)dt = \frac{1}{2} H_1(t)$$

$$\int H_1(t)dt = \frac{1}{4}\left[H_2(t) + 2H_0(t)\right]$$

$$\int H_2(t)dt = \frac{1}{6}H_3(t)$$

$$\int H_3(t)dt = \frac{1}{8}\left[H_4(t) + 12H_0(t)\right]$$

$$\int H_4(t)dt = \frac{1}{10}H_5(t)$$

$$\int H_5(t)dt = \frac{1}{12}\left[H_6(t) + 120H_0(t)\right]$$

The integration of the vector H(t) defined in eq.(3) may be approximated by

$$\int_0^t H(t)dt \cong P_{N+1}H(t) \tag{5}$$

Where P is the $N+1 \times N+1$ operational matrix for integration and from eq.(4) we can written this matrix as:

$$P = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & \cdots & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{2n!}{2(n+1)\left(\frac{n-1}{2}\right)!} & 0 & 0 & 0 & \cdots & 0 & \frac{1}{2(n+1)} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \frac{1}{2(n+1)} \end{bmatrix}$$

If N even, and

$$P = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & \cdots & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & \frac{1}{2(n+1)} & 0 \\ \frac{2n!}{2(n+1)\left(\frac{n-1}{2}\right)!} & 0 & 0 & 0 & \cdots & 0 & 0 & \frac{1}{2(n+1)} \end{bmatrix}$$

If N odd.

3. Application of Hermite Polynomials for Solving Variational Problem

Consider the problem of finding the extremum of the functional

$$J(x) = \int_0^1 F(t, x(t), \dot{x}(t))dt \tag{6}$$

with the boundary conditions

$$x(0) = x_0, \; x(1) = x_1 \tag{7}$$

The necessary condition for x(t) to extremize J(x) is that it should satisfy the Euler-Lagrange equation

$$\frac{\partial F}{\partial x} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}}\right) = 0$$

With appropriate boundary conditions, However, the above differential equation can be integrated easily only for simple cases. Thus numerical and direct methods such as the well-known Ritz and Galerkin methods [9], have been developed to solve variational problem. In this paper the Hermite polynomials are used to establish the direct method for the variational problems.

Suppose, the rate variable $\dot{x}(t)$ can be expressed approximately as:

$$\dot{x}(t) \cong \sum_{i=0}^{N} c_i H_i(t) = c^T H(t) \tag{8}$$

Integrating eq.(8) from 0 to t and using eq.(5), we represent $x(t)$ as:

$$x(t) = \int_0^t x(t)dt + x(0) \quad \cong c^T P_{N+1} H(t) + x(0) \tag{9}$$

we can also express t in terms of $H(t)$ as:

$$t \cong d^T H(t) \text{ where } d^T = [d_0, d_1, \dots, d_N] \tag{10}$$

subtitling eqs.(8-10) in eq.(6) the functional $J(x)$ becomes a functions $c_i, i = 0,1,\dots,N$ and we finally have

$$J = J(c_0, c_1, \dots, N) \tag{11}$$

Hence, to find the extremum of $J(x)$ we solve

$$\frac{\partial J}{\partial c_i} = 0, i = 0,1,\dots,n \tag{12}$$

The above procedure is now used to solve the following variational problems.

4. Numerical Examples

Example (1): Consider the problem of finding the extermal of the functional [7] [10]:

$$J(x) = \int_0^1 [\dot{x}^2(t) + t\dot{x}(t)]dt \tag{13}$$

The boundary conditions

$$x(0) = 0, x(1) = \frac{1}{4} \tag{14}$$

where the exact solution is obtained by using the Euler equation as:

$$\dot{x}(t) = \frac{1}{2}(1 - t), \quad x(t) = \frac{t}{2}(1 - \frac{t}{2})$$

using eqs.(8-10) in eq.(13) we get:

$$J(x) = \int_0^1 \left[c^T B(t) B^T(t) c + c^T B(t) B^T(t) d\right] dt \tag{15}$$

Let:

$$D = \int_0^1 H(t) H^T(t) dt \tag{16}$$

then:

$$J(x) = c^T D c + c^T D d \tag{17}$$

hence, the boundary conditions in eq.(14) substituted in eq.(9) yields

$$x(1) = c^T P H(1) = \frac{1}{4} \tag{18}$$

We minimize eq.(13) subject to eq.(18) using Lagrange multiplier. Suppose:

$$\tilde{J}(x) = J(x) + \lambda(c^T PH(1) - \frac{1}{4})$$

where λ is the Lagrange multiplier. Using eq.(12). We solve

$$\frac{\partial \tilde{J}}{\partial c} = 0, \quad \frac{\partial \tilde{J}}{\partial \lambda} = 0$$

By choosing N=2 we obtain

$$D = \begin{bmatrix} 1 & 1 & \frac{-2}{3} \\ 1 & \frac{4}{3} & 0 \\ \frac{-2}{3} & 0 & \frac{28}{15} \end{bmatrix}$$

and

$$d = \left[0, \frac{1}{2}, 0\right]^T H(t)$$

Therefore,

$$c = \left[\frac{1}{2}, \quad \frac{-1}{4}, \quad 0\right]^T$$

$$x(t) = [-0.125, 0.25, -0.0625]^T H(t)$$

then $J(x) = 0.166667$, Table(1) shows the approximate values of $x(t)$ using Hermite polynomials approach for $N = 2$ and 3.

Table (1): Estimated values for N=2 and 3 and exact values of $x(t)$

t	N=2	N=3	Analytical Solution	Absolute error
	$X(t)$	**$X(t)$**		**$\lvert x(t)_{exact} - X(t)_{app} \rvert$**
0	0.00000000	0.00000000	0.00000000	0.00000000
0.125	0.05859375	0.05859375	0.05859375	0.00000000
0.250	0.10937500	0.10937500	0.10937500	0.00000000
0.375	0.15234375	0.15234375	0.15234375	0.00000000
0.500	0.18750000	0.18750000	0.18750000	0.00000000
0.625	0.21484375	0.21484375	0.21484375	0.00000000
0.750	0.23437500	0.23437500	0.23437500	0.00000000
0.875	0.24609375	0.24609375	0.24609375	0.00000000
1	0.25000000	0.25000000	0.25000000	0.00000000

Example (2): Consider the same functional extremal of eq.(13) but with unspecified x(1),namely the boundary conditions [7] [10]:

$$x(0) = 0, x(1) = \text{unspecified} \tag{19}$$

The exact solution via Euler's equation is

$$\dot{x}(t) = -\frac{t}{2} \quad \text{and} \quad x(t) = -\frac{t^2}{4}$$

substituting eqs.(8-9) into eq.(13) with consider eq.(16) we get:

$$J(x) = c^T Dc + c^T Dd \tag{20}$$

The boundary condition eq.(19) is resulted [11]

$$F_x|_{t=1} = 0,\ \dot{x}(1) = -\frac{1}{2} \tag{21}$$

using eq.(8) in eq.(21) gives

$$c^T H(1) = -\frac{1}{2} \tag{22}$$

Now, we minimize eq.(20) subject to eq.(22) using Lagrange multiplier. Suppose

$$\tilde{J}(x) = J(x) + \lambda(c^T H(1) + \frac{1}{2})$$

where λ is the Lagrange multiplier. Using eq.(12). We solve

$$\frac{\partial \tilde{J}}{\partial c} = 0, \quad \frac{\partial \tilde{J}}{\partial \lambda} = 0$$

By choosing N=2, we get the same D and d previously, so we get

$$c = \left[0, -\frac{1}{4}, 0\right]^T$$

$$x(t) = [-0.125, 0, -0.0625]^T H(t)$$

then $J(x) = -0.833333$, Table (2) shows the approximate values of $x(t)$ using Hermite polynomials approach for $N = 2$ and 3.

Table (2): Estimated values for N=2 and 3 and exact values of $x(t)$

T	N=2	N=3	Analytical Solution	Absolute error
	$X(t)$	$X(t)$		$\lvert x(t)_{exact} - X(t)_{app}\rvert$
0	0.00000000	0.00000000	0.00000000	0.00000000
0.125	-0.00390652	-0.00390652	-0.00390652	0.00000000
0.250	-0.01625000	-0.01625000	-0.01625000	0.00000000
0.375	-0.03515625	-0.03515625	-0.03515625	0.00000000
0.500	-0.06250000	-0.06250000	-0.06250000	0.00000000
0.625	-0.09765625	-0.09765625	-0.09765625	0.00000000
0.750	-0.14062500	-0.14062500	-0.14062500	0.00000000
0.875	-0.19140625	-0.19140625	-0.19140625	0.00000000
1	-0.25000000	-0.25000000	-0.25000000	0.00000000

Example (3): Let us consider the problem of searching of exteremizing the functional [1] [2] [7]:

$$J(x)=\int_0^1\left[\dot{x}^2(t)+t\dot{x}(t)+x^2(t)\right]dt, \qquad (23)$$

With the following boundary conditions

$$x(0)=0\,,\; x(1)=\frac{1}{4} \qquad (24)$$

The exact solution of this problem is given by:

$$x(t)=\frac{-e^{-t}[(-1+e^{t})(e-2e^{2}-2e^{t}+e^{1+t})]}{4(-1+e^{2})}$$

Applying similar approach used in the previous problems led to

$$J(x)=c^{T}Dc+c^{T}Dd+c^{T}PDP^{T}c$$

$$\check{J}(x)=J(x)+\lambda(c^{T}PH(1)-\frac{1}{4})$$

By choosing N=3 we obtained

$$D=\begin{bmatrix} 1 & 1 & -\frac{2}{3} & -4 \\ 1 & \frac{4}{3} & 0 & -\frac{24}{5} \\ -\frac{2}{4} & 0 & \frac{28}{15} & \frac{4}{3} \\ -4 & -\frac{24}{5} & \frac{4}{3} & \frac{656}{35} \end{bmatrix}$$

And $d=\left[0,\frac{1}{2},0,0\right]^{T}H(t)$

therefore,

$$c=[0.546828,-0.290529,0.051598,-0.007025]^{T}$$

$$x(t)=[-0.057109,0.074033,-0.182898,0.011392]^{T}H(t)$$

then $J(x)=0.19759399$, Table (3) shows the approximate values of x(t) using Hermite polynomials approach for $N=3,4$ and 5.

Table (3): Estimated values for N =3, 4, and 5 and exact values of $x(t)$

t	N=3	N=4	N=5	Analytical Solution	Absolute error at N=4
	$X(t)$	$X(t)$	$X(t)$		$\lvert x(t)_{exact} - X(t)_{app} \rvert$
0	0.000000	0.000000	0.000000	0.000000	0.000000
0.1	0.041947	0.041951	0.041951	0.041951	0.000000
0.2	0.079319	0.079317	0.079317	0.079317	0.000000
0.3	0.112479	0.112473	0.112473	0.112473	0.000000
0.4	0.141755	0.141751	0.141751	0.141751	0.000000
0.5	0.1674423	0.167443	0.167443	0.167443	0.000000
0.6	0.189801	0.189807	0.189807	0.189807	0.000000
0.7	0.209060	0.209066	0.209066	0.209066	0.000000
0.8	0.225412	0.225413	0.225413	0.225413	0.000000
0.9	0.239017	0.239013	0.239013	0.239013	0.000000
1	0.250000	0.250000	0.250000	0.250000	0.000000

Example (4): Consider the following functional extremal problem when the functional is second order with two fixed and two free boundary conditions [7] [10]:

$$J(x) = \int_0^1 \left[\frac{1}{2} \ddot{x}^2(t) + 4(1-t)\, \dot{x}(t) \right] dt \qquad (25)$$

With

$x(0) = 0, \qquad \dot{x}(0) = 0$

$x(1) = \text{free}, \qquad \dot{x}(1) = \text{free}$

The exact solution via Euler's equation is

$\ddot{x}(t) = -2t^2 + 4t - 2$,

$\dot{x}(t) = -\frac{2}{3}t^3 + 2t^2 - 2t$,

$x(t) = -\frac{1}{6}t^4 + \frac{2}{3}t^3 - t^2.$

The natural boundary conditions are found from [11]

$$F_{\dot{x}} - \frac{d}{dt}(F_{\ddot{x}})\Big|_{t=1} = 0$$

$$\Longrightarrow 4(1-t) - \dddot{x}|_{t=1} = 0 \Longrightarrow \dddot{x}(1) = 0$$

$$F_{\ddot{x}}|_{t=1} = 0 \Longrightarrow \ddot{x}(1) = 0 \qquad (26)$$

Now we expand $\dddot{x}(t)$ into Hermit polynomials, we get

$$\dddot{x}(t) = \sum_{i=0}^{N} c_i H_i(t) = c^T H(t) \qquad (27)$$

Integrating eq.(27) three times gives

$$\ddot{x}(t) = c^T P H(t) + \ddot{x}(0), \qquad (28)$$
$$\dot{x}(t) = c^T P^2 H(t) + \ddot{x}(0)t, \qquad (29)$$
$$x(t) = c^T P^3 H(t) + \ddot{x}(0) d^T H(t). \qquad (30)$$

From eq.(26) and eq.(28) we get

$$\ddot{x}(0) = -c^T P H(1) \qquad (31)$$

Expressing $4 - 4t = f^T H(t)$, and substituting eqs. (28-31), in eq.(24) and using eq.(16) we get

$$J(x) = \frac{1}{2} c^T P D P^T c - 2c^T P^2 H(1) H^T(1) P^T + c^T P H(1) H^T(1) P^T c + f^T (P^2)^T c - c^T P H(1) d^T D f \qquad (32)$$

and solve eq.(32) using eq.(12) for N=2, we get:

$$D = \begin{bmatrix} 1 & 1 & \frac{-2}{3} \\ 1 & \frac{4}{3} & 0 \\ \frac{-2}{3} & 0 & \frac{28}{15} \end{bmatrix}$$

$$d = \left[0, \frac{1}{2}, 0\right]^T H(t)$$

Therefore,

$$c = [\,4\,, -2\,,\ 0]^T$$

$$x(t) = [-1\,, 2\,, -0.5]^T H(t)$$

then $J(x) = -0.4$, Table (4) shows the approximate values of $x(t)$ using Hermite Polynomials approach for $N = 2$ and 3.

Table (4): Estimated values for N=2 and 3 and exact values of $x(t)$

T	N=2	N=3	Analytical Solution	Absolute error
	$X(t)$	$X(t)$		$\lvert x(t)_{exact} - X(t)_{app} \rvert$
0	0.00000000	0.00000000	0.00000000	0.00000000
0.125	-0.01436068	-0.01436068	-0.01436068	0.00000000
0.250	-0.05273438	-0.05273438	-0.05273438	0.00000000
0.375	-0.10876465	-0.10876465	-0.10876465	0.00000000
0.500	-0.17708333	-0.17708333	-0.17708333	0.00000000
0.625	-0.25329589	-0.25329589	-0.25329589	0.00000000
0.750	-0.33398438	-0.33398438	-0.33398438	0.00000000
0.875	-0.41670736	-0.41670736	-0.41670736	0.00000000
1	-0.50000000	-0.50000000	-0.50000000	0.00000000

5. Conclusion

The numerical solutions of variational problem were introduced using Hermite polynomials. Some important formulas concerning the integration of Hermite polynomials as well as the operational matrix of integration had been derived which were essential to our numerical computations. Examples were solved and good results were achieved.

The uniform approximation capabilities of Hermite polynomials coupled with the fact that only a small number of polynomials (three - five to be precise) are needed to obtain satisfactory results that makes our method attractive comparing to other methods.

Tables (5,6,7and 8) shows the superiority of our method over the other existing methods for examples(1,2,3,and 4), the methods recently proposed by Hsiao[10], Razzaghi [1], Singh [7],and Ordokhani[2].

Table (5): Comparison between Hermite solution with Haar and Bernstein solutions for example (1)

t	Haar solution[10] (N=100)	Bernstein solution[7] (N=6)	Hermite solution (N=2)	Analytical solution
	X(t)	X(t)	X(t)	
0	0.0000	0.000000	0.00000000	0.00000000
0.125	0.0586	0.058594	0.05859375	0.05859375
0.375	0.1523	0.152344	0.15234375	0.15234375
0.625	0.2148	0.214844	0.21484375	0.21484375
0.875	0.2461	0.246094	0.24609375	0.24609375
1	0.2500	0.250000	0.25000000	0.25000000
$\mathbf{Max}(\lvert \mathbf{x(t)_{exact}} - \mathbf{X(t)_{app}} \rvert)$	3.475×10^{-4}	3.475×10^{-4}	0.00000000	

Table (6): Comparison between Hermite solution with Haar and Bernstein solutions for example (2)

t	Haar solution[1] (N=100)	Bernstein solution[7] (N=6)	Hermite solution (N=2)	Analytical solution
	X(t)	X(t)	X(t)	
0	0.0000	0.000000	0.00000000	0.00000000
0.125	-0.0039	-0.003906	-0.00390652	-0.00390652
0.375	-0.0352	-0.035156	-0.03515625	-0.03515625
0.625	-0.0977	-0.097656	-0.09765625	-0.09765625
0.875	-0.1914	-0.191406	-0.19140625	-0.19140625
1	-0.2539	-0.250000	-0.25000000	-0.25000000
$\mathbf{Max}(\lvert \mathbf{x(t)_{exact}} - \mathbf{X(t)_{app}} \rvert)$	3.9×10^{-3}	5.1999×10^{-7}	0.00000000	

Table(7): Comparison between Hermite solution with RH functions and Walsh-hybrid solutions for example (3)

t	RH function[1] (N=8) X(t)	Walsh-hybrid solution[2] (N=32) X(t)	Hermite solution (N=4) X(t)	Analytical solution
0	0.0000	0.00000	0.000000	0.000000
0.2	0.0761	0.07933	0.079317	0.079317
0.4	0.1482	0.14171	0.141751	0.141751
0.6	0.1817	0.18984	0.189807	0.189807
0.8	0.2267	0.22545	0.225413	0.225413
1	0.2515	0.25002	0.250000	0.250000
$\mathbf{Max}(\lvert \mathbf{x(t)_{exact}} - \mathbf{X(t)_{app}} \rvert)$	8.107×10^{-2}	4.1×10^{-5}	0.000000	

Table(8): Comparison between Hermite solution with Haar and Bernstein solutions for example (4)

t	Haar solution[10] (N=100) X(t)	Bernstein solution[7] (N=6) X(t)	Hermite solution (N=2) X(t)	Analytical solution
0	0.0000	0.000000	0.00000000	0.00000000
0.250	-0.0518	-0.052735	-0.05273438	-0.05273438
0.500	-0.1758	-0.177083	-0.17708333	-0.17708333
0.750	-0.3330	-0.333985	-0.33398438	-0.33398438
1	-0.4999	-0.500000	-0.50000000	-0.50000000
$\mathbf{Max}(\lvert \mathbf{x(t)_{exact}} - \mathbf{X(t)_{app}} \rvert)$	1.283×10^{-2}	6.1999×10^{-7}	0.00000000	

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هالة رحمن جابر
halah.jaber@yahoo.com
الجامعة المستنصرية - كلية العلوم - قسم الرياضيات

المستخلص

تم اقتراح خوارزمية لحل مسألة التغاير ذات الشروط الحدودية المقيدة والغير معلومة باستخدام متعددة حدود هيرمت. أستخدمت خصائص متعددة حدود هيرمت ومصفوفة العمليات التكاملية لاختزال مسألة التغايرالى حل معادلات جبرية. الطريقة حققت نتائج تقريبية دقيقة باستخدام اقل عدد من متعددات الحدود المذكورة انفا مقارنة مع الطرق الاخرى. امثلة عددية طبقت على الطريقة المذكورة.

الكلمات الرئيسية: مسألة التغاير، متعددة حدود هيرمت، مصفوفة العمليات التكاملية.