Jornal of kerbala university , vol. 10 no.3 scientific . 2012

Finding the optimal solution for fractional linear programming problems with fuzzy numbers

Aws Nidhal Dheyab
Al-Muthanna university – college of sciences-department of mathematics and computer applications
e-mail : natal.owse@yahoo.com

Abstract:
The fuzzy set and theory has been applied in many fields such as operations research , control theory and management sciences etc ,an application of this theory in decision making problem is linear fractional programming problem with fuzzy numbers , in this study we present a new method for solving fuzzy number fractional linear programming problems by use of linear ranking function
Key-wards : fractional linear programming , fuzzy numbers , simplex method

المستخلص:
المجموعات و النظريات الضبابية طبقت في العديد من المجالات مثل بحوث العمليات ونظرية السيطرة وعلوم الإدارة الخ , إن تطبيق هذه النظرية في مسائل اتخاذ القرار هو البرمجة الكسرية الخطية بالأعداد الضبابية , في هذه الدراسة سنقدم طريقة جديدة لحل مسائل البرمجة الكسرية الخطية ذوات المعاملات الضبابية باستخدام دالة تخفيض الرتبة

1- Introduction
Fuzzy set theory introduced by Zadeh [13] is generalization of conventional set theory to represent vagueness or imprecision in everyday life in strict mathematical framework[3] . There are many kinds of formulations to the objective function of problems, may be linear programming , quadratic programming [11] , sum objective , multi-objective[9] and fractional linear programming[13], and all of this kinds it is possible , that some coefficient of the problem in objective function ,technical-coefficient or decision making variable be fuzzy numbers[. In this work , we focus on fractional linear programming problem with fuzzy numbers in the objective function . Nassirie and etc [8] proposed an method for solving linear programming problem with fuzzy numbers , Here we first explain the comparison of fuzzy number by introducing ranking function[1] and then the method of transformation of fractional linear programming into linear programming with fuzzy numbers .

2- Definitions and notations
We will review the necessary background of fuzzy set theory [13]

2.1- Fuzzy decision
Let X be a given set of all possible solutions to a decision problem . A fuzzy number A is a fuzzy set on X characteristic by its membership function

$\mu_A : X \to [0,1]$

Where {0 , 1 } is called the valuation set [14] .
If the valuation set is allowed to be real interval [0,1] then A is called the fuzzy set $\mu_A(X)$ is the degree of membership of X in A [10] . A fuzzy number A is convex normalized fuzzy set on real line R such that
1) It exists at least one $x_o \in R$ with $\mu(x_o) = 1$
2) $\mu_A(x)$ is piecewise continues function .
Among the various types of fuzzy number ,triangular and Trapezoidal fuzzy numbers are the most important[7],[10] , in this study we only consider Trapezoidal fuzzy numbers . A fuzzy numbers is Trapezoidal fuzzy number if the membership function of it be in the following form :

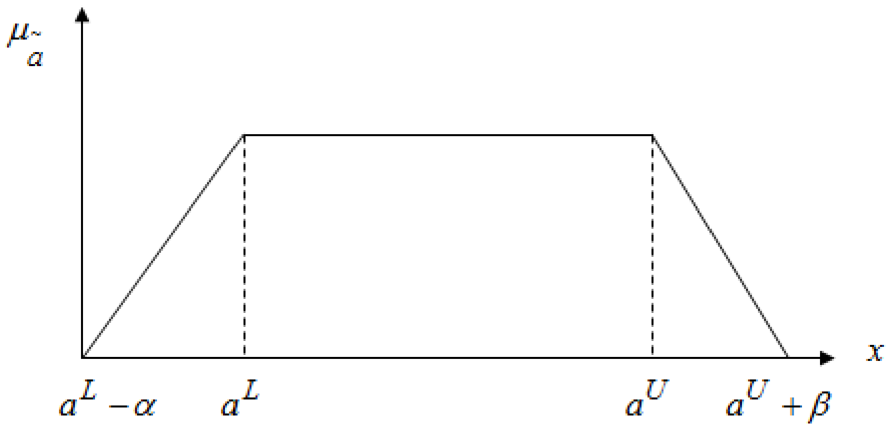

Figure 1 Trapezoidal fuzzy number

2.2- Arithmetic on fuzzy numbers

Let $a^{\sim}=(a^L,a^U,\alpha,\beta)$ and $b^{\sim}=(b^L,b^U,\xi,\pi)$ be two Trapezoidal fuzzy arithmetic on the Trapezoidal fuzzy numbers as shown below

$$image(a^{\sim})=-a^{\sim}=(-a^U,-a^L,\beta,\alpha)$$

$$addition: a^{\sim}+b^{\sim}=(a^L+b^L,a^U+b^U,\alpha+\xi,\beta+\pi)$$

Scalar multiplication

$$x>0, xa^{\sim}=(xa^L,xa^U,x\alpha,x\beta)$$

$$x<0, xa^{\sim}=(xa^U,xa^L x\beta,x\alpha)$$

[4],[6].

3- Ranking function

A convenient method for comparing of the fuzzy numbers is by use of the ranking functions[11] . A ranking function is map from F(R) into real line , where F(R) is fuzzy line, now we will define orders on F(R) as following :

$$a^{\sim}\underset{\Re}{\geq}b^{\sim} \text{ iff } \Re(a^{\sim})\geq\Re(b^{\sim}) \qquad (1)$$

$$a^{\sim}\underset{\Re}{>}b^{\sim} \text{ iff } \Re(a^{\sim})>\Re(b^{\sim}) \qquad (2)$$

$$a^{\sim}\underset{\Re}{=}b^{\sim} \text{ iff } \Re(a^{\sim})=\Re(b^{\sim}) \qquad (3)$$

Where $a^{\sim}$ and $b^{\sim}$ are in F(R) , It is obvious that we may write $a^{\sim}\underset{\Re}{\leq}b^{\sim}$ iff $b^{\sim}\geq a^{\sim}$ [6], since there are many ranking functions for comparing fuzzy numbers , we only apply linear ranking function ,so, it is obvious that if we suppose that $\Re$ by any linear ranking function then

$$i) a^{\sim}\underset{\Re}{\geq}b^{\sim} \text{ iff } a^{\sim}-b^{\sim}\geq 0 \text{ iff } -b^{\sim}\underset{\Re}{\geq}-a^{\sim}$$

$$ii) \text{ if } a^{\sim}\underset{\Re}{\geq}b^{\sim} \text{ and } c^{\sim}\underset{\Re}{\geq}d^{\sim} \text{, then } a^{\sim}+c^{\sim}\underset{\Re}{\geq}b^{\sim}+d^{\sim}$$

The ranking function we use in our paper is :

$$R(a^{\sim}) = a^L + a^U + \frac{1}{2}(\beta - \alpha) \qquad (4)$$

Where $a^{\sim} = (a^L, a^U, \alpha, \beta) \in$ F(R) [10].

4- Fuzzy fractional linear programming

In this section we introduce fuzzy fractional linear programming (FFLP) so ,we first define a fractional programming problems

4.1- Fractional linear programming

A fractional linear programming (FLP) is define as :

$$Max \quad z = \frac{cx + p}{dx + q}$$

$$s.to \qquad (5)$$

$$AX \le B$$

$$X \ge 0$$

Where $c=(c_1,c_2\ldots,c_n)$, $d=(d_1,d_2,\ldots,d_n)$, $B=(b_1.b_2,\ldots,b_m)^T$, $X \in R^n, x \in X$, p and q are scalar and $A=[a_{ij}]_{nxm}$. now if some parameter in (5) is fuzzy number then we obtain a fuzzy fractional linear programming problems (FFLP)

4.2- Fuzzy linear fractional programming(FLFP)

Suppose that in (FLP) some parameters be fuzzy ,hence it is possible that some coefficients the right-hand sides or decision making variable be fuzzy number in objective function[5] .

5- Fractional linear programming problems with fuzzy numbers

A fuzzy number fractional programming linear programming (FNFLP) problems is defined as following :

$$Max \quad \tilde{z} \underset{\Re}{=} \frac{\tilde{c}\,x + \tilde{p}}{\tilde{d}\,x + \tilde{q}}$$

$$s.to \qquad (6)$$

$$AX \le B$$

$$X \ge 0$$

Where $b \in R^m$ $, X \in R^n,\ x \in X , A \in R^{n\times m},\ c^{\sim}, d^{\sim}, p^{\sim}, q^{\sim} \in (F(R))$ and $\Re$ is the linear ranking function [8],[12].

Definition 5.1[10] - a vector $X \in R^n$ is called feasible solution to (6) iff X is satisfies the constraints of problem.

Definition 5.2[10] -A feasible solution x^* is an optimal solution for (6) if for all feasible solution x for (6) we have : $((c^{\sim}x^* + p^{\sim})/(d^{\sim}x^* + q^{\sim})) \ge ((c^{\sim}x + p^{\sim})/(d^{\sim}x + q^{\sim}))$

6-Complementry development method to solve fuzzy number fractional linear programming (CDFNFLP)

It is one of methods which used to fractional linear programming problem that we will use it in this paper to convert fuzzy number linear programming problem which give us the optimal solution by using the Simplex method with fuzzy numbers.

6.1-Algorthem of complementary development method

Step(1) : dividing objective function into two linear functions , the first function is represent the numerator function and the second is denominator function and the value of objective function to be maximum at possible it is must be the numerator function to be maximum ($\max z_1^{\sim}(x)$) and the denominator function be minimum ($\min z_2^{\sim}(x)$)

Step(2) : reclamation a function $\max z^{\sim *}(x)$ from subtracting the denominator function from nominator function and this function is putting in mathematical module made up of original restriction of problem in addition to nonnegative conditions and to solve this linear system we going to step(3)

Step(3) enervation the mathematical module to the standard form by adding slack variable (si) then solve the system by using simplex method and when we getting a solution for x_j we recompense in original objective function and then we reclamation the value of $\max z^{\sim *}(x)$

Step(4) :we compare the value resulted from $\max z^{\sim *}(x)$ with the value of $\max z^{\sim *}(x)$ in the last table . If the new value is greater than last value we keep going on solution else we stop and the value of x_j in the last table is optimal solution .

7- Numerical example

For an illustrate of the method above we solve (FNFLP) by using of (CDFNFLP)

$$Max \quad z^{\sim} \underset{\Re}{=} \frac{(4,6,8,10)x_1 + (4,7,8,13)x_2}{(0,1,3,4)x_1 + (1,2,-3,-2)x_2}$$

$$s.to:$$

$$3x_1 + 2x_2 \le 12$$

$$4x_1 + 5x_2 \le 27$$

$$x_1, x_2 \ge 0$$

Where (4,6,8,10) , (4,7,8,13) , (0,1,3,4) and (1,2,-3,-2) are fuzzy numbers .

We separate the objective function into sub-function with the same restricted

$$Max \quad z_1^{\sim} \underset{\Re}{=} (4,6,8,10)x_1 + (4,7,8,13)x_2$$

$$s.to:$$

$$3x_1 + 2x_2 \le 12$$

$$4x_1 + 5x_2 \le 27$$

$$x_1, x_2 \ge 0$$

and

$$Min \quad z_2^{\sim} \underset{\Re}{=} (0,1,3,4)x_1 + (1,2,-3,-2)x_2$$

$$s.to:$$

$$3x_1 + 2x_2 \le 12$$

$$4x_1 + 5x_2 \le 27$$

$$x_1, x_2 \ge 0$$

Now we construct $Maxz^{\sim *}$ from step (2) in (CDFNFLP)

$$Max \quad z^{\sim *} = (3,6,4,7)x_1 + (2,6,6,7)x_2$$
$$s.to:$$
$$3x_1 + 2x_2 \le 12$$
$$4x_1 + 5x_2 \le 27$$
$$x_1, x_2 \ge 0$$

We will transform the form above in standard form by adding slack variable to restriction

$$Max \quad z^{\sim *} = (3,6,4,7)x_1 + (2,6,6,7)x_2$$
$$s.to:$$
$$3x_1 + 2x_2 + x_3 = 12$$
$$4x_1 + 5x_2 + x_4 = 27$$
$$x_1, x_2 \ge 0$$

Now we building the first table of simplex method

Basis	x_1	x_2	x_3	x_4	R.H.S
$Z^{\sim}$	(-6,-3,7,4)	(-6,-2,10,6)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
x_3	3	2	1	0	12
x_4	4	5	0	1	27

$$(\Re(-6,-3,7,4), \Re(-6,-2,10,6)) = (-10.5,-10)$$

Since then x_1 will entry and x_3 we get out

Basis	x_1	x_2	x_3	x_4	R.H.S
$Z^{\sim}$	$\tilde{0}$	$(-5,0,\frac{34}{3},\frac{25}{3})$	$(1,2,\frac{4}{3},\frac{7}{3})$	$\tilde{0}$	(12,24,28,16)
x_1	1	$\frac{2}{5}$	$\frac{1}{3}$	0	4
x_4	0	$\frac{7}{3}$	$-\frac{4}{3}$	1	11

$$(\Re(\tilde{0}), \Re(-5,0,\frac{34}{3},\frac{25}{3})) = (0,-9)$$

Since the x_2 we entry and x_4 will leaving we have the following table

Basis	x_1	x_2	x_3	x_4	R.H.S
$Z^{\sim}$	$\tilde{0}$	$\tilde{0}$	$(6,2,\frac{164}{21},\frac{149}{21})$	$(0,\frac{15}{7},\frac{25}{7},\frac{34}{7})$	$(12,\frac{333}{7},\frac{1413}{21},\frac{1458}{21})$
x_1	1	0	$\frac{15}{21}$	$\frac{2}{7}$	$\frac{6}{7}$
x_2	0	1	$-\frac{4}{7}$	$\frac{7}{3}$	$\frac{33}{7}$

$$(\Re(6,2,\frac{164}{21},\frac{149}{21}),\Re(0,\frac{15}{7},\frac{25}{7},\frac{34}{7}))=(4.405,2.786)>0$$

and $(\Re(12,\frac{333}{7},\frac{1413}{21},\frac{1458}{21})=(60.6429)$

and now we use the optimality condition we find this base is the optimal solution for the problem

8- Conclusion

We considered the fractional linear programming problem with fuzzy numbers and introduce the basic feasible solution for this problem and we get an algorithm to solve this problems by using linear ranking function and simplex method

References

[1] Allahviranloo ,T , Lotfy, F.H, Kiasary ,M.Kh, Kiani, N.A. and Alizadeh, L " Solving fully fuzzy linear programming problem by ranking function" ,Applied Mathematical science, Vol. 2,No. 1, 2008 ,pp(19-32)

[2] Davar, S. and Ghasrodashti ,S, "Fuzzy linear multi-objective programming" , international conference at Puerto de La Cruz , Tanarif, November 30-December 2,2010

[3] Garcia, J.C.F , "linear programming with interval Type-2 Fuzzy right hand side parameter", IEEE,2008

[4] Gong, Z and Wang, L , " The numerical calculus of expectations of fuzzy random variables , Fuzzy sets and system , 2007 , pp(722-738).

[5] Gunes ,M and Umarsman ,N, " Fuzzy goal programming approach on computation of the fuzzy Arithmetic Mean" , mathematical and computational applications , vol. 10, No. 2,2005, pp(211-220)

[6] Kassem ,M.H, and EL-Kholy,N.A. " On fuzzy parametric programming" , international journal and Management systems , vol. 3, No. 1-2,2008,pp(101-125)

[7] Kumar,A and Gupta ,A,"Method for solving fully fuzzy assignment problems using Triangular fuzzy number", international journal of computer and information engineering , 2009, pp(231-234) .

[8] Liang ,T.F, , "Applying interactive fuzzy multi-objective linear programming to transportation planning decision" , journal of information and optimization sciences , Vol. 27 ,2006 , pp(107-126)

[9] Menez ,M,J Arenas , A.Bilbao ,M, and Rodriguez, M.V," Linear programming with fuzzy parameters : An interactive method resolution ", European journal of operation research, 2005

[10] Nasseri ,S.H, Adril ,E , Yazdani , Zaefarian ,A.,R, "Simplex method for solving linear programming with fuzzy variables " , World academy of science, Engineering and Technology ,2005, pp(284-288)

[11] Pramanik ,S and Dey, P.P , "Multi-objective quadratic programming : A priority based fuzzy goal programming " , international journal of computer Applications , vol. 26 ,No. 10, 2011, pp(30-35)

[12] Ravi ,V and Reddy ,P.J," Fuzzy linear fractional goal programming applied to refinery operations planning " , Fuzzy sets and system 96 , 1998, pp(173-182) .

[13] zadeh, L.A, "Fuzzy sets", information and control 8 , 1965,pp(338-353)

[14] Zemermann ,H.J, " Fuzzy programming and linear programming with several objective functions" , Science Direct, Fuzzy sets and systems, 1978, pp(45-55)