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A New Family Of Distributions: Exponential Power-X Family Of Distributions And Its Some Properties

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Abstract

In this study, it has been aimed to introduce a new family called as Exponential Power - X family of distributions by using the method suggested by Alzaatreh et al. (2013). Exponential Power Weibull (EP-W) distribution as a special sub model of this new family is discussed and its some statistical properties has been obtained. Moreover, the maximum likelihood estimators (MLEs) for unknown parameters of EP-W distribution have been derived and a simulation study based on MSEs and bias of this estimator for various sample sizes has been performed. Finally, an application with real data set has been presented.

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1. Introduction:

In recent years, many new statistical distributions are suggested to modelling lifetime and various dataset. These models are obtained using diverse methods. Some of these studies are listed as follows. Eugene et al. (2002) introduced a family of distributions generated by Beta distributions. Jones (2009), Cordeiro and Castro (2011) introduced the family of distributions generated by the Kumaraswamy distribution. The motivation of this study is method suggested by Alzaatreh et al. (2013). this method can be defined as followes .

Suppose $r(t)$, is probability density function of $T \in [a,b]$, $-\infty < a < b < \infty$ continuous random variable, G(x) is cumutative distribution function (cdf) of any X random variable and $W(G(x))$, is a function that provide the following properties.

a- $W(G(x)) \in [a,b]$

b- $W(G(x))$ is a derivable and monotone non-decreasing function.

c- While $x \to -\infty$, it is $W(G(x)) \to a$ and $x \to \infty$ while $W(G(x)) \to b$, it is

In this case the family of new distributions is defined as follows

$$F(x)=\int_a^{W(G(x))} r(t)dt = R\left(W\left(G(x)\right)\right) \tag{1}$$

where $R(t)$ denotes the cdf of random variable T. New distributions obtained using this method are called as $T-X$ distributions family. (Alzaatreh et al. 2013) has introduced Beta-Exponential-X and Weibull-X families of distributions. Then, many researchers have found new statistical distributions using this method. Some of these studies can be listed as follows; (Alzaghal et al. 2013) have suggested Exponentiated T-X Family of distribution, (Tahir et al. 2015) has introduced the odd generalized exponential family of distribution, The logistic-X family of distributions is proposed by (Tahir et al. 2016a) and A New Weibull family of distributions has been generated by (Tahir et al. 2016b). (Çelik and Guloksuz, 2017) have introduced a new lifetime distribution called "Uniform-Exponential Distribution" using $W\left(G(x)\right)=\dfrac{e^{-\theta F(x)}-1}{e^{-\theta}-1}$ in (1).

We introduce a new family of distributions called "Exponential Power-X family of distributions" using the method suggested by Alzaatreh et al. (2013). This study is organized as follows. In section 2, exponential power distribution is examined. In section 3, exponential power-X family of distributions are introduced. Also a special model of this new family of distributions named a EP-W distribution and its statistical properties are examined. Then, in estimation section, the unknown parameters of EP-W distribution are estimated by maximum likelihood method. In section 7, the MSE and biases of this estimator are computed by means of a Monte-Carlo simulation study. In section 8, a real data application is presented to show whether the real data set can be modelled by EP-W distribution. Finally, concluding remarks are given.

2. Exponential Power Distribution

EP distribution is introduced by (Smith and Bain, 1975). is used to modelling lifetime data. The cdf, pdf and hazard function of a X random variable having to this distribution with α and β parameters are given below.

$$F(x)=1-\exp\left(1-\exp\left(\frac{x}{\alpha}\right)^{\beta}\right) \tag{2}$$

$$f(x)=x^{\beta-1}\alpha^{-\beta}\beta\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\exp\left(1-\exp\left(\frac{x}{\alpha}\right)^{\beta}\right) \tag{3}$$

$$h(x)=x^{\beta-1}\alpha^{-\beta}\beta\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right) \tag{4}$$

, respectively.

3. Exponential Power-X Family of Distribution

In this section, we ıntroduce exponential power T-X family of distribution. This new family of distribution is obtained by using $W\left(G(x)\right)=-\log\left(1-G(x)\right)$ and $r(t)=t^{\beta-1}\alpha^{-\beta}\beta\exp\left(\left(\frac{t}{\alpha}\right)^{\beta}\right)\exp\left(1-\exp\left(\frac{t}{\alpha}\right)^{\beta}\right)$ in Eq(1) then . this new family can be written as follows.

$$F_X(x)=\int_0^{-\log(1-G(x))} t^{\beta-1}\alpha^{-\beta}\beta\exp\left(\left(\frac{t}{\alpha}\right)^{\beta}\right)\exp\left(1-\exp\left(\frac{t}{\alpha}\right)^{\beta}\right)dt \tag{5}$$

Where $r(t)$ is pdf of T random variable having to EP distribution. $G(x)$ denotes cdf of any distribution. .

4. Special Model: Exponential Power-Weibull Distribution

In this part, it is introduced a special model of EP-X family called Exponential Power-Weibull (EP-W) distribution. This model is obtained by taken $G(x)=1-\exp\left(-(\lambda x)^{\theta}\right)$ in Eq .(5). Where $G(x)$ is cdf of weibull distribution. The cdf and pdf of EP-W distribution are given by (6) and (7) respectively.

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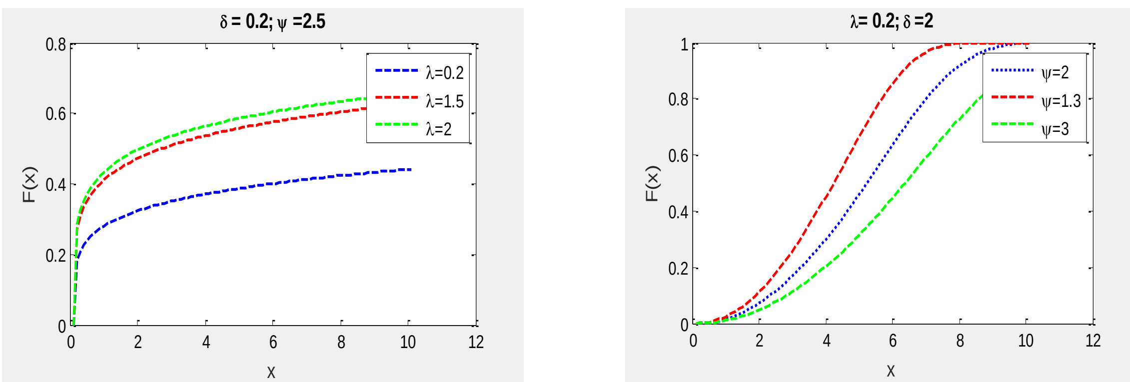
$$F(x;\lambda,\delta,\psi)=\int_{0}^{-\log(1-G(x))} t^{\beta-1}\alpha^{-\beta}\beta\exp\left(\left(\frac{t}{\alpha}\right)^{\beta}\right)\exp\left(1-\exp\left(\frac{t}{\alpha}\right)^{\beta}\right)dt$$

$$=1-\exp\left(1-\exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\right) \tag{6}$$

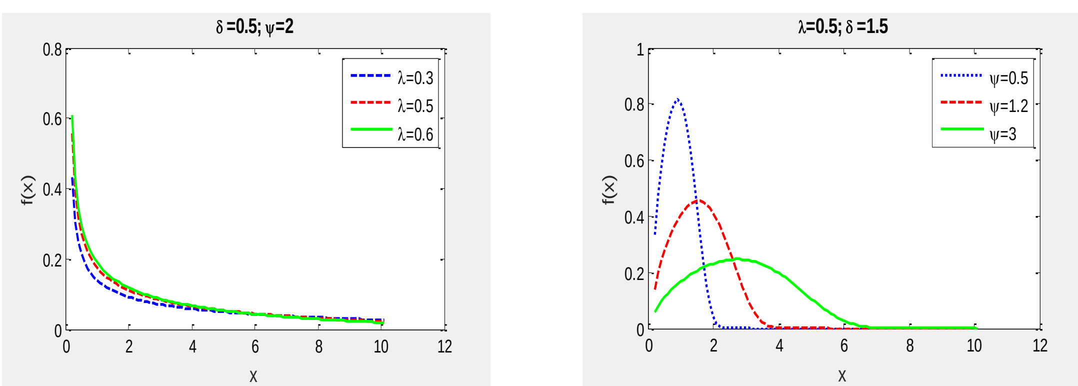
$$f(x;\lambda,\delta,\psi)=x^{\delta-1}\lambda^{\delta}\delta\psi^{-1}\exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\exp\left(1-\exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\right) \tag{7}$$

Here $\psi=\alpha^{\beta}>0$ and $\delta=\beta\theta>0$

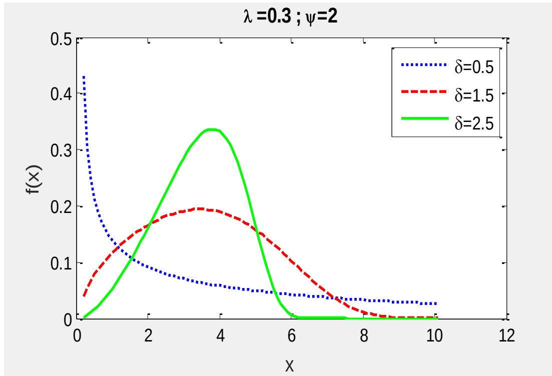


$\delta=0.2,\psi=2.5$ and $\lambda=0.2,1.5,2$ $\quad$ $\lambda=0.2,\delta=2$ and $\psi=2,1.3,3$

Figure 1. The cdf of the EP-W distribution



$\delta=0.5,\psi=2$ and $\lambda=0.3,0.5,0.6$ $\quad$ $\lambda=0.5,\delta=1.5$ and $\psi=0.5,1.2,3$



$\lambda = 0.3, \psi = 2$ and $\delta = 0.5, 1.5, 2.5$

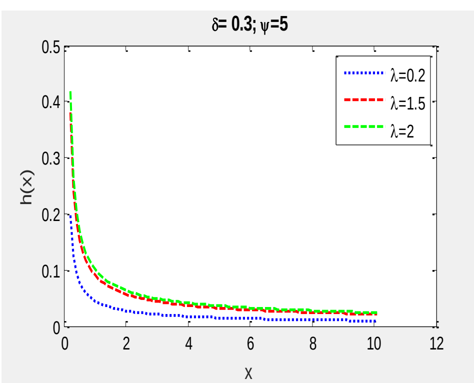
Figure 2. The pdf of the EP-W distribution

5. Some Statistical Properties for EP-W distribution

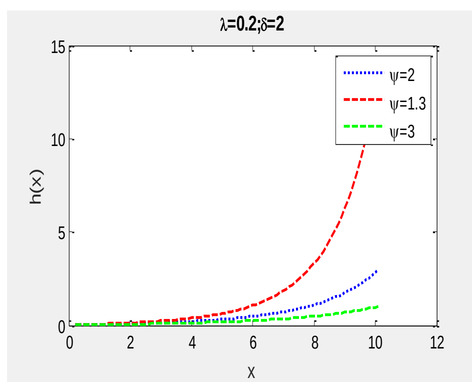
5.1. Hazard Function

The hazard function of EP-W distribution with λ, δ and ψ is given by

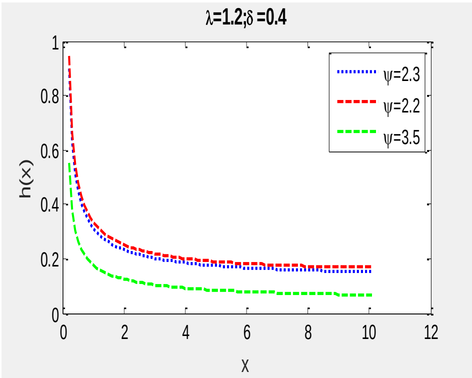
$$h(x;\lambda,\delta,\psi) = (x)^{\delta-1} \lambda^{\delta} \delta \psi^{-1} \exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right) \tag{8}$$



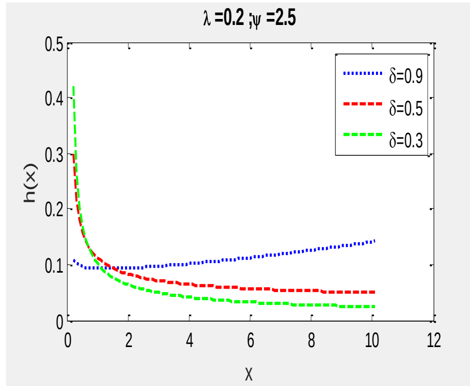
$\delta = 0.3, \psi = 5$ and $\lambda = 0.2, 1.5, 2$



$\lambda = 0.2, \delta = 2$ and $\psi = 2, 1.3, 3$



$\lambda = 1.2, \delta = 0.4$ and $\psi = 2.3, 2.2, 3.5$



$\lambda = 0.2, \psi = 2.5$ and $\delta = 0.9, 0.5, 0.3$

Figure 3. The hazard function of the EP-W distribution

In order to examine the behavior of the hazard function regarding the EP-W distribution, the derivative of the hazard function is needed. $h'(x)$ The derivative of x the hazard function denoted by is given below:

$$h'(x) = (\psi x)^{-2}\left(\left(\lambda x\right)^{\delta}\delta + \psi(\delta - 1)\right)(\lambda x)^{\delta}\,\delta \exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right) \tag{9}$$

for $\delta \geq 1$, $h'(x) > 0$ Since the hazard function is $\forall x > 0$ is said to be increased .

when taken $\delta < 1$ $x < \lambda^{-1}\left(-\psi\delta^{-1}(\delta - 1)\right)$ for $h'(x) < 0$ and $x < \lambda^{-1}\left(-\psi\delta^{-1}(\delta - 1)\right)$ for $h'(x) > 0$

The hazard regime for EP-W status is bathtub-shaped. Both cases are shown in Figure 3. seen from single graphs.

5.2. Random Numer Generator for EP-W Distribution

To generate random numers from EP-W distribution with λ, δ and ψ parameters it is used the method of inversion transformation as

$$F(x_u) = 1 - \exp\left(1 - \exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\right) = u, 0 < u < 1 \tag{10}$$

Solution of Eq. (9) is given by

$$x = \lambda^{-1}\left(\log\left(1 - \log(1 - u)\right)\psi\right)^{\left(\frac{1}{\delta}\right)} \tag{11}$$

where, u is a uniform random variable on the unit interval (0,1).

5.3. Moments of EP-W distribution

the r^{th} moment for EP-W distribution with λ, δ and ψ parameters is given by

$$m_r = E(X^r) = \int_0^{\infty} x^r f(x)\,dx \tag{12}$$

$$E(m^r) = \int_0^{\infty} x^{r+\delta-1}\lambda^{\delta}\delta\psi^{-1}\exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\exp\left(1 - \exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\right)dx$$

$$=\int_0^\infty \left(\frac{u\psi}{\lambda^\delta}\right)^{\frac{r}{\delta}} e^u e^{1-e^u}\, du$$

$$=\psi^{\frac{r}{\delta}}\lambda^{-r}\int_0^\infty u^{\frac{r}{\delta}} e^u e^{1-e^u}\, du$$

$$=\psi^{\frac{r}{\delta}}\lambda^{-r}\int_0^1 \left[\ln\left(1-\ln(s)\right)\right]^{\frac{r}{\delta}} ds \qquad (13)$$

from the equatıon (13) We have computed some statistical properties such as r^{th} moment coefficient of skewness and excess kurtosis for different values of parameters of EP-W distribution . this calculations are given in table 1 .

$$Skewness(SK)=\frac{E\left(X^3\right)-3E(X)E\left(X^2\right)+2\left(E(X)\right)^3}{\left(E\left(X^2\right)-\left(E(X)\right)^2\right)^{\frac{3}{2}}} \qquad (14)$$

$$Kurtosis(KU)=\frac{E\left(X^4\right)-4E(X)E\left(X^3\right)+6\left(E(X)\right)^2 E\left(X^2\right)-3\left(E(X)\right)^4}{\left[E\left(X^2\right)-\left(E(X)\right)^2\right]^2} \qquad (15)$$

Using equations (13), (14) and (15), the r^{th} moment, variance, skewness and kurtosis coefficients for different parameter values of the EP-W distribution are given in Table 1 Graphs regarding the skewness and kurtosis coefficients are presented in Figure 4.

Table 1.epresents an account r-moment ,varians skewness and kurtosis for different values of parameters.

(λ,ψ,δ)	$E(X)$	$E(X^2)$	$E(X^3)$	$E(X^4)$	$Var(X)$	SK	KU
(1.5,3,0.8)	1.4900	3.7800	12.3500	47.4000	1.5590	1.0610	3.8310
(1.5,3,1.5)	0.9220	1.0710	1.4180	2.0500	0.2210	0.2150	2.3660
(1.5,3,2)	0.8260	0.7950	0.8390	0.9460	0.1130	-0.0790	2.3560
(0.3,0.6,0.5)	0.6383	1.0400	2.5117	7.7343	0.6351	2.0670	8.4110
(0.3,0.6,1)	1.1927	2.1277	4.6455	11.5560	0.7052	0.7185	2.9830
(0.3,0.6,2)	1.8479	3.9757	9.3853	23.6414	0.5609	-0.0799	2.3560
(1,2,1.5)	1.0550	1.4040	2.1280	3.5210	0.2909	0.2152	2.3662
(1.5,2,1.5)	0.7036	0.6240	0.6304	0.6954	0.1289	0.2152	2.3663
(2.6,2,1.5)	0.4059	0.2077	0.1211	0.0770	0.0429	0.2152	2.3665
(0.5,0.5,0.6)	0.3385	0.2448	0.4469	0.3064	0.1302	1.6150	5.9573
(1.5,0.5,0.6)	0.1128	0.0272	0.0091	0.0038	0.0145	1.6152	5.9574
(3,0.5,0.6)	0.0564	0.0068	0.0011	0.0002	0.0036	1.6160	5.9580

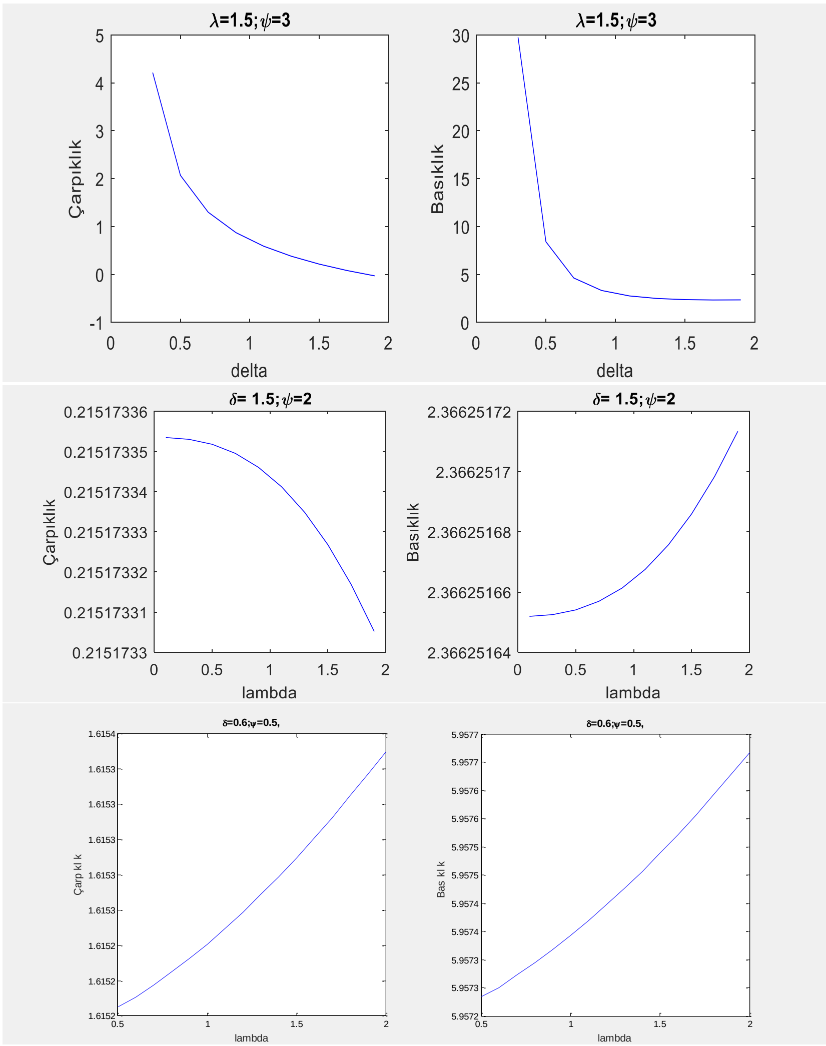


Figure 4.5. The plots of coefficient of Skewness and Kurtorsis for EP-Wdistribution .

Moment Generating Function for EP-W distribution

The moment generating function $M_x(t)$ of random variable X is given by

$$
\begin{aligned}
M_x(t) &= E\left(e^{tx}\right) \\
&= \int_0^\infty e^{tx} f(x)\,dx \\
&= \sum_{n=0}^{\infty} \frac{t^n}{n!} \int_0^\infty x^n f(x)\,dx \\
&= \sum_{n=0}^{\infty} \frac{t^n}{n!} E\left(x^n\right)
\end{aligned}
\tag{16}
$$

5.4. Order Statistics for EP-W distribution

Let $X_1, X_2 ..., X_n$ is a *iid* random sample from EP-W (λ, δ, ψ) distribution with cdf and pdf given by (6) and (7) respectively. Let $X_{(1:n)} \le X_{(2:n)} \ldots \le X_{(n:n)}$ denote the order statistics obtained from this sample. The pdf of the i^{th} order statistic for $i = 1, ..., n$ is simply as $f_{i:n}(x)$ and it is given by;

$$f_{i:n}(x) = \frac{1}{B(i, n-i+1)} f(x)\left[F(x, \underline{\mu})\right]^{i-1}\left[1-F(x, \underline{\mu})\right]^{n-i}$$

$$= \frac{1}{B(i, n-i+1)} x^{\delta-1}\delta\lambda^{\delta}\psi^{-1} \exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\exp\left(1-\exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\right)\left[F(x; \underline{\mu})\right]^{i-1}\left[1-F(x; \underline{\mu})\right]^{n-i} \tag{17}$$

$$= \frac{x^{\delta-1}\delta\lambda^{\delta}\psi^{-1} \exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\exp\left(1-\exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\right)}{B(i, n-i+1)}\left[1-e^{1-e^{\left(\frac{(\lambda x)^{\theta}}{\alpha}\right)^{\beta}}}\right]^{i-1}\left[e^{1-e^{\left(\frac{(\lambda x)^{\theta}}{\alpha}\right)^{\beta}}}\right]^{n-i}$$

where $\underline{\mu} = (k, \delta, \psi)$ and $F(x, \underline{\mu})$ and $f(x, \underline{\mu})$ are cdf and pdf of the EP –W distribution respectively and B(..) is the beta function. By using the binomial series expansion the order statistic function can expressed as follows;

$$f_{i:n}(x) = \sum_{k=0}^{i-1}(-1)^k \binom{i-1}{k}\frac{(x)^{\delta-1}\lambda^{\delta}\delta\psi^{-1}}{\beta(i, n-i+1)}\exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\left[\exp\left(1-\exp\left(\frac{(\lambda x)^{\delta}}{\psi}\right)\right)\right]^{n-i+k+1} \tag{18}$$

6. Parameter Estimation

We consider estimation of unknown parameters of exponential power weibull (EP-W) distribution are derived by using the maximum likelihood based on random sample.

6.1. Maximum Likelihood Estimation

Let $X_1, X_2, ..., X_n$ be be a random sample with size n from EP-W$(\underline{\mu}, x)$ distribution. where $\underline{\mu} = (k, \delta, \psi)$ is a vector parameters. The log-likelihood function is given by;

$$\begin{aligned}\ln L = l(\underline{\mu} \mid \underline{x}) &= n\,\delta\ln(\lambda) + (\delta-1)\sum_{i=1}^{n}\ln(x_i) + n\ln(\delta) - n\ln(\psi) \\ &+ \left(\sum_{i=1}^{n}\frac{(\lambda x_i)^{\delta}}{\psi}\right) + n + \left(\sum_{i=1}^{n}\left(-\exp\left(\frac{(\lambda x_i)^{\delta}}{\psi}\right)\right)\right)\end{aligned} \tag{19}$$

By diferentiating partially the log-likelihood function according to λ, δ and ψ parameters, and then equalizing them to zero we get.

$$\frac{\partial \ln L}{\partial \lambda} = \lambda^{-1}\psi^{-1}\delta\left(n\psi + \left(\sum_{i=1}^{n}(\lambda x_i)^{\delta}\right) - \left(\sum_{i=1}^{n}(\lambda x_i)^{\delta}\exp\left(\frac{(\lambda x_i)^{\delta}}{\psi}\right)\right)\right) \tag{20}$$

$$\frac{\partial \ln L}{\partial \delta}=n\ln(\lambda)+\left(\sum_{i=1}^{n}\ln(x_i)\right)+\frac{n}{\delta}+\left(\sum_{i=1}^{n}\frac{(\lambda x_i)^{\delta}\ln(\lambda x_i)}{\psi}\right)-\left(\sum_{i=1}^{n}\frac{(\lambda x_i)^{\delta}\ln(\lambda x_i)\exp\left(\frac{(\lambda x_i)^{\delta}}{\psi}\right)}{\psi}\right) \tag{21}$$

$$\frac{\partial \ln L}{\partial \psi}=-\frac{n}{\psi}+\left(\sum_{i=1}^{n}\left(-\frac{(\lambda x_i)^{\delta}}{\psi^{2}}\right)\right)+\left(\sum_{i=1}^{n}\frac{(\lambda x_i)^{\delta}\exp\left(\frac{(\lambda x_i)^{\delta}}{\psi}\right)}{\psi^{2}}\right) \tag{22}$$

MLE of λ,δ and ψ parameters are obtaind by simultaneous solutions of the equations (20)-(22) these nonlinear equations can be solved using Newton Raphson method .

7. **Simulation Study for** EP-W distribution

In this part, a Monte-Carlo simulation study based on 10000 replications for different sample sizes such as 25,50,100,200,500 and for different parameter values such as (1.6,0.9,0.7) ,(0.5,0.6.0.4) ,(0.7,0.6.1.2), (0.3,1.3,2) is performed to see The performances of MLE_S of unknown parameters of EP-W (λ,δ,ψ) distribution in terms of the bias and mean square error (MSE) . the simulation results are given in table 2.

Table 2. Bias and MSE for various values of $\hat{\lambda},\hat{\delta}$ and $\hat{\psi}$ parameters

Parameters			$\hat{\lambda}$		$\hat{\delta}$		$\hat{\psi}$	
	n		Yan	mse	yan	mse	yan	mse
(1.6,0.9,0.7)	25	MLE	0.0211	0.0118	0.0826	0.0685	-0.0205	0.0071
	50	MLE	0.0207	0.0057	0.0416	0.0294	-0.0041	0.0028
	100	MLE	0.0221	0.0029	0.0191	0.0128	0.0041	0.0012
	200	MLE	0.0204	0.0012	0.0090	0.0062	0.0076	0.0006
	500	MLE	0.0176	0.0008	0.0050	0.0024	0.0081	0.0003
(0.5,0.6.0.4)	25	MLE	0.0126	0.0018	0.0408	0.0148	-0.0213	0.0046
	50	MLE	0.0109	0.0010	0.0190	0.0063	-0.0088	0.0021
	100	MLE	0.0091	0.0005	0.0089	0.0027	-0.0022	0.0009
	200	MLE	0.0085	0.0003	0.0044	0.0013	0.0010	0.0005
	500	MLE	0.0083	0.0002	0.0020	0.0005	0.0023	0.0002
(0.7,0.6.1.2)	25	MLE	-0.0012	0.0079	0.0402	0.0149	-0.0257	0.0118
	50	MLE	0.0068	0.0040	0.0190	0.0061	-0.0083	0.0051
	100	MLE	0.0083	0.0020	0.0110	0.0028	0.0037	0.0022
	200	MLE	0.0105	0.0011	0.0051	0.0013	0.0072	0.0011
	500	MLE	0.0111	0.0006	0.0016	0.0005	0.0099	0.0005
(0.3, 1.3, 2)	25	MLE	-0.0011	0.0013	0.0883	0.0710	-0.0324	0.0243
	50	MLE	0.0013	0.0007	0.0401	0.0292	-0.0025	0.0091

	100	MLE	0.0024	0.0004	0..0205	0.0132	0.0120	0.0041
	200	MLE	0.0025	0.0002	0.0190	0.0065	0.0218	0.0025
	500	MLE	0.0029	0.0001	0.0046	0.0024	0.0241	0.0014

8. Real Data Analysis

In this section, A real data application is performed to examine the fit of the EP-W model in real life and to compare with other distributions. Real data set was used for these purposes. In order to compare the fits of the distributions, it has been considered the Akaike's Information Criterion (AIC), corrected Akaike's Information Criterion (AICc), the Bayesian Information Criterion (BIC) and -2×log-likelihood value by using these distributions . for three data sets. These measures are given by

$$AIC = -2\mathrm{l} + 2k \tag{23}$$

$$AICc = AIC + \left(\frac{2k(k+1)}{n-k-1}\right) \tag{24}$$

$$BIC = -2\mathrm{l} + k\log(n) \tag{25}$$

where k is number of parameters, n is sample size. l is the value of log–likelihood function.

The data set for total milk production rates for 107 beef living in the Camauba farm of Brazil Was used. for real data analysis Yousof et.get (2017), Cordeiro and Brito. (2012) Brito (2009). data set are given in Table3 ;

Tablo3. Real Data set

0,4365	0,6012	0,6789	0,5481	0,5627	0,4612	0,5349	0,6488
0,4260	0,1525	0,4576	0,1131	0,515	0,3188	0,3751	0,2747
0,5140	0,5483	0,3259	0,729	0,0776	0,216	0,1546	
0,6907	0,6927	0,2303	0,0168	0,3945	0,6707	0,4517	
0,7471	0,7261	0,7687	0,5529	0,4553	0,6220	0,2681	
0,2605	0,3323	0,4371	0,4530	0,4470	0,5629	0,4049	
0,6196	0,0671	0,3383	0,3891	0,5285	0,4675	0,5553	
0,8781	0,2361	0,6114	0,4752	0,5232	0,6844	0,5878	
0,4990	0,4800	0,348	0,3134	0,6465	0,3413	0,4741	
0,6058	0,5707	0,4564	0,3175	0,065	0,4332	0,3598	
0,6891	0,7131	0,7804	0,1167	0,8492	0,0854	0,7629	
0,5770	0,5853	0,3406	0,6750	0,8147	0,3821	0,5941	
0,5394	0,6768	0,4823	0,5113	0,3627	0,4694	0,6174	
0,1479	0,5350	0,5912	0,5447	0,3906	0,3635	0,6860	
0,2356	0,4151	0,5744	0,4143	0,4438	0,4111	0,0609	

The MLE(s) of the unknown parameters and standart errors for models are given in Table 4.

Tablo 4. Parameter estimaters (standart errors)

Distribution	MLE Estimaters
EP-Weibull	$\hat{\lambda}=0.0433(0.3781),\hat{\delta}=1.9788(0.1613),\hat{\psi}=0.0008(0.0147)$

Exponentiated exponential	$\hat{\theta} = 3.7139(0.5658), \hat{\alpha} = 4.2007(0.3728)$
Weibull	$\hat{\mu} = 2.6012(0.2098), \hat{\sigma} = 0.5236(0.0202)$
Exponential	$\hat{\theta} = 2.1329(0.3087)$

Tablo 5. Selection criteria statistics for data set1

Distribution	**-2LogL**	**AIC**	**AICc**	**BIC**	**K-S**	**p-values**
EP-W	-54,2257	-48,2257	-41,7597	-40,2072	0,066602	0,729563
Üstell Exponentiated exponential	-10,0775	-6,0775	-5,96212	-0,73184	0,147648	0,018834
Weibull	-42,695	-38,695	-38,5796	-33,3494	0,083244	0,448642
Exponential	51,90155	53,90155	53,93964	56,57438	0,319267	6,72E-10

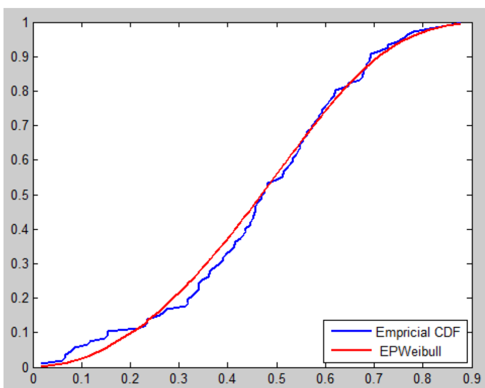


Figure 4. Empirical cdf and theoretical cdf.

Table6. Real Data set

0.0251	0.0886	0.0891	0.2501	0.3113	0.3451	0.4763	0.5650	0.5671	0.6566	0.6748	0.6751	0.6753
0.7696	0.8375	0.8391	0.8425	0.8645	0.8851	0.9113	0.9120	0.9836	1.0483	1.0596	1.0773	1.1733
1.2570	1.2766	1.2985	1.3211	1.3503	1.3551	1.4595	1.4880	1.5728	1.5733	1.7083	1.7263	1.7460

Tablo 7. Parameter estimaters (standart errors)

	MLE Estimaters
EP-Weibull	$\hat{\lambda} = 0.0273(0.4416)$, $\hat{\delta} = 1.5428(0.2168)$, $\hat{\psi} = 0.0063(0.1573)$
Exponentiated exponential	$\hat{\theta} = 2.1974(0.5054), \hat{\alpha} = 1.6428(0.2700)$

weibull	$\hat{\lambda}=1.9495(0.2649)$, $\hat{\theta}=1.9495(0.2649)$
Exponential	$\hat{\theta}=1.0616(0.4139)$
Transmuted exponentiated Exponential	$\hat{\theta}=1.8064(0.5263)$, $\hat{\alpha}=1.8684(0.2917)$, $\hat{\lambda}=-0.6665(0.2398)$

Tablo 8. Selection criteria statistics for data set1

Dağılım	-2LogL	AIC	AICc	BIC	K-S	p-değeri
EP-W	49.0556	53.0556	53.3889	56.3827	0.0882	0.9217
Exponentiated exponential	63.0761	67.0761	67.4094	70.4032	0.1705	0.2071
Weibull	54.9506	58.9506	59.2839	62.277	0.1174	0.6552
exponential	73.3353	75.3353	75.4434	76.9988	0.2716	0.0063
Transmuted exponentiated Exponential	59.5441	65.5441	66.2298	70.5348	0.1449	0.3863

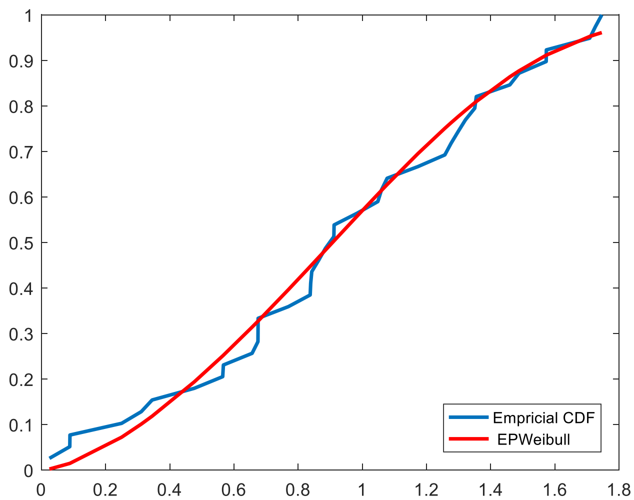


Figure 4. Empirical cdf and theoretical cd.

9. Concluding Remarks

In this study, we have introduced a new family of distributions called Exponential Power-X family of distributions using the method suggested by Alzaatreh et al. (2013). A special model called Exponential Power-Weibull (EP-W) is examined to illustrate the applicability of this new family of distributions. This new model can be used for skewed data and to model the data having increasing and decreasing hazard rate. Also, some statistical properties of EP-W model are obtained such as, moments, moment generating function, order statistics. Further, the maximum likelihood estimators (MLE) of unknown parameters are derived. An extensive Monte-Cario simulation study has been carried out to examine the performance of this estimator in terms of mean square error and bias .According to simulation study results, it has been observed that the estimators of model parameters provide estimation procedures. It has been compared the fits with EP-W model and other models via a real data application .According to real data analysis results, EP-W model is the best fitting model in real data analysis . These situation indicated the applicability of the EP-W model in real life.

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عائلة جديدة من التوزيعات: مجموعة التوزيعات القوة الأسية- X وبعض خصائصها

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الخلاصة :يهدف البحث إلى تقديم عائلة جديدة تسمى عائلة التوزيعات الأسية –X باستخدام الطريقة التي اقترحها Alzaatreh وآخرون. (2013). تمت مناقشة توزيع القوة الأسية ويبل (EP–W) كنموذج فرعي خاص لهذه العائلة الجديدة وتم الحصول على بعض خصائصه الإحصائية. علاوة على ذلك، تم استخلاص الإمكان الاعظم لمقدرات الاحتمالية (MLEs) للمعلمات غير المعروفة لتوزيع EP–W وتم إجراء دراسة محاكاة تعتمد على قيم المربعات الصغرى MSEs والتحيز لهذا المقدر لأحجام العينات المختلفة. وأخيراً، تم تقديم تطبيق يحتوي على مجموعة بيانات حقيقية.

الكلمات المفتاحية:توزيعات عائلة القوة الأسية -X، القوة الأسية توزيع ويبل (EP-W)، تقدير الاحتمال الإمكان الاعظم (MLE).