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on The characters table and rational valued characters table of the group ($Q_{2m}\times D_3$)when m is even number and m=2p, p is prime number

حول جدول الشواخص ذات القيم الاعتيادية وجدول الشواخص
ذات القيم النسبية للزمرة ($Q_{2m}\times D_3$ عندما m عدد زوجي و m=2p و p عدد اولي .

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Abstract

The main purpose is to determine valued characters table and rational valued characters table of the group($Q_{2m}\times D_3$)when m is even number and m=2p,p is prime number, and knowing relationship between of them to given minute result .

المستخلص

الهدف الرئيسي هو ايجاد جدول الشواخص ذات القيم الاعتيادية وجدول الشواخصذات القيم النسبية للزمرة ($Q_{2m}\times D_3$) عندما mعدد زوجي وm=2p, p عدد اولي ومعرفة العلاقة بينهما للتوصل الى ادق النتائج.

Introduction

Let G be a finite group ,two elements of G are said to be **Γ- conjugate** if the cyclic subgroups they generate are conjugate in G ; this defines an equivalence relation on G .Its classes are called **Γ- classes .**

The Z - valued class function on the group G , which is constant on the Γ- classes forms a finitely generated abelian group with operation point wise addition denoted by cf(G,Z) of a rank equal to the number of **Γ- classes**

In this search consists defined of Quaternion group and example, irreducible representations of the Quaternion group Q_{2m} when m is even number and m=2p, the character table of the Quaternion group Q_{2m} when m is even number and m=2p, the group D_3 and the character table of her, the defined group($Q_{2m}\times D_3$)when m is even number and m=2p ,example ,find the rational valued characters tables of the group ($Q_{2m}\times D_3$)when m is even number and m=2p,p is prime number.

The Generalized Quaternion Group Q_{2m}(1.1)[6]

For each positive integer m, the generalized quaternion group $\mathbf{Q_{2m}}$ of order 4m with two generators ***x*** and ***y*** such that

$\mathbf{Q_{2m}} = \{ x^k y^h ,0\leq k \leq 2m-1,h=0,1, x^m=y^2 , x^m=y^2 =I, y x^r y^{-1}= x^{-r} \}$

Irreducible Representations of the Quaternion Group Q_{2m} when m is an even number(1.2)[5]

There are four distinct irreducible representations R_1, R_2, R_3 and R_4 of degree 1, obtained by letting ±1 correspond to x and y in all possible ways. The representations R_1, R_2, R_3 and R_4 are given by the following table :

	χ^k	$\chi^k y$
R_1	1	1
R_2	1	-1
R_3	$(-1)^k$	$(-1)^k$
R_4	$(-1)^k$	$(-1)^{k+1}$

Table (1.1)

where $0 \le k \le 2m-1$,and there are $m-1$ distinct irreducible representations for Q_{2m}, of degree 2, we denote it by T_h **,**which T_h take the following form:

$$T_h(x) = \begin{bmatrix} \omega^h & 0 \\ 0 & \omega^{-h} \end{bmatrix},\ T_h(y) = \begin{bmatrix} 0 & \omega^{-hm} \\ 1 & 0 \end{bmatrix}$$

Now, for all elements of Q_{2m} the representations T_h is written as follows:

$$T_h(x^k) = \begin{bmatrix} \omega^{hk} & 0 \\ 0 & \omega^{-hk} \end{bmatrix},\ T_h(x^k y) = \begin{bmatrix} 0 & \omega^{h(k-m)} \\ \omega^{-hk} & 0 \end{bmatrix}$$

where $0 \le k \le 2m-1,\ 1 \le h \le m-1$ and $\omega = e^{2\pi i/2m}$.

The Character Table of the Quaternion Group Q_{2m} when m is an Even Number (1.3) [3]

There are two types of irreducible characters. One of them is the character of the linear representations R_1, R_2, R_3 and R_4 which are denoted by ψ_1, ψ_2, ψ_3 and ψ_4 respectively as in the following table:

	χ^k	$\chi^k y$
ψ_1	1	1
ψ_2	1	-1
ψ_3	$(-1)^k$	$(-1)^k$
ψ_4	$(-1)^k$	$(-1)^{k+1}$

Table (1.2)

where $0 \le k \le 2m-1$.

And the other characters of irreducible representations T_h of degree 2 are denoted by χ_h such that :

$$\chi_h(x^k) = \omega^{hk} + \omega^{-hk} = e^{\pi ihk/m} + e^{-\pi ihk/m} = 2\cos(\pi hk/m)$$

We are denoted to $(\omega^{hk}+\omega^{-hk})$ by V_{hk} ,Thus $V_{hk} = V_{2m-hk}$,$V_m = -2$,$V_{2m} = 2$, also we will write $V_{J(hk)}$ such that $J(hk) = \min\{hk \pmod{2m}, 2m-hk \pmod{2m}\}$ in the character table of the quaternion group Q_{2m} when m is an even number, such that:

$$V_{J(hk)} = 2\cos(\pi J(hk)/m)\ ,\ \chi_h(x^k y) = 0$$

where $0 \le k \le 2m-1,\ 1 \le h \le m-1$ and $\omega = e^{2\pi i/2m}$.

So, there are $m+3$ irreducible characters of Q_{2m} . Then the general form of the characters table of Q_{2m} when m is an even number is given in the followingtable:

$\equiv(Q_{4P})=$

CL_α	[I]	$[x^2]$	$[x^4]$	….	$[x^{2p-2}]$	$[x^{2p}]$	$[x]$	$[x^3]$	….	$[x^{2p-1}]$	$[y]$	$[xy]$
$\lvert CL_\alpha \rvert$	1	2	2	….	2	1	2	2	….	2	2p	2p
ψ_1	1	1	1	….	1	1	1	1	….	1	1	1
ψ_4	1	1	1	….	1	1	1-	1-	….	1-	-1	1
χ_2	2	$V_{J(4)}$	$V_{J(8)}$	….	$V_{J(4p-4)}$	2	V_2	$V_{J(6)}$	….	$V_{J(4p-2)}$	0	0
⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
χ_{2p-2}	2	$V_{J(4p-4)}$	$V_{J(8p-8)}$	….	$V_{J((2p-2)(2p-2))}$	2	$V_{J(2p-2)}$	$V_{J(6p-6)}$	….	$V_{J((2p-2)(2p-1))}$	0	0
χ_1	2	V_2	$V_{J(4)}$	….	$V_{J(2p-2)}$	-2	V_1	$V_{J(3)}$	….	$V_{J(2p-1)}$	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
χ_{2p-1}	2	$V_{J(4p-2)}$	$V_{J(8p-4)}$	….	$V_{J((2p-1)(2p-2))}$	-2	$V_{J(2p-1)}$	$V_{J(6p-3)}$	….	$V_{J((2p-1)(2p-1))}$	0	0
ψ_2	1	1	1	….	1	1	1	1	….	1	-1	-1
ψ_3	1	1	1	….	1	1-	1-	1-	….	1-	1	-1

Table (1.3)

The characters table of 2patrix fro2p degree *(2p+3)*, where $V_{J(hk)} = 2\cos(\pi J(hk)/2p)$

Theorem (1.4):[1]

Let T^1: G1→GL (n,F) and T^2: G2→GL(m,F) are two irreducible representations of the groups G1 and G2 with characters χ^1 and χ^2 respectively , then $T^1 \otimes T^2$ is irreducible representation of the group

G1× G2 with the character $\chi^1 \cdot \chi^2$.

The Group D_3(1.5):[4]

D_3 is the dihedral group of order 6 **D_3** = {I*, r, r^2,s,sr,sr^2}The characters table of **D_3** is given:

≡ **D_3** =

CL_α	[I]	$[r]$	$[S]$
$\lvert CL_\alpha \rvert$	1	2	3
χ'_1	1	1	1
χ'_2	1	1	-1
χ'_3	2	-1	0

Table (1.4)

The Group Q2m×D3(1.6)[2] :

The direct product group **Q_{2m}×D_3={(q,d): q∈ Q_{2m}, d∈ D_3}**, each irreducible character χ_i of **Q_{2m}** and χ_i' of **D_3**defines three characters $\chi_{(i,1)}$, $\chi_{(i,2)}$ and $\chi_{(i,3)}$ such that $\chi_{(i,1)} = \chi_i \ \chi'_1$, $\chi_{(i,2)} = \chi_i \ \chi'_2$ and $\chi_{(i,3)} = \chi_i \ \chi'_3$ of **Q_{2m}×D_3**.

Then **≡ (Q_{2m}× D_3) = ≡ (Q_{2m}) ⊗ ≡ (D_3)** .

Then, the general form of the characters table of **Q_{2m}× D_3**when m is an even number and m=2p is given in the following table:

≡($Q_{4P}\times D_3$) =

CL_{α}	[I,I*]	[I, r]	[I, s]	[x^2,I*]	[x^2,r]	[x^2,s]	….	[x^{2p-2},I*]	[x^{2p-2},r]	[x^{2p-2},s]	[x^{2p},I*]	[x^{2p},r]	[x^{2p},s]
$\lvert CL_{\alpha}\rvert$	1	2	3	2	4	6	….	2	4	6	1	2	3
$\lvert C_{Q_{2m}\times D_3}(CL_{\alpha})\rvert$	48p	24p	16p	24p	12p	8p	….	24p	12p	8p	48p	24p	16p
$\psi_{(1,1)}$	1	1	1	1	1	1	….	1	1	1	1	1	1
$\psi_{(1,2)}$	1	1	-1	1	1	-1	….	1	1	-1	1	1	-1
$\psi_{(1,3)}$	2	-1	0	2	-1	0	….	2	-1	0	2	-1	0
$\psi_{(4,1)}$	1	1	1	1	1	1	….	1	1	1	1	1	1
$\psi_{(4,2)}$	1	1	-1	1	1	-1	….	1	1	-1	1	1	-1
$\psi_{(4,3)}$	2	-1	0	2	-1	0	….	2	-1	0	2	-1	0
$\chi_{(2,1)}$	2	2	2	*VJ(4)*	*VJ(4)*	*VJ(4)*	….	*VJ(4p-4)*	*VJ(4p-4)*	*VJ(4p-4)*	2	2	2
$\chi_{(2,2)}$	2	2	-2	*VJ(4)*	*VJ(4)*	*- VJ(4)*	….	*VJ(4p-4)*	*VJ(4p-4)*	*-VJ(4p-4)*	2	2	-2
$\chi_{(2,3)}$	4	-2	0	*2(VJ(4))*	*- VJ(4)*	*0*	….	*2(VJ(4p-4))*	*-VJ(4p-4)*	0	4	-2	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\chi_{((2p-2),1)}$	2	2	2	*VJ(4p-4)*	*VJ(4p-4)*	*VJ(4p-4)*	….	*VJ((2p-2)(2p-2))*	*VJ((2p-2)(2p-2))*	*VJ((2p-2)(2p-2))*	2	2	2
$\chi_{((2p-2),2)}$	2	2	-2	*VJ(4p-4)*	*VJ(4p-4)*	*- VJ(4p-4)*	….	*VJ((2p-2)(2p-2))*	*VJ((2p-2)(2p-2))*	*-VJ((2p-2)(2p-2))*	2	2	-2
$\chi_{((2p-2),3)}$	4	2	0	2(*VJ(4p-4)*)	*- VJ(4p-4)*	*0*		2(*VJ((2p-2)(2p-2))*)	*-VJ((2p-2)(2p-2))*	*0*	4	2	0

Table (1.5)

$[x ,I^*]$	$[x ,r]$	$[x ,s]$	….	$[x^{2p-1},I^*]$	$[x^{2p-1},r]$	$[x^{2p-1},s]$	$[y ,I^*]$	$[y ,r]$	$[y ,s]$	$[xy ,I^*]$	$[xy ,r]$	$[xy ,s]$
2	4	6	….	2	4	6	*2p*	4p	6p	*2p*	4p	6p
24p	12p	8p	….	24p	12p	8p	24	12	8	24	12	8
1	1	1	….	1	1	1	1	1	1	1	1	1
1	1	-1	….	1	1	-1	1	1	-1	1	1	-1
2	-1	0	….	2	-1	0	2	-1	0	2	-1	0
-1	-1	-1	….	-1	-1	-1	-1	-1	-1	1	1	1
-1	-1	1	….	-1	-1	1	-1	-1	1	1	1	-1
-2	1	0	….	-2	1	0	-2	1	0	2	-1	0
V1	*V1*	*V1*	….	*VJ(2p-1)*	*VJ(2p-1)*	*VJ(2p-1)*	0	0	0	0	0	0
V1	*V1*	*- V1*	….	*VJ(2p-1)*	*VJ(2p-1)*	*VJ(2p-1)*	0	0	0	0	0	0
2(V1)	*- V1*	*0*	….	2(*VJ(2p-1)*	*VJ(2p-1)*	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$V_{J(2p-2)}$	$V_{J(2p-2)}$	$V_{J(2p-2)}$	….	*VJ((2p-2)(2p-1))*	*VJ((2p-2)(2p-1))*	*VJ((2p-2)(2p-1))*	0	0	0	0	0	0
$V_{J(2p-2)}$	$V_{J(2p-2)}$	$-V_{J(2p-2)}$	….	*VJ((2p-2)(2p-1))*	*VJ((2p-2)(2p-1))*	*- VJ((2p-2)(2p-1))*	0	0	0	0	0	0
$2(V_{J(2p-2)})$	$-V_{J(2p-2)}$	0	….	*2(VJ((2p-2)(2p-1)))*	*- VJ((2p-2)(2p-1))*	0	0	0	0	0	0	0

Table(1.5)

$\chi_{(1,1)}$	2	2	2	V_2	V_2	V_2	….	$V_{J(2p-2)}$	$V_{J(2p-2)}$	$V_{J(2p-2)}$	-2	-2	-2
$\chi_{(1,2)}$	2	2	-2	V_2	V_2	$-V_2$	….	$V_{J(2p-2)}$	$V_{J(2p-2)}$	$-V_{J(2p-2)}$	-2	-2	2
$\chi_{(1,3)}$	4	-2	0	$2(V_2)$	$-V_2$	0	….	$2(V_{J(2p-2)}$	$-V_{J(2p-2)}$	0	-4	2	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\chi_{((2p-1),1)}$	2	2	2	$V_{J(4p-2)}$	$V_{J(4p-2)}$	$V_{J(4p-2)}$	….	$V_{J((2p-1)(2p-2))}$	$V_{J((2p-1)(2p-2))}$	$V_{J((2p-1)(2p-2))}$	-2	-2	-2
$\chi_{((2p-1),2)}$	2	2	-2	$V_{J(4p-2)}$	$V_{J(4p-2)}$	$-V_{J(4p-2)}$	….	$V_{J((2p-1)(2p-2))}$	$V_{J((2p-1)(2p-2))}$	$-V_{J((2p-1)(2p-2))}$	-2	-2	2
$\chi_{((2p-1),3)}$	4	-2	0	$2(V_{J(4p-2))}$	$-V_{J(4p-2)}$	0	….	$2(V_{J((2p-1)(2p-2))}$	$-V_{J((2p-1)(2p-2))}$	0	-4	2	0
$\psi_{(2,1)}$	1	1	1	1	1	1	….	1	1	1	1	1	1
$\psi_{(2,2)}$	1	1	-1	1	1	-1	….	1	1	-1	1	1	-1
$\psi_{(2,3)}$	2	-1	0	2	-1	0	….	2	-1	0	2	-1	0
$\psi_{(3,1)}$	1	1	1	1	1	1	….	1	1	1	-1	-1	-1
$\psi_{(3,2)}$	1	1	-1	1	1	-1	….	1	1	-1	-1	-1	1
$\psi_{(3,3)}$	2	-1	0	2	-1	0	….	2	-1	0	-2	1	0

Table(1.5)

V_1	V_1	V_1	….	$V_{J\,(2p-1)}$	$V_{J\,(2p-1)}$	$V_{J\,(2p-1)}$	0	0	0	0	0	0
V_1	V_1	$-V_1$	….	$V_{J\,(2p-1)}$	$V_{J\,(2p-1)}$	$-V_{J\,(2p-1)}$	0	0	0	0	0	0
$2(V_1$	$-V_1$	0	….	$2(V_{J\,(2p-1)}$	$-V_{J\,(2p-1)}$	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$V_{J\,(2p-1)}$	$V_{J\,(2p-1)}$	$V_{J\,(2p-1)}$	….	$V_{J((2p-1)(2p-1))}$	$V_{J((2p-1)(2p-1))}$	$V_{J((2p-1)(2p-1))}$	0	0	0	0	0	0
$V_{J\,(2p-1)}$	$V_{J\,(2p-1)}$	$-V_{J\,(2p-1)}$	….	$V_{J((2p-1)(2p-1))}$	$V_{J((2p-1)(2p-1))}$	$-V_{J((2p-1)(2p-1))}$	0	0	0	0	0	0
$2(V_{J\,(2p-1)})$	$-V_{J\,(2p-1)}$	0	….	$2(V_{J((2p-1)(2p-1))}$	$-V_{J((2p-1)(2p-1))}$	0	0	0	0	0	0	0
1	1	1	….	1	1	1	-1	-1	-1	-1	-1	-1
1	1	-1	….	1	1	-1	-1	-1	1	-1	-1	1
2	-1	0	….	2	-1	0	-2	1	0	-2	1	0
-1	-1	-1	….	-1	-1	-1	1	1	1	-1	-1	-1
-1	-1	1	….	-1	-1	1	1	1	-1	-1	-1	1
-2	1	0	….	-2	1	0	2	-1	0	-2	1	0

Table(1.5)

Where $V_{J(hk)} = 2\cos(\pi J(hk)/2p)$, $V_{4p}=2$, $V_{2p}=-2w = e^{2\pi i/2m}, w^m = -1$

Example(1.7):

Q_{20}={1,x,x^2,x^3,x^4,x^5, x^6,x^7, x^8, x^9,x^{10},x^{11} , x^{12}, x^{13}, x^{14} , x^{15} , x^{16}, x^{17}, x^{18}, x^{19} , y, xy, x^2y, x^3y, x^4y, x^5y, x^6y, x^7y, x^8y ,x^9y , $x^{10}y$, $x^{11}y$, $x^{12}y$, $x^{13}y$, $x^{14}y$, $x^{15}y$, $x^{16}y$, $x^{17}y$, $x^{18}y$, $x^{19}y$}

There are therteen conjugate classes in Q_{20},

[1], [x], [x^2], [x^3], [x^4], [x^5], [x^6],[x^7],[x^8],[x^9],[x^{10}],[y] and [xy]

[1] = {1}, [x] = {x, x^{19}}, [x^2] = {x^2, x^{18}}, [x^3] = {x^3, x^{17}},[x^4] = {x^4, x^{16}},

[x^5] = {x^5, x^{15}}, [x^6] ={ [x^6],[x^{14}]} , [x^7]= {x^7, x^{13}},[x^8]= {x^8, x^{12}},

[x^9]= {x^9, x^{11}},[x^{10}]= { x^{10}},[y] = {y, x^2y, x^4y, x^6y, x^8y, $x^{10}y$, $x^{12}y$, $x^{14}y$, $x^{16}y$, $x^{18}y$ } and

[xy] = {xy, x^3y, x^5y, x^7y, x^9y, $x^{11}y$, $x^{13}y$, $x^{15}y$, $x^{17}y$, $x^{19}y$ }.

And it has therteen non-equivalent irreducible representations,

Then we can write the character table of Q_{20} as follows:

$$\equiv(Q_{20}) =$$

CL_α	[1]	[x^2]	[x^4]	[x^6]	[x^8]	[x^{10}]	[x]	[x^3]	[x^5]	[x^7]	[x^9]	[y]	[xy]
$\|CL_\alpha\|$	1	2	2	1	2	2	2	2	2	2	2	6	6
ψ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
ψ_4	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1
χ_2	2	V_4	V_8	V_8	V_4	2	V_2	V_6	-2	V_6	V_2	0	0
χ_4	2	V_4	V_4	V_4	V_8	2	V_4	V_8	2	V_8	V_4	0	0
X_6	2	V_8	V_4	V_2	V_8	2	V_6	V_2	-2	V_2	V_6	0	0
X_8	2	V_4	V_8	V_2	V_4	2	V_8	V_4	2	V_4	V_8	0	0
χ_1	2	V_2	V_4	V_6	V_8	-2	V_1	V_3	0	V_7	V_8	0	0
χ_3	2	V_6	V_8	V_2	V_4	-2	V_3	V_9	0	V_1	V_7	0	0
χ_5	2	-2	2	-2	2	-2	0	0	0	0	0	0	0
X_7	2	V_6	V_8	V_2	V_6	-2	V_7	V_1	0	V_9	V_3	0	0
X_9	2	V_2	V_4	V_6	V_8	-2	V_9	V_7	0	V_3	V_1	0	0
ψ_2	1	1	1	1	1	1	1	1	1	1	1	1-	1-
ψ_3	1	1	1	1	1	1	-1	-1	-1	-1	-1	1	-1

Table (1.6)

where $V_j = 2\cos(\pi j/10)$, $V_{2m}=2$, $V_m=-2$,$V_5=2\cos(5\pi/10)=0$

the characters table of $\mathbf{Q_{20} \times D_3}$ can be written as follows :

$\equiv(\mathbf{Q_{20} \times D_3}) = \equiv(\mathbf{Q_{20}}) \otimes \equiv(\mathbf{D_3})$ Then**:**

$\equiv(Q_{20}\times D_3)$ =

CL_α	[1,1]	[1,r]	[1,s]	$[x^2,I]$	$[x^2,r]$	$[x^2,s]$	$[x^4,I]$	$[x^4,r]$	$[x^4,s]$	$[x^6,I]$	$[x^6,r]$	$[x^6,s]$	$[x^8,I]$	$[x^8,r]$	$[x^8,s]$	$[x^{10},I$	$[x^{10},r]$	$[x^{10},s$	[x,I]
$\lvert CL_\alpha\rvert$	1	2	3	2	4	6	2	4	6	2	4	6	2	4	6	1	2	3	2
$\lvert C_{Q_{2m}}(CL_\alpha)\rvert$	240	120	80	120	60	40	120	60	40	120	60	40	120	60	40	240	120	80	120
Ψ_{11}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ψ_{12}	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
ψ_{13}	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2
Ψ_{41}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1
Ψ_{42}	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1
Ψ_{43}	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	-2
χ_{21}	2	2	2	V_4	V_4	V_4	V_8	V_8	V_8	V_8	V_8	V_8	V_4	V_4	V_4	2	2	2	V_2
χ_{22}	2	2	-2	V_4	V_4	$-V_4$	V_8	V_8	$-V_8$	V_8	V_8	$-V_8$	V_4	V_4	$-V_4$	2	2	-2	V_2
χ_{23}	4	-2	0	$2V_4$	$-V_4$	0	$2V_8$	$-V_8$	0	$2V_8$	$-V_8$	0	$2V_4$	$-V_4$	0	4	-2	0	$2V_2$
X_{41}	2	2	2	V_8	V_8	V_8	V_4	V_4	V_4	V_4	V_4	V_4	V_8	V_8	V_8	2	2	2	V_4
X_{42}	2	2	-2	V_8	V_8	$-V_8$	V_4	V_4	$-V_4$	V_4	V_4	$-V_4$	V_8	V_8	$-V_8$	2	2	-2	V_4
X_{43}	4	-2	0	$2V_8$	$-V_8$	0	$2V_4$	$-V_4$	0	$2V_4$	$-V_4$	0	$2V_8$	$-V_8$	0	4	-2	0	$2V_4$
X_{61}	2	2	2	V_8	V_8	V_8	V_4	V_4	V_4	V_4	V_4	V_4	V_8	V_8	V_8	2	2	2	V_6
X_{62}	2	2	-2	V_8	V_8	$-V_8$	V_4	V_4	$-V_4$	V_2	V_4	$-V_4$	V_8	V_8	$-V_8$	2	2	-2	V_6
X_{63}	4	-2	0	$2V_8$	$-V_8$	0	$2V_4$	$-V_4$	0	$2V_2$	$-V_4$	0	$2V_8$	$-V_8$	0	4	-2	0	$2V_6$
X_{81}	2	2	2	V_4	V_4	V_4	V_8	V_8	V_8	V_8	V_8	V_8	V_4	V_4	V_4	2	2	2	V_8
X_{82}	2	2	-2	V_4	V_4	$-V_4$	V_8	V_8	$-V_8$	V_2	V_8	$-V_8$	V_4	V_4	$-V_4$	2	2	-2	V_8
X_{83}	4	-2	0	$2V_4$	$-V_4$	0	$2V_8$	$-V_8$	0	$2V_2$	$-V_8$	0	$2V_4$	$-V_4$	0	4	-2	0	$2V_8$
X_{11}	2	2	2	V_2	V_2	V_2	V_4	V_4	V_4	V_6	V_6	V_6	V_8	V_8	V_8	-2	-2	-2	V_1

Table (1.7)

$\equiv(Q_{20}\times D_3) =$

[x,r]	*[x,s]*	$[x^3,I]$	$[x^3,r]$	$[x^3,s]$	$[x^5,I]$	$[x^5,r]$	$[x^5,s]$	$[x^7,I]$	$[x^7,r]$	$[x^7,s]$	$[x^9,I]$	$[x^9,r]$	$[x^9,s]$	*[y,I]*	*[y,r*	*[y,s]*	*[xy,I*	*[xy,r*	*[xy,s*
4	6	2	4	6	2	4	6	2	4	6	2	4	6	10	20	30	10	20	30
60	40	120	60	40	120	60	40	120	60	40	120	60	40	24	12	8	24	12	8
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1
-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1
-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	1	1	-1
1	0	-2	1	0	-2	1	0	-2	1	0	-2	1	0	-2	1	0	2	-1	0
V_2	V_2	V_6	V_6	V_6	-2	-2	-2	V_6	V_6	V_6	V_2	V_2	V_2	0	0	0	0	0	0
V_2	$-V_2$	V_6	V_6	$-V_6$	-2	-2	2	V_6	V_6	$-V_6$	V_2	V_2	$-V_2$	0	0	0	0	0	0
$-V_2$	0	$2V_6$	$-V_6$	0	-4	2	0	$2V_6$	$-V_6$	0	$2V_2$	$-V_2$	0	0	0	0	0	0	0
V_4	V_4	V_8	V_8	V_8	2	2	2	V_8	V_8	V_8	V_4	V_4	V_4	0	0	0	0	0	0
V_4	$-V_4$	V_8	V_8	$-V_8$	2	2	-2	V_8	V_8	$-V_8$	V_4	V_4	$-V_4$	0	0	0	0	0	0
$-V_4$	0	$2V_8$	$-V_8$	0	4	-2	0	$2V_8$	$-V_8$	0	$2V_4$	$-V_4$	0	0	0	0	0	0	0
V_6	V_6	V_2	V_2	V_2	-2	-2	-2	V_2	V_2	V_2	V_6	V_6	V_6	0	0	0	0	0	0
V_6	$-V_6$	V_2	V_2	$-V_2$	-2	-2	2	V_2	V_2	$-V_2$	V_6	V_6	$-V_6$	0	0	0	0	0	0
$-V_6$	0	$2V_2$	$-V_2$	0	-4	2	0	$2V_2$	$-V_2$	0	$2V_6$	$-V_6$	0	0	0	0	0	0	0
V_8	V_8	V_4	V_4	V_4	2	2	2	V_4	V_4	V_4	V_8	V_8	V_8	0	0	0	0	0	0
V_8	$-V_8$	V_4	V_4	$-V_4$	2	2	-2	V_4	V_4	$-V_4$	V_8	V_8	$-V_8$	0	0	0	0	0	0
$-V_8$	0	$2V_4$	$-V_4$	0	4	-2	0	$2V_4$	$-V_4$	0	$2V_8$	$-V_8$	0	0	0	0	0	0	0
V_1	V_1	V_3	V_3	V_3	0	0	0	V_7	V_7	V_7	V_9	V_9	V_9	0	0	0	0	0	0
V_1	$-V_1$	V_3	V_3	$-V_3$	0	0	0	V_7	V_7	$-V_7$	V_9	V_9	$-V_9$	0	0	0	0	0	0

Table (1.7)

$\equiv(Q_{20}\times D_3)$ =

X_{12}	2	2	-2	V_2	V_2	$-V_2$	V_4	V_4	$-V_4$	V_6	V_6	$-V_6$	V_8	V_8	$-V_8$	-2	-2	2	V_1
X_{13}	4	-2	0	$2V_2$	$-V_2$	0	$2V_4$	$-V_4$	0	$2V_6$	$-V_6$	0	$2V_8$	$-V_8$	0	-4	2	0	$2V_1$
X_{31}	2	2	2	V_6	V_6	V_6	V_8	V_8	V_8	V_2	V_2	V_2	V_4	V_4	V_4	-2	-2	-2	V_3
X_{32}	2	2	-2	V_6	V_6	$-V_6$	V_8	V_8	$-V_8$	V_2	V_2	$-V_2$	V_4	V_4	$-V_4$	-2	-2	2	V_3
X_{33}	4	-2	0	$2V_6$	$-V_6$	0	$2V_8$	$-V_8$	0	$2V_2$	$-V_2$	0	$2V_4$	$-V_4$	0	-4	2	0	$2V_3$
X_{51}	2	2	2	-2	-2	-2	2	2	2	-2	-2	-2	2	2	2	-2	-2	-2	0
X_{52}	2	2	-2	-2	-2	2	2	2	-2	-2	-2	2	2	2	-2	-2	-2	2	0
X_{53}	2	-2	0	-4	2	0	4	-2	0	-4	2	0	4	-2	0	-4	2	0	0
X_{71}	2	2	2	V_6	V_6	V_6	V_8	V_8	V_8	V_2	V_2	V_2	V_4	V_4	V_4	-2	-2	-2	V_7
X_{72}	2	2	-2	V_6	V_6	$-V_6$	V_8	V_8	$-V_8$	V_2	V_2	$-V_2$	V_4	V_4	$-V_4$	-2	-2	2	V_7
X_{73}	4	-2	0	$2V_6$	$-V_6$	0	$2V_8$	$-V_8$	0	$2V_2$	$-V_2$	0	$2V_4$	$-V_4$	0	-4	2	0	$2V_7$
X_{91}	2	2	2	V_2	V_2	V_2	V_4	V_4	V_4	V_6	V_6	V_6	V_8	V_8	V_8	-2	-2	-2	V_9
X_{92}	2	2	-2	V_2	V_2	$-V_2$	V_4	V_4	$-V_4$	V_6	V_6	$-V_6$	V_8	V_8	$-V_8$	-2	-2	2	V_9
X_{93}	4	-2	0	$2V_2$	$-V_2$	0	$2V_4$	$-V_4$	0	$2V_6$	$-V_6$	0	$2V_8$	$-V_8$	0	-4	2	0	$2V_9$
Ψ_{21}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Ψ_{22}	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
Ψ_{23}	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2
Ψ_{31}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1
Ψ_{32}	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1
Ψ_{33}	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	-2

Table (1.7)

$\equiv(Q_{20}\times D_3)$ =

$-V_1$	0	$2V_3$	$-V_3$	0	0	0	0	$2V_7$	$-V_7$	0	$2V_9$	$-V_9$	0	0	0	0	0	0	0
V_3	V_3	V_9	V_9	V_9	0	0	0	V_1	V_1	V_1	V_7	V_7	V_7	0	0	0	0	0	0
V_3	$-V_3$	V_9	V_9	$-V_9$	0	0	0	V_1	V_1	$-V_1$	V_7	V_7	$-V_7$	0	0	0	0	0	0
$-V_3$	0	$2V_9$	$-V_9$	0	0	0	0	$2V_1$	$-V_1$	0	$2V_7$	$-V_7$	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V_7	V_7	V_1	V_1	V_1	0	0	0	V_9	V_9	V_9	V_3	V_3	V_3	0	0	0	0	0	0
V_7	$-V_7$	V_1	V_1	$-V_1$	0	0	0	V_9	V_9	$-V_9$	V_3	V_3	$-V_3$	0	0	0	0	0	0
$-V_7$	0	$2V_1$	$-V_1$	0	0	0	0	$2V_9$	$-V_9$	0	$2V_3$	$-V_3$	0	0	0	0	0	0	0
V_9	V_9	V_7	V_7	V_7	0	0	0	V_3	V_3	V_3	V_1	V_1	V_1	0	0	0	0	0	0
V_9	$-V_9$	V_7	V_7	$-V_7$	0	0	0	V_3	V_3	$-V_3$	V_1	V_1	$-V_1$	0	0	0	0	0	0
$-V_9$	0	$2V_7$	$-V_7$	0	0	0	0	$2V_3$	$-V_3$	0	$2V_1$	$-V_1$	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1-	1-	1-	1-	1-	1-
1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1-	1-	1	1-	1-	1
-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	-2	1	0	-2	1	0
-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	1	1	1	-1	-1	1
-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	1	1	-1	-1	-1	1
1	0	-2	1	0	-2	1	0	-2	1	0	-2	1	0	2	-1	0	-2	1	0

Table (1.7)

To calculate the rational valued character table of $Q_{20}\times D_3$(1.8):

$\theta_{11}=\Psi_{11}$, $\theta_{12}=\Psi_{12}$, $\theta_{13}=\Psi_{13}$, $\theta_{21}=\Psi_{41}$, $\theta_{22}=\Psi_{42}$, $\theta_{23}=\Psi_{43}$,
$\theta_{71}=\Psi_{21}$, $\theta_{72}=\Psi_{22}$, $\theta_{73}=\Psi_{23}$, $\theta_{81}=\Psi_{31}$, $\theta_{82}=\Psi_{32}$, $\theta_{83}=\Psi_{33}$
$\theta_{51}=\chi_{51}$, $\theta_{52}=\chi_{52}$, $\theta_{53}=\chi_{53}$,

The elements of $Gal(\chi_{1i})/Q$, *are* :$\{\sigma_{1i}, \sigma_{2i}, \sigma_{3i}, \sigma_{4i}, \sigma_{6i}, \sigma_{7i}, \sigma_{8i}, \sigma_{9i}\}$
Where $\sigma_{1i}(\chi_{1i})=\chi_{1i}$, $\sigma_{2i}(\chi_{1i})=\chi_{2i}$, $\sigma_{3i}(\chi_{1i})=\chi_{3i}$, $\sigma_{4i}(\chi_{1i})=\chi_{4i}$, $\sigma_{6i}(\chi_{1i})=\chi_{6i}$, $\sigma_{7i}(\chi_{1i})=\chi_{7i}$, $\sigma_{8i}(\chi_{1i})=\chi_{8i}$, $\sigma_{9i}(\chi_{1i})=\chi_{9i}$, and $i=1,2,3$.
$\Theta_{6i}=\sigma_{1i}(\chi_{1i})+\sigma_{3i}(\chi_{1i})+\sigma_{7i}(\chi_{1i})+\sigma_{9i}(\chi_{1i})$:

1- if i=1

$\theta_{61}=\sigma_{11}(\chi_{11})+\sigma_{31}(\chi_{11})+\sigma_{71}(\chi_{11})+\sigma_{91}(\chi_{11})$
$\theta_{61}([I,I^*])=\theta_{61}([I,r])=\theta_{61}([I,s])=2+2+2+2=8$
$\theta_{61}([x^2,I^*])=\theta_{61}([x^2,r])=\theta_{61}([x^2,s])=V_2+V_6+V_2+V_6=2$
$\theta_{61}([x^4,I^*])=\theta_{61}([x^4,r])=\theta_{61}([x^4,s])=V_4+V_8+V_8+V_4=-2$
$\theta_{61}([x^6,I^*])=\theta_{61}([x^6,r])=\theta_{61}([x^6,s])=V_6+V_2+V_2+V_6=2$
$\theta_{61}([x^8,I^*])=\theta_{61}([x^8,r])=\theta_{61}([x^8,s])=V_8+V_4+V_4+V_8=-2$
$\theta_{61}([x^{10},I^*])=\theta_{61}([x^{10},r])=\theta_{61}([x^{10},s])=-2+-2+-2+-2=-8$
$\theta_{61}([x,I^*])=\theta_{61}([x,r])=\theta_{61}([x,s])=V_1+V_3+V_7+V_9=0$
$\theta_{61}([x^3,I^*])=\theta_{61}([x^3,r])=\theta_{61}([x^3,s])=V_3+V_9+V_1+V_7=0$
$\theta_{61}([x^5,I^*])=\theta_{61}([x^5,r])=\theta_{61}([x^5,s])=0+0+0+0=0$
$\theta_{61}([x^7,I^*])=\theta_{61}([x^7,r])=\theta_{61}([x^7,s])=V_7+V_1+V_9+V_3=0$
$\theta_{61}([x^9,I^*])=\theta_{61}([x^9,r])=\theta_{61}([x^9,s])=V_9+V_7+V_3+V_1=0$
$\theta_{61}([y,I^*])=\theta_{61}([y,r])=\theta_{61}([y,s])=0$
$\theta_{61}([xy,I^*])=\theta_{61}([xy,r])=\theta_{61}([xy,s])=0$

$\theta_{4i}=\chi_{2i}+\chi_{6i}$ and $i=1,2,3$.

$\theta_{41}=\chi_{21}+\chi_{61}$
$\theta_{41}([I,I^*])=\theta_{41}([I,r])=\theta_{41}([I,s])=2+2=4$
$\theta_{41}([x^2,I^*])=\theta_{41}([x^2,r])=\theta_{41}([x^2,s])=V_4+V_8=-1$
$\theta_{41}([x^4,I^*])=\theta_{41}([x^4,r])=\theta_{41}([x^4,s])=V_8+V_4=-1$
$\theta_{41}([x^6,I^*])=\theta_{41}([x^6,r])=\theta_{41}([x^6,s])=V_8+V_4=-1$
$\theta_{41}([x^8,I^*])=\theta_{41}([x^8,r])=\theta_{41}([x^8,s])=V_4+V_8=-1$
$\theta_{41}([x^{10},I^*])=\theta_{41}([x^{10},r])=\theta_{41}([x^{10},s])=2+2=4$
$\theta_{41}([x,I^*])=\theta_{41}([x,r])=\theta_{41}([x,s])=V_2+V_6=1$
$\theta_{41}([x^3,I^*])=\theta_{41}([x^3,r])=\theta_{41}([x^3,s])=V_6+V_2=1$
$\theta_{41}([x^5,I^*])=\theta_{41}([x^5,r])=\theta_{41}([x^5,s])=(-2)+(-2)=-4$
$\theta_{41}([x^7,I^*])=\theta_{41}([x^7,r])=\theta_{41}([x^7,s])=V_6+V_2=1$
$\theta_{41}([x^9,I^*])=\theta_{41}([x^9,r])=\theta_{41}([x^9,s])=V_2+V_6=1$
$\theta_{41}([y,I^*])=\theta_{41}([y,r])=\theta_{41}([y,s])=0$
$\theta_{41}([xy,I^*])=\theta_{41}([xy,r])=\theta_{41}([xy,s])=0$

$\theta_{3i}=\chi_{4i}+\chi_{8i}$ and $i=1,2,3$.

$\theta_{31}=\chi_{41}+\chi_{81}$
$\theta_{31}([I,I^*])=\theta_{31}([I,r])=\theta_{31}([I,s])=2+2=4$
$\theta_{31}([x^2,I^*])=\theta_{31}([x^2,r])=\theta_{31}([x^2,s])=V_8+V_4=-1$
$\theta_{31}([x^4,I^*])=\theta_{31}([x^4,r])=\theta_{31}([x^4,s])=V_4+V_8=-1$
$\theta_{31}([x^6,I^*])=\theta_{31}([x^6,r])=\theta_{31}([x^6,s])=V_4+V_8=-1$
$\theta_{31}([x^8,I^*])=\theta_{31}([x^8,r])=\theta_{31}([x^8,s])=V_8+V_4=-1$
$\theta_{31}([x^{10},I^*])=\theta_{31}([x^{10},r])=\theta_{31}([x^{10},s])=2+2=4$

$\theta_{31}([x,I^*])= \theta_{31}([x,r])= \theta_{31}([x,s])= V_4+V_8=-1$
$\theta_{31}([x^3,I^*])= \theta_{31}([x^3,r])= \theta_{31}([x^3,s])= V_8+V_4=-1$
$\theta_{31}([x^5,I^*])= \theta_{31}([x^5,r])= \theta_{31}([x^5,s])=2+2=4$
$\theta_{31}([x^7,I^*])= \theta_{31}([x^7,r])= \theta_{31}([x^7,s])= V_8+V_4=-1$
$\theta_{31}([x^9,I^*])= \theta_{31}([x^9,r])= \theta_{31}([x^9,s])= V_4+V_8=-1$
$\theta_{31}([y,I^*])= \theta_{31}([y,r])= \theta_{31}([y,s])=0$
$\theta_{31}([xy,I^*])= \theta_{31}([xy,r])= \theta_{31}([xy,s])=0$

2- if i=2

$\theta_{62}= \sigma_{12}(\chi_{12})+\sigma_{32}(\chi_{12})+\sigma_{72}(\chi_{12})+\sigma_{92}(\chi_{12})$
$\theta_{62}([I,I^*])= \theta_{62}([I,r])= 2+2+2+2=8= -\theta_{62}([I,s])$
$\theta_{62}([x^2,I^*])= \theta_{62}([x^2,r])= V_2+V_6+V_2+V_6=2=-\theta_{62}([x^2,s])$
$\theta_{62}([x^4,I^*])= \theta_{62}([x^4,r])= V_4+V_8+V_8+V_4=-2=-\theta_{62}([x^4,s])$
$\theta_{62}([x^6,I^*])= \theta_{62}([x^6,r])= V_6+V_2+V_2+V_6=2=-\theta_{62}([x^6,s])$
$\theta_{62}([x^8,I^*])= \theta_{62}([x^8,r])= V_8+V_4+V_4+V_8=-2= -\theta_{62}([x^8,s])$
$\theta_{62}([x^{10},I^*])= \theta_{62}([x^{10},r])= -2+-2+-2+-2=-8=-\theta_{62}([x^{10},s])$
$\theta_{62}([x,I^*])= \theta_{62}([x,r])= V_1+V_3+V_7+V_9=0=-\theta_{62}([x,s])$
$\theta_{62}([x^3,I^*])= \theta_{62}([x^3,r])=V_3+V_9+V_1+V_7=0=-\theta_{62}([x^3,s])$
$\theta_{62}([x^5,I^*])= \theta_{62}([x^5,r])= 0+0+0+0=0=-\theta_{62}([x^5,s])$
$\theta_{62}([x^7,I^*])= \theta_{62}([x^7,r])= V_7+V_1+V_9+V_3=0=-\theta_{62}([x^7,s])$
$\theta_{62}([x^9,I^*])= \theta_{62}([x^9,r])= V_9+V_7+V_3+V_1=0=-\theta_{62}([x^9,s])$
$\theta_{62}([y,I^*])= \theta_{62}([y,r])= 0=-\theta_{62}([y,s])$
$\theta_{62}([xy,I^*])= \theta_{62}([xy,r])= 0= -\theta_{62}([xy,s])$

$\boldsymbol{\theta_{42}= \chi_{22}+\chi_{62}}$
$\theta_{42}([I,I^*])= \theta_{42}([I,r])= 2+2=4= -\theta_{42}([I,s])$
$\theta_{42}([x^2,I^*])= \theta_{42}([x^2,r])= V_4+V_8=-1 = -\theta_{42}([x^2,s])$
$\theta_{42}([x^4,I^*])= \theta_{42}([x^4,r])= V_8+V_4=-1= -\theta_{42}([x^4,s])$
$\theta_{42}([x^6,I^*])= \theta_{42}([x^6,r])= V_8+V_4=-1= -\theta_{42}([x^6,s])$
$\theta_{42}([x^8,I^*])= \theta_{42}([x^8,r])= V_4+V_8=-1= -\theta_{42}([x^8,s])$
$\theta_{42}([x^{10},I^*])= \theta_{42}([x^{10},r])= 2+2= 4= -\theta_{42}([x^{10},s])$
$\theta_{42}([x,I^*])= \theta_{42}([x,r])= V_2+V_6=1= -\theta_{42}([x,s])$
$\theta_{42}([x^3,I^*])= \theta_{42}([x^3,r])= V_6+V_2=1= -\theta_{42}([x^3,s])$
$\theta_{42}([x^5,I^*])= \theta_{42}([x^5,r])= (-2)+(-2)=-4= -\theta_{42}([x^5,s])$
$\theta_{42}([x^7,I^*])= \theta_{42}([x^7,r])= V_6+V_2=1= -\theta_{42}([x^7,s])$
$\theta_{42}([x^9,I^*])= \theta_{42}([x^9,r])= V_2+V_6=1= -\theta_{42}([x^9,s])$
$\theta_{42}([y,I^*])= \theta_{42}([y,r])= 0= -\theta_{42}([y,s])$
$\theta_{42}([xy,I^*])= \theta_{42}([xy,r])= 0= -\theta_{42}([xy,s])$

$\boldsymbol{\theta_{32}= \chi_{42}+\chi_{82}}$
$\theta_{32}([I,I^*])= \theta_{32}([I,r])= 2+2=4= -\theta_{32}([I,s])$
$\theta_{32}([x^2,I^*])= \theta_{32}([x^2,r])= V_8+V_4=-1= -\theta_{32}([x^2,s])$
$\theta_{32}([x^4,I^*])= \theta_{32}([x^4,r])= V_4+V_8=-1= -\theta_{32}([x^4,s])$
$\theta_{32}([x^6,I^*])= \theta_{32}([x^6,r])= V_4+V_8=-1= -\theta_{32}([x^6,s])$
$\theta_{32}([x^8,I^*])= \theta_{32}([x^8,r])= V_8+V_4=-1= -\theta_{32}([x^8,s])$
$\theta_{32}([x^{10},I^*])= \theta_{32}([x^{10},r])= 2+2=4= -\theta_{32}([x^{10},s])$
$\theta_{32}([x,I^*])= \theta_{32}([x,r])= V_4+V_8=-1= -\theta_{32}([x,s])$
$\theta_{32}([x^3,I^*])= \theta_{32}([x^3,r])= V_8+V_4=-1= -\theta_{32}([x^3,s])$
$\theta_{32}([x^5,I^*])= \theta_{32}([x^5,r])= 2+2=4= -\theta_{32}([x^5,s])$
$\theta_{32}([x^7,I^*])= \theta_{32}([x^7,r])= V_8+V_4=-1= -\theta_{32}([x^7,s])$

$\theta_{32}([x^9, I^*]) = \theta_{32}([x^9, r]) = V_4 + V_8 = -1 = -\theta_{32}([x^9, s])$
$\theta_{32}([y, I^*]) = \theta_{32}([y, r]) = 0 = -\theta_{32}([y, s])$
$\theta_{32}([xy, I^*]) = \theta_{32}([xy, r]) = 0 = -\theta_{32}([xy, s])$

3- if i=3

$\theta_{63} = \sigma_{13}(\chi_{13}) + \sigma_{23}(\chi_{13}) + \sigma_{73}(\chi_{13}) + \sigma_{93}(\chi_{13})$
$\theta_{63}(2[I, I^*]) = \theta_{63}([I, r]) = 2+2+2+2=8$, $\theta_{63}([I, s])=0$
$\theta_{63}(2[x^2, I^*]) = \theta_{63}([x^2, r]) = V_2 + V_6 + V_2 + V_6 = 2$, $\theta_{63}([x^2, s]) = 0$
$\theta_{63}(2[x^4, I^*]) = \theta_{63}([x^4, r]) = V_4 + V_8 + V_8 + V_4 = -2$, $\theta_{63}([x^4, s]) = 0$
$\theta_{63}(2[x^6, I^*]) = \theta_{63}([x^6, r]) = V_6 + V_2 + V_2 + V_6 = 2$, $\theta_{63}([x^6, s]) = 0$
$\theta_{63}(2[x^8, I^*]) = \theta_{63}([x^8, r]) = V_8 + V_4 + V_4 + V_8 = -2$, $\theta_{63}([x^8, s]) = 0$
$\theta_{63}(2[x^{10}, I^*]) = \theta_{63}([x^{10}, r]) = -2+-2+-2+-2=-8$, $\theta_{63}([x^{10}, s]) = 0$
$\theta_{63}(2[x, I^*]) = \theta_{63}([x, r]) = V_1 + V_3 + V_7 + V_9 = 0$, $\theta_{63}([x, s]) = 0$
$\theta_{63}(2[x^3, I^*]) = \theta_{63}([x^3, r]) = V_3 + V_9 + V_1 + V_7 = 0$, $\theta_{63}([x^3, s]) = 0$
$\theta_{63}(2[x^5, I^*]) = \theta_{63}([x^5, r]) = 0+0+0+0=0$, $\theta_{63}([x^5, s]) = 0$
$\theta_{63}(2[x^7, I^*]) = \theta_{63}([x^7, r]) = V_7 + V_1 + V_9 + V_3 = 0$, $\theta_{63}([x^7, s]) = 0$
$\theta_{63}(2[x^9, I^*]) = \theta_{63}([x^9, r]) = V_9 + V_7 + V_3 + V_1 = 0$, $\theta_{63}([x^9, s]) = 0$
$\theta_{63}(2[y, I^*]) = \theta_{63}([y, r]) = 0$, $\theta_{63}([y, s]) = 0$
$\theta_{63}(2[xy, I^*]) = \theta_{63}([xy, r]) = =0$, $\theta_{63}([xy, s]) = 0$

$\theta_{43} = \chi_{23} + \chi_{63}$
$\theta_{43}([I, I^*]) = \theta_{43}([I, r]) = 2+2=4$, $\theta_{43}([I, s])=0$
$\theta_{43}([x^2, I^*]) = \theta_{43}([x^2, r]) = V_4 + V_8 = -1$, $\theta_{43}([x^2, s]) = 0$
$\theta_{43}([x^4, I^*]) = \theta_{43}([x^4, r]) = V_8 + V_4 = -1$, $\theta_{43}([x^4, s]) = 0$
$\theta_{43}([x^6, I^*]) = \theta_{43}([x^6, r]) = V_8 + V_4 = -1$, $\theta_{43}([x^6, s]) = 0$
$\theta_{43}([x^8, I^*]) = \theta_{43}([x^8, r]) = V_4 + V_8 = -1$, $\theta_{43}([x^8, s]) = 0$
$\theta_{43}([x^{10}, I^*]) = \theta_{43}([x^{10}, r]) = 2+2=4$, $\theta_{43}([x^{10}, s]) = 0$
$\theta_{43}([x, I^*]) = \theta_{43}([x, r]) = V_2 + V_6 = 1$, $\theta_{43}([x, s]) = 0$
$\theta_{43}([x^3, I^*]) = \theta_{43}([x^3, r]) = V_6 + V_2 = 1$, $\theta_{43}([x^3, s]) = 0$
$\theta_{43}([x^5, I^*]) = \theta_{43}([x^5, r]) = (-2)+(-2)=-4$, $\theta_{43}([x^5, s]) = 0$
$\theta_{43}([x^7, I^*]) = \theta_{43}([x^7, r]) = V_6 + V_2 = 1$, $\theta_{43}([x^7, s]) = 0$
$\theta_{43}([x^9, I^*]) = \theta_{43}([x^9, r]) = V_2 + V_6 = 1$, $\theta_{43}([x^9, s]) = 0$
$\theta_{43}([y, I^*]) = \theta_{43}([y, r]) = 0$, $\theta_{43}([y, s]) = 0$
$\theta_{43}([x^2, I^*]) = \theta_{43}([x^2, r]) = 0$, $\theta_{43}([xy, s]) = 0$

$\theta_{33} = \chi_{43} + \chi_{83}$
$\theta_{33}([I, I^*]) = \theta_{33}([I, r]) = 2+2=4$, $\theta_{33}([I, s])=0$
$\theta_{33}([x^2, I^*]) = \theta_{33}([x^2, r]) = V_8 + V_4 = -1$, $\theta_{33}([x^2, s]) = 0$
$\theta_{33}([x^4, I^*]) = \theta_{33}([x^4, r]) = V_4 + V_8 = -1$, $\theta_{33}([x^4, s]) = 0$
$\theta_{33}([x^6, I^*]) = \theta_{33}([x^6, r]) = V_4 + V_8 = -1$, $\theta_{33}([x^6, s]) = 0$
$\theta_{33}([x^8, I^*]) = \theta_{33}([x^8, r]) = V_8 + V_4 = -1$, $\theta_{33}([x^8, s]) = 0$
$\theta_{33}([x^{10}, I^*]) = \theta_{33}([x^{10}, r]) = 2+2=4$, $\theta_{33}([x^{10}, s]) = 0$
$\theta_{33}([x, I^*]) = \theta_{33}([x, r]) = V_4 + V_8 = -1$, $\theta_{33}([x, s]) = 0$
$\theta_{33}([x^3, I^*]) = \theta_{33}([x^3, r]) = V_8 + V_4 = -1$, $\theta_{33}([x^3, s]) = 0$
$\theta_{33}([x^5, I^*]) = \theta_{33}([x^5, r]) = 2+2=-4$, $\theta_{33}([x^5, s]) = 0$
$\theta_{33}([x^7, I^*]) = \theta_{33}([x^7, r]) = V_8 + V_4 = -1$, $\theta_{33}([x^7, s]) = 0$
$\theta_{33}([x^9, I^*]) = \theta_{33}([x^9, r]) = V_4 + V_8 = -1$, $\theta_{33}([x^9, s]) = 0$
$\theta_{33}([y, I^*]) = \theta_{33}([y, r]) = 0$, $\theta_{33}([y, s]) = 0$
$\theta_{33}([xy, I^*]) = \theta_{33}([xy, r]) = 0$, $\theta_{33}([xy, s]) = 0$

Then, the rational characters table of $Q_{20} \times D_3$ is given in the following table :

$\equiv^{*}(Q_{20}\times D_3)$ =

CL	$[I, I^*]$	$[I, r]$	$[I, s]$	$[X^2, I^*]$	$[X^2, r]$	$[X^2, s]$	$[X^4, I^*]$	$[X^4, r]$	$[X^4, s]$	$[X^{10}, I^*]$	$[X^{10}, r]$	$[X^{10}, s]$	$[X^5, I^*]$	$[X^5, r]$	$[X^5, s]$	$[x, I^*]$
$\lvert CL_\alpha \rvert$	1	2	3	2	4	6	2	4	6	1	2	3	2	4	6	2
$\lvert C_{Q_{2m}\times D_3}(CL_\alpha)\rvert$	240	120	80	120	60	40	120	60	40	240	120	80	120	60	40	120
$\theta_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\theta_{(1,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
$\theta_{(1,3)}$	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2
$\theta_{(2,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
$\theta_{(2,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1
$\theta_{(2,3)}$	2	-1	0	2	-1	0	2	-1	0	2	-1	0	-2	1	0	-2
$\theta_{(3,1)}$	4	4	4	-1	-1	-1	-1	-1	-1	4	4	4	4	4	4	-1
$\theta_{(3,2)}$	4	4	-4	-1	-1	1	-1	-1	1	4	4	-4	4	4	-4	-1
$\theta_{(3,3)}$	8	-4	0	-2	1	0	-2	1	0	8	-4	0	8	-4	0	-2
$\theta_{(4,1)}$	4	4	4	-1	-1	-1	-1	-1	-1	4	4	4	-4	-4	-4	1
$\theta_{(4,2)}$	4	4	-4	-1	-1	1	-1	-1	1	4	4	-4	-4	-4	4	1
$\theta_{(4,3)}$	8	-4	0	-2	1	0	-2	1	0	8	-4	0	-8	4	0	2
$\theta_{(5,1)}$	2	2	2	-2	-2	-2	2	2	2	-2	-2	-2	0	0	0	0
$\theta_{(5,2)}$	2	2	-2	-2	-2	2	2	2	-2	-2	-2	2	0	0	0	0
$\theta_{(5,3)}$	4	-2	0	-4	2	0	4	-2	0	-4	2	0	0	0	0	0
$\theta_{(6,1)}$	8	8	8	2	2	2	-2	-2	-2	-8	-8	-8	0	0	0	0
$\theta_{(6,2)}$	8	8	-8	2	2	-2	-2	-2	2	-8	-8	8	0	0	0	0
$\theta_{(6,3)}$	16	-8	0	4	-2	0	-4	2	0	-16	8	0	0	0	0	0
$\theta_{(7,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1
$\theta_{(7,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1
$\theta_{(7,3)}$	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	-2
$\theta_{(8,1)}$	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1
$\theta_{(8,2)}$	1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	1	-1
$\theta_{(8,3)}$	2	-1	0	2	-1	0	2	-1	0	-2	1	0	-2	1	0	-2

Table (1.8)

$[x, r]$	$[x, s]$	$[y, I^*]$	$[y, r]$	$[y, s]$	$[xy, I^*]$	$[xy, r]$	$[xy, s]$
4	**6**	**10**	**20**	**30**	**10**	**20**	**30**
60	**40**	**24**	**12**	**8**	**24**	**12**	**8**
1	1	1	1	1	1	1	1
1	-1	1	1	-1	1	1	-1
-1	0	2	-1	0	2	-1	0
-1	-1	-1	-1	-1	1	1	1
-1	1	-1	-1	1	1	1	-1
1	0	-2	1	0	2	-1	0
-1	-1	0	0	0	0	0	0
-1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0
1	-1	0	0	0	0	0	0
-1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
1	1	-1	-1	-1	-1	-1	-1
1	-1	-1	-1	1	-1	-1	1
-1	0	-2	1	0	-2	1	0
-1	-1	1	1	1	-1	-1	-1
-1	1	1	1	-1	-1	-1	1
1	0	2	-1	0	-2	1	0

Table (1.8)

Then , the general form of the rational characters table of $Q_{2m} \times D_3$ when m is an even number and m=2p is given in the following table :

$^*(Q_{4p} \times D_3) =$

CL	$[I, I^*]$	$[I, r]$	$[I, s]$	$[X^2, I^*]$	$[X^2, r]$	$[X^2, s]$	$[X^4, I^*]$	$[X^4, r]$	$[X^4, s]$	$[X^{2p}, I^*]$	$[X^{2p}, r]$	$[X^{2p}, s]$	$[X^p, I^*]$	$[X^p, r]$	$[X^p, s]$	$[x, I^*]$
$\lvert CL_\alpha \rvert$	1	2	3	2	4	6	2	4	6	1	2	3	2	4	6	2
$\lvert C_{Q_{2m} \times D_3}(CL_\alpha) \rvert$	24p	12p	8p	12p	6p	4p	12p	6p	4p	24p	12p	8p	12p	6p	4p	12p
$\theta_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\theta_{(1,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
$\theta_{(1,3)}$	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2
$\theta_{(2,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
$\theta_{(2,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1
$\theta_{(2,3)}$	2	-1	0	2	-1	0	2	-1	0	2	-1	0	-2	1	0	-2
$\theta_{(3,1)}$	(p-1)	(p-1)	(p-1)	-1	-1	-1	-1	-1	-1	(p-1)	(p-1)	(p-1)	(p-1)	(p-1)	(p-1)	-1
$\theta_{(3,2)}$	(p-1)	(p-1)	-(p-1)	-1	-1	1	-1	-1	1	(p-1)	(p-1)	-(p-1)	(p-1)	(p-1)	-(p-1)	-1
$\theta_{(3,3)}$	2(p-1)	-(p-1)	0	-2	1	0	-2	1	0	2(p-1)	-(p-1)	0	2(p-1)	-(p-1)	0	-2
$\theta_{(4,1)}$	(p-1)	(p-1)	(p-1)	-1	-1	-1	-1	-1	-1	(p-1)	(p-1)	(p-1)	-(p-1)	-(p-1)	-(p-1)	1
$\theta_{(4,2)}$	(p-1)	(p-1)	-(p-1)	-1	-1	1	-1	-1	1	(p-1)	(p-1)	-(p-1)	-(p-1)	-(p-1)	(p-1)	1
$\theta_{(4,3)}$	2(p-1)	-(p-1)	0	-2	1	0	-2	1	0	2(p-1)	-(p-1)	0	-2(p-1)	(p-1)	0	2
$\theta_{(5,1)}$	2	2	2	-2	-2	-2	2	2	2	-2	-2	-2	0	0	0	0
$\theta_{(5,2)}$	2	2	-2	-2	-2	2	2	2	-2	-2	-2	2	0	0	0	0
$\theta_{(5,3)}$	4	-2	0	-4	2	0	4	-2	0	-4	2	0	0	0	0	0
$\theta_{(6,1)}$	2(p-1)	2(p-	2(p-	2	2	2	-2	-2	-2	-2(p-1)	-2(p-1)	-2(p-1)	0	0	0	0
$\theta_{(6,2)}$	2(p-1)	2(p-	-2(p-	2	2	-2	-2	-2	2	-2(p-1)	-2(p-1)	2(p-1)	0	0	0	0
$\theta_{(6,3)}$	4(p-1)	-2(p-	0	4	-2	0	-4	2	0	-4(p-1)	2(p-1)	0	0	0	0	0
$\theta_{(7,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1
$\theta_{(7,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1
$\theta_{(7,3)}$	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	-2
$\theta_{(8,1)}$	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1
$\theta_{(8,2)}$	1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	1	-1
$\theta_{(8,3)}$	2	-1	0	2	-1	0	2	-1	0	-2	1	0	-2	1	0	-2

Table (1.9)

$[x,r]$	$[x,s]$	$[y,I^*]$	$[y,r]$	$[y,s]$	$[xy,I^*]$	$[xy,r]$	$[xy,s]$
4	6	2p	4p	6p	2p	4p	6p
6p	4p	12	6	4	12	6	4
1	*1*	*1*	*1*	*1*	*1*	*1*	*1*
1	*-1*	*1*	*1*	*-1*	*1*	*1*	*-1*
-1	*0*	*2*	*-1*	*0*	*2*	*-1*	*0*
-1	*-1*	*-1*	*-1*	*-1*	*1*	*1*	*1*
-1	*1*	*-1*	*-1*	*1*	*1*	*1*	*-1*
1	*0*	*-2*	*1*	*0*	*2*	*-1*	*0*
-1	*-1*	*0*	*0*	*0*	*0*	*0*	*0*
-1	*1*	*0*	*0*	*0*	*0*	*0*	*0*
1	*0*	*0*	*0*	*0*	*0*	*0*	*0*
1	*1*	*0*	*0*	*0*	*0*	*0*	*0*
1	*-1*	*0*	*0*	*0*	*0*	*0*	*0*
-1	*0*	*0*	*0*	*0*	*0*	*0*	*0*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
1	*1*	*-1*	*-1*	*-1*	*-1*	*-1*	*-1*
1	*-1*	*-1*	*-1*	*1*	*-1*	*-1*	*1*
-1	*0*	*-2*	*1*	*0*	*-2*	*1*	*0*
-1	*-1*	*1*	*1*	*1*	*-1*	*-1*	*-1*
-1	*1*	*1*	*1*	*-1*	*-1*	*-1*	*1*
1	*0*	*2*	*-1*	*0*	*-2*	*1*	*0*

Table (1.9)

Theorem (1.9):-

The rational valued characters table of the group $Q_{2m}\times D_3$ when m is an even number and m=2p is given as follows:

$$\equiv^*(Q_{4p}\times D_3) = \equiv^*(Q_{4p}) \otimes \equiv^*(D_3)$$

Proof:-

Since $D_3 = \{1^*, r , r^2, S, Sr, Sr^2\}$

Since

$\equiv D_3 = \equiv^* D_3 =$

	h'_1	h'_2	h'_3
χ'_1	1	1	1
χ'_2	1	1	-1
χ'_3	2	-1	0

table(1.9)

Where $h'_1=\{(I^*)\}$, $h'_2=\{r, r^2\}$, $h'_3=\{S, Sr, Sr^2\}$ and

From the definition of $Q_{2m}\times D_3$, theorem(1.4)

$$(\equiv Q_{4p}\times D_3) = (\equiv Q_{4p}) \otimes (\equiv D_3)$$

each element in $Q_{4p}\times D_3$

$h_{ng} = h_n \cdot h'_g \quad \forall h_n \in Q_{4p}$, $h'_g \in D_3$, $n = 1,2,3,\ldots,4m$, $\mathbf{g} \in \{I^*, r , r^2, S, Sr, Sr^2\}$.

And each irreducible character of $Q_{4p}\times D_3$ is

$$\chi_{(i,j)} = \chi_i . \chi'_j$$

where χ_i is an irreducible character of Q_{4p} and χ'_j is the irreducible character of D_3 ,then

$$\chi_{(i,j)}(h_{ng}) = \begin{cases} \chi_i(h_n) & if \quad j=1 \quad and \quad g \in D_3 \\ \chi_i(h_n) & if \quad j=2 \quad and \quad g \in \{I, r, r^2\} \\ -\chi_i(h_n) & if \quad j=2 \quad and \quad g \in \{S, Sr, Sr^2\} \\ 2\chi_i(h_n) & if \quad j=3 \quad and \quad g \in \{I^*\} \\ -\chi_i(h_n) & if \quad j=3 \quad and \quad g \in \{r, r^2\} \\ 0 & if \quad j=3 \quad and \quad g \in \{S, Sr, Sr^2\} \end{cases}$$

From proposition (2.3.2)

$$\theta_{(i,j)} = \sum_{\sigma \in Gal(Q(\chi_{(i,j)})/Q)} \sigma(\chi_{(i,j)})$$

where $\theta_{(i,j)}$ is the rational valued character of $Q_{4p}\times D_3$

Then, $\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_{(i,j)}(h_{ng}))/Q)} \sigma(\chi_{(i,j)}(h_{ng}))$

(I) If j=1 and $g \in D_3$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n)\ .\ 1 = \theta_i(h_n)\ .\ \theta'_j(h'_g)$$

where θ_i is the rational valued character of Q_{4p} .

(II) (a) If j=2 and $g \in \{I, r, r^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n)\ .\ 1 = \theta_i(h_n)\ .\ \theta'_j(h'_g)$$

(b) If j=2 and $g \in \{S, Sr, Sr^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(-\chi_i(h_n)) = -\sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))\ .\ \text{-}1 = \theta_i(h_n)\ .\text{-}1 = \theta_i(h_n)\ .\theta'_j(h'_g)\ .$$

(III) (a) If j=3 and $g \in \{1^*\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(2\chi_i(h_n)) = 2\sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))\ .\ 2 = \theta_i(h_n)\ .2 = \theta_i(h_n)\ .\theta'_j(h'_g)\ .$$

(b) If j=3 and $g \in \{r, r^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(-\chi_i(h_n)) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) \cdot -1$$

$$= \theta_i(h_n)\ .\theta'_j(h'_g)\ .$$

(c) If j=3 and $g \in \{S, Sr, Sr^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(0 \cdot \chi_i(h_n)) = 0 \cdot \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))\ .\ 0 = 0$$

$$= \theta_i(h_n)\ .\theta'_j(h'_g)\ .$$

From [I] , [II] and [III] we have

$$\theta_{(i,j)} = \theta_i\ .\theta'_j\ .$$

Then $\equiv^*(Q_{4p} \times D_3) = \equiv^*(Q_{4p}) \otimes \equiv^*(D_3)$.

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