TOPOLOGICAL ENTROPY OF A SEQUANCE OF $\beta-$IRRESOLUTE ON $\beta-$COMPACT TOPOLOGICAL SPACE

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Abstract

In 1963 ,R.L.Adler and another [9] define the topological entropy for open cover .in this paper we introduce new classes of the topological entropy of a sequence of $\beta-$irresolute on $\beta-$compact topological space with given some a new definitions , results ,topological conjugate and show that the topological entropy has the same entropy in the commutative

الملخّص

في 1963 , R. L. Adler والآخرون [9] عرّفُوا الانتروبي التبولوجي للغطاءِ المفتوحِ ،في هذا البحث نُقدّمُ أنماطاً جديدةَ للانتروبي التبولوجية لمتتابعة من الدوال المتحيرة على الفضاءِ التبولوجي المرصوص مَعَ البعضِ التعاريف الجديدة والنَتائِج، والمرافق التبولوجي وبيان بان للانتروبي التبولوجي في الحالة الابدالية يكون متساوياً

1-Introduction

The modern theory of topological dynamical systems studies (continuous) maps $f: X \to X$ of topological spaces (indeed, in a more general sense it also studies flows, yet in this paper we concentrate upon maps only). An important problem here is to describe the limit behavior of trajectories of points (the trajectory of a point x is the sequence $(x, f(x), f^2(x),..)$. Apart from studying the trajectories of individual points, it is reasonable to also study the limit behavior of all points of X and estimate the variety of the behaviors exhibited by them. widely recognized parameter of a dynamical system which can serve as a quantitative counter-part of the term "variety" used above is the topological entropy of a map, usually denoted $h(\varphi)$ where φ is the map. Originally [9] it was introduced for compact spaces *X*, but can be extended onto non-compact spaces as well [8]. Therefore, it is important to estimate the topological entropy of a map. Therefore the topological entropy is positive if and only if there exists a power of the map which has positive topological entropy. We know introduce some definition as classes and new of it.

Definition1-1 [7,2]

A subset A of (X,τ) is called a $\beta-$0pen [7](semipreopen [2]) if $A \subset cl\,\mathrm{int}(A)$.

Definition1-2 [10]

A space X is said to be $\beta-$compact if every cover of X by $\beta-$open sets has a finite subcover.

Definition1-3 [10]

A function $f: X \to Y$ is said to be $\beta-$irresolute if for each $V \in \beta O(Y, f(x))$, there exists $U \in \beta O(X,x)$ such that $f(cl\,U) \subset V$.

Definition1-4 [2,7]

A function $f : X \to Y$ is said to be $\beta-$continuous function if $f^{-1}(V)$ is $\beta-$open set in X for each open set V in Y.

Definition1-5 [7]

A bijective function $f : X \to Y$ is said to be $\beta-$homeomorphism if f is $\beta-$irresolute and $\beta-$open set.

Let X be $\beta-$compact topological space and let $\varphi_{1,\infty} = \{\varphi_i\}_{i=1}^{\infty}$ be sequence of $\beta-$irresolute function from X to X, let u be $\beta-$open cover of X if their union is X .for $\beta-$open covers $u_1, u_2, \cdots u_n$ of X we denoted $\bigvee_{i=1}^{n} u_i = \{u_1 \vee u_2 \vee \dots \vee u_n\} = \{A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n : A_1 \subset u_1, A_2 \subset u_2, A_3 \subset u_3, \dots, A_n \subset u_n\}$ is also $\beta-$open covers of X since $\vee$ be the join operation .for $\beta-$open covers $\varphi_i^{-n}(u) = \{\varphi_i^{-n}(A) : A \in u\}$ and $u_i^{-n}(u) = \bigvee_{j=0}^{n-1} \varphi_i^{-j}(u)$ for each j , $\varphi_i^{-n}(u)$ is $\beta-$open cover of X ,since φ_i is $\beta-$irresolute so $u_i^{-n}(u)$ is also $\beta-$open cover of X ,let $\mathrm{N}(u)$ be the minimal cardinality of subcover chosen form u then $h(\varphi_{1,\infty}, u) = \lim_{n\to\infty} \max \frac{1}{n} \log \aleph\left(\bigvee_{j=0}^{n-1} \varphi_i^{-j}(u) \right) = \lim_{n\to\infty} \max \frac{1}{n} \log \aleph\left(u_i^{-n}(u)\right)$ is said to be topological entropy of a sequence of $\beta-$irresolute function $\varphi_{1,\infty}$ on $\beta-$open cover of $\beta-$compact topological space.

The topological entropy of a sequence of $\beta-$irresolute function $\varphi_{1,\infty}$ is define by : $h(\varphi_{1,\infty}) = \max\{(\varphi_{1,\infty}, u) : u \text{ is } \beta\text{-open cover of } X\}$

3- Topological entropy of a sequence of $\beta-$irresolute function

Let u, α are two $\beta-$open cover of $\beta-$compact topological space X , is also a $\beta-$open cover of X . Let $u \prec \alpha$ such that the symbol $\prec$ mane that α is refinement of u ,if every $\beta-$open set in α is $\beta-$open subset in u

We recall the properties of topological entropy:

1. $\aleph(u \vee \alpha) \le \aleph(u) \cdot \aleph(\alpha)$
2. if $\varphi_i^{-j}(u)$ be $\beta-$irresolute function from $\beta-$compact topological space X onto X , then $\aleph\left(\varphi_i^{-j}(u)\right) \le \aleph(u)$
3. if the last inequality of $\beta-$irresolute function with surjective we get $\aleph\left(\varphi_i^{-j}(u)\right) = \aleph(u)$,
4. if $g^{-1} : X \to X$ be $\beta-$homeomorphism $g^{-1}(u \vee \alpha) = g^{-1}(u) \vee g^{-1}(\alpha)$,
5. if $u \prec \alpha$ then $\aleph(u) \le \aleph(\alpha)$ and $H(u) \le H(\alpha)$,
6. If $u_i^n \prec \alpha_i^n$ then $\aleph(u_i^n) \le \aleph(\alpha_i^n)$,

7. if $u \prec \alpha$ then $\aleph(u \vee \alpha) = \aleph(\alpha)$ and $H(u \vee \alpha) = H(\alpha)$
8. If $\varphi_{1,\infty}$ be a sequence of $\beta-$irresolute function on $\beta-$compact topological space X then $h(\varphi_{1,\infty}, u) \le H(u)$.
9. If $u \prec \alpha$ then $h(\varphi_{1,\infty}, u) \le h(\varphi_{1,\infty}, \alpha)$.

Since X is $\beta-$compact topological space in the definition of $h(\varphi_{1,\infty})$. it is sufficient to take the maximum only over all $\beta-$open finite covers .

Now we introduce some basic properties of topological entropy of $\beta-$compact topological space, let $\varphi_{1,\infty} = \{\varphi_i\}_{i=1}^{\infty}$ be $\beta-$irresolute function from $\beta-$compact topological space X to itself and the following some new proposition suppose that the sets K_i are not need to be $\beta-$closed or invariant .

Proposition 3-1

Let X be $\beta-$compact topological space and let $\varphi_{1,\infty}$ be sequence of a $\beta-$irresolute function on X to itself if $X = \bigcup_{i=1}^{n} K_i$ then $h(\varphi_{1,\infty}) = \max_i h(\varphi_{1,\infty}; K_i)$.

Proof:

Supposes $K_i \ne \phi$ then the inequality $h(\varphi_{1,\infty}) \ge \max_i h(\varphi_{1,\infty}; K_i)$ it trivial ,

Let take a β-open cover u of X , let $\alpha_1, \alpha_2, ..., \alpha_n$ be subcover chosen from the β-open covers $(u_1^n | K_1),..., (u_1^n | K_i)$,receptivity .then each element of $\alpha = \alpha_1 \cup \alpha_2 \cup ... \cup \alpha_n$ is contained in some element of u_1^n and α is a β-open cover of X , then we get $\aleph(u_1^n) \le \sum_{i=1}^{k} \aleph(A_1^n | K_i), \forall n$

Now use induction form to prove

$\lim_{n\to\infty} \max \frac{1}{n} \log \sum_{i=1}^{k} a_{n,i} = \max_{1\le i\le k} \lim_{n\to\infty} \max \frac{1}{n} \log a_{n,i}$,where $a_{n,i}$,i =1.2,..,k, n=0,1,2,…are positive numbers [3] we get

$$h(\varphi_{1,\infty}, u) = \lim_{n\to\infty} \max \frac{1}{n} \log \mathrm{N}\left(\bigvee_{j=0}^{n-1} \varphi_i^{-j} u \right) = \lim_{n\to\infty} \max \frac{1}{n} \log \mathrm{N}(u_1^n)$$

$$\le \lim_{n\to\infty} \max \frac{1}{n} \log \sum_{i=1}^{k} \mathrm{N}(A_1^n | K_i) = \max_i \lim_{n\to\infty} \max \frac{1}{n} \log \mathrm{N}(u_1^n | K_i)$$

$$= \max_i h(\varphi_{1,\infty}; K_i, u) \le \max_i h(\varphi_{1,\infty}; K_i)$$

Then $h(\varphi_{1,\infty}) = \max_i h(\varphi_{1,\infty}; K_i)$

Proposition 3-2

Let $\varphi_{1,\infty}$ be a sequence of a β -continuous on a β -compact topological space X to itself. Then $h(\varphi_{1,\infty}^n) \le n \cdot h(\varphi_{1,\infty})$ for every $n \ge 1$.

Proof:

Fix n .for any β -open cover u of X , by $u \prec \alpha$ then $\aleph(u) \le \aleph(\alpha)$ that

$$h(\varphi_{1,\infty},u)=\lim_{n\to\infty}\max\frac{1}{k}\log\aleph(u_1^n)\geq\lim_{m\to\infty}\max\frac{1}{mn}\log\aleph(u_1^{mn})=\lim_{m\to\infty}\max\frac{1}{mn}\log\aleph\left(u\vee\varphi_1^{-1}u\vee\ldots\vee\varphi_1^{-(mn-1)}u\right)$$
$$\geq\lim_{m\to\infty}\max\frac{1}{mn}\log\aleph\left(u\vee\varphi_1^{-n}u\vee\varphi_1^{-2n}u\vee\ldots\vee\varphi_1^{-(m-1)n}u\right)$$
$$=\frac{1}{n}\lim_{m\to\infty}\max\frac{1}{m}\log\aleph\left(u\vee\left(\varphi_1^n\right)^{-1}u\vee\left(\varphi_1^n\right)^{-1}\circ\left(\varphi_{n+1}^n\right)^{-1}u\vee\ldots\vee\left(\varphi_1^n\right)^{-1}\circ\left(\varphi_{n+1}^n\right)^{-1}\circ\ldots\circ\left(\varphi_{(m-2)n+1}^n\right)^{-1}u\right)$$
$$=\frac{1}{n}h(\varphi_{1,\infty}^n,u)$$

Hence $h(\varphi_{1,\infty}^n)\leq n\cdot h(\varphi_{1,\infty})$

Proposition:3-3

Let $\varphi_{1,\infty}$ be a sequence of a β-continuous on a β-compact topological space X to itself. let $\varphi_{1,\infty}$ periodic with period n .Then $h(\varphi_{1,\infty}^n)=n\cdot h(\varphi_{1,\infty})$ for every $n\geq 1$.

Proposition:3-4

Let $\varphi_{1,\infty}$ be a sequence of a β-homeomorphism on a β-compact topological space X to itself .Then $h(\varphi_{1,\infty}^n)=n\cdot h(\varphi_{1,\infty})$ for every $n\geq 1$.

Proof:

The proof is similarly as Proposition 3-2. $h(\varphi_{1,\infty}^n)\leq n\cdot h(\varphi_{1,\infty})$

Proposition:3-5

Let $\varphi_{1,\infty}$ be a sequence of a β-continuous on a β-compact topological space X to itself .Then for every $1\leq i\leq j<\infty$,and a β-open cover u of X $h(\varphi_{i,\infty},u)\leq h(\varphi_{j,\infty},u)$, $h(\varphi_{i,\infty})\leq h(\varphi_{j,\infty})$.

Proof:

For any a β-open cover u of X and $i\geq 1$ by using the property $g^{-1}(u\vee\alpha)=g^{-1}(u)\vee g^{-1}(\alpha)$ we have

$$u_1^n=\left(u\vee\varphi_i^{-1}u\vee\varphi_i^{-2}u\vee\ldots\vee\varphi_i^{-(n-1)}u\right)$$
$$=\left(u\vee\varphi_i^{-1}\left(u\vee\varphi_{+1i}^{-1}u\vee\ldots\vee\varphi_{+1i}^{-(n-2)}u\right)\right)$$
$$=\left(u\vee\varphi_i^{-1}(u)_{i+1}^{n-1}\right)$$

By $\aleph(u\vee\alpha)\leq\aleph(u)\cdot\aleph(\alpha)$ and $\aleph(\varphi_i^{-n}u)\leq\aleph(u)$ we get

$$h(\varphi_{i,\infty},u)=\lim_{n\to\infty}\max\frac{1}{n}\log\aleph(u_1^n)\leq\lim_{n\to\infty}\max\frac{1}{n}\log\left(\aleph(u)\cdot\aleph(u_{i+1}^{n-1})\right)$$
$$\leq\lim_{n\to\infty}\max\frac{1}{n-1}\log\aleph(u_{i+1}^{n-1})$$
$$=h(\varphi_{i+1,\infty},u)$$
$$\Rightarrow h(\varphi_{i,\infty},u)\leq h(\varphi_{i+1,\infty},u)\ \Rightarrow h(\varphi_{i,\infty},u)\leq h(\varphi_{j,\infty},u)\ \Rightarrow h(\varphi_{i,\infty})\leq h(\varphi_{j,\infty})$$

Let φ,ψ are two β-continuous on a β-compact topological space X ,Y respectively ,we give properties about commutative of topological entropy definition for β-compact topological space

Proposition 3-6

For any φ,ψ are two β-continuous on a β-compact topological space X into itself, then $h(\psi\circ\varphi)=h(\varphi\circ\psi)$

Proof:

From Proposition 3-5

$h(\psi\circ\varphi\circ\psi\circ\varphi\circ\ldots)\le h(\varphi\circ\psi\circ\varphi\circ\psi\circ\ldots)\le h(\psi\circ\varphi\circ\psi\circ\ldots)$ and so

$h(\psi\circ\varphi\circ\psi\circ\varphi\circ\ldots)=h(\varphi\circ\psi\circ\varphi\circ\psi\circ\ldots)$ by Proposition 3-3

$h(\psi\circ\varphi)=h(\psi\circ\varphi,\psi\circ\varphi,\ldots)=2.h(\varphi,\psi,\varphi,\psi,\ldots)=h(\varphi\circ\psi,\varphi\circ\psi,\ldots)=h(\varphi\circ\psi)$

Proposition 3-7

Let φ_i be a sequence of a β-continuous self map on a β-compact topological space X .Then for every $1\le i\le n$, $h(\varphi_n\circ\ldots\circ\varphi_1)=h(\varphi_{i-1}\circ\ldots\circ\varphi_2\circ\varphi_1\circ\varphi_n\circ\ldots\circ\varphi_i)$

Proof:

$$h(\varphi_n\circ\ldots\circ\varphi_i\circ\varphi_{i-1}\circ\ldots\circ\varphi_2\circ\varphi_1)=h((\varphi_n\circ\ldots\circ\varphi_i)\circ(\varphi_{i-1}\circ\ldots\circ\varphi_2\circ\varphi_1))$$
$$=h((\varphi_{i-1}\circ\ldots\circ\varphi_1)\circ(\varphi_n\circ\ldots\circ\varphi_i))=h(\varphi_{i-1}\circ\ldots\circ\varphi_2\circ\varphi_1\circ\varphi_n\circ\ldots\circ\varphi_i)$$

Proposition 3-8

For any φ,ψ are two β-continuous on a β-compact topological space X into itself, if ψ is conjugate to φ then φ,ψ have the same topological entropy.

Proof:

Let $\sigma: X\to X$ be conjugate between φ,ψ .then

$$h(\psi)=h(\sigma\circ\varphi\circ\sigma^{-1})$$
$$=h((\sigma\circ\varphi)\circ\sigma^{-1})$$
$$=h(\sigma^{-1`}(\sigma\circ\varphi))$$
$$=h((\sigma^{-1`}\circ\sigma)\circ\varphi)$$
$$=h(\mathrm{id}_X\circ\varphi)$$
$$=h(\varphi)$$

Proposition 3-9

For any $\varphi_{1,\infty},\psi_{1,\infty}$ are two β-continuous on a β-compact topological space X into itself, if $\psi_{1,\infty}$ is conjugate to $\varphi_{1,\infty}$ then $\varphi_{1,\infty},\psi_{1,\infty}$ have the same topological entropy .

Proof:

Is similarity to Proposition 3-8 with add $\sigma_{1,\infty}: X\to X$ be sequence conjugate between $\varphi_{1,\infty},\psi_{1,\infty}$ .then $h(\varphi_{1,\infty})=h(\psi_{1,\infty})$

References:

[1] A.M.Rajab, On Topological Entropy, master thesis, Tikrit Univ., 2005

[2] D.Andrijevic,Semipreopen sets,Mat.Vesnik 38 (1986),24-32

[3] J.Franks and M.Misiurewicz ,Topological methods in dynamics ,Handbook of dynamical systems ,vol1,(2002).

[4] LI.Alseda,S.F.Kolyado and L.Snoha, On Topological Entropy of triangular maps of the square ,Bull.Austral.Math..Soc 48(1993)

[5] LI.Alseda,S.F.Kolyado, J.Llibre and L.Snoha, Entropy and periodic points transitive maps , Trans.Amer.Math.Soc.315(4)(1999),1551-1573.

[6] M.E.Abd El-Monsef and A.M.Kozae,Some generalized forms of compactness and closedness ,Delta J.Sci.9(2)(1985),257-269.

[7] M.E.Abd El-Monsef,S.NEl-Deeb and R.A.Mahmoud , β-open sets and β-continuous mappings,Bull.Fac.Sci.Assiut Univ.12(1)(1983),77-90.

[8] R. Bowen, Topological entropy for noncompact sets, Trans. Amer. Math. Soc. 184 (1973),(125-136).

[9] R.L.Addler,A.G.Konieim and M.H.McAndrew ,Topological Entropy ,Trans. Amer. Math. Soc. 114(1965),309-319.

[10] S.Jafari and T.Noiri ,On β-quasi-irresolute functions, Mem. Fac. Sci. Kochi. univ. (math) 21 (2000),53-62.