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# Devising a mathematical model for displacement estimation of the wetting pattern center from the emitter location resulting from the soil surface slope

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## A B S T R A C T

This research aims to study the effect of land surface slope on the shape and size of the wetting pattern in the soil resulting from a dripping source, especially when there is a local surface runoff resulting from a water application rate higher than the soil infiltration capacity. As a result, it leads to a noticeable displacement of the center of the wetting pattern from the emitter position. The present research included finding this displacement mathematically and verifying its validity with the results of laboratory data. The studied data included 85 overlapping cases of the change in the water application rate and the slope of the soil surface for two levels of initial moisture and two soils of different textures. The results showed that the first devising equation could be used to estimate the wetting pattern center displacement from the emitter, resulting from the slope of the soil surface. The usage of the first devising equation was based on the of half the maximum horizontal surface advance (a), the maximum vertical advance under the emitter (b), and the soil surface slope, especially when the ratio (a/b) was within (0.9-1.1). The second devising equation can estimate the wetting pattern center displacement from the emitter resulting from the soil surface slope, regardless of the (a/b) ratio. That is, there was a considerable convergence between the displacement values of the center of the wetting pattern from the emitter position estimated by the first devising equation and the second devising equation when the value of (a/b) was close to one, which represents 87% of the 85 cases covered in this research.

## استنباط نموذج رياضي لتخمين إزاحة مركز نمط الابتلال عن موقع المنقط نتيجة لانحدار سطح التربة

مركز بحوث السدود والموارد المائية / جامعة الموصل / الموصل - العراق.  
قسم هندسة السدود والموارد المائية / كلية الهندسة / جامعة الموصل / الموصل - العراق.

محمد طارق محمود  
حقي إسماعيل ياسين

### الخلاصة

انحدار سطح التربة من العوامل التي تؤثر على شكل وحجم نمط الابتلال في التربة الناتج من مصدر تنقيط وخاصةً عندما يكون هنالك سيح سطحي موقعي ناتج عن زيادة معدل إضافة الماء إلى قابلية التربة على الارتشاح فهذا يؤدي إلى إزاحة ملحوظة لمركز نمط الابتلال عن موقع المنقط. تضمن البحث الحالي إيجاد هذه الإزاحة رياضياً والتحقق من صحتها مع نتائج بيانات مختبرية شملت 85 حالة متراكبة لتغير كل من معدل إضافة الماء وانحدار سطح التربة لمستويين من الرطوبة الابتدائية ولتربتين مختلفتي النسجة. وبين البحث إمكانية استخدام المعادلة المستنبطة الأولى في تخمين إزاحة مركز نمط الابتلال عن موقع المنقط الناتجة من انحدار سطح التربة اعتماداً على قيم كل من نصف أقصى تقدم أفقي سطحي (a) وأقصى تقدم عمودي تحت المنقط (b) وانحدار سطح التربة خاصةً عندما تكون النسبة (a/b) بحدود (0.9-1.1). ويمكن استخدام المعادلة المستنبطة الثانية في تخمين إزاحة مركز نمط الابتلال عن موقع المنقط الناتجة من انحدار سطح التربة بغض النظر عن نسبة (a/b). أي أن هنالك تقارب كبير جداً بين قيم الإزاحة لمركز نمط الابتلال عن موقع المنقط المخمّنة بالمعادلة المستنبطة الأولى والمعادلة المستنبطة الثانية عندما تكون قيمة (a/b) مقارنة للواحد وهي تمثل نسبة 87% من الـ 85 حالة التي تم تغطيتها بهذا البحث.

الكلمات الدالة: الري بالتنقيط، نمط الابتلال، انحدار سطح التربة.

### 1. INTRODUCTION

The shape and size of the wetting pattern in soil resulting from a trickle source are affected by several factors, including the water application rate, the volume of applied water, soil texture, initial soil moisture, method of water application—continuous or intermittent—the slope of the soil surface, the bulk density of the soil, temperatures of water and soil, and water and soil salts' quantity [1-8]. The effect of the soil surface slope on the wetting pattern includes two cases. The first is when there is a local surface runoff resulting from the water application rate that is higher than the soil infiltration capacity, as there is an apparent change in the shape of the wetting pattern and a large displacement of the center of the wetting pattern from the emitter. The second case is when the water application rate from the emitter is low compared to the soil infiltration capacity. Therefore, there is no local surface runoff; however, the slope of the soil surface insignificantly affects the shape of the wetting pattern and the displacement of the center of the wetting pattern from the emitter position [3,9,10]. The values of the wetting front diameter, depth, and total area for different cases of soil surface slope and water application rate help determine the distance between the emitters and laterals. Also, to reduce the deep percolation losses and locate the plant from the emitter where the plant location should be downstream the emitter position, Solat et al. [11] and Alekseev et al. [12] noted that the shape of the wetting front was unaffected by the inclination of the earth's surface at the beginning of irrigation. However, with ongoing water adding, the wetting front commenced to be affected by the surface slope and water loss possibility in the deep percolation. The wetting

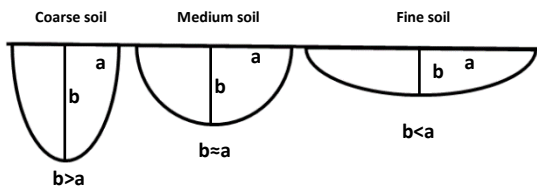
pattern shape change displaced the wetting pattern center from the emitter position [13,14]. The best direction for laterals was in the direction of the short slope, and the trickle irrigation system was affected by the earth's surface slope, especially the slopes with a large inclination, Khalifa [15]. From the above, it was noted that many factors affect the wetting pattern. Also, the previous studies adopted all of these factors to clarify the soil surface slope effect without adopting the wetting pattern in the case of the flat surface. Thus, the present study mathematically investigated the wetting pattern center displacement from the emitter position resulting from the soil surface mathematically when there was a local surface runoff depending on the wetting pattern in the case of the flat surface. Then, the validity of the proposed model was verified by comparing its estimate of the displacement of the wetting pattern center from the emitter position with the results of published laboratory data.

### 2. THE DISPLACEMENT OF THE CENTER OF THE WETTING PATTERN FROM THE EMITTER POSITION

#### 2.1. The shape of the wetting pattern is semi-ellipse

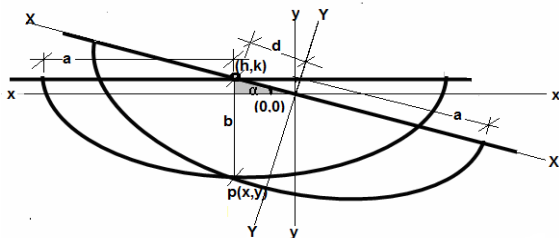
The shape of the wetting pattern under a trickle line source can be one of the patterns shown in Fig. 1. In general, the patterns depend on the type of soil. The shape of the wetting pattern resulting from a trickle line source in the soil sections, when a certain amount of water is added, can be approximated by a semi of an ellipse [16-18]. Half of the maximum horizontal surface advance is a, and the maximum vertical advance under the emitter is b, assuming that the intersection of the wetting pattern in a flat

surface soil with the wetting pattern in soil with a sloped surface is downstream of the emitter position.



**Fig. 1** Wetting patterns under a trickle source for different types of soils.

Fig. 2 shows the intersection of two patterns of wetting resulting from adding water with a trickle line source in soil with two surfaces. One surface is flat, and the other is sloped at an angle of ( $\alpha$ ). Both surfaces are semi-ellipse and symmetrical, as was mentioned above. The distance ( $d$ ) represents the displacement of the wetting pattern center in case the soil surface is sloped from the emitter position. The displacement of the wetting pattern center in the case of the soil surface is sloping from the emitter position by adopting Fig. 2.



**Fig. 2** Intersection of two wetting patterns forms a semi-ellipse in soil.

The wetting pattern equation is expressed in soil with a flat surface according to the (x-x) and (y-y) axes as follows:

$$[x-h]^2/a^2 + [y-k]^2/b^2 = 1 \quad (1)$$

The wetting pattern equation in soil with a sloped surface according to the (X-X) and (Y-Y) axes is:

$$X^2/a^2 + Y^2/b^2 = 1 \quad (2)$$

The relationship between the (x-x) and (y-y) axes and the (X-X) and (Y-Y) axes is as follows:

$$X = x \cos \alpha + y \sin \alpha \quad (3)$$

$$Y = y \cos \alpha - x \sin \alpha \quad (4)$$

Substituting the value of (X) from Eq. (3) and (Y) from Eq. (4) into Eq. (2) results:

$$[x \cos \alpha + y \sin \alpha]^2/a^2 + [y \cos \alpha - x \sin \alpha]^2/b^2 = 1 \quad (5)$$

By substituting the value of ( $x=h$ ) and ( $y=k-b$ ) the coordinates of the point  $p(x,y)$  in Eq. (5), the following equations result:

$$[h \cos \alpha + (k-b) \sin \alpha]^2/a^2 + [(k-b) \cos \alpha - h \sin \alpha]^2/b^2 = 1 \quad (6)$$

$$\sin^2 \alpha [h \cos \alpha / \sin \alpha + (k-b)]^2/a^2 + \cos^2 \alpha [(k-b) - h \sin \alpha / \cos \alpha]^2/b^2 = 1 \quad (7)$$

From the shaded triangle in Fig. 2 ( $\tan \alpha = k/h$ ) and by substituting ( $\sin \alpha / \cos \alpha$ ) with ( $k/h$ ) and ( $\cos \alpha / \sin \alpha$ ) with ( $h/k$ ) in Eq. (7), the following results:

$$\sin^2 \alpha [h^2/k + k - b]^2/a^2 + \cos^2 \alpha [k - b - k]^2/b^2 = 1 \quad (8)$$

$$\sin^2 \alpha [h^2/k + (k-b)]^2/a^2 + \cos^2 \alpha = 1 \quad (9)$$

$$[h^2/k + (k-b)]^2 = a^2 \quad (10)$$

$$h^2/k + (k-b) = a \quad (11)$$

$$h^2 = k[(a+b) - k] \quad (12)$$

$$h^2 + k^2 = k(a+b) \quad (13)$$

From the shaded triangle in Fig. 2, ( $\sin \alpha = k/d$ ), meaning that ( $k = d \sin \alpha$ ) and ( $d^2 = h^2 + k^2$ ) and by substituting that in Eq. (13), results:

$$d^2 = (a+b) d \sin \alpha \quad (14)$$

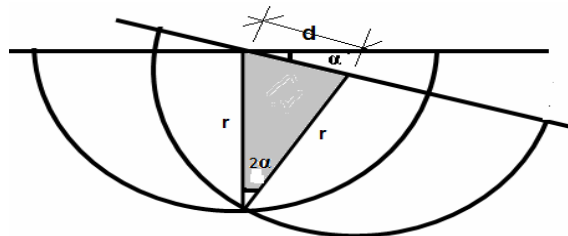
$$d = (a+b) \sin \alpha \quad (15)$$

Since ( $\tan \alpha = \text{land slope}$ ) and ( $\alpha = \tan^{-1}(\text{land slope})$ ), then Eq. (15) becomes:

$$d = (a+b) \sin \{\tan^{-1}(\text{land slope})\} \quad (16)$$

## 2.2. The shape of the wetting pattern is semi-circle

The above derivation can also be clarified for a particular case when ( $a = b$ ), that is, the shape of the wetting pattern is a semi-circle, as shown in Fig. 3.



**Fig. 3** Intersection of the two wetting patterns in the form of a semi-circle in soil with two surfaces, one flat and the other sloping.

Applying the cosine law to the shaded triangle in Fig. 3 results:

$$d^2 = r^2 + r^2 - 2 r^2 \cos 2\alpha \quad (17)$$

$$d^2 = 2r^2(1 - \cos 2\alpha) \quad (18)$$

and since ( $1 - \cos 2\alpha$ ) equals ( $2 \sin^2 \alpha$ ), then Eq. (18) becomes:

$$d^2 = 4r^2 \sin^2 \alpha \quad (19)$$

$$d = 2r \sin \alpha \quad (20)$$

Eq. (20) matches Eq. (15) when ( $a$ ) and ( $b$ ) equal ( $r$ ).

### 3.RESULTS AND DISCUSSIONS

Appendix presents the surface advance downstream the emitter, the surface advance upstream the emitter, the vertical advance downstream the emitter for a trickle line, source at a specific time since the beginning of water application, initial volumetric moisture of the soil, water application rate, and soil surface slope adopted from Sulaiman and Yasin [9] in clay loam soil and sandy loam soil. In addition, the data includes the wetting pattern center displacement from the actual emitter position (da), which was calculated by Eq. 21, and the values of the displacement of the wetting pattern center from the predicted emitter position (dp), which was calculated by Eq. 16.

$$da = 0.5 * (\text{surface advance downstream the emitter} - \text{surface advance upstream the emitter}) \quad (21)$$

Appendix includes 85 cases that covered a wide area of change in the water application rate and the soil surface slope for two levels of initial soil moisture and two soils of different textures. The wetting pattern was affected by many factors, including the properties of the soil, its structure and texture, the initial moisture of the soil, applied water volume, the applied water rate from the emitter, or the application's sustainability. To obtain the desired wetting pattern, there was an agreement between the mentioned factors' overlap and the soil surface slope, especially in the case of local surface runoff. By comparing the values of the displacement of the wetting pattern center from the actual emitter position (da) and the values of the wetting pattern center displacement from the predicted emitter position (dp), there was an agreement up to the level of a correlation coefficient of 0.858. Fig. 4 shows the size of the agreement that exists between the actual displacement and estimated by Eq. (16) values from knowing the dimensions (a and b) of a flat soil surface at any hypothetical slope. In order to increase the correlation coefficient, the developed values were found for the wetting pattern center displacement from the emitter position (ddp). The developed values were the product of the wetting pattern center displacement from the predicted emitter position (dp) and the square of the ratio (a/b). As a result, an increase to 0.966 was gained in the correlation coefficient between the advanced of the wetting pattern center displacement from the emitter position (ddp) and the wetting pattern center displacement

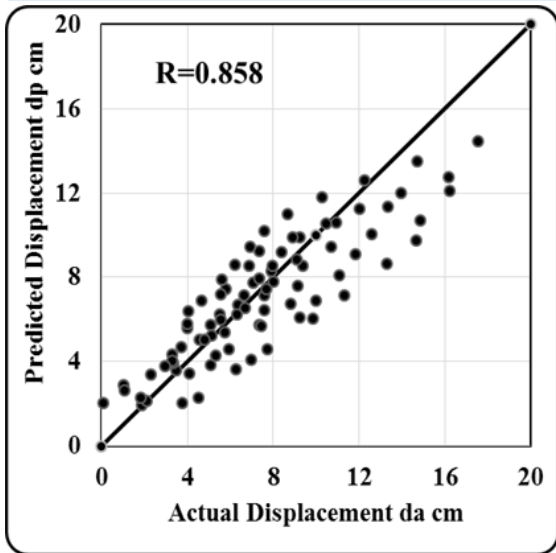
values from the actual emitter position (da), as shown in Fig. 5. To verify the above equations' accuracy for estimating the values of displacement (dp) and the developed displacement (ddp), Table.1 presents a summary of the statistical criteria values resulting from the comparison between the actual and predicted displacement values. Table.1 tabulates the coefficient of determination (R2), root mean square error (RMSE), and model efficiency (EF). The displacement values that correspond to the values of (a/b) from 0.9 to 1.1, which was close to one, had an insignificant effect. There was a considerable convergence between the center displacement values of the wetting pattern from the predicted emitter position (dp) and the developed center displacement values of the wetting pattern from the emitter position (ddp). These cases represented 87% of the 85 cases covered in this research. Table.2 presents the values of (da), (dp), developed displacement of the center of the wetting pattern from the predicted emitter position (ddp), and the ratio b/a, i.e., the shaded part represents the aforementioned states. In addition to the possibility of adopting the (ddp) values, Eq. (16) becomes:

$$ddp = (a+b) * (a/b)^2 * \sin\{\tan^{-1}(\text{land slope})\} \quad (22)$$

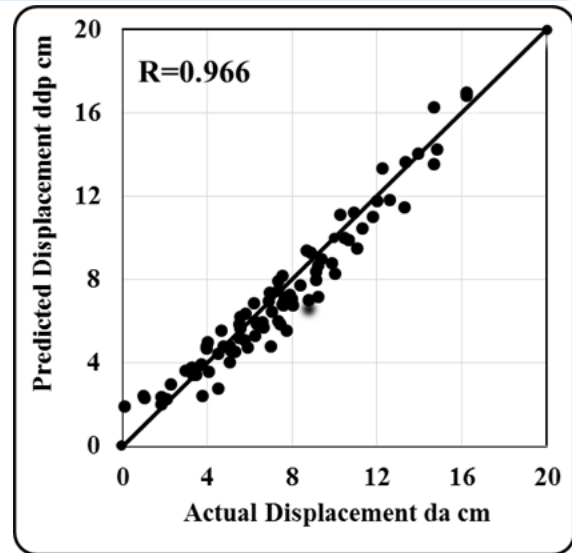
**Table.1** Results of the statistical criteria for comparing the predicted and the actual values of the emitter position.

Statistical Standard	dp	ddp
R <sup>2</sup> Coefficient of determination	0.736	0.933
RMSE Root mean square error	1.494	0.941
EF Efficiency of the model	0.785	0.915

The present research estimated the displacement depending on the wetting pattern in the case of zero slope (flat soil surface), while the previous studies, such as Sulaiman and Yasin [9], estimated the displacement depending on the water application rate, soil texture, initial moisture, surface slope and application time.



**Fig. 4** Comparison between the center displacement of the wetting pattern from the actual trickler position ( $d_a$ ) and the center displacement of the wetting pattern from the predicted trickler position ( $d_p$ ).



**Fig. 5** Comparison between the center displacement of the wetting pattern from the location of the actual trickler position ( $d_a$ ) and the center displacement of the wetting pattern from the location of the predicted trickler position ( $dd_p$ ).

**Table.2** The values of the center displacement of the wetting pattern from the actual emitter position ( $d_a$ ), the center displacement of the wetting pattern from the predicted emitter position ( $d_p$ ), and the center displacement of the wetting pattern from the predicted emitter position ( $dd_p$ ) and the ratio  $a/b$ .

$a/b$	$dp*(a/b)^2$	$dp$	$da$	$a/b$	$dp*(a/b)^2$	$dp$	$da$	$a/b$	$dp*(a/b)^2$	$dp$	$da$
1.15	11.5	8.63	13.3	1.03	11.2	10.6	10.9	1.02	4.73	4.57	5.91
1.15	14.3	10.7	14.9	1.03	13.3	12.6	12.3	1.02	5.88	5.67	7.45
1.15	17	12.8	16.2	0.97	3.42	3.6	3.5	1.02	7.01	6.76	8.8
1.08	2.41	2.05	3.78	0.97	5.11	5.37	5.76	0.97	1.9	2.01	0.09
1.08	4.8	4.09	6.98	0.97	6.77	7.13	7.58	0.97	3.78	4.01	3.29
1.08	7.17	6.11	9.25	0.97	8.42	8.86	9.12	0.97	5.66	6	5.56
1.08	9.51	8.1	11.1	0.97	10	10.6	10.5	0.97	7.5	7.96	7.37
1.08	11.8	10.1	12.6	0.94	2.96	3.37	2.28	0.97	9.32	9.89	8.91
1.08	14.1	12	14	0.94	4.42	5.03	4.55	0.97	11.1	11.8	10.3
1.02	2.01	1.92	1.87	0.94	5.87	6.67	6.36	0.93	4.8	5.6	3.98
1.02	4.02	3.83	5.07	0.94	7.29	8.3	7.9	0.93	6.37	7.42	5.8
1.02	6.01	5.73	7.33	0.94	8.69	9.88	9.25	0.93	7.91	9.23	7.34
1.02	7.97	7.6	9.15	0.92	3.93	4.69	3.71	0.93	9.42	11	8.69
1.02	9.9	9.45	10.7	0.92	5.21	6.23	5.52	0.9	5.54	6.9	4.68
1.02	11.8	11.3	12	0.92	6.48	7.74	7.06	0.9	6.89	8.58	6.22
1.18	13.5	9.76	14.7	0.92	7.72	9.22	8.41	0.9	8.2	10.2	7.57
1.18	16.8	12.1	16.2	0.91	2.41	2.91	1	0.89	4.99	6.37	4.05
1.18	20	14.4	17.6	0.91	3.61	4.35	3.27	0.89	6.21	7.92	5.59
1.1	2.78	2.31	4.54	0.91	4.78	5.77	5.09	0.89	7.39	9.43	6.94
1.1	5.55	4.6	7.74	0.91	5.95	7.17	6.62	0.9	4.71	5.79	3.99
1.1	8.29	6.87	10	0.91	7.08	8.54	7.97	0.9	5.86	7.19	5.53
1.1	11	9.12	11.8	0.93	2.31	2.65	1.05	0.9	6.98	8.57	6.88
1.1	13.7	11.3	13.4	0.93	3.46	3.96	3.32	0.98	3.62	3.78	2.96
1.1	16.3	13.5	14.7	0.93	4.59	5.25	5.14	0.98	4.8	5.02	4.78
1.03	2.28	2.15	2.1	0.93	5.7	6.53	6.68	0.98	5.96	6.24	6.32
1.03	4.55	4.3	5.3	0.93	6.79	7.77	8.02	0.98	7.1	7.43	7.67
1.03	6.8	6.42	7.57	1.02	2.39	2.3	1.83	1.21	5.32	3.65	6.27
1.03	9.02	8.52	9.39	1.02	3.57	3.44	4.1	1.21	8.78	6.02	9.86
								1.21	10.5	7.17	11.3

#### 4. CONCLUSIONS

From the results obtained, it can be concluded that equation:  $d = (a+b) \sin \{\tan^{-1}(\text{land slope})\}$  can be used to estimate the wetting pattern center displacement from the emitter position, which resulted from the soil surface. The above equation usage depended on (a) and (b) values, and the soil surface slope, especially when the ratio (a/b) was in the range of (0.9-1.1). In general, the equation:  $ddp = (a+b) * (a/b) 2 \sin \{\tan^{-1}(\text{land slope})\}$  can be used in estimating the center displacement of the wetting pattern from the emitter position resulting from the slope of the soil surface regardless of the (a/b) ratio. Note that the effect of the initial soil moisture, the rate of water application, and the volume of applied water implicitly in the values of (a) and (b). Changing the factors that affected the wetting pattern in the experimental data showed the validity of the deduced mathematical model. Also, it may give more data than or develop the same method proposed in the present research to cover those data in future studies.

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**APPENDIX**

Data of surface advance downstream and upstream of the emitter, vertical advance under the emitter at a specific time since the beginning of water application, initial soil volumetric moisture, water application rate, soil surface slope for a trickle line source in clay loam soil and sandy loam soil, the values of the displacement of the center of the wetting pattern from the actual emitter position ( $d_a$ ), and the center displacement values of the wetting pattern from the predicted emitter position ( $d_p$ ).

Specific time since the start of water application (minutes)	Water application rate (ml /min/ cm)	Soil surface slope (cm/ cm)	Initial volumetric moisture of the soil	Surface advance downstream in the emitter (cm)	Surface advance upstream the emitter (cm)	Vertical advance under the emitter (cm)	a+b (cm)	$d_a$ (cm)	$d_p$ (cm)
120	4.5	0.15	0.05	45.9	19.2	26.2	58.2	13.3	8.63
120	4.5	0.1875	0.05	47.7	18	26	58.2	14.9	10.7
120	4.5	0.225	0.05	49.3	16.9	25.9	58.2	16.2	12.8
120	4	0.0375	0.05	32.2	24.6	26.2	54.6	3.78	2.05
120	4	0.075	0.05	36	22	25.8	54.6	6.98	4.09
120	4	0.1125	0.05	38.7	20.2	25.6	54.6	9.25	6.11
120	4	0.15	0.05	40.9	18.7	25.4	54.6	11.1	8.1
120	4	0.1875	0.05	42.7	17.5	25.2	54.6	12.6	10.1
120	4	0.225	0.05	44.3	16.4	25	54.6	14	12
120	3.5	0.0375	0.05	27.8	24.1	25.3	51.3	1.87	1.92
120	3.5	0.075	0.05	31.6	21.5	25	51.3	5.07	3.83
120	3.5	0.1125	0.05	34.3	19.7	24.7	51.3	7.33	5.73
120	3.5	0.15	0.05	36.5	18.2	24.5	51.3	9.15	7.6
120	3.5	0.1875	0.05	38.3	16.9	24.3	51.3	10.7	9.45
120	3.5	0.225	0.05	39.9	15.9	24.2	51.3	12	11.3
120	4.5	0.15	0.1	51.7	22.3	29.4	65.8	14.7	9.76
120	4.5	0.1875	0.1	53.5	21.1	29.2	65.8	16.2	12.1
120	4.5	0.225	0.1	55.1	20	29	65.8	17.6	14.4
120	4	0.0375	0.1	36.7	27.6	29.3	61.5	4.54	2.31
120	4	0.075	0.1	40.5	25.1	28.9	61.5	7.74	4.6
120	4	0.1125	0.1	43.2	23.2	28.7	61.5	10	6.87
120	4	0.15	0.1	45.4	21.8	28.5	61.5	11.8	9.12
120	4	0.1875	0.1	47.2	20.5	28.3	61.5	13.4	11.3
120	4	0.225	0.1	48.9	19.4	28.1	61.5	14.7	13.5
120	3.5	0.0375	0.1	31.2	27	28.3	57.5	2.1	2.15
120	3.5	0.075	0.1	35.1	24.5	28	57.5	5.3	4.3
120	3.5	0.1125	0.1	37.8	22.6	27.7	57.5	7.57	6.42
120	3.5	0.15	0.1	39.9	21.2	27.5	57.5	9.39	8.52
120	3.5	0.1875	0.1	41.8	19.9	27.3	57.5	10.9	10.6
120	3.5	0.225	0.1	43.4	18.8	27.2	57.5	12.3	12.6
120	3	0.075	0.05	27.8	20.8	24	48.1	3.5	3.6
120	3	0.1125	0.05	30.5	19	23.7	48.1	5.76	5.37
120	3	0.15	0.05	32.7	17.6	23.5	48.1	7.58	7.13
120	3	0.1875	0.05	34.5	16.3	23.3	48.1	9.12	8.86
120	3	0.225	0.05	36.2	15.2	23.2	48.1	10.5	10.6
120	2.5	0.075	0.05	24.7	20.1	22.9	45	2.28	3.37
120	2.5	0.1125	0.05	27.4	18.3	22.6	45	4.55	5.03
120	2.5	0.15	0.05	29.5	16.8	22.4	45	6.36	6.67
120	2.5	0.1875	0.05	31.4	15.6	22.2	45	7.9	8.3
120	2.5	0.225	0.05	33	14.5	22.1	45	9.25	9.88
120	2	0.1125	0.05	24.8	17.4	21.3	42	3.71	4.69

120	2	0.15	0.05	27	15.9	21.1	42	5.52	6.23
120	2	0.1875	0.05	28.8	14.7	20.9	42	7.06	7.74
120	2	0.225	0.05	30.4	13.6	20.8	42	8.41	9.22
120	1.5	0.075	0.05	20.2	18.1	20	38.9	1	2.91
120	1.5	0.1125	0.05	22.9	16.3	19.7	38.9	3.27	4.35
120	1.5	0.15	0.05	25	14.9	19.5	38.9	5.09	5.77
120	1.5	0.1875	0.05	26.9	13.6	19.3	38.9	6.62	7.17
120	1.5	0.225	0.05	28.5	12.5	19.2	38.9	7.97	8.54
120	1	0.075	0.05	18.8	16.7	17.9	35.4	1.05	2.65
120	1	0.1125	0.05	21.5	14.8	17.7	35.4	3.32	3.96
120	1	0.15	0.05	23.7	13.4	17.5	35.4	5.14	5.25
120	1	0.1875	0.05	25.5	12.1	17.3	35.4	6.68	6.53
120	1	0.225	0.05	27.1	11	17.1	35.4	8.02	7.77
120	0.5	0.075	0.05	18	14.3	14.9	30.8	1.83	2.3
120	0.5	0.1125	0.05	20.7	12.5	14.6	30.8	4.1	3.44
120	0.5	0.15	0.05	22.8	11	14.4	30.8	5.91	4.57
120	0.5	0.1875	0.05	24.7	9.78	14.2	30.8	7.45	5.67
120	0.5	0.225	0.05	26.3	8.69	14.1	30.8	8.8	6.76
120	3	0.0375	0.1	26.5	26.3	27.2	53.7	0.09	2.01
120	3	0.075	0.1	30.4	23.8	26.9	53.7	3.29	4.01
120	3	0.1125	0.1	33.1	21.9	26.6	53.7	5.56	6
120	3	0.15	0.1	35.2	20.5	26.4	53.7	7.37	7.96
120	3	0.1875	0.1	37.1	19.2	26.2	53.7	8.91	9.89
120	3	0.225	0.1	38.7	18.1	26.1	53.7	10.3	11.8
120	2.5	0.1125	0.1	29.1	21.1	25.4	50.1	3.98	5.6
120	2.5	0.15	0.1	31.3	19.7	25.2	50.1	5.8	7.42
120	2.5	0.1875	0.1	33.1	18.4	25	50.1	7.34	9.23
120	2.5	0.225	0.1	34.7	17.3	24.8	50.1	8.69	11
120	2	0.15	0.1	28.1	18.7	23.7	46.5	4.68	6.9
120	2	0.1875	0.1	29.9	17.5	23.5	46.5	6.22	8.58
120	2	0.225	0.1	31.5	16.4	23.4	46.5	7.57	10.2
120	1.5	0.15	0.1	25.6	17.5	22	43	4.05	6.37
120	1.5	0.1875	0.1	27.5	16.3	21.8	43	5.59	7.92
120	1.5	0.225	0.1	29.1	15.2	21.6	43	6.94	9.43
120	1	0.15	0.1	23.9	15.9	19.7	39	3.99	5.79
120	1	0.1875	0.1	25.7	14.7	19.5	39	5.53	7.19
120	1	0.225	0.1	27.4	13.6	19.4	39	6.88	8.57
120	0.5	0.1125	0.1	20.7	14.8	16.5	33.9	2.96	3.78
120	0.5	0.15	0.1	22.9	13.4	16.3	33.9	4.78	5.02
120	0.5	0.1875	0.1	24.7	12.1	16.1	33.9	6.32	6.24
120	0.5	0.225	0.1	26.4	11	16	33.9	7.67	7.43
120	0.5	0.1125	0.1	25.4	12.8	13.8	32.7	6.27	3.65
120	0.5	0.1875	0.1	29.7	9.98	13.2	32.7	9.86	6.02
120	0.5	0.225	0.1	31.4	8.83	13	32.7	11.3	7.17