Calculations Of Coulomb Collisions Time For Plasma Particles inside fusion Reactors

حسابات زمن التصادمات الكولومي لجسيمات البلازما داخل مفاعلات الاندماج

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Abstract:

In this paper study the coulomb collisions among plasma particles, using Maxwellian Distribution to reach of collisions time equations for electron, ion and transfer energy between the electron and ion inside plasma by indication of Coulomb Logarithm (Ln Λ). The coulomb collisions time calculations are necessary for the fusion reactors because Collisions between ions and electrons give rise to an electrical resistance which leads to ohmic heating of the plasma. Because of the collisions the particles and energy transfer leading to losses in the plasma. In order to control on the fusion reaction which happen inside the ionization medium (the plasma) must be control the temperatures to reach the max. possible period of energy confinement.

الخلاصة:

في هذا البحث تم دراسة التصادمات الكولومية بين جسيمات البلازما، وباستخدام معادلة توزيع ماكسويل تم التوصل الى معادلات زمن التصادم للإلكترون وللايون وزمن انتقال الطاقة بينهما داخل البلازما بدلالة لوغاريتم كولوم (LnΛ). ان عملية حساب هذا الزمن ضروري في مفاعلات الاندماج بسبب كون التصادمات بين الايونات والالكترونات تعطي زيادة للمقاومة الكهربائية والتي تؤدي الى التسخين الاومي للبلازما. بسبب التصادمات الجسيمات والطاقة ستنتقل فتقود الى خسائر في البلازما. من اجل السيطرة على التفاعل الاندماجي الحاصل داخل الوسط المؤين (البلازما) يجب علينا السيطرة على درجات الحرارة للوصول الى أطول فترة ممكنة لاحتواء الطاقة.

Introduction:

A plasma is an ionized gas composed of ions, electrons and neutral particles. These two components are strongly coupled because any substantial separation of charges within the plasma leads to a very large restoring force. Such separation can therefore only occur over short lengths. Small charge separations do arise as results of thermal fluctuations. In a plasma with electron density (n_e) and temperature (T_e) the thermal energy density of electrons per degree of freedom is $(1/2\ n_e\ T_e)$. if this compared to the electrostatic energy density resulting from a separation of charge over a length (d), that is $(1/2\ \varepsilon_0 E^2 \approx 1/2\ \varepsilon_0 (n_e\ ed/\ \varepsilon_0)^2$, its seen that substantial separation of charge can only occur over length up to $d \approx \lambda_D$ where (λ_D) Is the Debye length. Over lengths much larger than this the average electron and ion charge densities are held almost equal the electric field of individual particles is shielded over distances much larger than (λ_D), the process being called Debye shielding. For $(T_e=1keV)$ and $(n_e=10^{20}\ m^{-3})$ the Debye length $(\lambda_D=0.024\ mm)$.

When the plasma is in a magnetic field the individual particles are constrained in their motion. They are free to move parallel to the magnetic field but perpendicular to the field they gyrate in Larmor orbits. In fusion reactor the ion orbits typically have a radius of a few millimeters and the electron orbits are smaller by the square root of the mass ratio. Although the precise

behavior of the plasma is determined by the motion of the individual particles in the local electromagnetic field the constraints on the particle motions described above give the plasma fluid-like properties on lengths larger than the Larmor radii **[1]**.

Many processes in the plasma are determined by particle collisions. Typically ion collision times are in the range *(1-100 ms)*. Electron collision times are shorter by the square root of the mass ratio. Collision times increase with increasing temperature varying as *($T^{3/2}$)*. As a consequence ohmic heating becomes inefficient at high temperatures. On the other hand Collisional plasma losses are reduced **[1]**.

The basic behavior of fusion reactor plasma is poorly understood. The energy loss substantially exceeds that predicated on the basis of simple collisions and this not explained. Its widely believed that the anomaly is due to small scale plasma instabilities. The plasma behavior is also strongly influenced by impurities which enter the plasma from the surrounding material. In this work we studying coulomb collisions between plasma particles, by use of Maxwellian Distribution to reach of collisions time equations for electron, ion and transfer energy between the electron and ion inside plasma by indication of Coulomb Logarithm *(Ln Λ)*. The calculations of coulomb collision times, electron collision time *(τ_e),* ion collision time *(τ_i),*exchanging time *(τ_{ie}),*proton collision time *(τ_P),*tritium collision time *(τ_t),*deuterium collision time*(τ_d).* procedure is necessary for the fusion reactors because Collisions between ions and electrons give rise to an electrical resistance which leads to ohmic heating of the plasma. Because of the collisions the particles and energy transfer leading to losses in the plasma. In order to control on the fusion reaction which happen inside the ionization medium (the plasma) must be control the temperatures to reach the max. possible period of energy confinement **[2]**. The results arranged in tables *(1,2,3,4)*. As a comparison we compare between the calculated collision times *(Theo.)* and the collision times values taken from reference *(ref.**[6]**)*, the results was arranged in table *(5)*.

Theory:

If we consider a test particle with mass *(m_1)*, velocity ($\vec{v}_1$), charge *(e_1)*, and momentum ($\vec{P} = m_1\vec{v}_1$) interacting with field particles having mass *(m_2)*, velocity ($\vec{v}_2$), charge *(e_2)* and momentum ($\vec{P}* = m_2\vec{v}_2$). By definition, **[2]**:

Relative velocity $$\vec{u} \equiv \vec{v}_1 - \vec{v}_2 \quad \ldots\ldots(1)$$

Reduced mass $$m_r \equiv \frac{m_1 m_2}{m_1 + m_2} \quad \ldots\ldots(2)$$

Center-of-mass velocity $$\vec{v}_C \equiv \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} \quad \ldots\ldots(3)$$

The rate of change of test particle momentum may be written:

$$\frac{d\vec{P}}{dt} = (\textit{collision frequency})(\textit{change in } \vec{P} \textit{ per collision}) \ldots\ldots (4)$$

In small differential volume (dv_2) of field particle velocity space, this may be written:

$$\frac{d\vec{P}}{dt} = (dn* \quad u d\sigma)(\delta\vec{P}) \ldots\ldots(5)$$

Where [$dn* = f(\vec{v}_2)d\vec{v}_2$] is the number of field particles per unit volume, *($d\sigma$)* is the differential cross section for coulomb scattering, and ($\delta\vec{P}$) is the change in momentum per collision. The

differential scattering cross section is a function of scattering angle and relative velocity *(u)*. Expressing the cross section in terms of differential solid angle *(dΩ):*

$$d\sigma = \frac{d\sigma}{d\Omega} d\Omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

the total rate of change of momentum by integrating over all field particle velocities and scattering angles **[2]**:

$$\frac{d\vec{P}}{dt} = \int d\vec{v}_2 \int d\Omega \; f(\vec{v}_2)\, u \frac{d\sigma}{d\Omega} \delta\vec{P} \ldots\ldots\ldots\ldots\ldots (7)$$

Similarly, the rate of change of test particle kinetic energy is given by:

$$\frac{dW}{dt} = \int d\vec{v}_2 \int d\Omega \; f(\vec{v}_2)\, u \frac{d\sigma}{d\Omega} \delta W \ldots\ldots\ldots\ldots (8)$$

Where *(δW)* is the change due to a single collision. These are the basic equations for momentum and energy change due to coulomb collisions. We will derive expressions for $(\delta\vec{P}), (\delta W)$, and the coulomb scattering cross section $(\frac{d\sigma}{d\Omega})$, then evaluate the integrals. From Fig.1 (P-11), to evaluate the differential area *(dA)* on the surface of the sphere and use it in the definition of differential ***Solid Angle(dΩ)*** **[2]*:***

$$d\Omega \equiv \frac{dA}{r^2} = \frac{(r\sin\theta d\phi)(r d\theta)}{r^2} = \sin\theta \, d\theta \, d\phi \ldots\ldots\ldots (9)$$

If azimuthally symmetry exists, then $\int d\phi = 2\pi, \quad and \; d\Omega = 2\pi \sin\theta d\theta.$

To Evaluation Of $(\delta\vec{P}), (\delta W)$***:-*** after a collision the final velocities of the test particle and field particle will be $(\vec{v}_1 + \delta\vec{v}_1) and (\vec{v}_2 + \delta\vec{v}_2)$. From momentum conservation:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1(\vec{v}_1 + \delta\vec{v}_1) + m_2(\vec{v}_2 + \delta\vec{v}_2) \rightarrow m_1\delta\vec{v}_1 = -m_2\delta\vec{v}_2 \ldots (10)$$

In the coordinate system in which *(m_2)* is at rest the velocity of the test particle is equal to $(\vec{u})$, the relative velocity .

From conservation of kinetic energy **[2]**:

$$\frac{1}{2}mu^2 = \frac{1}{2}m(\vec{u} + \delta\vec{u})^2 \rightarrow \vec{u}.\delta\vec{u} + \frac{1}{2}\delta\vec{u}.\delta\vec{u} = 0 \ldots\ldots\ldots (11)$$

Which shows that the magnitude of *(u)* remains unchanged although its direction changes during the collision as illustrated in Fig.2.

From Eqs. (1) and (10) :

$$\delta\vec{u} = \delta\vec{v}_1 - \delta\vec{v}_2 = \delta\vec{v}_1(1 + m_1/m_2) = \delta\vec{v}_1(m/m_r) \rightarrow \delta\vec{v}_1 = (m_r/m)\delta\vec{u} \ldots . (12)$$

Then

$$\delta\vec{P} = m \; \delta\vec{v} = m_r \; \delta\vec{u} \ldots\ldots\ldots\ldots (13)$$

Let the subscripts $||$ *and* $\perp$ denote components parallel and perpendicular to the original direction of motion. Because the scattering is symmetric in azimuthally angle the $(\delta P_\perp)$ component will average to zero during integration over $(d\phi)$, and we only need to find $(\delta P_{||})$.

From Fig.2 (P-11) we find:

$$\delta u_{||} = u\cos\theta - u = -u(1-\cos\theta) = -2u\sin^2(\frac{1}{2}\theta).....(14)$$

From Eqs. *(13) and (14):*

$$\delta\vec{P}_{||} = -2m_r\vec{u}\sin^2(\frac{1}{2}\theta).....(15)$$

The change in kinetic energy is **[2]**:

$$\delta W = \frac{1}{2}m_1(\vec{v}_1+\delta\vec{v}_1)^2 - \frac{1}{2}m_1{v_1}^2 = m_1\vec{v}_1\cdot\delta\vec{v}_1 + \frac{1}{2}m_1\delta\vec{v}_1\cdot\delta\vec{v}_1$$

$$= \frac{m_1(m_1\vec{v}_1+m_2\vec{v}_2)\cdot\delta\vec{v}_1 + m_1m_2(\vec{v}_1-\vec{v}_2).\delta\vec{v}_1}{m_1+m_2} + \frac{1}{2}m_1\delta\vec{v}_1\cdot\delta\vec{v}_1$$

$$= m_1\vec{v}_C\cdot\delta\vec{v}_1 + m_r\vec{u}\cdot\delta\vec{v}_1 + \frac{1}{2}m_1\delta\vec{v}_1\cdot\delta\vec{v}_1$$

$$= m_1\vec{v}_C\cdot\delta\vec{v}_1 + (m_r^2/m_1)(\vec{u}\cdot\delta\vec{u} + \frac{1}{2}\delta\vec{u}\cdot\delta\vec{u})$$

$$= m_1\vec{v}_C\cdot\delta\vec{v}_1$$

$$\Rightarrow \delta W = \vec{v}_C\cdot\delta\vec{P}.....(16)$$

Where Eqs. *(1), (2), (3), (11), (12) and (13)* have been used since ($\vec{v}_C$) is independent of scattering angle it may be taken outside of the ($d\Omega$) integral sign in Eq.(8). As before the ($\delta P_{\perp}$) component averages to Zero, and only ($\delta P_{||}$) needs to be taken into account.

If we consider the coordinate system in which *(m_2)* is at rest as illustrated in Fig.3. The differential scattering cross section for scattering into angles between *(θ)* and *($\theta+d\theta$)* is simply the target area of the ring between *(b) and (b+db):*

$$d\sigma = 2\pi b\,db.....(17)$$

In view of the azimuthally symmetry **[3]**:

$$\frac{d\sigma}{d\Omega} = \left|\frac{2\pi b\,db}{2\pi\sin\theta\,d\theta}\right| = \left|\frac{b\;db}{\sin\theta\,d\theta}\right|.....(18)$$

The absolute value signs have been added to keep ($\frac{d\sigma}{d\Omega}$) positive since *($\frac{db}{d\theta}$ < 0)*, we will derive a relation between the impact parameter *(b)* and the scattering angle *(θ)*. From Newton's law (non-relativistic) and the coulomb force:

$$m_1\frac{d\vec{v}_1}{dt} = \frac{e_1e_2}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3} = -m_2\frac{d\vec{v}_2}{dt}.....(19)$$

Where (r) is defined in Fig.4, then:

$$\frac{d\vec{u}}{dt}=\frac{d\vec{v}_1}{dt}-\frac{d\vec{v}_2}{dt}=\frac{e_1e_2\,\vec{r}}{4\pi\varepsilon_0 r^3}\left[\frac{1}{m_1}+\frac{1}{m_2}\right]=\frac{e_1e_2\,\vec{r}}{4\pi\varepsilon_0 m_r r^3}\text{.....(20)}$$

By symmetry ($\delta\vec{u}$) is in the z-direction and we only need to consider components of the coulomb force in that direction to calculate ($\delta\vec{u}$), then **[2,3]**:

$$\delta u=\int dt\left[\frac{du}{dt}\right]_Z=\int_{-a_0}^{a_0}da\,\frac{dt}{da}\,\frac{du}{dt}\cos a\text{.....(21)}$$

From conservation of angular momentum around the point (m_2):

$$m_1ub=m_1r^2\quad\frac{da}{dt}\rightarrow\frac{dt}{da}=\frac{r^2}{ub}\text{.....(22)}$$

By using Eqs. *(22),* and *(20),* in *(21)*:

$$\delta u=\int_{-a_0}^{a_0}da\,\frac{r^2}{bu}\,\frac{e_1e_2}{4\pi\varepsilon_0 m_r r^2}\cos a=\frac{e_1e_2}{4\pi\varepsilon_0 m_r bu}2\sin a_0\text{.....(23)}$$

From Fig.4 (P-11):

$$2a_0+\theta=\pi\ \rightarrow\ a_0=\frac{\pi}{2}-\frac{\theta}{2}\text{.....(24)}$$

With the aid of a trigonometric identity Eq. (*23)* becomes :

$$\delta u=\frac{e_1e_2}{4\pi\varepsilon_0 m_r bu}2\cos(\frac{1}{2}\theta)\text{.....(25)}$$

From Fig.5 :

$$\frac{1}{2}\delta u=u\sin(\frac{1}{2}\theta)\ \rightarrow\ \delta u=2u\sin(\frac{1}{2}\theta)\text{.....(26)}$$

From Eqs. *(25)* and *(26)*:

$$b=\frac{e_1e_2\ \cos(\frac{1}{2}\theta)}{4\pi\varepsilon_0 m_r u^2}\quad,\quad db=\frac{-e_1e_2\ d\theta}{8\pi\varepsilon_0 m_r u^2\sin^2(\frac{1}{2}\theta)}\text{.....(27)}$$

After substituting these values into Eq.*(18)* and using the identity: *[Sinθ=2sin(1/2 θ) Cos(1/2 θ)]*
We obtain the ***((Rutherford Scattering Cross Section)):***

$$\frac{d\sigma}{d\Omega}=\left[\frac{e_1e_2}{8\pi\varepsilon_0 m_r u^2\sin^2(\frac{1}{2}\theta)}\right]^2\text{.....(28)}$$

To Find Coulomb Logarithm: by symmetry *[(dΩ) =2πsinθ dθ]*. Substituting Eqs. *(15)* and *(28)* into Eq. *(7)*, **[2,3]**:

$$\frac{d\vec{P}}{dt}=\int d\vec{v}_2\int_{\theta_{\min}}^{\pi}2\pi\sin\theta\,d\theta\,f(\vec{v}_2)u\left[\frac{e_1e_2}{8\pi\varepsilon_0 m_r u^2\sin^2(\frac{1}{2}\theta)}\right]^2\left[-2m_r\vec{u}\sin^2(\frac{1}{2}\theta)\right]^2$$

$$=\frac{-e_1^2e_2^2}{4\pi\varepsilon_0^2 m_r}\int d\vec{v}_2\frac{f(\vec{v}_2)\vec{u}}{u^3}\int_{\theta_{\min}}^{\pi}d(\frac{1}{2}\theta)\cot(\frac{1}{2}\theta)$$

$$=\frac{-e_1^2e_2^2}{4\pi\varepsilon_0^2 m_r}\int d\vec{v}_1\frac{f(\vec{v}_2)\vec{u}}{u^3}\ln\frac{1}{\sin(\frac{1}{2}\theta_{\min})}\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

The minimum scattering angle corresponds to the maximum impact parameter.
From Eq.*(27)*:

$$\tan(\frac{1}{2}\theta_{\min})=\frac{e_1e_2}{4\pi\varepsilon_0 m_r u^2 b_{\max}}=\sin(\frac{1}{2}\theta_{\min})\ldots\ldots\ldots\ldots(30)$$

Since the angle is very small, the electrostatic potential in plasma is self-shielded by charged particle motions and decays exponentially with a characteristic scale length called the ***Debye Length- (λ_D)***. therefore its logical to choose:

$$b_{max.}=\lambda_D\ldots\ldots\ldots\ldots\ldots(31)$$

Because of the logarithm function the numerical result would vary little if we chose *($2\lambda_D$)* instead of *(λ_D)*. the logarithmic function is called the ***Coulomb Logarithm(Ln Λ)*** in the form**[4]*:***

$$(\ Ln\Lambda=\frac{4\pi\varepsilon_0 m_e v_p^{\ 2}}{Z_P^2 e^4 N Z_i}S\ )$$ or in the form:

$$Ln\Lambda\equiv\ln\frac{1}{\sin(\frac{1}{2}\theta_{\min})}\approx\ln\left[\frac{4\pi\varepsilon_0 m_r u^2\lambda_D}{e_1e_2}\right]\ldots\ldots\ldots\ldots(32)$$

from above relation: $$\left[b=\frac{e_1e_2}{4\pi\varepsilon_0 m_r u^2}\right]\ldots\ldots\ldots\ldots(33)$$

Then Eq. (32) will become:

$$\ln\Lambda\approx\ln\left[\frac{\lambda_D}{b}\right]\ldots\ldots\ldots\ldots\ldots(34)$$

Introducing the temperature through:

$$\frac{1}{2}m_r u^2=\frac{3}{2}T\ldots\ldots\ldots\ldots\ldots(35)$$

The expression required for Eq. (34) is **[2,4]**:

$$\lambda_D = \left(\frac{\varepsilon_0 T}{n_e e^2} \right)^{1/2} \text{.....(36)}$$

If $(e_1=e_2=e)$, equation (34) needs modification if b is less than $(\lambda/2\pi)$ where (λ) is the **De Broglie Wavelength** (h/mv)**,** this occurs for:

$$\frac{v}{c} \geq |Z_1 Z_2| \alpha \text{.....(37)}$$

Where$(Z_1=e_1/e)$,$(Z_2=e_2/e)$, and the fine structure constant $(\alpha=e^2/2\varepsilon_0 hc=1/137)$.
Using relation (35) the condition (37) for the applicability of the classical formula for singly charged particles become: $(T \leq \frac{m_r c^2}{5\times 10^4})$, since the rest mass energy of electrons,$(m_e c^2 = 0.5MeV)$

Collision Times: the characteristic times for the collision relaxation processes are functions of the velocity of the test particle. Typical values of the times involved may be obtained by taking the test particle to have the average thermal velocity.
Thus putting $(u=v_T)$ and using $(mv_T^2=T)$ its seen that all of the resulting characteristic collision times are proportional to a time having the form:$(\frac{\varepsilon_0^2 m^{1/2} T^{3/2}}{ne^4 \ln\Lambda})$. Test particles have a **Maxwellian Distribution** characterized by temperature (T) we can average over that distribution to find the rate of energy transfer between the two maxwellian distributions if we define **Equilibration Time or *Heat Exchange Time*** (τ_{eq}) by the equation, Let us consider $(m_1 \equiv m_e)$ and $(m_2 \equiv m_i)$ we get **[2,4]**:

$$\frac{dT}{dt} \cong \frac{T_i - T_e}{\tau_{eq}} \Rightarrow \tau_{eq} = \frac{(2\pi^3)^{1/3} 3\varepsilon_0^2 m_i m_e}{n_i e^2 \ln\Lambda} \left[\frac{KT_e}{m_e} + \frac{KT_i}{m_i} \right]^{3/2} \text{.....(38)}$$

Where (m_i) is the ion mass , (m_e) is the electron mass , (T_e) is the electron temperature in (KeV) and (n) is the plasma density (m^{-3}) k-Boltzmann constant.
And its convenient to define collision times with the numerical factors which occur in the calculation of macroscopic quantities such as the electrical conductivity. For a plasma with ions of charge (Z) this lead to an:
Electron Collision Time **[2,5]*:***

$$\tau_e = 3(2\pi)^{3/2} \frac{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}{n_i Z^2 e^4 \ln\Lambda} \text{.....(39)}$$

And an ***Ion Collision Time*** **[2,5]*:***

$$\tau_i = 12\pi^{3/2} \frac{\varepsilon_0^2 m_i^{1/2} T_i^{3/2}}{n_i Z^4 e^4 \ln\Lambda} \text{.....(40)}$$

Where Z-atomic number, The ratio of these times, (τ_e / τ_i) is of order $(m_e/m_i)^{1/2}$ reflecting the faster thermal velocity of the electrons compared with that of the ions. A further characteristic time is that for heat exchange between the electron and ion components of the plasma. The frequency of the collisions involved is determined by the faster component, the electrons. However because of the difference in mass the energy transfer is inefficient only a fraction of order (m_e/m_i) of the electron energy being transferred to the ions. A precise calculation leads to a ***Heat Exchange Time***:

$$(\tau_{ie} = \frac{m_i}{2m_e} \tau_e).$$

From Eqs (32),(34): Some of the references use (($\lambda_e = Ln\Lambda$)) , where (λ_e) -collisional mean free path for electron or (λ_i) for ion. **The Precise Calculations** For These Equations After Calculate The Constants, lead us to the following formulae :

Electron-Electron Collisions ($T_e \geq 10eV$):

$$\ln\Lambda = 14.9 - \frac{1}{2}\ln(n_e/10^{20}) + \ln T_e \quad .....(41)$$

$$where T_e \; in \; KeV$$

From Ref.**[5]**: $Ln\Lambda_{ee} = 23.5 - Ln(n_e^{1/2} T_e^{-5/4}) - [10^{-5} + (LnT_e - 2)^2/16]^{1/2}$ .....(42)

Electron-Ion Collisions ($T_e \geq 10eV$):

$$\ln\Lambda = 15.2 - \frac{1}{2}\ln(n_e/10^{20}) + \ln T_e \quad .....(43)$$

$$where T_e \; in \; KeV$$

From Ref.**[5]**: $Ln\Lambda_{ei} = 23 - Ln(n_e^{1/2} Z T_e^{-3/2})$ .....(44)

Ion-Ion Collisions: singly charged ions, ($T_i \geq 25KeV$):

$$\ln\Lambda = 17.3 - \frac{1}{2}\ln(n_e/10^{20}) + \frac{3}{2}\ln T_i \quad .....(45)$$

$$where T_i \; in \; KeV$$

Expressing *(T)* in (*KeV)* **by calculate the constants** to get the collision times from Eqs. (39),(40) in new forms:

$$\left.\begin{aligned} \tau_e\,(second) &= 1.09\times10^{16}\,\frac{T_e^{3/2}}{n_i Z^2 \ln\Lambda} \\ \tau_i\,(second) &= 6.60\times10^{17}\,\frac{(m_i/m_p)^{1/2}\; T_i^{3/2}}{n_i Z^4 \ln\Lambda} \\ \tau_{ie}\,(second) &= 0.99\times10^{19}\,\frac{(m_i/m_p) T_e^{3/2}}{n_i Z^2 \ln\Lambda} \end{aligned}\right\} ...(46)$$

Where *T* in *KeV*.

The coulomb logarithm for ions is approximated by ***(Ln Λ_i =1.1 Ln Λ),*** to within 10% over the range covered by Fig.5 (P-11), using relations in Eq.(46) give the following approximations when *(T_i=T_e)* from ref.**[6]***:*

$$\left.\begin{aligned} ions(Z=1) \quad & \tau_i \cong \frac{1}{1.1}\left(\frac{2m_i}{m_e}\right)^{1/2}\tau_e \\ protons \quad & \tau_p \cong 55\tau_e \\ deutrons \quad & \tau_d \cong 78\tau_e \\ tritons \quad & \tau_t \cong 95\tau_e \end{aligned}\right\} .....(47)$$

Calculations and Results:

By using the equations (41, 43, 45) the calculation of the coulomb logarithm *(Ln Λ)* for high density *($n_e=10^{19}$, 10^{20}) m^{-3} –Densities Of Fusion Reactors Plasma* ***[7])*** and temperatures reach *(100keV)* for electron-electron collisions, electron-ion collisions and ion-ion collisions as shown in tables (1,2,3). Then from calculate *(Ln Λ)* we can calculate the collision times in side fusion plasma by using the relations in equations (46,47) as shown in table (4) at ionic charge *(Z=1)* and plasma temperature reach *(10keV)*.

Table: (1) For Electron-Electron Collisions ($T_e \geq 10eV$) , The Calculated (Ln Λ)

$n_e = 10^{19}$ (m^{-3})		$n_e = 10^{20}$ (m^{-3})	
T_e (eV)	Ln Λ	T_e (eV)	Ln Λ
10	*11.45*	10	*10.3*
100	*13.75*	100	*12.6*
1000	*16.10*	1000	*14.9*
10000	*18.40*	10000	*17.2*
100000	*20.70*	100000	*19.5*

Table: (2) For Electron-Ion Collisions ($T_e \geq 10eV$) , The Calculated (Ln Λ)

$n_e = 10^{19}$ (m^{-3})		$n_e = 10^{20}$ (m^{-3})	
T_e (eV)	Ln Λ	T_e (eV)	Ln Λ
10	*11.74*	10	*10.6*
100	*14.05*	100	*12.9*
1000	*16.35*	1000	*15.2*
10000	*18.65*	10000	*17.5*
100000	*20.96*	100000	*19.8*

Table:(3) For Ion-Ion Collisions ($T_i \geq 25KeV$), The Calculated (Ln Λ)

$n_e = 10^{19}$ (m^{-3})		$n_e = 10^{20}$ (m^{-3})	
T_i (keV)	Ln Λ	T_i (keV)	Ln Λ
25	*23.28*	25	*22.13*
50	*24.32*	50	*23.17*
75	*24.93*	75	*23.78*
100	*25.35*	100	*24.20*

Table:(4) The Calculated Collisions Times at Z=1

	$n_e = 10^{19}$ m^{-3} [8]				$n_e = 10^{20}$ m^{-3} [8]		
T (eV)	100	1000	10000	T(eV)	100	1000	10000
τ_e	*2.36 μs*	*58.0 μs*	*1.7 ms*	τ_e	*0.27 μs*	*7.20 μs*	*0.20 ms*
τ_i	*0.24 ms*	*0.46 ms*	*0.1 s*	τ_i	*21.0 μs*	*0.54 ms*	*14.0 ms*
τ_{ie}	*4.0 ms*	*0.090 s*	*3.7 s*	τ_{ie}	*0.49 ms*	*13 m s*	*0.37 s*
τ_p	*0.13 ms*	*3.70 ms*	*0.1 s*	τ_p	*14.9 μs*	*0.39 ms*	*11.0 ms*
τ_d	*0.19 ms*	*5.20 ms*	*0.15 s*	τ_d	*21.1 μs*	*0.56 ms*	*15.6 ms*
τ_t	*0.23 ms*	*6.40 ms*	*0.18 s*	τ_t	*25.7 μs*	*0.68 ms*	*0.019 s*

As a comparison we compare between the calculated collision times (Theo.) and the collision times values taken from reference (ref.), the results was arranged in table (5).

Table (5): Theoretical Collision Times (Theo.) Compare With Reference Collision Times Values (Ref.) at ($n_e = 10^{19} m^{-3}$)

T (eV)	τ_e		τ_i		τ_{ie}	
	Theo.	**Ref. [7]**	**Theo.**	**Ref. [6]**	**Theo.**	**Ref. [6]**
100	*2.36 μs*	***2.4 μs***	*0.24 ms*	***0.2 ms***	*4.09 ms*	***4.40 ms***
1000	*58.9 μs*	***67 μs***	*0.46 ms*	***0.5 ms***	*0.090 s*	***0.12 s***
10000	*1.74 ms*	***1.9 ms***	*0.109 s*	***0.13 s***	*3.70 s*	***3.40 s***

Conclusions:

1. Coulomb logarithm ***(Ln Λ)*** depending on temperature and density of electrons as shown in tables (1,2), so coulomb logarithm ***(Ln Λ)*** decrease when density increase, increase when temperature increase, and so that the collisions times increasing when temperature reach 10keV but collision times decrease when density increase as shown in table (4) .
2. From table (3) we see also Coulomb logarithm ***(Ln Λ)*** increase when ion temperature increase, we conclude that Coulomb logarithm ***(Ln Λ)*** constant when the plasma at corona equilibrium ***($T_i = T_e = T_{Plasma}$), ($n_e = n_i = n_{Particles}$).***
3. Electron temperature and density effective more than the ions on the calculations of collisions times where ***($\tau_e < \tau_i$)***, because of the small mass of electron and the higher kinetic energy.
4. Most of the collisions times depending on electron collision time so when***(τ_e)*** increase all the collisions times increase too as shown in table (4).
5. From table (5) the comparison prove that our calculations are so reasonable.
6. From this work we conclude that the temperatures in fusion reactors should be controlled to reach max. plasma (energy) confinement.

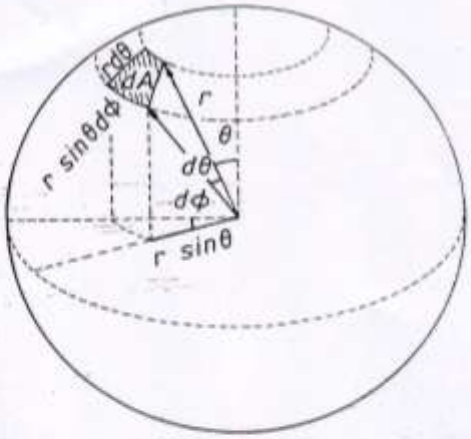

Fig. 1. A differential area on the surface of a sphere subtended by the differential angle dθ . [2]

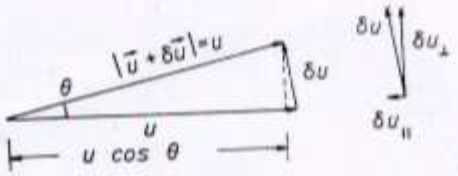

Fig. 2. During a collision the relative velocity $\vec{u}$ changes direction, but not magnitude. [2]

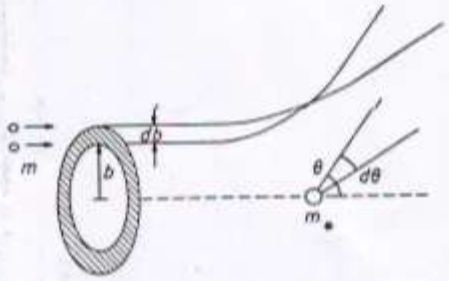

Fig. 3. Hyperbolic trajectories of test particles incident at impact parameters b and b+db, in a coordinate system with m_* at rest. Particles incident at larger radii are scattered through smaller angles θ. [2]

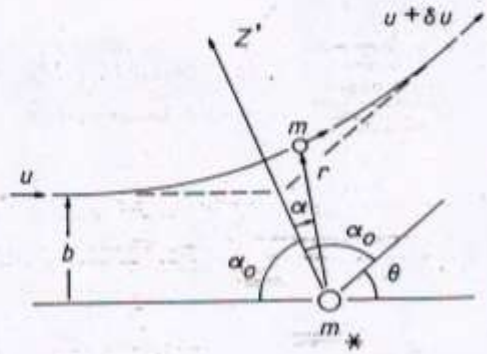

Fig. 4. Definition of radius vector $\vec{r}$, angle α, symmetry direction z', scattering angle θ, and limiting values of α equal to ± α_0. [2]

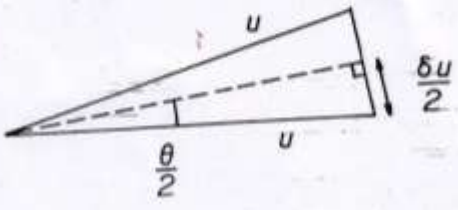

Fig. 5. Vector diagram relating $\delta\vec{u}$ to $\vec{u}$ and θ.

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