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LINE REGULAR FUZZY SEMIGRAPHS

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Abstract:

This paper introduce two types of edge degrees (line degree and near line degree) and total edge degrees (total line degree and total near line degree) of an edge in a fuzzy semigraph, where a fuzzy semigraph is defined as (V, σ, μ, η) defined on a semigraph G^* in which $\sigma : V \rightarrow [0, 1]$, $\mu : VxV \rightarrow [0, 1]$ and $\eta : X \rightarrow [0, 1]$ satisfy the conditions that for all the vertices u, v in the vertex set, $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ and $\eta(e) = \mu(u_1, u_2) \wedge \mu(u_2, u_3) \wedge \ldots \wedge \mu(u_{n-1}, u_n) \leq \sigma(u_1) \wedge \sigma(u_n)$, if $e = (u_1, u_2, \ldots, u_n)$, $n \geq 2$ is an edge in the semigraph G^*, in which a semigraph is defined as a pair of sets (V, X) in which the vertex set V is a non - empty set and edge set X is a set of n – tuples for various $n \geq 2$, of distinct elements of V with the properties that, any two elements in the edge set X has at most one vertex in common and for any two edges $(a_1, a_2,\ldots, a_n)$ and $(b_1, b_2,\ldots, b_m)$ in the edge set X are equal if, and only if, n = m and either one of the conditions $a_j = b_j$ or $a_j = b_{n-j+1}$ occur for j where the value of j lies between 1 and n. In addition to that edge regularities (line regular and near line regular) and total edge regularities (total line regular and total near line regular) of the corresponding edge degrees and total edge degrees are studied, their properties are examined and a few results connecting vertex regularity and edge regularity of a fuzzy semigraph are obtained.
2020 Mathematics Subject Classification: 05C72, 05C07.

Keywords: Fuzzy semigraph, Line degree, Line regular, Near line degree, Semigraph, Total line regular.

Introduction:

The concept of fuzzy graph was pioneered by A. Rosenfeld[1]. A. Nagoor Gani and K. Radha[2] studied a branch of fuzzy graph theory that deals with the regularity and total regularity of vertices in fuzzy graphs, while T. Nusantara et al.[3] explored the idea of edge degree in fuzzy graphs. In order to improve graph theory and include more scenarios, E. Sampathkumar introduced the concept of semigraph theory[4] . A semigraph[4] G with a vertex set V and an edge set X is defined as the pair of sets (V, X) in which $V \neq \varphi$ and X is a set of n - tuples, for various $n \geq 2$, of distinct elements of V satisfy the followings,

1. Any two elements in X has atmost one common vertex,
2. Any two edges $(a_1, a_2,\ldots, a_n)$ and $(b_1, b_2,\ldots, b_m)$ are equal if, and only if, n = m and either one of the below conditions occur for j such that $1 \leq j \leq n$
   a. $a_j = b_j$
   b. $a_j = b_{n-j+1}$

A partial edge[4] in a semigraph G is a subedge of an edge in G in which the consecutive vertices in the edge is again consecutive in that subedge. Semigraphs can be of different kinds. A semigraph in which the cardinality of each edge is same is called a uniform semigraph[4]. Integrating the concepts of fuzzy graph theory and semigraph theory K. Radha and P. Renganathan[5] introduced a novel idea called fuzzy semigraph. Let $G^* = (V, X)$ be a semigraph. Then (V, σ, μ, η) be the fuzzy semigraph[5] defined on G^* in which $\sigma : V \rightarrow [0, 1]$, $\mu : VxV \rightarrow [0, 1]$ and $\eta : X \rightarrow [0, 1]$ satisfy the conditions that

1. $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all the vertices u, v in V,
2. $\eta(e) = \mu(u_1, u_2) \wedge \mu(u_2, u_3) \wedge \ldots \wedge \mu(u_{n-1}, u_n) \leq \sigma(u_1) \wedge \sigma(u_n)$,

if $e = (u_1, u_2, \ldots, u_n)$, $n \geq 2$ is an edge in G^*. Note that $\wedge$ represents the minimum.

The authors introduced various degrees to the vertices of a fuzzy semigraph and communicated the paper to a journal. That paper discusses, edge degree of a vertex u_1 in a fuzzy semigraph (V, σ, μ, η), denoted by $d_e(u_1)$ defined to be $\sum\eta(e)$ where the addition is taken over all of the edges e with the vertex u_1 no matter whether u_1 is an end vertex or a middle vertex. Consecutive adjacent degree of a vertex u_2, denoted by $d_{ca}(u_2)$ defined to be $\sum\mu(u_2, u_3)$ where the addition is taken over all u_3 in V which is consecutively adjacent with u_2 in (V, σ, μ, η). By adding σ value of a vertex to a particular kind of degree of the same vertex gives the total degree of that vertex of the same kind. (V, σ, μ, η) is edge regular (total edge regular) if edge (total edge) degrees are same for each vertex where as the edge degree[4] of an edge e denoted by ed(e) is the number of edges which share a vertex in common with e. This work mainly follows[6] for the terminologies and preliminaries in graph theory, [7] for fuzzy set concepts and [8-11] for fuzzy graph theory.

Results:

Two types of edge degrees, namely line degrees and near line degrees in a fuzzy semigraph are defined.

Definition 1: Consider a fuzzy semigraph G = (V, σ, μ, η) and an edge E = $(u_1, u_2, \ldots, u_m)$ with cardinality m in G. Then the line degree of E, denoted by $d_l(E)$ is the total membership values of the edges which are adjacent to E.

The addition of membership value of the edge E to the line degree of E defines the total line degree of the edge E, denoted by $d_{tl}(E)$.

The total membership values of the partial edges of cardinality 2 which are consecutively adjacent to the edge E is interpreted as the near line degree of the edge E, denoted by $d_{nl}(E)$.

The addition of membership value of the partial edges of cardinality 2 of E to the near line degree of E defines the total near line degree of the edge E, denoted by $d_{tnl}(E)$.

Example 1: Consider the edge E = (u_1, u_2, u_3, u_4) in the fuzzy semigraph G given in Fig. 1. Note that the edges which are adjacent to the edge E are (u_1, u_5, u_6), (u_2, u_7) and (u_8, u_3, u_9) with membership values 0.3, 0.4 and 0.4 respectively and the partial edges of cardinality 2 which are consecutively adjacent to the edge E are (u_1, u_5), (u_2, u_7), (u_3, u_8) and (u_3, u_9) with membership values 0.3, 0.4, 0.4 and 0.5 respectively. The line degree and near line degree of the edge E are,

$d_l(E) = 0.3 + 0.4 + 0.4 = 1.1$ and $d_{nl}(E) = 0.3 + 0.4 + 0.4 + 0.5 = 1.6$

The total line degree and total near line degree of the edge E are,
$d_{tl}(E) = 1.1 + 0.2 = 1.3$ and $d_{tnl}(E) = 1.6 + (0.2 + 0.3 + 0.4) = 2.5$

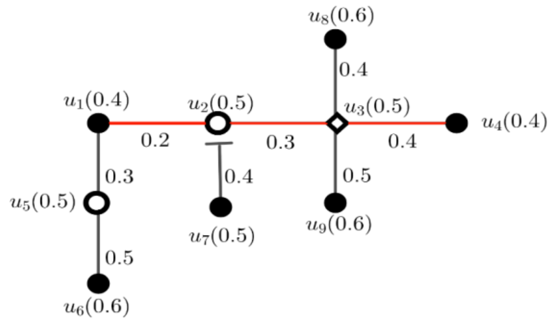

Figure 1. A fuzzy semigraph G

Definition 2: Consider a fuzzy semigraph G = (V, σ, μ, η). If the line degree (near line degree) of each edge in G is same then G is called a line regular (near line regular) fuzzy semigraph.

Similarly if the total line degree (total near line degree) of each edge in G is same, then G is called a total line regular (total near line regular) fuzzy semigraph.

Example 2: Consider the Fig. 2. Here G_1 is line regular and total line regular with regularity 0.8 and 1 respectively in which η value of each edge is 0.2. The fuzzy semigraph G_2 in Fig. 2 is line regular and near line regular with regularity 0.4.

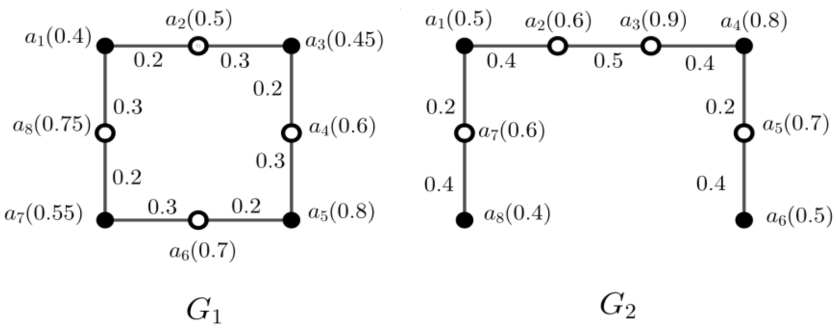

Figure 2. Fuzzy semigraphs G_1 and G_2

Observation 1: Consider an edge $E = (u_1, u_2, \ldots, u_m)$ of cardinality m in a fuzzy semigraph $G = (V, \sigma, \mu, \eta)$ with $G^* = (V, X)$ as the underlying semigraph. Then

1. $d_l(E) = \sum_{v\epsilon E} (d_e(v) - \eta(E))$, where $d_e(v)$ represents the edge degree of the vertex v in G.
2. $d_{nl}(E) = \sum_{v\epsilon E} d_{ca}(v) - 2\sum_i \mu(u_i, u_{i+1})$, where $d_{ca}(v)$ represents the consecutive adjacent degree of the vertex v in G, $1 \le i \le m-1$.
3. $d_{tl}(E) = d_l(E) + \eta(E)$.
4. $d_{tnl}(E) = d_{nl}(E) + \sum_i \mu(u_i, u_{i+1})$, $1 \le i \le m-1$.
5. $d_{tnl}(E) \ge d_{nl}(E) + (m-1)\eta(E)$.
6. It follows from the definitions that there does not exist any general relation among the different types of vertex regularity, line regularity, total vertex regularity and total line regularity of a fuzzy semigraph.
7. $\sum_{E\epsilon X} d_l(E) = \sum_{E\epsilon X} ed(E)\, \eta(E)$.
8. $\sum_{E\epsilon X} d_{tl}(E) = \sum_{E\epsilon X} d_l(E) + S(G)$.

Theorem 1: Consider a line regular fuzzy semigraph G with regularity k where the underlying semigraph of G is $G^* = (V, X)$. Then
$\sum_{E\epsilon X} (\sum_{v\epsilon E} d_e(v)) = \sum_{v\epsilon V} d_e(v) + |X|k$.

Proof: Since G is a line regular fuzzy semigraph with regularity k, $d_l(E) = k$ for each edge E in G. Thus

$\sum_{v\epsilon E} (d_e(v) - \eta(E)) = k$.
This implies $\sum_{v\epsilon E} d_e(v) = k + \sum_{v\epsilon E} \eta(E)$. Taking summation over all the edges E in G,

$$\sum_{E\epsilon X} \sum_{v\epsilon E} d_e(v) = \sum_{E\epsilon X} k + \sum_{E\epsilon X} \sum_{v\epsilon E} \eta(E)$$

$$\sum_{E\epsilon X} \sum_{v\epsilon E} d_e(v) = |X|k + \sum_{E\epsilon X} |E|\eta(E).$$

Thus
$\sum_{E\epsilon X} \sum_{v\epsilon E} d_e(v) = |X|k + \sum_{v\epsilon V} d_e(v)$.
Hence the result.

Theorem 2: Consider a fuzzy semigraph $G = (V, \sigma, \mu, \eta)$ in which the function η is constant and the underlying semigraph is uniform. Suppose G is edge regular fuzzy semigraph then G is both line regular and total line regular.

Proof: Consider G be a k - edge regular fuzzy semigraph whose underlying semigraph is an r - uniform. Let $\eta(E) = c$ for any edge E in G where c is a constant need not be an integer. Let $E = (u_1, u_2, \ldots, u_r)$ be an edge in G. Then

$d_l(E) = d_l(u_1, u_2, \ldots, u_r) = \sum_{v\epsilon E} (d_e(v) - \eta(E)) = r(k - c)$.

Thus, for any edge E in G the line degree is $r(k - c)$. Hence G is a line regular fuzzy semigraph. Since η is constant and G is line regular, the total line degree is $r(k - c) + c$ for any edge in G. Thus G is a total line regular fuzzy semigraph. Hence the result.

The conditions in Theorem 2 is not a sufficient condition for a total edge regular fuzzy semigraph to be line regular or total line regular fuzzy semigraph.

For, consider a 2 - uniform semigraph (a semigraph in which each edge is a 2 - tuple) $G^* = (V, X)$ with $V = \{x_1, x_2, x_3, x_4, x_5\}$ and $X = \{(x_1, x_2), (x_1, x_3), (x_2, x_4), (x_3, x_4), (x_3, x_5), (x_4, x_5)\}$ as in Fig. 3. Now define $\sigma(x_1) = 0.4 = \sigma(x_2) = \sigma(x_5)$, $\sigma(x_4) = 0.2 = \sigma(x_3)$ and membership value, η of each edge is 0.2. Here the 2 – uniform fuzzy semigraph (a fuzzy semigraph in which each edge is a 2 - tuple) is total edge regular with regularity 0.8 but neither line regular nor total line regular.

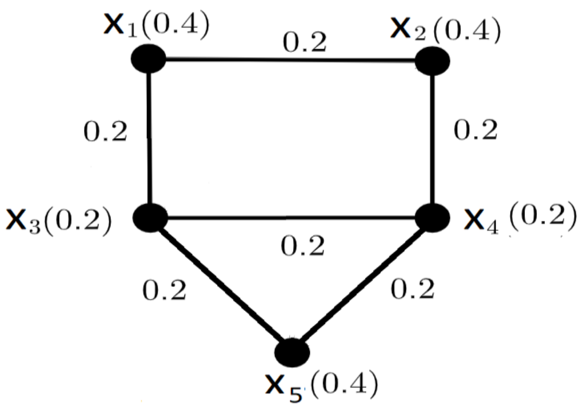

Figure 3. 2 – Uniform fuzzy semigraph

Corollary 1: Consider a fuzzy semigraph $G = (V, \sigma, \mu, \eta)$ which is both edge regular and total edge regular in which each of its edges are effective and the underlying semigraph is uniform. Then G is line regular as well as total line regular.

Proof: Since G is an edge regular and a total edge regular fuzzy semigraph, the function σ is constant. That is G is an effective fuzzy semigraph with a constant σ function. Hence the function η is also constant. Then by Theorem 2 the fuzzy semigraph G is line regular as well as total line regular fuzzy semigraph.

Corollary 2: Consider a fuzzy semigraph $G = (V, \sigma, \mu, \eta)$ in which $G^* = (V, X)$ is the underlying semigraph. In addition G^* is an edge regular uniform semigraph. Then G is both line regular and edge regular fuzzy semigraph if, and only if, the function η is constant.

Proof: Assume that η is a constant function. Since G^* is an edge regular semigraph, G also an edge regular fuzzy semigraph. Then by Theorem 2 the fuzzy semigraph G is line regular.

Conversely assume that G is a k_1 - line regular and k_2 - edge regular fuzzy semigraph. Then for any edge E_i in G,

$d_l(E_i) = \sum_{v\epsilon E_i} (d_e(v) - \eta(E_i)) = k_1$

$\sum_{v\epsilon E_i} (k_2 - \eta(E_i)) = k_1$

Assume that G^* is an r - uniform semigraph. Then

$rk_2 - r\,\eta(E_i) = k_1$

$\eta(E_i) = k_2 - (k_1/r)$.

Here k_1, k_2 and r are fixed for any edge in G. Thus, η is a constant function in G.

Theorem 3: Consider a fuzzy semigraph G = (V, σ, μ, η). Then the following condition are analogous,

1. G is a line regular fuzzy semigraph
2. G is a total line regular fuzzy semigraph

if, and only if, η is a constant function.

Proof: Assume that η is a constant function. Let η(E) = c for an edge E in G where c is a constant need not be an integer. Assume G is line regular. Thus, for any edges E_i and E_j in G, $d_l(E_i) = d_l(E_j)$. Consequently $d_l(E_i) + \eta(E_i) = d_l(E_j) + \eta(E_j)$. That is $d_{tl}(E_i) = d_{tl}(E_j)$ for any edges E_i and E_j. Hence G is a total line regular fuzzy semigraph. Next assume G is total line regular. Hence for any edges E_i and E_j, $d_{tl}(E_i) = d_{tl}(E_j)$, which implies $d_l(E_i) = d_l(E_j)$. That is G is a line regular fuzzy semigraph.

Conversely assume that the given statements are equivalent. Suppose that η is a non-constant function. That is one can find atleast a pair of edges E_i and E_j in G such that $\eta(E_i) \neq \eta(E_j)$. Assume G is line regular. So that the edges E_i and E_j satisfies $d_l(E_i) = d_l(E_j)$. But $d_l(E_i) + \eta(E_i) \neq d_l(E_j) + \eta(E_j)$, which gives G is not a total line regular fuzzy semigraph. Reached a contradiction. Now assume G is total line regular. Here the edges E_i and E_j satisfies $d_{tl}(E_i) = d_{tl}(E_j)$, which implies $d_l(E_i) + \eta(E_i) = d_l(E_j) + \eta(E_j)$. This hold only if $d_l(E_i) \neq d_l(E_j)$, which shows G is not a line regular fuzzy semigraph, again reached a contradiction. Thus, η must be a constant function.

Theorem 4: Consider a fuzzy semigraph G = (V, σ, μ, η). Suppose G is a line regular and a total line regular fuzzy semigraph then the function η is constant. Moreover, the constant value is the difference between regularity and total regularity of G.

Proof: Consider G be a k_1 - line regular and k_2 - total line regular fuzzy semigraph. Then for any edges E_1 and E_2 in G,

$d_{tl}(E_1) = d_l(E_1) + \eta(E_1) = k_2 = d_{tl}(E_2) = d_l(E_2) + \eta(E_2)$.

Also note that $d_l(E_1) = k_1 = d_l(E_2)$. Thus $\eta(E_1) = \eta(E_2) = k_2 - k_1$, where k_1 and k_2 are fixed. Thus η is a constant function and the constant value is the difference between the line regularity and total line regularity of G.

The converse is not true. For, consider the fuzzy semigraph G in the Fig. 4 where the function η, which is the membership value of the edges has a constant value 0.2. Here G is neither line regular nor total line regular.

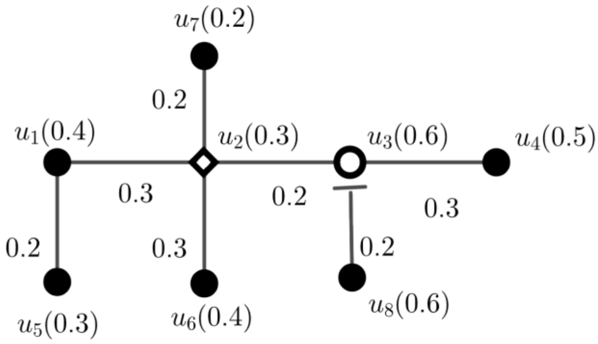

Figure 4. A fuzzy semigraph G

The following corollary is obvious in view of Theorem 4.

Corollary 3: Consider a fuzzy semigraph G = (V, σ, μ, η) in which G is both line regular and total line regular fuzzy semigraph. Then the underlying semigraph of G is edge regular if, and only if, the fuzzy semigraph G itself is edge regular.

Theorem 5: Consider a semigraph G^* = (V, X) and a fuzzy semigraph G = (V, σ, μ, η) defined on G^*. Suppose that G is a k_1 - line regular and k_2 - total line regular fuzzy semigraph. Then size of G is $|X|(k_2 - k_1)$, where size of G is $S(G) = \sum_{E\epsilon X} \eta(E)$.

Proof: Note that $d_{tl}(E) = d_l(E) + \eta(E)$. Taking summation on both sides gives the size of G as,

$S(G) = \sum_{E\epsilon X} \eta(E) = \sum_{E\epsilon X} d_{tl}(E) + \sum_{E\epsilon X} d_l(E) = |X|k_2 - |X|k_1 = |X|(k_2 - k_1)$.

Hence the result.

With the help of 6. and 7. in the Observation 1, the below result holds.

Theorem 6: Consider a fuzzy semigraph G = (V, σ, μ, η) which is k_1 line regular and k_2 total line regular with the k_3 edge regular underlying semigraph G^* = (V, X). Then

$S(G) = |X|(k_1/k_3) = |X|(k_2/(k_3 + 1))$.

Proof: Since $d_l(E) = k_1$ for each edge in G and $\sum_{E\epsilon X} d_l(E) = k_3S(G)$, note that $\sum_{E\epsilon X} k_1 = k_3S(G)$. Thus $S(G) = |X|(k_1/k_3)$. Also $\sum_{E\epsilon X} d_{tl}(E) = (k_3 +1)S(G)$. Hence $\sum_{E\epsilon X} k_2 = (k_3 +1)S(G)$. That is $S(G) = |X|(k_2/(k_3+1))$. Hence the result.

The above can be restructured as,

$K_1 = k_2k_3/(k_3 + 1)$.

Conclusion:

In this research, the line degree, near line degree, total line degree, and total near line degree of a fuzzy semigraph are introduced. Additionally, relevant regularities and total regularities are investigated. Despite the fact that the regularities cannot be generally contrasted, an analysis is carried out by restricting the properties of the fuzzy semigraph.

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Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Besides, the Figures and images, which are not ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Calicut.

Authors’ contributions statement:

A S. wrote the manuscript with support from P K (Supervisor of the results of this work). All authors discussed the results and contributed to the final manuscript.

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شبه البيانيات الضبابية المنتظمة الخطية

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الخلاصة:

تقدم هذه الورقة نوعين من درجات الحافة (الدرجة الخطية والدرجة قريب الخطية) ودرجات الحافة الكلية (الدرجة الكلية الخطية ودرجة قريب الخطية الكلية) للحافة في شبه البيان ضبابي، حيث يُعرّف الخط الضبابي بأنه (V, σ, μ, η) مُعرَّفة على شبه البيان * G حيث $\sigma : V \to [0, 1], \mu : VxV \to [0, 1]$ و $\eta : X \to [0, 1]$ تفي بالشروط التي تناسب جميع الرؤوس u ، v في مجموعة الرأس، $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ و $\eta(e) = \mu(u_1, u_2) \wedge \mu(u_2, u_3) \wedge \ldots \wedge \mu(u_{n-1}, u_n) \leq \sigma(u_1) \wedge \sigma(u_n)$ اذا كانت $e = (u_1, u_2, \ldots, u_n)$, $n \geq 2$ هي حافة في نصف الرسم البياني* G ، حيث يتم تعريف النصف البياني كزوج من المجموعات (V, X) حيث تكون مجموعة الرأس V مجموعة غير فارغة ومجموعة الحافة X عبارة عن مجموعة من n - tuples لمختلف $n \geq 2$ ، من العناصر المميزة لـ V مع الخصائص التي، أي عنصرين في حافة المجموعة X لها رأس مشترك واحد تقريبًا ولأي حافتين $(a_1, a_2, \ldots, a_n)$ و $(b_1, b_2, \ldots, b_m)$ في مجموعة الحافة X متساوية إذا، وفقط إذا، $n = m$ وأيًا منهما أحد الشروط $a_j = b_j$ أو $a_j = b_{n-j+1}$ تتحقق ل r لـ j حيث تقع قيمة j بين 1 و n. بالإضافة إلى انتظام الحواف (الخط المنتظم والخط القريب المنتظم) وإجمالي انتظام الحواف (الخط الكلي المنتظم والإجمالي القريب من الخط المنتظم) لدرجات الحافة المقابلة ودرجات الحافة الكلية التي تمت دراستها، تم فحص خصائصها وتم الحصول على نتائج قليلة تربط انتظام الرأس وانتظام الحافة لشبه البيان الضبابي .

الكلمات المفتاحية: شبه البيان الضبابي، الدرجة الخطية، منتظم خطي، درجة قريب الخطية، شبه بياني، منتظم خطي كلي.