Calculation of the Compton (Incoherent) and Rayleigh (Coherent) Differential Cross sections of Scattering for Vanadium $^{51}V_{23}$, Cobalt $^{59}Co_{27}$ and Nickel $^{59}Ni_{28}$ by employing CSC model.

حساب مساحة المقطع العرضي التفاضلي لاستطارتي كومبتن (غير المتشاكهه) ورايلي (المتشاكه) لعناصر الفناديوم $^{51}V_{23}$ و الكوبالت $^{59}Co_{27}$ والنيكل $^{59}Ni_{28}$ باستخدام نموذج CSC .

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Abstract

In this work, the differential cross section for the elements $^{51}V_{23}$, $^{59}Co_{27}$ and $^{59}Ni_{28}$ has been calculated by using the Mathematical Model named Cross Section Calculations (CSC) .Based on the programming the Klein-Nashina and Rayleigh Equations, atomic form factors as well as the coherent Functions in Fortran90 language Machine proved very fast an accurate results to certain extent and the possibility of application of such model to obtain the total coefficient for any elements or compounds.

الخلاصة :

في هذا البحث تم حساب مساحات المقطع العرضي التفاضلي للعناصر $^{59}Ni_{28}$, $^{59}Co_{27}$, $^{51}V_{23}$ من خلال ستخدام النموذج الرياضي (حسابات مساحات المقطع العرضي)وبالاعتماد على برمجة معادلتي استطارة كلين-ناشينية و رايلي ومعاملي التركيب الذري و الدوال المتشاكه بلغة فورتران 90 والذي اثبت دقة وسرعة عالية في الحصول على النتائج بدقة معينة وامكانية تطبيقه لحصول على معاملات التوهين الكلية لاي عنصر اوسبيكة .

Introduction

There has been more interest in obtaining reliable values of cross section for elements and compounds as well as alloys because its required in Varity of applications in radiography, tomography ,space physics ,plasma physics and etc [1].A review of literature show that the studies concern Geswar [2] measured the total cross section for various elements in energy range of 4.5 to 20keV.Theoretical photoelectric cross section for Z=13 are calculated by Brysk and Zerby [3] in energy range 1 to 150KeV , they employed bound-state wave function that's obtained by Lieberman and Cromer [4] .Hubbell and Berger [5] reported pair production cross sections in the fields of atomic nucleus for 11 elements over range 15 to 100 MeV these values were calculated by using Bon approximation with Bethe –Hitler high energy approximation. In present research, we use the mathematical model Cross section Calculation (CSC) which has more applications.

Theoretical Aspects of CSC model

The various methods of photons interactions with matters allows to utilizes the scattered photons or transmitted through material to determine and calculating the information about the material itself [6] .The probabilities of photons interactions with matter are functions for both incident energy and atomic number Z for material; these probabilities are expressed as a parameter calls cross section express in barns unit(1barn=10^{-28} meter^2) and the Mechanisms of Photon Interaction are described as following:

I- The Coherent (Rayleigh) Scattering differential cross section:

In Raleigh scattering event, a photon scattered off atomic electrons where energies of incident and scattered photons are identical, in this process the recoil energy of absorber atom is negligible and the process occurs at small angle, thus the cross section of Rayleigh depends upon the photon energy E and atomic number Z of the absorber [7]:

$$\sigma_{Rayleigh} \; \alpha \, (Z^{8/3} / E^2) \quad ....(1)$$

And differential cross section is given by [8]:

$$\frac{d\sigma_{Rayleigh}}{d\Omega} = \frac{d\sigma_{Thomson}}{d\Omega} \left[\left(F(x, Z) \right) \right]^2 \quad \ldots\ldots\ldots\ldots (2)$$

is defined as $\frac{d\sigma_{Thomson}}{d\Omega}$ Where $d\Omega$ is the solid angle and the term

The differential cross section of free electron or Thomson cross section [8]:

$$\frac{d\sigma_{Thomson}}{d\Omega} = \frac{1}{2} r_e^{\,2} \left(1 + \cos^2 \phi \right) \quad .... (3)$$

And r_e is the classical radius of electron ($r_e = 2.817\times10^{-15}$m) and F(x,Z) is the atomic form factor will be discussed in next section .So equation (2) can be written as following [9]:

$$\frac{d\sigma_{Thomson}}{d\Omega} = \frac{r^2_{\,e}}{2} \left(1 + \cos^2 \phi \right) \left(F^2(x, Z) \right) \quad \ldots\ldots\ldots\ldots (4)$$

II-Compton (Incoherent) differential Cross section

Compton scattering is an inelastic scattering process in which a photon imparts some of its energy to atomic electrons and is deflected through angle of ,the differential cross section of Compton is proportional to atomic number Z of the absorber and given by Klein-Nashina equation [10]:

$$\frac{d_{Incohernt}}{d\Omega} = \frac{d\sigma_{Klein-Nashina}}{d\Omega} s(q, Z) \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Where $\frac{d\sigma_{Klein-Nashina}}{d\Omega}$ is defined as

$$\frac{d\sigma_{Klein-Nashina}}{d\Omega} = Zr^2_{\,e} \left(\frac{1}{1+\alpha(1-\cos\phi)} \right) \left(\frac{1+\cos^2\phi}{2} \right) \left[1 + \frac{\alpha(1-\cos\phi)^2}{(1+\cos^2\phi)(1+\alpha(1-\cos\theta)} \right] \ldots\ldots (6)$$

These equation gives the probability that a photon is deflected at given angle and transfer some momentum to the electron a thought it were free and $\alpha=hf/m_oc^2$ ((h is Planks constant, f photon frequency and m_oc^2 is the rest mass energy of electron equal to 0.511MeV)) and the term S(q, Z)represent the incoherent scattering function will discussed in next section. The relation between probability and incident energy .

III-The Atomic Form Factor F(x, Z):

The Atomic Form Factor F(x, Z) defined as the amplitude of electron oscillation when interact with incident radiation, the electron in turn oscillates with the same frequency of the incoming wave train and since an accelerating electric charge emits electromagnetic radiation, scattered photon appears to emerged from the interaction site with the same frequency of the oscillating electron, consequently the scattered amplitude (as function of the scattering angle of Z electron. Is not equal to Z times the free electron amplitude; but its modified by atomic form factor F(x, Z) where x is the momentum transfer parameter measured in units of $1/A^0$ defined as :

$$x = (1/\lambda)\sin(\phi/2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Where λ is the incident wavelength and ϕ is the scattering angle the function F(x, Z) is related to the ratio of the amplitude of scattered wave form to scattered amplitude from free electron and given by:

$$[F(x,Z)]^2 = \frac{d\sigma / d\Omega_{Atom}}{d\sigma / d\Omega_{Thomson}} \quad \ldots\ldots\ldots\ldots\ \ldots..(8)$$

The values of F(x,Z) for atoms up to Z=100 published by Hubbell et als [11].By employing the relativistic Hartee-Fock wave function [12] to constructing a table of atomic form Factor.

IV-The Coherent Scattering function S(q,Z):

The function S(q, Z) represent the probability that an atom will be raised to the excited state or ionized state when a photon imparts a recoil momentum (q) to any one of the atomic electrons , Cromer and Mann[13] have calculated the incoherent scattering function S(q,Z) for all spherically symmetric free electrons by using Hartree –Fock -Slater wave function with exchanging term .The values of S(q,Z) and F(x, Z) are feed into series subroutine of Fortran 90 program which all the equations are written in Fortran machine language which gives the results in a very short in of running.

VI-Results and Discussion

This research never contains a comparison the obtained results with any experimental works for similar elements ,this due to that in scattering process the atoms are assumed to be isolated from the influences of the neighbors atoms this is in contrast to the reality because there are unavoidable interactions between various atoms as the molecular and chemical effects are not taken into account thus for example in Compton scattering ,the electron assumed to be bound and not free also initially at rest however in reality the bound electrons in material their momentum gives raise to range of possible energies which is referred as Doppler Broadening and the quantum theory refers that even at zero Kelvin the atoms vibrates in their equilibrium Positions. Hubbell et als [14] estimated the magnitude of the discrepancy between theoretical and experimental K-edge cross section for various elements and compound from Ti to Zn to be in the range of 3% to 12%. The coherent differential cross section has been calculated for the from eq.(2) and their values are given in tables (1,2,3)as shown in figs. 2,4,6 for ${}^{51}V_{23}$, ${}^{59}Co_{27}$ and ${}^{59}Ni_{28}$, respectively by comparison these values we conclude increasing the values as Z increases and decreasing as photons energy increases in accordance with eq.(1) This means that the Coherent scattering differential cross section of Rayleigh never dominates the total cross section but at small angles scattering ((ϕ less than 10^0)) and at photon energies less than 100KeV because the scattering angle become greater and for the incoherent differential cross section which obtained from eq.(5) also proportional to the atomic number Z and the photon incident energy. By noting values of coherent and incoherent differential cross sections in tables (1,2,and 3) the probabilities of coherent decreases with energy increases and vise versa for incoherent (equation1).

Table (1): The values of differential cross section of Coherent and Incoherent for vanadium $^{51}V_{23}$.

E (MeV)	Coh(barn/atom)	Incoh(barn/atom)
0.001	323.3	0.9062
0.015	298.1	1.52
0.002	273.2	2.059
0.003	228.9	3.041
0.004	192	3.936
0.005	161.7	4.734
0.006	137.4	5.442
0.008	103	6.61
0.01	81.26	7.497
0.015	51.5	8.95
0.02	35.44	9.846
0.03	19.27	10.8
0.04	12.17	11.16
0.05	8.453	11.26
0.06	6.236	11.24
0.08	3.793	11.03
0.1	2.54	10.72
0.15	1.196	9.9
0.2	0.692	9.164
0.3	0.316	8.044
0.4	0.1797	7.236
0.5	0.1157	6.622
0.6	0.0806	6.133
0.8	0.04548	5.354
1.00	0.02916	4.854

Table (2): The values of differential cross section of Coherent and Incoherent for Cobalt $^{59}Co_{27}$.

E (MeV)	Coh(barn/atom)	Incoh(barn/atom)
0.001	456	0.786
0.015	428	1.387
0.002	398.3	1.941
0.003	341.7	2.955
0.004	291.7	3.9
0.005	248.7	4.777
0.006	212.7	5.571
0.008	159.1	6.941
0.01	123.6	8.052
0.015	76.17	9.945
0.02	52.76	11.08
0.03	29.19	12.03
0.04	18.4	12.82
0.05	12.74	13
0.06	9.392	13
0.08	5.737	12.79
0.1	3.861	12.46
0.15	1.83	11.54
0.2	1.061	10.71
0.3	0.86	9.42
0.4	0.277	8.481
0.5	0.1785	7.764
0.6	0.1245	7.194
0.8	0.07032	6.33
1.00	0.0451	5.696

Table (3): The values of differential cross section of Coherent and Incoherent for Nickel $^{59}Ni_{28}$.

E (MeV)	Coh(barn/atom)	Incoh(barn/atom)
0.001	492.4	0.7613
0.015	464	1.355
0.002	433.4	1.91
0.003	374	2.927
0.004	321	3.883
0.005	274.7	4.771
0.006	235.6	5.581
0.008	176.5	6.99
0.01	137	8.146
0.015	83.82	10.15
0.02	57.96	11.35
0.03	32.2	12.66
0.04	20.3	13.22
0.05	14.03	13.42
0.06	10.34	13.40
0.08	6.318	13.23
0.1	4.256	12.89
0.15	2.019	11.95
0.2	1.172	11.1
0.3	0.537	9.76
0.4	0.3063	8.791
0.5	0.1975	8.04
0.6	0.1377	7.45
0.8	0.0778	6.564
1.00	0.0499	5.907

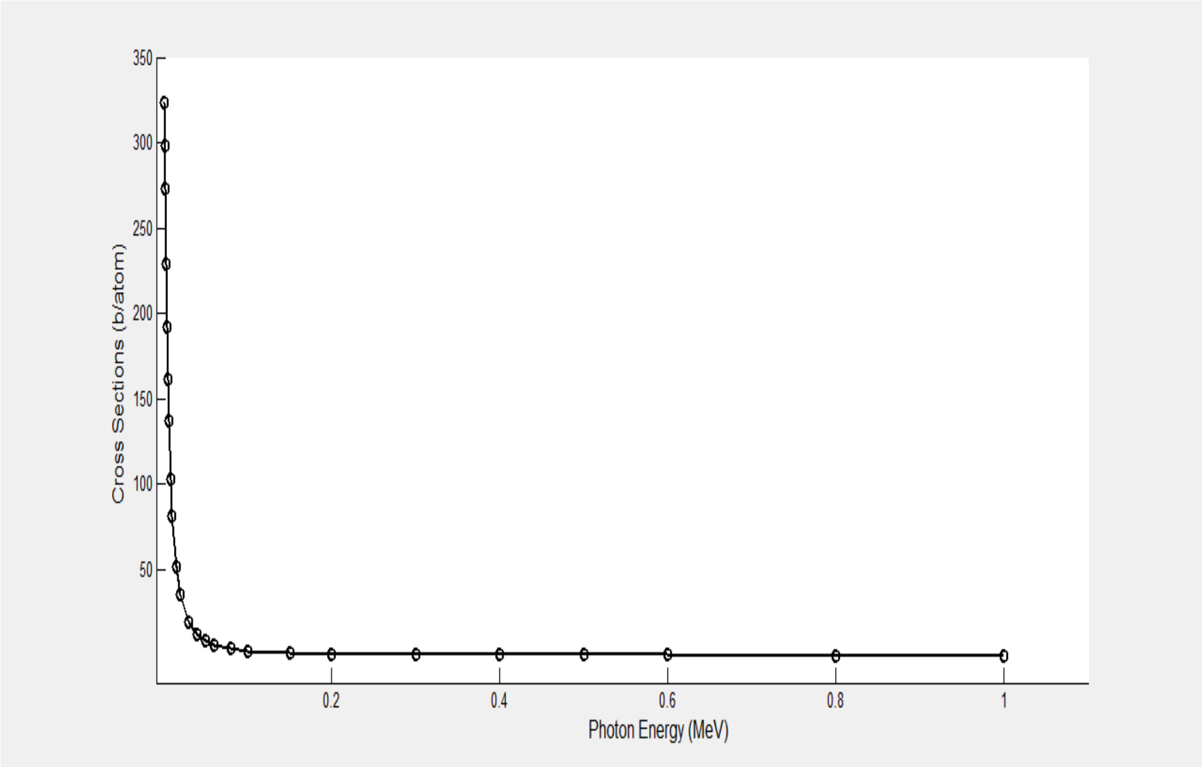

Fig.(2):show the coherent differential cross section versus the photon energies for Vanadium $^{51}V_{23}$.

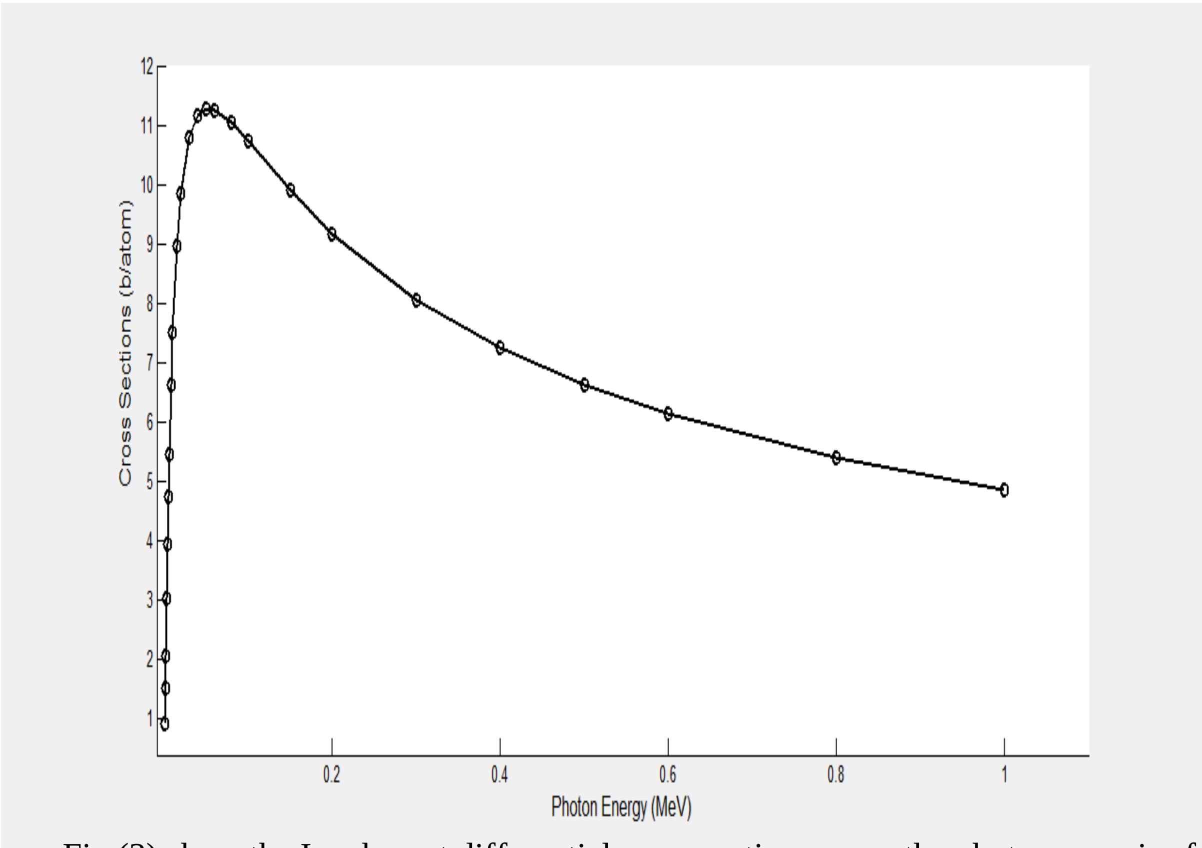

Fig.(3):show the Incoherent differential cross section versus the photon energies for Vanadium $^{51}V_{23}$.

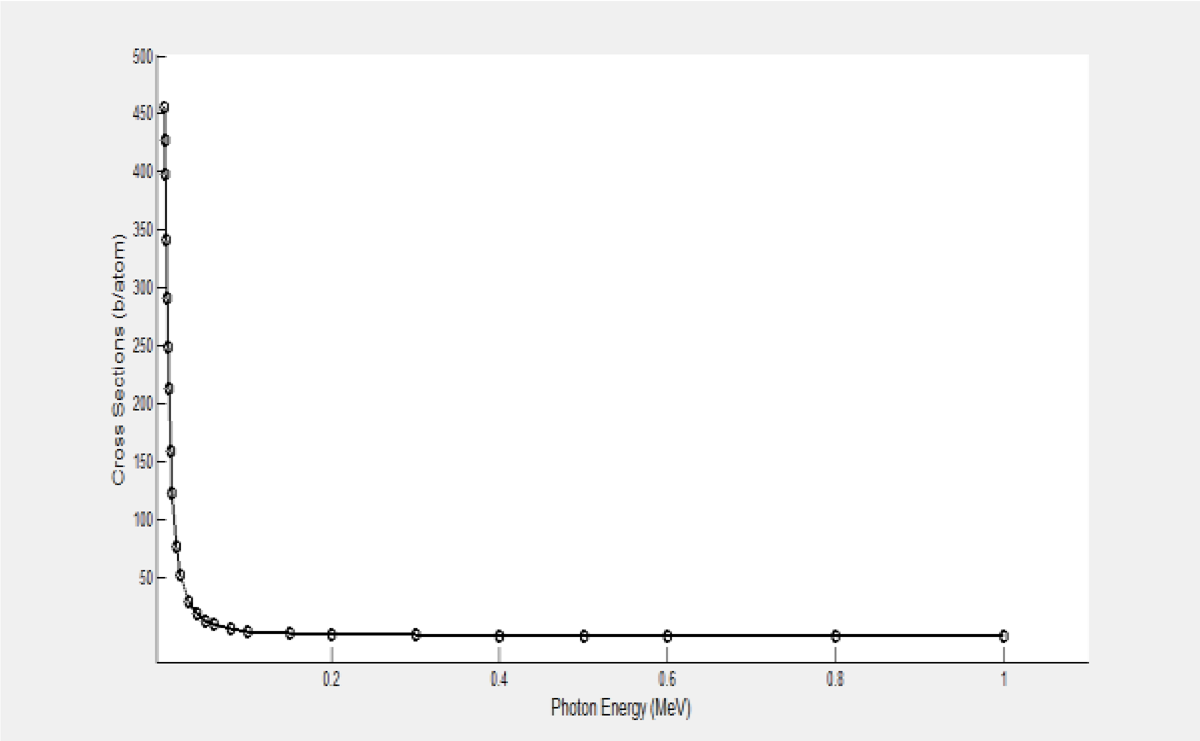

Fig.(4):show the coherent differential cross section versus the photon energies for Cobalt $^{59}Co_{27}$.

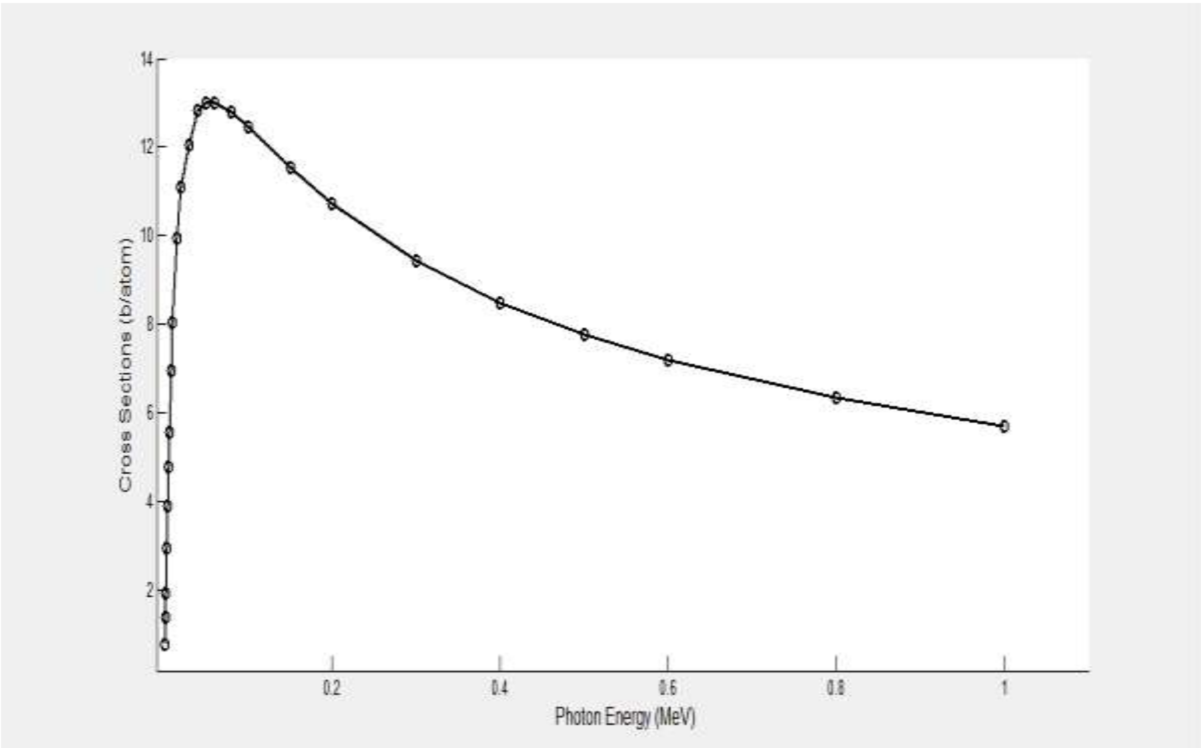

Fig.(5):show the Incoherent differential cross section versus the photon energies for Cobalt $^{59}Co_{27}$.

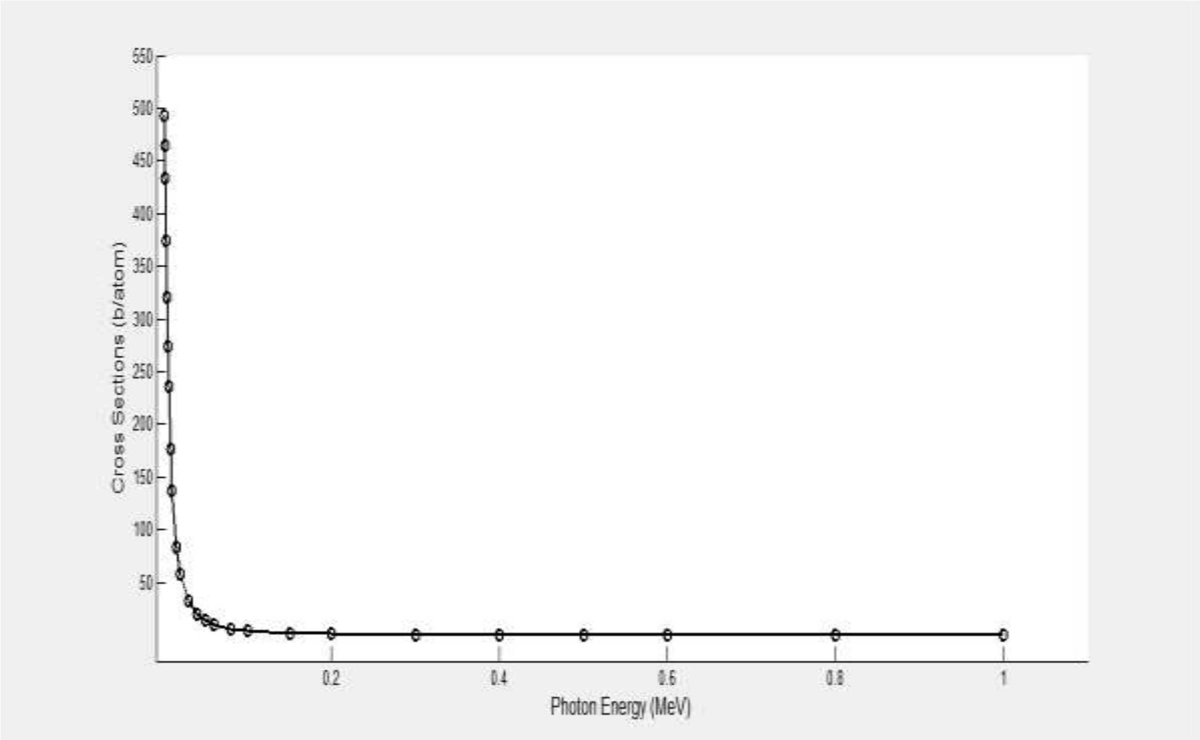

Fig.(6):show the coherent differential cross section versus the photon energies for Nickel $^{59}Ni_{28}$.

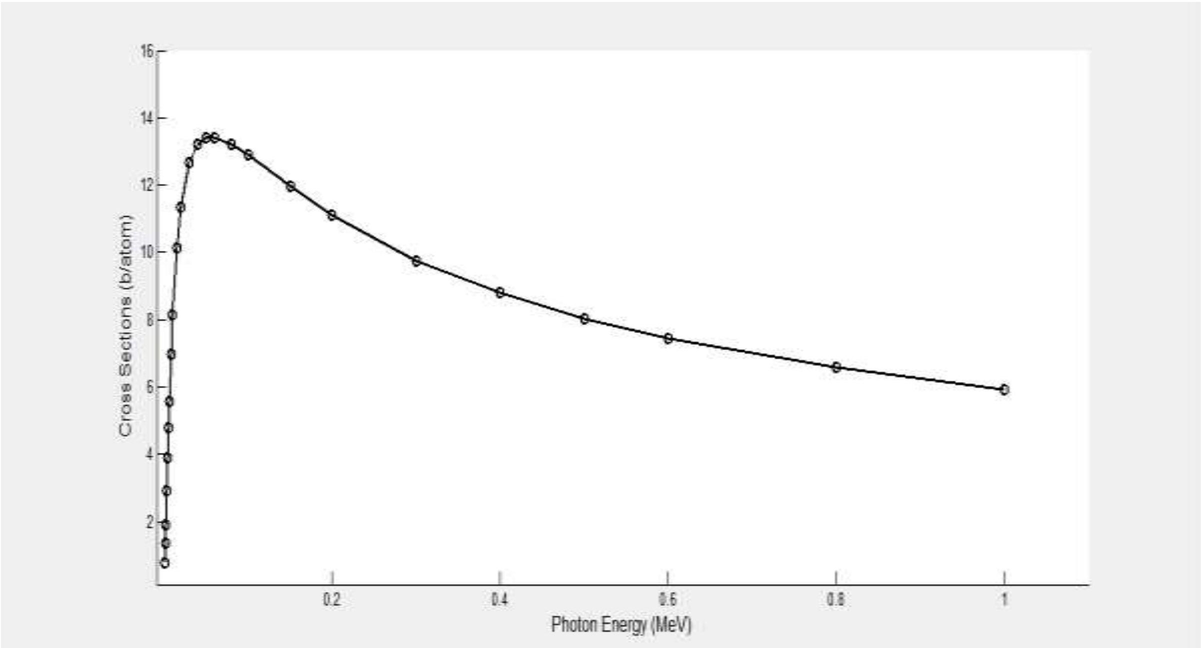

Fig.(7):show the Incoherent differential cross section versus the photon energies for Nickel $^{59}Ni_{28}$.

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