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# حول مقدرات الاختبار الأولي المقلصة ذي المرحلتين المعدلة لتقدير معالم أنموذج الانحدار الخطي البسيط

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## الخلاصة

في هذا البحث اقتراح مقدر الاختبار الأولي المقلص المعدل ذو المرحلتين لتقدير معالم أنموذج الانحدار الخطي البسيط وذلك باختيار عامل تقلص موزون $\psi(\cdot)$ في المرحلة الاولى، فضلا عن اقتراح تقنية معدلة في المرحلة الثانية، واشتقت معادلات التحيز ، متوسط مربعات الخطأ ، والكفاية النسبية ، وحجم العينة المتوقع واحتمالية تجنب المرحلة الثانية (التوقف في المرحلة الاولى) للمقدرات المقترحة، واعطيت بعض النتائج العددية الخاصة بنسبة التحيز $[B(\cdot)]$ والكفاية النسبية $[R.Eff(\cdot)]$ للمقدرات المذكورة ولمختلف الثوابت فيها، ثم اعطيت بعض الخواص لهذه المقدرات وقورنت مع المقدرات الكلاسيكية وكذلك قورنت مع المقدرات المقترحة من بعض الباحثين لبيان كفايتها.

**الكلمات المفتاحية:** – التقدير ، معالم، التقلص، معلومات مسبقة، المقدر ذو المرحلتين، الانحدار الخطي البسيط

# On Modified Pre-Test Double Stage Shrunken Estimators for Estimate the Parameters of Simple Linear Regression Model

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## Abstract

In this paper, we proposed modified preliminary test double stage Shrunken estimator to estimate the parameters of simple linear regression model, using shrinkage weight function $\psi(\cdot)$, in first stage and proposed modify technique in second stage.

The expression for bias, mean squared error, relative efficiency, expected sample size, probability for avoiding the second sample are derived for considered estimators. Numerical results and conclusions concern bias ratio [B(·)] and relative efficiency [R.Eff(·)] are reported for different constants involved in the expressions mentioned above.

**Key words:** Estimation, Parameters, Shrinkage, Prior information, Double stage estimator, Linear Regression

## Introduction

Assume the following simple linear regression model;

$y_i = A + Bx_i + e_i$, $i = 1,2,\ldots,n$ …(1)

Where A and B are parameters of the above model, $x_i$ is an independent variable, $y_i$ is a response variable and $e_i$ is a random error that distribute normally with zero mean and $cov(e_i,e_j)=0$, $i \neq j$, that is; $e_i \sim N(0,\sigma^2)$.

To estimate the parameter $\theta$ ($\theta$ may be refer to A or to B) when a prior estimate $\theta_0$ about $\theta$ available using double stage-Shrinkage technique, we observe $n_1$ sample and calculate the classical estimator $\hat{\theta}_1$ based on the $n_1$ observations. If $\hat{\theta}_1$ implies that our prior estimate was reasonable ($\hat{\theta}_1 \in R$), we stop sampling and shrink $\hat{\theta}_1$ toward $\theta_0$. Otherwise, we observe second sample of size $n_2$ ($n = n_1 + n_2$) and calculate the classical estimator $\hat{\theta}_2$. Furthermore, calculate polling estimator $\hat{\theta}_p$ based on all n observation.

Hence, we introduce a new double stage Shrinkage estimators (modified double stage shrinkage estimator with the following form;-

$$\tilde{\theta} = \begin{cases} \psi(\hat{\theta})(\hat{\theta}_1 - \theta_0) + \theta_0 & , \text{if } \hat{\theta}_1 \in R \\ (\hat{\theta}_p + \theta_0)/2 & , \text{if } \hat{\theta}_1 \notin R \end{cases} \quad \ldots(2)$$

where $\hat{\theta}_p = \dfrac{n\hat{\theta}_1 + n_2\hat{\theta}_2}{n}$, and R is a pre-test region of acceptance of sizes $\alpha$ for testing the hypothesis $H_0 : \theta = \theta_0$ against the hypothesis $H_1 : \theta \neq \theta_0$.

The aim of this paper is to use a preliminary test double stage shrunken estimator define in (2) to estimate the parameters A and B for simple linear regression model (1).

The expressions for bias, mean square error and relative efficiency, expected sample size and probability of avoiding the second sample of considered estimator are given.

Numerical results are presented for above expressions. These results are compared with the last studies in the sense of MSE and relative efficiency.

Noted that, the general double stage shrinkage estimator (DSSE) has the following form:

$$\tilde{\theta} = \begin{cases} \psi_1(\hat{\theta}_1)(\hat{\theta}_1 - \theta_0) + \theta_0 & , \text{if } \hat{\theta}_1 \in R \\ \hat{\theta}_p & , \text{if } \hat{\theta}_1 \notin R \end{cases} \quad \ldots(3)$$

Many authors have studied the double stage shrunken estimator (DSSE) in (3); for example see Katti [1962], Al-Bayyati and Arnold [1972], Waikar Schuurmann and Raghunathan [1984], Kambo, Handa, and Al-Hemyari [1988], Al-Hemyari [1990], Al-Nazzal [1996], Al-Kanane [1997], and Kalaf [2007].

**Modified Preliminary Test Double Stage Shrunken Estimators $\tilde{A}$ and $\tilde{B}$**

In this section, we have to estimate the parameters A and B using modified method (2). The preliminary double stage shrunken estimators $\tilde{A}$ and $\tilde{B}$ (for estimate A and B respectively), have the forms below:

$$\tilde{A} = \begin{cases} k_1(\hat{A}_1 - A_0) + A_0 & , \text{if } \hat{A}_1 \in R_1 \\ \frac{1}{2}(\hat{A}_p + A_0) & , \text{if } \hat{A}_1 \notin R_1 \end{cases} \quad \ldots(4)$$

and

$$\tilde{B} = \begin{cases} k_2(\hat{B}_1 - B_0) + B_0 & , \text{if } \hat{B}_1 \in R_2 \\ \frac{1}{2}(\hat{B}_p + B_0) & , \text{if } \hat{B}_1 \notin R_2 \end{cases} \quad \ldots(5)$$

Where $k_i$ (i =1,2) is shrinkage weight factor, such that $0 \le k_i \le 1$, which is found by minimizing the MSE of $\tilde{A}$ and $\tilde{B}$ respectively.

$\hat{A}_p = \dfrac{n_1\hat{A}_1 + n_2\hat{A}_2}{n_1 + n_2}$, $\hat{B}_p = \dfrac{n_1\hat{B}_1 + n_2\hat{B}_2}{n_1 + n_2}$; $A_0$ and $B_0$ are prior information about A and B respectively, $R_1$ and $R_2$ are pre-test regions of acceptance of size ($\alpha$) for testing the hypothesis $H_{01}: A = A_0$ against the hypothesis $H_{11}: A \neq A_0$ and $H_{02}: B = B_0$ against the hypothesis $H_{12}: B \neq B_0$ respectively,
in that,

$$R_1 = \left[ A_0 - t_{1-\frac{\alpha}{2}, n_1-2}\sqrt{\frac{\sigma^2 \sum x_{1i}^2}{n_1 SS_{x_1}}}, A_0 + t_{1-\frac{\alpha}{2}, n_1-2}\sqrt{\frac{\sigma^2 \sum x_{1i}^2}{n_1 SS_{x_1}}} \right] \quad \ldots(6)$$

$$R_2 = \left[ B_0 - t_{1-\frac{\alpha}{2}, n-2}\sqrt{\frac{\sigma^2}{SS_{x_1}}}, B_0 + t_{1-\frac{\alpha}{2}, n-2}\sqrt{\frac{\sigma^2}{SS_{x_1}}} \right] \quad \ldots(7)$$

It is known that:

$\hat{A}_1 = \bar{y} - \hat{B}_1\bar{x}$ and $\hat{B}_1 = \dfrac{\sum_{j=1}^{n_1}(x_{1j} - \bar{x})(y_i - \bar{y})}{\sum_{j=1}^{n_1}(x_{1j} - \bar{x})^2}$ are unbiased estimators of A and B respectively see [1], [3].

Also,

$$\hat{A}_1 \sim N(A, \frac{\sigma^2 \sum_{i=1}^{n_1} x_{1i}^2}{n_1 SS_x}), \hat{B}_1 \sim N(B, \frac{\sigma^2}{SS_x})$$

where $SS_x = \sum_{i=1}^{n}(x_i - \bar{x})^2$ , $SS_{x_1} = \sum_{j=1}^{n_1}(x_{1j} - \bar{x})^2$ and $SS_{x_2} = \sum_{j=1}^{n_2}(x_{2j} - \bar{x})^2$ see [1], [3].

The expressions for bias and MSE of estimators $\tilde{A}$ and $\tilde{B}$ respectively given by:

$$\text{Bias}(\tilde{A}|A,R_1) = E(\tilde{A} - A)$$

$$= \int_{\hat{A}_2=-\infty}^{\infty} \int_{\hat{A}_1 \in R} \left[k_1(\hat{A}_1 - A_0) + A_0 - A\right] f_1(\hat{A}_1) f_2(\hat{A}_2) d\hat{A}_1 d\hat{A}_2 +$$

$$\int_{\hat{A}_2=-\infty}^{\infty} \int_{\hat{A}_1 \notin R} \left[\frac{1}{2}(\hat{A}_p + A_0) - A\right] f_1(\hat{A}_1) f_2(\hat{A}_2) d\hat{A}_1 d\hat{A}_2$$

$$= \sqrt{\text{var}(\hat{A}_1)}\left[\lambda_1(1-k_1) + \frac{1}{2}\lambda_1 - \frac{n_1}{2n}J_1(a,b) - \frac{1}{2}\lambda_1 J_0(a,b)\right], 0 \le k_1 \le 1 \quad \ldots(8)$$

where $f(\hat{A}_i) = \frac{1}{\sqrt{2\pi\left(\frac{\sigma^2 \sum_{i=1}^{n_1} x_i^2}{n_1 SS_x}\right)}} \text{Exp}\left[-\left(\frac{n_1 SS_x (\hat{A}_i - A)^2}{2\sigma^2 \sum_{i=1}^{n_1} x_i^2}\right)\right]$ ,for $-\infty < \hat{A} < \infty$

and

$$\text{Bias}(\tilde{B}|B,R_2) = E(\tilde{B} - B)$$

$$= \sqrt{\text{var}(\hat{B}_1)}\left[\lambda_2(1-k_2) + \frac{1}{2}\lambda_2 - \frac{n_1}{2n}J_1^*(c,d) - \frac{1}{2}\lambda_2 J_0^*(c,d)\right], 0 \le k_2 \le 1 \quad \ldots(9)$$

where

$$J_i(a,b) = \int_a^b \frac{1}{\sqrt{2\pi}}(Z)^i e^{-\frac{z^2}{2}} dZ, i = 0,1,2 \quad \ldots(10)$$

$$J_i^*(c,d) = \int_c^d \frac{1}{\sqrt{2\pi}}(Z^*)^i e^{-\frac{z^{*2}}{2}} dZ^*, i = 0,1,2 \quad \ldots(11)$$

$Z = (\hat{A}_1 - A)\Big/\sqrt{\text{var}(\hat{A}_1)}$ , $Z^* = (\hat{B}_1 - B)\Big/\sqrt{\text{var}(\hat{B}_1)}$ and

$a = -\lambda_1 - t_{1-\frac{\alpha}{2}, n-2}$,

$b = -\lambda_1 - t_{1-\frac{\alpha}{2}, n-2}$,

$c = -\lambda_2 - t_{1-\frac{\alpha}{2}, n-2}$,

$d = -\lambda_2 - t_{1-\frac{\alpha}{2}, n-2}$.

and $R_1^* = [a,b]$, and $R_2^* = [c,d]$.

Bias Ratio (B.R.) of $\tilde{A}$:

$$B.R(\tilde{A}|A,R_1)=\frac{Bias(\tilde{A}|A,R_1)}{\sqrt{var(\hat{A}_1)}}$$

and

Bias Ratio (B.R.) of $\tilde{B}$:

$$B.R(\tilde{B}|B,R_2)=\frac{Bias(\tilde{B}|B,R_2)}{\sqrt{var(\hat{B}_1)}}.$$

The expressions for mean squared error of $\tilde{A}$ and $\tilde{B}$ are respectively given as below:

$$MSE(\tilde{A}|A,R_1)=E\left[(\tilde{A}-A)^2\right]$$

$$=var(\hat{A}_1)\Big\{\Big[k_1^2J_2(a,b)+\Big\{\Big[k_1^2\lambda_1J_0(a,b)-2k_1^2\lambda_1J_1(a,b)+\lambda_1^2J_0(a,b)+2k_1\lambda_1J_1(a,b)-$$

$$2k_1\lambda_1^2J_0(a,b)\Big]+\frac{1}{4}\left(\frac{n_1}{n}\right)^2+\frac{1}{4}\left(\frac{n_2}{n}\right)^2w+\frac{1}{4}\lambda_1^2-\left[\left(\frac{n_1}{n}\right)^2J_2(a,b)+\right.$$

$$\left.\left(\frac{n_2}{n}\right)^2J_0(a,b)w+\lambda_1^2J_0(a,b)+2\left(\frac{n_1}{n}\right)\lambda_1J_1(a,b)\right]\Big\} \quad ...(12)$$

here

$$w=\frac{var(\hat{A}_2)}{var(\hat{A}_1)}=\frac{\sigma^2\sum_{j=1}^{n_2}x_{2j}^2/n_2SS_{x_2}}{\sigma^2\sum_{j=1}^{n_1}x_{1j}^2/n_1SS_{x_1}}=\frac{\sum_{j=1}^{n_2}x_{2j}^2}{\sum_{j=1}^{n_1}x_{1j}^2}\cdot\frac{SS_{x_2}}{SS_{x_1}}\cdot\frac{n_1}{n_2}=\frac{d_2}{d_1}\cdot\frac{h_1}{h_2}\cdot\frac{1}{u},\qquad d_p=\sum_{i=1}^{n_p}x_{pi}^2,\ h_p=SS_{x_p},$$

$SS_{x_p}=\sum_{i=1}^{n_p}(x_{p_i}-\bar{x})^2,\ p=1,2$. Also, $\frac{n_2}{n_1}=u,\ \frac{n_1}{n}=\frac{1}{1+u},\ \frac{n_2}{n}=\frac{u}{1+u}.$

Therefore;

$$MSE(\tilde{A}|A,R_1)=var(\hat{A}_1)\Big\{\ k_1^2J_2(a,b)+k_1^2\lambda_1J_0(a,b)-2k_1^2\lambda_1J_1(a,b)+\lambda_1^2J_0(a,b)+2k_1\lambda_1J_1(a,b)-$$

$$2k_1\lambda_1^2J_0(a,b)+\frac{1}{4}\left(\frac{1}{1+u}\right)^2+\frac{1}{4}\left(\frac{u}{1+u}\right)^2w+\frac{1}{4}\lambda_1^2-\left[\left(\frac{1}{1+u}\right)^2J_2(a,b)+\right.$$

$$\left.\left(\frac{u}{1+u}\right)^2J_0(a,b)w+\lambda_1^2J_0(a,b)+2\left(\frac{1}{1+u}\right)\lambda_1J_1(a,b)\right]\Big\} \quad ...(13)$$

Similarly,

$$MSE(\tilde{B}|B,R_2)=E[(\tilde{B}-B)^2]$$

$$=var(\hat{B}_1)\Big\{\ k_2^2J_2^*(c,d)+k_2^2\lambda_2J_0^*(c,d)-2k_2^2\lambda_2J_1^*(c,d)+\lambda_2^2J_0^*(c,d)+2k_2\lambda_2J_1^*(c,d)-$$

$$2k_2\lambda_2^2J_0^*(c,d)+\frac{1}{4}\left(\frac{1}{1+u}\right)^2+\frac{1}{4}\left(\frac{u}{1+u}\right)^2w^*+\frac{1}{4}\lambda_2^2-\left[\left(\frac{1}{1+u}\right)^2J_2^*(c,d)+\right.$$

$$\left.\left(\frac{u}{1+u}\right)^2J_0^*(c,d)w^*+\lambda_2^2J_0^*(c,d)+2\left(\frac{1}{1+u}\right)\lambda_2J_1^*(c,d)\right]\Big\} \quad ...(14)$$

where $w^*=\frac{var(\hat{B}_2)}{var(\hat{B}_1)}=\frac{\sigma^2}{SS_{x_2}}\Big/\frac{\sigma^2}{SS_{x_1}}=\frac{SS_{x_1}}{SS_{x_2}}=\frac{h_1}{h_2}.$

Also, the value of $k_i$ (i=1,2) can be found by minimizing the mean squared of $\tilde{A}$ and $\tilde{B}$ respectively,

i.e. $\frac{d}{dK_1} MSE(\tilde{A}|A, R_1) = 0$ and $\frac{d}{dK_2} MSE(\tilde{B}|B, R_2) = 0$

and by simple calculation we have:

$$K_1 = \frac{2\lambda_1^2 J_0(a,b) - 2\lambda_1 J_1(a,b)}{2J_2(a,b) + 2\lambda_1 J_0(a,b) - 4\lambda_1 J_1(a,b)}; \text{ w.r.t.} \tilde{A}$$

and,

$$K_2 = \frac{2\lambda_2^2 J_0^*(c,d) - 2\lambda_2 J_1^*(c,d)}{2J_2^*(c,d) + 2\lambda_2 J_0^*(c,d) - 4\lambda_2 J_1^*(c,d)}; \text{ w.r.t.} \tilde{B} \cdot$$

**Remark:** To be ensure that:

**(i)** $K_i$ (i=1,2) $\in [0,1]$, we suggest the value of $K_i$ (i=1,2) as follows:

$$K = \begin{cases} 0 & , \text{if } K_i < 0 \\ K_i & , \text{if } 0 \le K_i \le 1 \\ 1 & , \text{if } K_i > 1 \end{cases}$$

**(ii)** $K_i$ is minimize MSE($\tilde{\theta}$), we test the second derivatives for the MSE($\tilde{\theta}$) with respect to $K_i$,

i.e. $\frac{d^2}{dK_i^2} MSE(\tilde{\theta}|\theta, R) > 0$

where $\theta$ may be denoted to A or B and the region R may be denoted to $R_1$ or $R_2$.

The relative efficiency of both $\tilde{A}$ and $\tilde{B}$ are respectively given as:

$$R.Eff(\tilde{A}|A, R_1) = \frac{MSE(\hat{A}|A)}{MSE(\tilde{A}|A, R_1)} \cdot \left( \frac{n}{E(n|A,R)} \right) = \frac{var(\hat{A}_1)}{MSE(\tilde{A}|A, R_1) \cdot \{E\}}$$

where $E = \frac{E(n|A, R_1)}{n}$;

$E(n|A, R_1) = n(1 - \frac{u}{1+u} J_0(a,b))$ denote to expected sample size w.r.t.A.

Therefore;

$$R.Eff(\tilde{A}|A, R_1) = \left[ \left[ \left\{ k_1^2 J_2(a,b) + k_1^2 \lambda_1 J_0(a,b) - 2k_1^2 \lambda_1 J_1(a,b) + \lambda_1^2 J_0(a,b) + 2k_1 \lambda_1 J_1(a,b) - \right. \right. \right.$$
$$2k_1 \lambda_1^2 J_0(a,b) + \frac{1}{4}\left(\frac{1}{1+u}\right)^2 + \frac{1}{4}\left(\frac{u}{1+u}\right)^2 w + \frac{1}{4}\lambda_1^2 - \left[ \left(\frac{1}{1+u}\right)^2 J_2(a,b) + \right.$$
$$\left. \left. \left. \left(\frac{u}{1+u}\right)^2 J_0(a,b) w + \lambda_1^2 J_0(a,b) + 2\left(\frac{1}{1+u}\right)\lambda_1 J_1(a,b) \right] \right\} \cdot \left(1 - \frac{u}{1+u} J_0(a,b)\right) \right]^{-1} \quad ...(15)$$

and

$$R.Eff(\tilde{B}|B, R_2) = MSE(\hat{B}|B) / MSE(\tilde{B}|B, R_2) \cdot E(n|B, R_2) = \frac{var(\hat{B})}{MSE(\tilde{B}|B, R_2) \cdot \{E^*\}}$$

where $E^* = \frac{E^*(n|B, R_2)}{n}$;

$E^*(n|B,R_2) = n(1-\frac{u}{1+u}J_0^*(c,d))$ reffer to expected sample size w.r.t.B.

Therefore;

$$R.Eff(\tilde{B}|B,R_2) = \Bigg[\Bigg\{ k_2^2J_2^*(c,d) + k_2^2\lambda_2J_0^*(c,d) - 2k_2^2\lambda_2J_1^*(c,d) + \lambda_2^2J_0^*(c,d) + 2k_2\lambda_2J_1^*(c,d) - 2k_2\lambda_2^2J_0^*(c,d) + \frac{1}{4}\left(\frac{1}{1+u}\right)^2 + \frac{1}{4}\left(\frac{u}{1+u}\right)^2 w^* + \frac{1}{4}\lambda_2^2 - \Bigg[\left(\frac{1}{1+u}\right)^2 J_2^*(c,d) + \left(\frac{u}{1+u}\right)^2 J_0^*(c,d)w^* + \lambda_2^2J_0^*(c,d) + 2\left(\frac{1}{1+u}\right)\lambda_2J_1^*(c,d)\Bigg]\Bigg\}\left(1-\frac{u}{1+u}J_0^*(c,d)\right)\Bigg]^{-1} \quad ...(16)$$

where $\lambda_1 = (A-A_0)\big/\sqrt{var(\hat{A}_1)}$, $\lambda_2 = (B-B_0)\big/\sqrt{var(\hat{B}_1)}$, and

**Numerical Results and Conclusions**

**1-** $\tilde{A}$ and $\tilde{B}$ are consistence estimators.

i.e. $\lim_{n\to\infty} MSE(\tilde{A}|A,R_1) = 0$, and $\lim_{n\to\infty} MSE(\tilde{B}|B,R_2) = 0$.

**2-** $\tilde{A}$ and $\tilde{B}$ are dominates to $\hat{A}$ and $\hat{B}$ respectively with the large sample size (n).

i.e. $\lim_{n\to\infty}\left[MSE(\tilde{A}|A,R_1) - MSE(\hat{A}|A)\right] \le 0$, and $\lim_{n\to\infty}\left[MSE(\tilde{B}|B,R_2) - MSE(\hat{B}|B)\right] \le 0$.

**3-** The estimators $\tilde{A}$ and $\tilde{B}$ are biased when $A = A_0$ and $B = B_0$ respectively.

**4-** The R.Eff. of $\tilde{A}$ and $\tilde{B}$ are even function with $\lambda_1$ and $\lambda_2$ respectively.

**5-** The computation of relative efficiency R.Eff(·) and bias ratio B(·) were used for the estimators $\tilde{A}$ and $\tilde{B}$, these computation were performed for $\alpha$=0.01, 0.05, 0.1 and $\lambda_i$=0.0(0.1)1, i=1,2 and n=4,8,12,20. Some of these computations are given in the table leads to the following conclusions:

**i.** The relative efficiency of $\tilde{A}$ and that of $\tilde{B}$ are adversely proportional with small values of $\alpha$ and those of n.

**ii.** The relative efficiency of $\tilde{A}$ and that of $\tilde{B}$ are maximum when $A \approx A_0$ and $B \approx B_0$ respectively (i.e. $\lambda_1 \approx 0$ and $\lambda_2 \approx 0$).

**iii.** The bias ratio $\left[B(\cdot) = Bias(\tilde{\theta}|\theta,R)\big/\sqrt{var(\hat{\theta})}\right]$ of $\hat{A}$ and $\hat{B}$ are reasonably small when $A \approx A_0$ and $B \approx B_0$ respectively, and maximum when $\lambda_1$ and $\lambda_2$ are maximum.

**iv.** The bias ratio B(·) of $\tilde{A}$ and $\tilde{B}$ are reasonably small with small sample size (n).

**6-** The considered estimators $\tilde{A}$ and $\tilde{B}$ are better than the classical estimators $\hat{A}$ and $\hat{B}$, also than the estimators of Al-Bayyati and Arnold [1972] and Al-Kanane [1997].

**7-** The relative efficiency [R.Eff(·)] and bias ratio [B(·)] are decreasing function w.r.t.($n_1$), and the bias ratio [B(·)] of considered estimators are decreasing functions w.r.t. $\alpha$.

**8-** The considerd estimators are increasing function w.r.t. (u) especially when ($\lambda_1 \approx 0$) and ($\lambda_2 \approx 0$).

**9-** The expected sample size closed to $n_1$ especially when ($\lambda_1 \approx 0$) and ($\lambda_2 \approx 0$).

**10-** The probability for avoiding the second stage is very heigh when $A \approx A_0$ and $B \approx B_0$.

**11-** The effective intervals [The value of $\lambda_i$ which make the R.Eff. greater than 1] of $\tilde{A}$ is [-1,1] and [-1,1] for $\tilde{B}$ for all $k_i$, u, $\alpha$ and n.

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