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Strongly Essentially Quasi-Dedekind Modules

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Abstract

Let *R* be a commutative ring with unity. In this paper we introduce and study the concept of strongly essentially quasi-Dedekind module as a generalization of essentially quasi-Dedekind module. A unitary *R*-module *M* is called a strongly essentially quasi-Dedekind module if $Hom(M/N, M) = 0$ for all semiessential submodules *N* of *M*. Where a submodule *N* of an *R*-module *M* is called semiessential if , $N \cap p \neq 0$ for all nonzero prime submodules *P* of *M* .

Key Words: Essentially quasi-Dedekind Modules; Strongly essentially quasi-Dedekind Modules, Semiessential submodules, Multiplication Modules.

1. Introduction

Let *R* be a commutative ring with unity and *M* be an *R*-module. Mijbass A.S in [7] introduced and studied the concept of quasi-Dedekind, where an *R*-module *M* is called quasi-Dedekind if, $Hom(M/N, M) = 0$ for all nonzero submodules *N* of *M*. Ghawi Th.Y. in [4] introduced and studied the concept of essentially quasi-Dedekind, where an *R*-module *M* is called essentially quasi-Dedekind if, $Hom(M/N, M) = 0$ for all essential submodules *N* of *M* ($N \leq_e M$). In this paper we give a generalization of essentially quasi-Dedekind which we call it strongly essentially quasi-Dedekind, where an *R*-module *M* is called strongly essentially quasi-Dedekind if, $Hom(M/N, M) = 0$ for all $N \leq_{se} M$. In fact a submodule *N* of *M* is called semiessential in *M* and denoted by ($N \leq_{se} M$) if, $N \cap p \neq 0$ for all nonzero prime submodules *P* of *M* [1], provided that *M* has nonzero prime submodule. In this paper we present the basic properties of strongly essentially quasi-Dedekind and some relationships with other modules.

Next throughout this paper, M has a nonzero prime submodules.

1.1 Definition

An *R*-module *M* is called strongly essentially quasi-Dedekind if, $Hom(M/N, M) = 0$ for all semiessential submodules *N* of *M*.

1.2 Remarks and Examples:

1- It is clear that if M is a strongly essentially quasi-Dedekind R-module, then M is an essentially quasi-Dedekind R-module, since every essential submodule is semiessential submodule.

2- Every quasi-Dedekind R-module is a strongly essentially quasi-Dedekind R-module, but the converse is not true in general, for example: Z_6 as Z- module is strongly essentially quasi-Dedekind, but it is not quasi-Dedekind, since $Hom(Z_6/(\bar{2}), Z_6) \cong Z_2 \neq 0$.

3- Each of Z, Z_6, Z_{10} is strongly essentially quasi-Dedekind as Z-module.

4- Each of Z_4, Z_8, Z_{12}, Z_{16} is not strongly essentially quasi-Dedekind as Z-module.

5- Z_{p^∞} is not strongly essentially quasi-Dedekind as Z-module ,for all prime numbers p.

6- $Z \oplus Z_2$ is not essentially quasi-Dedekind as Z-module, see [4, Remark 1.2.14], so it is not strongly essentially quasi-Dedekind as Z-module.

7- Let $N \leq M$ and M/N is a strongly essentially quasi-Dedekind R-module, then it is not necessarily that M is a strongly essentially quasi-Dedekind R-module; For example :

Let $M = Z_{12}$ as Z-module and let $N = (\bar{6}) \leq Z_{12}$, then $Z_{12}/N \cong Z_6$ is a strongly essentially quasi-Dedekind Z-module, but Z_{12} is not strongly essentially quasi-Dedekind as Z-module.

Recall that a nonzero R-module M is called semi-uniform, if every nonzero R-submodule of M is a semiessential submodule of M [1].

1.3 Proposition:

Let M be a semi-uniform R-module. Then M is a quasi-Dedekind R-module if and only if M is a strongly essentially quasi-Dedekind R-module.

Proof : It is clear. □

1.4 Corollary:

Let M be a uniform R-module .The following statements are equivalent:

1- M is a quasi-Dedekind R-module.
2- M is a strongly essentially quasi-Dedekind R-module.
3- M is an essentially quasi-Dedekind R-module.

Proof : It is clear . □

The following is a characterization of strongly essentially quasi-Dedekind module.

1.5 Theorem:

Let M be an R-module M is strongly essentially quasi-Dedekind if and only if for each $f \in End_R(M)$, $f \neq 0$ implies $Kerf \not\leq_{se} M$.

Proof :$\Rightarrow$) Suppose that M is a strongly essentially quasi-Dedekind R-module .Let $f \in End_R(M)$, $f \neq 0$. To prove that $Kerf \not\leq_{se} M$. Assume that $Kerf \not\leq_{se} M$, define $g : M/Kerf \longrightarrow M$ by $g(m+Kerf) = f(m)$ for all $m \in M$. It is clear that g is well-defined and $g \neq 0$, hence $Hom(M/Kerf, M) \neq 0$ which is a contradiction.

$\Leftarrow$) Assume that there exists $h : M/N \longrightarrow M$, $h \neq 0$, for some $N \leq_{se} M$. Consider the following : $M \xrightarrow{\pi} M/N \xrightarrow{h} M$, where π is the natural projective mapping, then

$\phi = ho\pi \in End_R(M)$ and $\phi \neq 0$. Since $N \subseteq Ker\phi$ and $N \leq_{se} M$, thus $Ker\phi \leq_{se} M$. Since (for any prime submodule P of M, $P \cap N \neq (0)$). But this is contradiction. □

1.6 Proposition:

Let M be an R-module and let $\overline{R} = R/J$, $J \subseteq ann_R M$, then M is a strongly essentially quasi-Dedekind R-module if and only if M is a strongly essentially quasi-Dedekind $\overline{R}$-module.

Proof : Since $Hom_{\overline{R}}(M/N, M) = Hom_R(M/N, M)$ for all $N \leq M$, by [6, p.51] the result follows easily. □

Recall that an injective R-module $E(M)$ is called an injective hull (injective envelope) of an R-module M if, there exists a monomorphism $f: M \longrightarrow E(M)$ such that $Imf \leq_e E(M)$ [6, p.142]. And recall that a quasi-injective R-module $\overline{M}$ is called a quasi-injective hull (quasi-injective envelope) of an R-module M if, there exists a monomorphism $g: M \longrightarrow \overline{M}$ such that $Img \leq_e \overline{M}$ [11].

To prove the next result, we state and prove the following lemma:

1.7 Lemma:

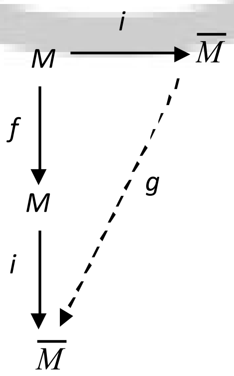
Let M be an R-module and let $A \leq M$, $B \leq M$. If $A \leq_{se} B \leq_{se} M$ then $A \leq_{se} M$.

Proof : Let P be a nonzero prime submodule in M, then $0 \neq P \cap B$ is prime in B and to show this: Let $x \in B$, $r \in R$. If $rx \in P \cap B$, then $rx \in P$ and $rx \in B$. Now $rx \in P$ implies either $x \in P$ or $r \in [P:M]$, since P is prime in M. If $x \in P$, then $x \in P \cap B$. And if $r \in [P:M]$, then $rM \subseteq P$, but $rB \subseteq rM \subseteq P$, then $rB \subseteq P$ and also $rB \subseteq B$, hence $rB \subseteq P \cap B$. Thus $r \in [P \cap B:B]$, so that $P \cap B$ is prime in B. It follows that $A \cap (P \cap B) \neq 0$ and hence $A \cap P \neq 0$. Therefore $A \leq_{se} M$. □

1.8 Proposition:

Let M be an R-module. If $\overline{M}$ is a strongly essentially quasi-Dedekind R-module, then M is a strongly essentially quasi-Dedekind R-module.

Proof : Let $f \in End_R(M)$, $f \neq 0$. To prove that $Kerf \leq_{se} M$. Since $\overline{M}$ is quasi- injective R-module, then there exists $g : \overline{M} \longrightarrow \overline{M}$, $g \neq 0$ such that $g \circ i = i \circ f$ (where i is the inclusion mapping).



But $\overline{M}$ is a strongly essentially quasi-Dedekind *R*-module, so $Kerg \leq_{se} \overline{M}$, but $Kerf \subseteq kerg$, then $Kerf \not\leq_{se} M$, and since $M \leq_e \overline{M}$; that is $M \leq_{se} \overline{M}$, then by (Lemma 1.7) $Kerf \leq_{se} \overline{M}$ which implies $Kerg \leq_{se} \overline{M}$ which is a contradiction. Thus *M* is a strongly essentially quasi-Dedekind *R*-module. □

The following results follow directly by (Prop. 1.8).

1.9 Corollary:

Let *M* be a strongly essentially quasi-Dedekind and quasi-injective *R*-module. If $N \leq_{se} M$ then *N* is a strongly essentially quasi-Dedekind *R*-module.

1.10 Corollary:

Let *M* be an *R*-module. If *E(M)* is a strongly essentially quasi-Dedekind *R*-module then *M* is a strongly essentially quasi-Dedekind *R*-module .

The converse of (Coro. 1.10) is not true in general, as the following example shows:

1.11 Example:

It is well known that Z_2 as *Z*-module is a strongly essentially quasi-Dedekind. But $E(Z_2) = Z_{2^\infty}$ is not strongly essentially quasi-Dedekind as *Z*-module .

1.12 Remark:

Let *M* be an *R*-module. If $N \leq M$ is a strongly essentially quasi-Dedekind *R*-module then it is not necessarily that *M/N* is a strongly essentially quasi-Dedekind *R*-module, consider the following example:

1.13 Example:

Let $M = Z$ as *Z*-module, and let $N = 4Z \leq Z = M$. It is clear that $N \leq_{se} M$ and *M* is strongly essentially quasi-Dedekind and quasi-injective as *Z*-module, so by (Coro.1.9) *N* is strongly essentially quasi-Dedekind as *Z*-module, but $M/N = Z/4Z \cong Z_4$ is not strongly essentially quasi-Dedekind as *Z*-module (see, Rem.and.Ex(1.2)(4)) .

Recall that a nonzero *R*-module *M* is called compressible if, *M* embedded in each of its nonzero submodules [2].

1.14 Proposition:

Let *M* be a multiplication *R*-module, $N \lneqq M$. If *N* is a prime *R*-submodule of *M*, then *M/N* is a strongly essentially quasi-Dedekind *R*-module.

Proof: Since *N* is a prime submodule of *M*, so by [12, Coro. 4.18, ch.1] *M/N* is a compressible *R*-module, thus by [7,Prop 2.6, p.30] *M/N* is a quasi-Dedekind *R*-module. Therefore by (Rem.and.Ex(1.2)(2)) *M/N* is a strongly essentially quasi-Dedekind *R*-module. □

To prove our next result, we need the following lemma:

1.15 Lemma:

Let M, N be an R-modules, let $f: M \longrightarrow N$ be a monomorphism. Let $K \leq M$, $A \leq N$, then:

(1) $K \leq_{se} M$ implies $f(K) \leq_{se} N$.

(2) $A \leq_{se} N$ implies $f^{-1}(A) \leq_{se} M$, if f is an epimorphism and $Kerf \subseteq$ P, where P is any prime submodule of M.

Proof :

(1) Suppose that there exists a nonzero prime submodule W of N such that $f(K) \cap W = 0$. But $K = f^{-1}(f(K))$, since f is a monomorphism. Hence $K \cap f^{-1}(W) = f^{-1}(f(K)) \cap f^{-1}(W) = f^{-1}(f(K) \cap W) = f^{-1}(0) = Kerf = \{0\}$. But $f^{-1}(W)$ is a nonzero prime submodule of M, so $K \not\leq_{se} M$ which is a contradiction.

(2) The proof is similarly. □

1.16 Proposition:

Let $M \cong N$. Then M is a strongly essentially quasi-Dedekind R-module if and only if N is a strongly essentially quasi-Dedekind R-module.

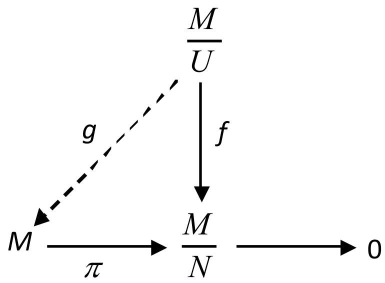
Proof : $\Rightarrow$) Let $\phi: M \longrightarrow N$ be an isomorphism. Suppose that M is a strongly essentially quasi-Dedekind R-module. Let $f \in End_R(N)$, $f \neq 0$. To prove that $Kerf \not\leq_{se} N$, consider the following: $M \xrightarrow{\phi} N \xrightarrow{f} N \xrightarrow{\phi^{-1}} M$, let $h = \phi^{-1} o\ f\ o \phi \in End_R(M)$, $h \neq 0$ since $h(M) = \phi^{-1} \circ f \circ \phi(M) \subseteq \phi^{-1}(f(N)) \subseteq \phi^{-1}(N) \neq 0$ .Then $Kerh \not\leq_{se} M$, since M is a strongly essentially quasi-Dedekind R-module. We claim that $Kerf = \{y \in N: \phi^{-1}(y) \in Kerh\}$. To prove our assertion. Let $y \in Kerf$, then $f(y) = 0$. $h(\phi^{-1}(y)) = \phi^{-1} \circ f \circ \phi(\phi^{-1}(y)) = \phi^{-1} \circ f(y) = \phi^{-1}(0) = 0$. Thus for each $y \in Kerf$, then $\phi^{-1}(y) \in Kerh$ and hence $\phi^{-1}(Kerf) \subseteq Kerh \not\leq_{se} M$ this implies $\phi^{-1}(Kerf) \not\leq_{se} M$, so by (Lemma.(1.16)(2)) $Kerf \not\leq_{se} N$. Therefore N is a strongly essentially quasi-Dedekind R-module.

$\Leftarrow$) The proof of the converse is similarly. □

1.17 Theorem:

Let M be an R-module such that M/V is projective R-module, for all $V \leq_{se} M$. If M is a strongly essentially quasi-Dedekind R-module, then M/N is a strongly essentially quasi-Dedekind R-module for all $N \leq M$. Provided $N \leq_{se} M$.

Proof: To prove that M/N is strongly essentially quasi-Dedekind, we must prove that $Hom(\frac{M/N}{U/N}, \frac{M}{N}) = 0$ for all $U/N \leq_{se} M/N$. By 3rd isomorphism theorem $\frac{M}{N} \Big/ \frac{U}{N} \cong \frac{M}{U}$, so its enough to show that $Hom(M/U, M/N) = 0$. Let $f \in Hom(M/U, M/N)$, $f \neq 0$ .Hence there exists $g: M/U \longrightarrow M$ such that $\pi o g = f$, since M/U is projective.



So $g \neq 0$, thus $Hom(M/U, M) \neq 0$. $U \leq_{se} M$, because $U \supseteq N$. Thus M is not strongly essentially quasi-Dedekind R-module, so we get a contradiction .Thus M/N must be a strongly essentially quasi-Dedekind R-module. □

To prove the next theorem we need the following lemma:

1.18 Lemma:

Let M_1, M_2 be R-modules. If $A \leq_{se} M_1$, $B \leq_{se} M_2$ then $A \oplus B \leq_{se} M_1 \oplus M_2$.

Proof : Let P be prime in $M_1 \oplus M_2$, then by [5] $P = P_1 \oplus P_2$, such that either p_1, p_2 prime in M_1, M_2 respectively , so $(A \oplus B) \cap (P_1 \oplus P_2) = (A \cap P_1) \oplus (B \cap P_2) \neq 0$.

Or , $P = P_1 \oplus M_2$, then $(A \oplus B) \cap (P_1 \oplus M_2) = (A \cap P_1) + (B \cap M_2) = A \cap P_1 \oplus B \neq 0$.

Or , $P = M_1 \oplus P_2$, then $(A \oplus B) \cap (M_1 \oplus P_2) = (A \cap M_1) + (B \cap P_2) = A \oplus B \cap P_2 \neq 0$. □

1.19 Theorem:

A direct summand of a strongly essentially quasi-Dedekind R-module is a strongly essentially quasi-Dedekind R-module.

Proof : Let $M = M_1 \oplus M_2$. To prove M_1 is a strongly essentially quasi-Dedekind R-module. Let $f \in End_R(M_1), f \neq 0$, we have the following diagram:

$$M_1 \oplus M_2 \xrightarrow{\rho} M_1 \xrightarrow{f} M_1 \xrightarrow{i} M_1 \oplus M_2$$

$i \circ f \circ \rho \in End_R(M)$ .If $i \circ f \circ \rho(M) = i \circ f(M_1) = i(f(M_1)) = f(M_1) \neq 0$,then $Ker(i \circ f \circ \rho) \not\leq_{se} M$. $Ker(i \circ f \circ \rho) = \{m_1 + m_2 : i \circ f \circ \rho(m_1, m_2) = 0\} = \{m_1 + m_2 : i \circ f(m_1) = 0\} = \{m_1 + m_2 : f(m_1) = 0\} = Kerf \oplus M_2 \not\leq_{se} M_1 \oplus M_2$. But $M_2 \leq_{se} M_2$, so $Kerf \not\leq_{se} M_1$, by (Lemma 1.18). □

The converse of (Theorem 1.19) is not true in general ,consider the following example:

1.20 Example:

We know that each of Z, Z_6 as Z-module is strongly essentially quasi-Dedekind. But $Z \oplus Z_6$ is not strongly essentially quasi-Dedekind as Z-module, since $Z \oplus Z_6$ is not essentially quasi-Dedekind.

Recall that a nonzero submodule N of an R-module M is called quasi-invertible if Hom(M | N,M) = 0, [7].

1.21 Proposition:

If M be a strongly essentially quasi-Dedekind R-module. Then $ann_R M = ann_R N$ for all $N \leq_{se} M$.

Proof : Suppose that M is a strongly essentially quasi-Dedekind R-module, then $Hom(M/N, M) = 0$ for all $N \leq_{se} M$, hence N is a quasi-invertible submodule of M, for all $N \leq_{se} M$. Thus by [7, Prop.1.4] $ann_R M = ann_R N$ for all $N \leq_{se} M$. □

To prove the following proposition, we need to prove the following lemma:

1.22 Lemma:

Let M be a faithful multiplication R-module. Then $N \leq_{se} M$ if and only if $[N:M] \leq_{se} R$.

Proof : $\Rightarrow$) If $N \leq_{se} M$. Let P be any nonzero prime ideal in R. Then by [3, Lemma 2.10] PM is a nonzero prime submodul in M, hence $N \cap PM \neq 0$; that is $[(N:M)M] \cap PM \neq 0$, and since M is a faithful multiplication R-module, $[(N:M) \cap P] M \neq 0$, by [3]. Thus $[N:M] \cap P \neq 0$, so $[N:M] \leq_{se} R$.

$\Leftarrow$) If $[N:M] \leq_{se} R$ .Let P be any nonzero prime submodule in M, then by [3, Prop.2.8,ch1] $[P:M]$ is prime ideal in R, and since $[N:M] \leq_{se} R$, we have $[N:M] \cap [P:M] \neq 0$ which implies $([N:M] \cap [P:M])M \neq 0$, so that by [3] $[N:M]M \cap [P:M] M \neq 0$, thus $N \cap P \neq 0$; that is $N \leq_{se} M$. □

1.23 Proposition:

Let M be a faithful multiplication R-module. If M is a strongly essentially quasi-Dedekind R-module, then R is a strongly essentially quasi-Dedekind R-module.

Proof : Let $f: R \longrightarrow R$, $f \neq 0$. For any $r \in R$, $f(r) = r f(1) = ra$, where $a = f(1)$. Define $g: M \longrightarrow M$ by $g(m) = am$ for each $m \in M$. g is well-defined and $g \neq 0$, hence $Ker g \nleq_{se} M$. But $Ker g = [Ker g:M]M$, since M is a multiplication R-module. However we can show that $[Ker g:M] = Ker f$ as the following: Let $r \in [Ker g:M]$ implies $rM \subseteq Ker g$, so $g(rM) = 0$, hence $arM = 0$; that is $ar \in ann_R M = 0$, thus $f(r) = ar = 0$, hence $r \in Ker f$. Now, let $r \in Ker f$, then $ar = f(r) = 0$, so $arM = 0$; that is $g(rM) = 0$, thus $rM \subseteq Ker g$ and hence $r \in [Ker g:M]$. Therefore $[Ker g:M] = Ker f$. But $Ker g \nleq_{se} M$, implies by (Lemma (1.22)) $[Ker g:M] \nleq_{se} R$, thus $Ker f \nleq_{se} R$ and hence R is a strongly essentially quasi-Dedekind R-module. □

Recall that an R-module M is called scalar if for each $f \in End_R(M)$, there exists $r \in R$ such that $f(a) = ar$ for all $a \in M$ [10, p.8].

1.24 Proposition:

Let M be a finitely generated faithful multiplication R-module. If R is a strongly essentially quasi-Dedekind R-module, then M is a strongly essentially quasi-Dedekind R-module.

Proof : Since M is a finitely generated multiplication R-module, then by [9,Th.2.3] M is a scalar R-module, so for each $f \in End_R(M)$, there exists $r \in R$ such that $f(m) = rm$, for all $m \in M$. Define $g : R \longrightarrow R$ by $g(a) = ra$, for all $a \in R$, $Kerg \not\leq_{se} R$, since R is a strongly essentially quasi-Dedekind R-module. But $Kerf = [Kerf: M]\ M$, also by the same argument of the proof of (Prop.1.23), we get $Kerg = [Kerf: M]$, but $Kerg \not\leq_{se} R$, so $[Kerf:M] \not\leq_{se} R$ which implies $Kerf \not\leq_{se} M$, by (Lemma 1.22). Thus M is a strongly essentially quasi-Dedekind R-module. □

By combining (Prop 1.23) and (Prop 1.24) , we get the following result:

1.25 Corollary:

Let M be a finitely generated faithful multiplication R-module. M is a strongly essentially quasi-Dedekind R-module if and only if R is a strongly essentially quasi-Dedekind R-module.

We end this paper with the following corollary:

1.26 Corollary:

Let M be a finitely generated faithful multiplication R-module. If R is a strongly essentially quasi-Dedekind R-module, then $End_R(M)$ is a strongly essentially quasi-Dedekind ring.

Proof : Since M is a finitely generated multiplication R-module , then by [9,I.2.3] M is a scalar R-module. Then by [8, Lemma 6.2, ch.3] $End_R(M) \cong R/ann_R M \cong R$, but R is a strongly essentially quasi-Dedekind ring ,thus by Prop.1.16 $End_R(M)$ is a strongly essentially quasi-Dedekind ring. □

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المقاسات شبه ـ ديديكاندية الواسعة بقوة

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الخلاصة

لتكن R حلقة أبدالية ذا عنصر محايد . في هذا البحث قدّمنا ودرسنا مفهوم المقاسات شبه – ديديكاندية الواسعة بقوة كأعمام إلى المقاسات شبه – ديديكاندية الواسعة، اذ يسّمى المقاس M على R مقاساً شبه– ديديكاندي واسع بقوة إذا كان $Hom(M/N, M) = 0$ لكل مقاس جزئي شبه واسع N في M . يطلق على مقاس جزئي N من مقاس Mعلى R شبه واسع إذا كان $N \cap p \neq 0$ لكل مقاس جزئي أولي غير صفري P في M. على شرط ان M لها مقاسات أولية غير صفرية.

الكلمات المفتاحية: المقلسات شبه الديدكانديه الواسعة، المقلسات شبه الديدكاندية الواسعة بقوة، المقلسات الجزئية شبه الواسعة، المقاسات الجدائية.