The Datums Transformation for GPS Navigation Measurements Correction

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Abstract

The GPS navigation measurements become more widely used in many civilian and scientific application. All GPS navigation data holds many errors, the main error sources arise from the geodetic Datum variation when user apply the GPS measurements with the map. Geodetic datums define the size and shape of the earth and the origin and orientation of the coordinate systems used to map the earth surface. In this paper, the Datum transformation was evaluated in two mathematical methods to overcome the errors due to the difference between the WGS-84 and our country Datum Clarck-1880. The results was evaluated and investigated using Carmin GPS device for GCPs comparison, topographic map for Hilla city, mid Iraq 1:100000 scale, and two georefrencing ETM+ & TM satellite images. The spatial transformation error was less than 10 meter for UTM projection & less than 1 sec for (Φ, λ) projection, which can considered as suitable results in transformation calculation All results were overcome using two written program for each method with the help of Matlab facility.

GPS: <u>G</u>lobal <u>P</u>ositioning <u>S</u>ystem, **GCPs: <u>G</u>round <u>C</u>ontrol <u>P</u>oints**

1. Introduction

Geodetic datums define the size and shape of the earth and the origin and orientation of the coordinate systems used to map the earth. Hundreds of different datums have been used to frame position descriptions. Datums have evolved from those describing a spherical earth to ellipsoidal models derived from years of satellite measurements, **[1]**. Modern geodetic datums range from flat-earth models used for plane surveying to complex models used for international applications which completely describe the size, shape, orientation, gravity field, and angular velocity of the earth. While cartography, surveying, navigation, and astronomy all make use of geodetic datums, **[2]**.

Referencing geodetic coordinates to the wrong datum can result in position errors of hundreds of meters, **[3]**. Different nations and agencies use different datums as the basis for coordinate systems used to identify positions in geographic information systems, precise positioning systems, and navigation systems. The diversity of datums in use today and the technological advancements that have made possible global positioning measurements with sub-meter accuracies requires careful datum selection and careful conversion between coordinates in different datums, **[4]**. The differencing between different Earth Datum depend on the values of major axis of Earth ellipsoid ***a***. This axis value generate different values of the flattening f and eccentricity e, which can be given as following, **[5]**;

$$f = 1 - \frac{b}{a} \qquad \boldsymbol{e} = \sqrt{\mathbf{1} - \frac{\boldsymbol{b}^2}{\boldsymbol{a}^2}} \qquad \mathbf{1}$$

where; b is the minor Earth axis.

table 1 shows the values of a, and f for most famous Datum, according to **[3]**.

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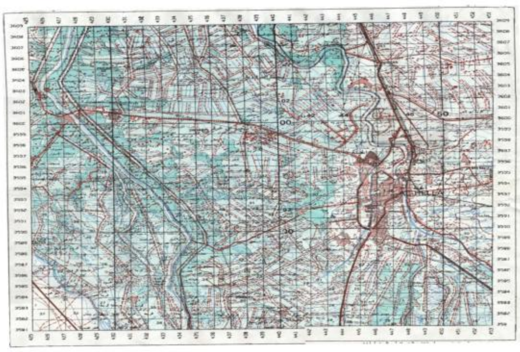
Table: 1 Some Famous Datum Major Axis and Flattening Values

Datum	*a* (meter)	*f*
Everest 1830	6377304	1/300.8
Bessel 1841	6377397	1/299.2
Clarke 1866	6378206	1/295.0
Clarke 1880	6378249	1/293.5
Hayford 1910	6378388	1/297.0
Krasovskii 1938	6378245	1/298.3
Hough 1956	6378270	1/297.0
Fischer 1960	6378166	1/298.3
Kauln 1961	6378165	1/298.3
Ficher 1968	6378150	1/298.3
WGS-84	6378137	1/298.2

2. Rejoin of Interest and Available Data

The rejoin of interest represent the Hilla city center of area 1344 sq. Km, Upper Left Point, LL 32° 38′ 59.28″, 44° 8′ 16.8″, Lower Right Point 32° 18′ 10.31″, 44° 44′ 48.01″. The available data are two satellite images for above region ETM+ & TM sensors, the first was georefrencing due to UTM projection, and the second image was georefrencing due to geographic projection, (Φ, λ). Both images was registered according to WGS-84 Datum, which was coincide with the GPS Datum measurements. Also, a map of scale 1:100000 represent Hilaa city center was used to pick up the ground control points in both UTM , and (Φ, λ) projections. The reasons for selection this region of interest was referred to the data availability. The two images and topographic map are shown in figure 1.



A

B

C

Figure: 1 region of interest, A, topographic map, B, ETM+ image, and C, TM image

3. GPS Navigation System Overview

GPS was funded and controlled by the U. S. Department of Defense (DOD). While there are many thousands of civil users of GPS world-wide, the system was designed and operated by the U. S. military. GPS provides specially coded satellite signals that can be processed in a GPS receiver, enabling the receiver to compute position, velocity and time, **[6]**. The GPs system consist of three segments which can describe as following; **1. (Space**

Segment), The Space Segment of the system consists of the GPS satellites. These Space Vehicles (SVs) send radio signals from space. The nominal GPS Operational Constellation consists of 24 satellites that orbit the earth in 12 hours. There are often more than 24 operational satellites as new ones are launched to replace older satellites. The satellite orbits repeat almost the same ground track (as the earth turns beneath them) once each day. The orbit altitude is such that the satellites repeat the same track and configuration over any point approximately each 24 hours (4 minutes earlier each day). There are six orbital planes (with nominally four SVs in each), equally spaced (60 degrees apart), and inclined at about fifty-five degrees with respect to the equatorial plane. This constellation provides the user with between five and eight SVs visible from any point on the earth. **2. (Control Segment),** The Control Segment consists of a system of tracking stations located around the world. The Master Control facility is located at Schriever Air Force Base (formerly Falcon AFB) in Colorado. These monitor stations measure signals from the SVs which are incorporated into orbital models for each satellites. The models compute precise orbital data (ephemeris) and SV clock corrections for each satellite. The Master Control station uploads ephemeris and clock data to the SVs. The SVs then send subsets of the orbital ephemeris data to GPS receivers over radio signals. **3. (User Segment),** The GPS User Segment consists of the GPS receivers and the user community. GPS receivers convert SV signals into position, velocity, and time estimates. Four satellites are required to compute the four dimensions of *X, Y, Z* (position) and Time. GPS receivers are used for navigation, positioning, time dissemination, and other research, according to [1].

The navigation of point on an Earth surface denoted as (X_u, Y_u, Z_u) depend on the concept of "Time of Arrival TOA". All GPS measurements customaries in the WGS-84 Datum. This system considered the center of Earth as the system origin, and any point in the space, on Earth surface, or under the earth surface (X_u, Y_u, Z_u) can be measured in this system coordinates with respect to origin, [6]. The GPS navigation equation for finding the Coordinates of any point (X_u, Y_u, Z_u), (user position) in the WGS-84 system can given as, [6];

$$(X_1 - X_u)^2 + (Y_1 - Y_u)^2 + (Z_1 - Z_u)^2 = (R_1 - C_b)^2$$

$$(X_2 - X_u)^2 + (Y_2 - Y_u)^2 + (Z_2 - Z_u)^2 = (R_2 - C_b)^2 \quad \mathbf{2}$$

$$(X_3 - X_u)^2 + (Y_3 - Y_u)^2 + (Z_3 - Z_u)^2 = (R_3 - C_b)^2$$

$$(X_4 - X_u)^2 + (Y_4 - Y_u)^2 + (Z_4 - Z_u)^2 = (R_4 - C_b)^2$$

Where; $(X_1, Y_1 Z_1), (X_2, Y_2, Z_2) \ldots\ldots\ldots (X_4, Y_4, Z_4)$; represent the coordinates of the four navigation space satellites in the WGS-84 system.

(X_u, Y_u, Z_u) ;the user position coordinates.

$R_1 \ldots\ldots\ldots R_4$; the distance between the GPS antenna and each space satellite.

C_b ; is time correction factor.

4. Mathematical Transformation and Results Analysis

The Datum transformation was evaluated in two mathematical methods, the methods can be summarized as following;

4.1. Direct Transformation, the direct equations between the GPS antenna coordinates in the Datum, WGS-84 and geographic coordinates in any Datum system was used to overcome the transformation. The GPS antenna vector ***u*** (X_u, Y_u, Z_u) in WGS-84 Datum can be calculated from any direct (Φ, λ), (Latitude , Longitude), and highest *h* can be given as, **[3]**;

$$u = \begin{bmatrix} \dfrac{a \cos\lambda}{\sqrt{1 + (1 - e^2)\tan^2\Phi}} + b\cos\lambda\cos\Phi \\ \dfrac{a \sin\lambda}{\sqrt{1 + (1 - e^2)\tan^2\Phi}} + b\sin\lambda\cos\Phi \\ \dfrac{a(1 - e^2)\sin\Phi}{\sqrt{1 - e^2\sin^2\Phi}} + b\sin\Phi \end{bmatrix} \quad \mathbf{3}$$

for each GPS navigation measurement, the ***u*** vector was found, therefore, the transformation for new (Φ, λ), and *h* in any new Datum system can be found according to the following equations set with different values of a for each Datum system , see table 1, according to **[3].**

$$\Phi = \arctan\left(\frac{Z_u + e^2 Z_u}{r}\right)$$

$$\lambda = \arctan\left(\frac{Y_u}{X_u}\right) \qquad h = U(1 - \frac{b^2}{aV})$$

$$r = \sqrt{X_u^2 + Y_u^2} \qquad U = \sqrt{(r - e^2 r_o)^2 + (1-e^2)Z_u^2}$$

$$V = \sqrt{(r - e^2 r_o) + (1-e^2)Z_u^2}$$

$$r_o = \frac{pe^2 r}{1+\Omega} + \sqrt{\frac{1}{2}a^2(1+\frac{1}{\Omega}) - \frac{p(1-e^2)Z_u^2}{\Omega(1+\Omega)} - \frac{1}{2}pr^2}$$

$$\Omega = \sqrt{1 + 2e^4 p}$$

$$p = \frac{F}{3(S + \frac{1}{S} + 1)^2 G^2}$$

$$s = \sqrt[3]{1 + c + \sqrt{c^2 + 2c}}$$

$$C = \frac{e^4 F r^2}{G^4} \qquad 4$$

where; G,F, and E²* *can be given as following;

$$G = r^2 + (1-e^2)Z_u^2 - e^2 E^2$$

$$F = 54b^2 Z_u^2 \qquad E^2 = a^2 - b^2 \qquad 5$$

The results obtained using this method were hold some second errors when it compare with the GCPs coordinates from the map, (Reference Data), after transformation of the TM GCPs coordinates, (Input Data). *Note that the 1 sec. in (Φ, λ) projection equal to 25 meter on ground in our country.* The TM image of Hilaa was used to pick up the GCPs coordinates (Φ, λ) in WGS-84 for many Road intersection and River boundary in the city, see table 2, the WGS-84 GPS, Inp. The TM image was covered all Iraq area, (Mosaic Image), therefore, the coordinates system (Φ, λ) of this image was investigated with Garmin GPS device in many navigation points taken in Baghdad University area. The results of image and GPS was the same, this was true because they are in the same Datum. The transform was applied for our country Datum Clarck 1880, with **a=6378249** meter, and ***f*=1/293.5**. The result was given in table 2, Calculated Coordinates, and it appear there are some error in using this method, but this method give a direct and easy calculation, it was evaluated using the written Matlab program.

Table: 2, The GCPs Editor for Direct Transform

GCPs Editor No. 1						
GCP#	**WGS-84 GPS, Inp. (Φ, λ)**		**Clarck 1880 Map, Ref. (Φ, λ)**		**Calculated Coordinates (Φ, λ)**	
	Φ N Deg. Min. Sec.	**λ E Deg. Min. Sec.**	**Φ N Deg. Min. Sec.**	**λ E Deg. Min. Sec.**	**Φ N Deg. Min. Sec.**	**λ E Deg. Min. Sec.**
1	32˚ 29′ 55.54″	44˚ 25′ 16.83″	32˚ 29′ 54.31″	44˚ 25′ 16.82″	32˚ 29′ 42.63″	44˚ 25′ 33.50″
2	32˚ 26′ 36.86″	44˚ 25′ 4.46″	32˚ 26′ 36.86″	44˚ 27′ 8.47″	32˚ 26′ 54.95″	44˚ 27′ 19.63″
3	32˚ 27′ 19.85″	44˚ 21′ 42.78″	32˚ 27′ 19.0″	44˚ 21′ 42.67″	32˚ 27′ 10.19″	44˚ 21′ 27.81″
4	32˚ 30′ 11.53″	44˚ 21′ 42.66″	32˚ 31′ 28.04″	44˚ 21′ 51.80″	32˚ 31′ 40.22″	44˚ 21′ 38.72″
5	32˚ 32′ 18.30″	44˚ 24′ 45.32″	32˚ 32′ 17.53″	44˚ 24′ 45.36″	32˚ 32′ 61.17″	44˚ 24′ 63.73″
6	32˚ 23′ 57.84″	44˚ 19′ 28.38″	32˚ 23′ 57.19″	44˚ 19′ 28.7″	32˚ 23′ 23.32″	44˚ 19′ 75.44″

4.2. Transformation using Least Mean Square Criteria

A first order polynomial transformation was applied for both UTM, and (Φ, λ) projections by use the GCPs fitting Least Mean square Criteria. For the same region of interest, the TM image was used to pick up the GCPs coordinates (Φ, λ), the ETM+ image was used to pick up the UTM coordinates for the same GCPs. These UTM & (Φ, λ) coordinates system considered as GPS input data because the two image were rectify according to WGS-84 Datum. The topographic map was used to pick up both UTM, & (Φ, λ) coordinates system considered as reference data, see table, 3.

Mathematically, the LMSC algorithm can be summarize using the matrix notation for n GCPs or measurements according to **[7]**. For first order polynomial transform and for 2-D data, the transform equation given as;

$$C1_o = a_o + a_1 * C1_{in} + a_2 * C2_{in}$$

$$C2_o = b_o + b_1 * C1_{in} + b_2 * C2_{in} \qquad \textbf{5}$$

where; $R = [a_o \quad a_1 \quad a_2]$, $Q = [b_o \quad b_1 \quad b_2]$ are the transformation matrix coefficients.

$C1_o C2_o$, are the output coordinates system (calculated), and $C1_{in} C2_{in}$, are the input coordinates system.

For the purpose of LMSC algorithm, the transform was evaluated using matrix notation as following, **[8]**;

$$R = (D^T * D)^{-1} * D^T * K$$

$$Q = (D^T * D)^{-1} * D^T * L \qquad \textbf{6}$$

where; D is input matrix, (from Images), given for ***n*** GCPs as following;

$$D = \begin{bmatrix} 1 & X1_{in1} & X2_{in1} \\ 1 & X1_{in2} & X2_{in2} \\ 1 & X1_{inofn} & X2_{inofn} \end{bmatrix} \qquad \textbf{7}$$

K, and *L*, are the reference vector matrices, (from Map), given for ***n*** GCPs as;

$$K = \begin{bmatrix} X1_{ref1} \\ X1_{ref2} \\ X1_{refn} \end{bmatrix} \qquad L = \begin{bmatrix} X2_{ref1} \\ X2_{ref2} \\ X2_{refn} \end{bmatrix} \qquad \textbf{8}$$

Note: the Input GPS and Reference Map data are given in the following table, 3;

Table: 3, The GCPs Editor for Transformation using Least Mean Square Criteria

GCPs Editor No. 2								
	WGS-84 GPS, Inp.				Clarck 1880 Map, Ref.			
GCP#	(Φ, λ)		UTM		(Φ, λ)		UTM	
	Φ N Deg. Min. Sec.	λ E Deg. Min. Sec.	E meter	N meter	Φ N Deg. Min. Sec.	λ E Deg. Min. Sec.	E meter	N meter
1	32° 29″ 55.54″	44° 25″ 16.83″	445640	3595866	32° 29″ 54.31″	44° 25″ 16.82″	445900	3595850
2	32° 26″ 36.86″	44° 25″ 4.46″	445284	3589753	32° 26″ 36.86″	44° 27″ 8.47″	445700	3589800
3	32° 27″ 19.85″	44° 21″ 42.78″	440025	3591106	32° 27″ 19.0″	44° 21″ 42.67″	440400	3590800
4	32° 30″ 11.53″	44° 21″ 42.66″	440054	3596393	32° 31″ 28.04″	44° 21″ 51.80″	440550	3598550
5	32° 32″ 18.30″	44° 24″ 45.32″	444842	3600269	32° 32″ 17.53″	44° 24″ 45.36″	445800	3600050
6	32° 23″ 57.84″	44° 19″ 28.38″	436477	3584908	32° 23″ 57.19″	44° 19″ 28.7″	436300	3588300

The transform was applied using 6 GCPs with accurate selection which represented the main Road and River boundary in the city. The reason for select this number of GCPs referred to order of transform, i.e. if the No. of GCPs increase than the twice minimum No. of transform, (only 3GCPs), the fitting go to generate large error in transformation process. The transform was applied for the two coordinates system using special program in Matlab facility. The output result in both UTM and (Φ, λ) are given in table 4. The total root mean square error was less than 10 meter for UTM, and less than one sec. for (Φ, λ), see table 4.

Table: 4 Transformation Result and Errors

GCP#	Calculated Coordinates				Errors			
	UTM		(Φ, λ)		UTM		(Φ, λ)	
	E meter	N meter	Φ N Deg. Min. Sec.	λ E Deg. Min. Sec.	E meter	N meter	Φ N Sec.	λ E Sec.
1	445909	3595856	32˚ 29″ 61.22″	44˚ 25″ 17.54″	+9	+6	6.91	1.72
2	445712	3589802	32˚ 26″ 36.22″	44˚ 27″ 8.11″	+12	+2	-0.64	-0.36
3	440413	3590800	32˚ 27″ 22.05″	44˚ 21″ 55.12″	+13	0	3.05	12.45
4	440540	3598532	32˚ 31″ 30.25″	44˚ 21″ 48.77″	-10	-18	2.21	-3.03
5	445800	3600054	32˚ 32″ 12.21″	44˚ 24″ 35.15″	0	+4	-5.32	-10.21
6	436308	3588285	32˚ 23″ 50.20″	44˚ 19″ 28.75″	+8	-15	-6.99	0.05

Total RMSE for UTM =5.83 meter
Total RMSE for (Φ, λ) =0.98 sec.

5. Conclusion

The following points summarize the main features on this paper;

1. The geodetic Datum describe the mathematical model for projection the global surface to flat one.Therefore, work with different Datum Make some hundred meter error in measurement. The aim of this paper is to estimate methods to transform the Datum to avoid errors.

2. The GPS navigation measurements was built on the WGS-84 Datum, where the topographic Maps of our country was built on the Clack 1880 Datum.

3. The work involve estimate two methods for datum transformation, the direct method, for transform (Φ, λ) projection only, second was the transformation according to LMSC & GCPs fitting, for both UTM and (Φ, λ) projections. The methods were evaluated through two written program in Matlab facility.

4. The first method hold some few seconds error, where each second represent 25 meter in our country, but it is simple and direct in use.

5. The second one, was low amount of errors. The spatial transformation error was less than 10 meter for UTM projection and less than 1 sec for (Φ, λ) projection.

6. The last transform method involve the best and carefully selection of GCPs among the image and map.

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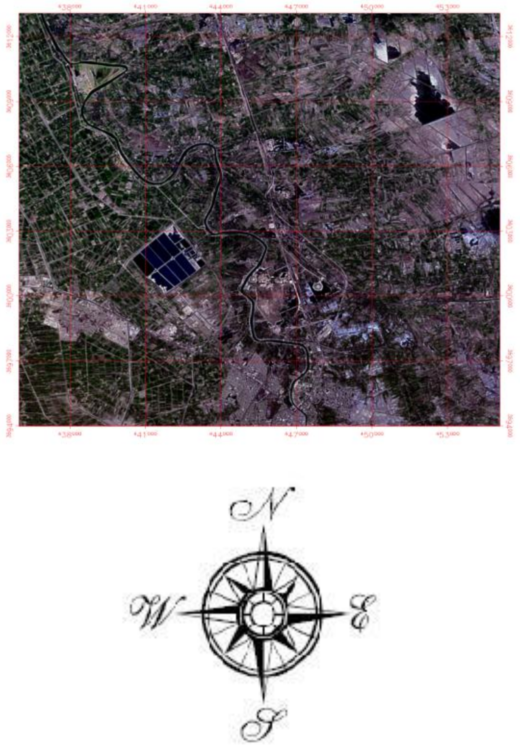
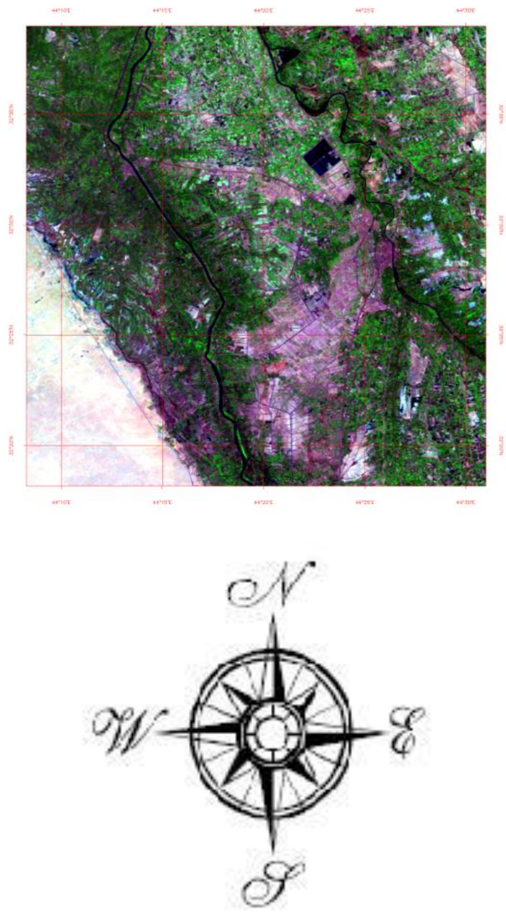


ETM+ Photo-Map, Projection UTM, Datum WGS84



TM Photo-Map, Projection (Φ, λ) , Datum WGS84

تحويل المجسمات لأغراض تصحيح قياسات نظام التموضع العالمي

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الخلاصة:

تعد القياسات المستخرجة بواسطة نظام التموضع العالمي GPS من التطبيقات المهمة والواسعة الاستخدام في الوقت الحاضر. غالبا ما ترافق هذه القياسات أخطاء متنوعة وكثيرة لعل أهمها الخطأ الناتج من اختلاف النموذج الرياضي للإسقاط الهندسي (مجسم الجيوئيد). يتضمن تعرف مجسم الجيوئيد وصف حجم وشكل الأرض ونظام الإحداثيات المستخدم لتمثيل سطح الأرض الكروي بشكل مستوي. في هذا البحث، تم استحداث طريقتين رياضيتين لغرض تحويل المجسمات وإيجاد معاملات التحويل وتصحيح الأخطاء الناتجة بين مجسمات نظام التموضع العالمي WGS-84 ومجسم الخرائط الطبوغرافية المحلية والذي هو Clarck-1880. لقد أخرجت النتائج واختبرت اعتمادا على استخدام جهاز Garmin GPS لغرض مقارنة نقاط الضبط الأرضي، وصورتين فضائيتين ذات المعلومات الجيوديسية، وخارطة طبوغرافية مقياس 1:100000 لمنطقة الحلة وسط العراق. أن خطأ التحويل الحيزي كان اقل من عشرة أمتار للمسقط UTM واقل من ثانية للمسقط (Φ, λ) وهذه نتيجة مقبولة يمكن اعتمادها في الحسابات التحويلية. تم إنجاز نتائج البحث بواسطة كتابة برنامجين منفصلين لكل طريقة وباستخدام بيئة Matlab.