Al-Qadisiyah Journal for Administrative and Economic Sciences **ISSN$_{Online}$: 2312-9883**
***QJAE*, Volume 24, Issue 4 (2022)** **ISSN$_{Print}$: 1816-9171**
Manuscript ID:

Simulation Study for the Bayesian Reciprocal Adaptive Lasso Quantile Rgeression

By

Zahraa Awaid Lafta
Department of Statistics
College of Administration and Economic
University of Al-Qadisiyah, Iraq

Dr. Ahmad Naeem Flaih
Department of Statistics
College of Administration and Economics
University of Al-Qadisiyah, Iraq

Admin.stat21.9@qu.edu.iq

Abstract

Regression analyses have two major purposes, explanation and the prediction. The explanation concept of the regression model can be capture by introducing the more interpretable model via variable selection procedure, while the prediction ability of the regression model can be capture by balancing between the bias and the variance of the interest parameter estimates. This paper explores the Bayesian adaptive lasso method, a shrinkage method that provides estimation and variable selection procedure, also this method yields more interpretable model with more prediction accuracy. The reciprocal lasso has favorable properties comparing with lasso and for that we utilize two scale mixture formulation, the first one is the scale mixture of normals and the scale mixture of uniforms. New hierarchical prior model and full conditional posterior distribution with the Gibbs sampler algorithm have developed. Two simulation scenarios conducted to test the performance of the proposed Bayesian methods in quantile regression. The results explained that the proposed models are comparable with some methods.
Keywords: Reciprocal adaptive lasso, Gibbs sampler, Simulation, full conditional posterior distribution.

Introduction

In recent years, quantile regression analysis has been very common used in the field of statistics, because it serve as a general linear regression model for the relationship between the response variable and predictor variables, Chatterjee and Hadi (2013). Also, quantile regression model can be viewed as method that detect more precise relationships (models) between response variable and predictor variables and assign for possibility hetrogenity. Consequently, quantile regression is a robust method. just like the median regression , that is mean the coefficients regression estimates is not very much affected by the outliers unlike the estimates of the ordinary least squares method . However; in same data analysis, the researchers are not interested in the mean estimate of relationship of the response variable (y) and the predictor variables (x) which is named mean regression, or the assumptions of the mean regression do not hold; such as the distribution of the error term. So, if the error term distribution is not specified or the there are violation of mean regression assumptions, the modeling of other quantities (quantiles) might be more reality in specifying the correct model . In section 1.1 we will review the most important studies related to the quantile regression same regularization method with a focus on Bayesian estimation. Koenker and Bassett in 1978 introduced the regression quantiles. Marasinghe (2014) stated that the quantile regression estimators are robust and does not require the condition that imposes on the distribution of the error term. In 1987 Koenker and Dorey modified and developed an efficient computing algorithm for estimating the quantile regression parameter estimates. Tibshirani (1996) introduced the lasso method as variable selection procedure with frequents estimation methods. In 1999 Koenker and Machado developed a goodness of fit test for quantile regression model by using the coefficient of determination. In 2001 Yu and Moyeed discussed using of asymmetric Laplace distribution as likelihood function in Bayesian quantile regression model. In 2006 Zou proposed new penalized function that adds to the residual sum of squares and named adaptive lasso. In 2007 Yu and Stander discussed the Bayesian reference for Tobit quantile regression. In 2008 Li and Zhu studied the variable selection in lasso quantile regression. In 2010 Leng introduced the Bayesian adaptive lasso assuming that tuning parameter takes different values. Kozumi and Kobayashi (2011) proposed new Gibbs sampler algorithm in Bayesian quantile regression assuming tht the asymmetric Laplace distribution can be represents as scale mixture of normal- exponential density. In 2017 Alhusseini introduced the variable selection in Bayesian lasso quantile regression by assuming that scale mixture unifroms. In 2020 Almusaedi and Flaih studied the Bayesian parameter estimation of the quantile regression based on asymmetric Laplace distribution. In 2021 Almusaedi and Flaih Studied the Penalized Bayesian Elastic Net Quantile Regression. In 2020, mallick et al. proposed two reciprocal lasso regression models based on the scale mixture of normal and the scale mixture of uniform. In 2021, Alhamzawi and mallick introduced the reciprocal lasso quantile regression in Bayesian estimation. This paper have new simple and efficient Gibbs sampler algorithm to generate the samples from the target posterior distributions. The simulation results showed that the proposed methods are comparable with other methods.

Bayesian Reciprocal Adaptive Lasso Quantile Regression

In this section, we proposed a new Gibbs Sampler algorithms based on new hierarchical model for Bayesian reciprocal adaptive lasso quantile regression (BrALqr). This Bayesian model has developed through using the proposed scale mixtures of Mallick et al. (2020) that represents the prior distribution of interested parameters as the inverse Laplace distribution. The inverse Laplace distribution takes the following prior form,

$$\pi(\beta) = \prod_{j=1}^{k} \frac{\lambda}{2\beta_j^2} e^{-\frac{\lambda}{|\beta_j|}} \quad I\left(\beta_j \neq 0\right) \cdots \cdots \cdots (1)$$

Mallick et al (2020) proved that the Bayesian reciprocal lasso method is more efficient in computation algorithms that provides efficient convergence in implementing to generates samples from the posterior distribution of the interested parameters, for more details check out Song (2014) and Song and Liang (2015). We will employ the Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms referred to as (BRALQRU) and Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of normals referred to as (BRALQRN), Alhamzawi and Mallick (2020). We can write the Bayesian minimization problem of the reciprocal lasso quantile regression as follows:

$$\min_{\beta} \sum_{i=1}^{n} \rho_\tau(y_i - x_i'\beta) + \sum_{j=1}^{p} \frac{\lambda_j}{|\beta_j|} I\{\beta_j \neq 0\}. \ldots (2)$$

Next subsections discuss the hierarchical prior models and the full conditional posterior distributions for implementing the Gibbs sampling algorithms.

Hierarchical Priors models based on the scale mixture of Uniform

Alhamzawi and Mallick (2020) introduced the parameter estimation in Bayesian reciprocal adaptive Lasso quantile regression by using the scale mixture of uniforms as representation for the prior distribution (1),

$$\frac{\lambda_j}{2\beta_j^2} e^{-\frac{\lambda_j}{|\beta_j|}} = \int_{u_j > \frac{1}{|\beta_j|}} \frac{1}{2u_j\beta_j^2} \frac{\lambda_j^2}{\Gamma(2)} u_j^{2-1} e^{-\lambda_j u_j} du_j, \lambda_j > 0 \quad \ldots \quad (3)$$

The hierarchical model of BRALQRU based on (1) and (2) can be defined as follows,

$$y_i = x_i^T \beta_\tau + \theta_\tau v_i + \alpha_\tau \sqrt{\sigma v_i} z_i,$$

$$y_i | x, \beta, \sigma, v \sim \prod_{i=1}^{n} N(x_i^T \beta_\tau + \theta_\tau v_i\ , \alpha_\tau^2 \sigma v_i),$$

$$v^{n\times 1} \mid \sigma \sim \prod_{i=1}^{n} Exp\ (\sigma),$$

$$\beta^{p\times 1} \mid u \sim \prod_{j=1}^{p} \frac{1}{Uniform\left(-u_j, u_j\right)} \quad , \qquad \dots (4)$$

$$u^{p\times 1} \mid \lambda_j \sim \prod_{j=1}^{p} Gamma(2, \lambda_j),$$

$$\sigma \sim \sigma^{-a-1} \exp\left(-\frac{b}{\sigma}\right),$$

$$\lambda_j \sim {\lambda_j}^{c-1} \exp\left(-d\lambda_j\right).$$

Now we can employ the above hierarchical prior model to write down the full conditional posterior distribution.

Full conditional posterior distributions of BRALQRU

The hierarchical model (4) can be employed with a Gibbs sampler algorithm. Gibbs sampling algorithm is a Markov Chain Monte Carlo (MCMC) tool that draws iteratively samples from the conditional posterior distribution of a specific variable conditioned on all other variables. The hierarchical model (4) utilized in such a way that we can formulate the full conditional posterior distributions that easy to simulate from. The full joint distribution defined as follows:

$$f(y \mid \beta, \sigma, v)\, \pi(\sigma) \prod_{j=1}^{p} \pi(\beta_j|u_j, \lambda_j)\, \pi\left(v_j\right) \pi(u_j)\pi(\lambda_j)$$

The conditional distribution of y is defined by:

$$y_i|x, \beta, \sigma, v \sim \prod_{i=1}^{n} N(x_i^T \beta_\tau + \theta_\tau v_i \,, \alpha_\tau^2 \sigma v_i).$$

The full conditional posterior distribution of β is defined by:

$$\beta \mid y, X, v, u, \lambda, \sigma \sim N_p\left(\hat{\beta}, 2\sigma(X'V^{-1}X)^{-1}\right) \prod_{j=1}^{p} I\left\{\left|\beta_j\right| > \frac{1}{u_j}\right\}.$$

Where $\hat{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}(y - \theta v)$, and $V = diag(v_1, \dots, v_n)$.

The full conditional posterior distribution of v_i is defined by:

$$v_i \mid y, X, \beta, \sigma, u, \lambda \sim \text{GIG}\left(\frac{1}{2}, \frac{(y_i - x_i'\beta)^2}{2\sigma}, \frac{1}{2\sigma}\right)$$

Where, GIG is generalized inverse Gaussian.

The full conditional posterior distribution of σ is defined by:

$$\sigma \mid y, X, \beta, v, u, \lambda \sim \text{IG}\left(a + \frac{3n}{2}, b + \frac{1}{2}(y - X\beta - \theta v)'V^{-1}(y - X\beta - \theta v)\right).$$

Where, IG is inverse gamma.

The full conditional posterior distribution of u is defined by:

$$u \mid y, X, \beta, v, \lambda, \sigma \sim \prod_{j=1}^{p} \text{Exp}(\lambda)\, I\left\{u_j > \frac{1}{|\beta_j|}\right\}.$$

The full conditional posterior distribution of λ is defined by:

$$\lambda \mid y, X, \beta, v, u, \sigma \sim \text{Gamma}\left(c + 2p, d + \sum_{j=1}^{p} \frac{1}{|\beta_j|}\right).$$

Hierarchical Priors models based on the scale mixture of Normal

Mallick et al. (2020), Alhamzawi and Mallick (2020) introduced the following proposition based on the work of Armagan et al. (2013):

$\beta \sim N(0, \gamma) I\{|\beta| > \eta\}, \gamma \sim \exp(\zeta^2/2), \ \zeta \sim \exp(\eta)$, and $\eta \sim$ Inverse Gamma $(2, \lambda)$, then β distributed according to inverse Laplace distribution with parameter λ.

From this proposition, the inverse Laplace distribution can be represents as scale mixture of truncated normal. Now based on the above proposition, minimization problem (2), and the quantile regression model , the hierarchical prior model defines as follows:

$$y_i | x, \beta, \sigma, v \sim \prod_{i=1}^{n} N(x_i^T \beta_\tau + \theta_\tau v_i \,, \alpha_\tau^2 \sigma v_i).$$

$$\beta^{p\times 1} \mid \gamma, u \sim \prod_{j=1}^{p} N(0, \gamma_j^2) I\left\{|\beta_j| > \frac{1}{u_j}\right\}$$

$$\gamma^{p\times 1} \mid \zeta \sim \prod_{j=1}^{p} \mathrm{Ex\,p}\left(\zeta_j^2\right)$$

$$\zeta^{p\times 1} \mid u \sim \prod_{j=1}^{p} \mathrm{Ex\,p}\left(\frac{1}{u_j}\right), \text{Where } u = \frac{1}{\eta}$$

$$u^{p\times 1} \mid \lambda \sim \prod_{k=1}^{p} \mathrm{Gamma}\ (2, \lambda) \qquad \dots (5)$$

$$\sigma \sim \sigma^{-a-1} \mathrm{ex\,p}\left(-\frac{b}{\sigma}\right)$$

$$\lambda \sim \lambda^{c-1} \exp\ (-d\lambda)$$

BRALQRN computation:

Calculation of MCMC iterations for drawing randomly samples from the full conditional posterior distributions can be done by the following algorithm steps:

1- Sampling y_i: this can be done by drawing samples from truncated normal with mean $x_i^T \beta_\tau + \theta_\tau v_i$ and variance $\alpha_\tau^2 \sigma v_i$.

2- Sampling v^{-1}: this can be done by drawing samples from inverse Gaussian:

$$v^{-1} \backslash . \sim \prod_{i=1}^{n} \text{Inverse} - \text{Gaussian}\left(\frac{1}{2}, \frac{1}{|\mathrm{y_i} - \mathrm{x_i'}\beta|}, \frac{1}{2\sigma}\right)$$

3- Sampling u_i: this can be done by drawing samples from

$$u \backslash . \sim \prod_{k=1}^{p} \text{Exponential}(\lambda) I\left\{u_k > \frac{1}{|\beta_k|}\right\}$$

4- Sampling τ^{-1}: this can be done by drawing samples from inverse Gaussian

$$\tau^{-1} \backslash . \sim \prod_{k=1}^{p} \text{Inverse} - \text{Gaussian}\left(\frac{1}{2}, \sqrt{\frac{\zeta_k^2}{\beta_k^2}}, \zeta_k^2\right).$$

5- Sampling ζ: this can be done by drawing samples from gamma distribution

$$\zeta \backslash . \sim \prod_{k=1}^{p} \text{Gamma}\left(2, \left(|\beta_k| + \frac{1}{u_k}\right)\right).$$

6- Sampling β: this can be done by drawing samples from truncated multivariate normal distribution:

$$N_P\left((X'\Omega^{-1}X + T^{-1})^{-1} X'\Omega^{-1}(\mathrm{y} - \theta \mathrm{v}), (X'\Omega^{-1}X + T^{-1})^{-1}\right) \prod_{k=1}^{p} I\left\{|\beta_k| > \frac{1}{u_k}\right\}.$$

7- Sampling σ: this can be done by drawing samples from inverse gamma distribution

$$\text{Inverse} - \text{Gamma}\left(\text{a} + \frac{3n}{2}, b, \frac{1}{4}(y - X\beta - \theta v)'V^{-1}(y - X\beta - \theta v)\right)$$

8- Sampling λ: this can be done by drawing samples from gamma distribution

$$\text{Gamm a}\left(c + 2p, d + \sum_{k=1}^{p} \frac{1}{|\beta_k|}\right).$$

Simulation Study Analysis

We carry out simulation studies and real data analysis to demonstrate the performance of the proposed approaches (Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms referred to as 'BrALqr.U' and Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of normals referred to as 'BrALqr.N'). The proposed approaches are compared with some existing Bayesian and non-Bayesian approaches. The approaches in this comparison include:

- Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms (BrALqr.U).
- Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms (BrALqr.N).
- Bayesian reciprocal Lasso quantile regression using scale mixture of uniforms (BrLqr.N).
- Bayesian Lasso quantile regression (BLqr).
- Bayesian bridge quantile regression (BBqr).
- Lasso regression (lasso).
- Quantile regression (qr).
- Quantile regression with L1 penalty (qrL1).

We consider four simulation studies:

- Simulation study 1 (sparse case): $\beta = (2,2,0,0,2,0,0,0,0,0)$.
- Simulation study 2 (dense case): $\beta = (1,1,1,1,1,1,1,1,1,1)$.

The data in the simulation examples were generated by

$$y_i = X_i'\beta + e_i, i = 1,2, \dots, n$$

We setup the error distribution e_i so that the q-th quantile equal to 0. Following Li, et. al (2010), we consider four error distributions:

- $N(\mu, 9)$, we setup μ so that the qth quantile equal to zero.
- $0.1N(\mu, 1) + 0.9\, N(\mu, 5)$, , we setup μ so that the qth quantile equal to zero.
- Laplace distribution, Laplace (μ; b = 3), we setup μ so that the qth quantile equal to zero.
- Mixture of two Laplace distribution, 0.1 Laplace (μ; b = 1) + 0:9 Laplace (μ; b $=\sqrt{5}$), we setup μ so that the qth quantile equal to zero. (Li et al. ,2010).

For the first three simulations (Simulation study 1, Simulation study 2 and Simulation study 3), the rows of the design matrix X were generated from $\mathrm{N}(0, \sum)$,where $\sum$ has an autoregressive correlated matrix, where $\sum_{ij} = 0.5^{|i-j|}$ for all $1 \leq i \leq j \leq p$. The data for Simulation 4 is following the setup of Zou (2006), where the $\mathrm{cor}(x_i, x_j) = -0.39$ for i < j < 4 and $\mathrm{cor}(x_1, x_4) = 0.23$, i < 4 . In each simulation study, we run 100 replications. For each replication, we simulate 20 observations as a training set and 200 observations as a testing set. We run the Bayesian algorithms for 13000 iterations discarding the first 1000 iteration as a burn-in. Approaches are compared using median of mean absolute deviation (MMAD):

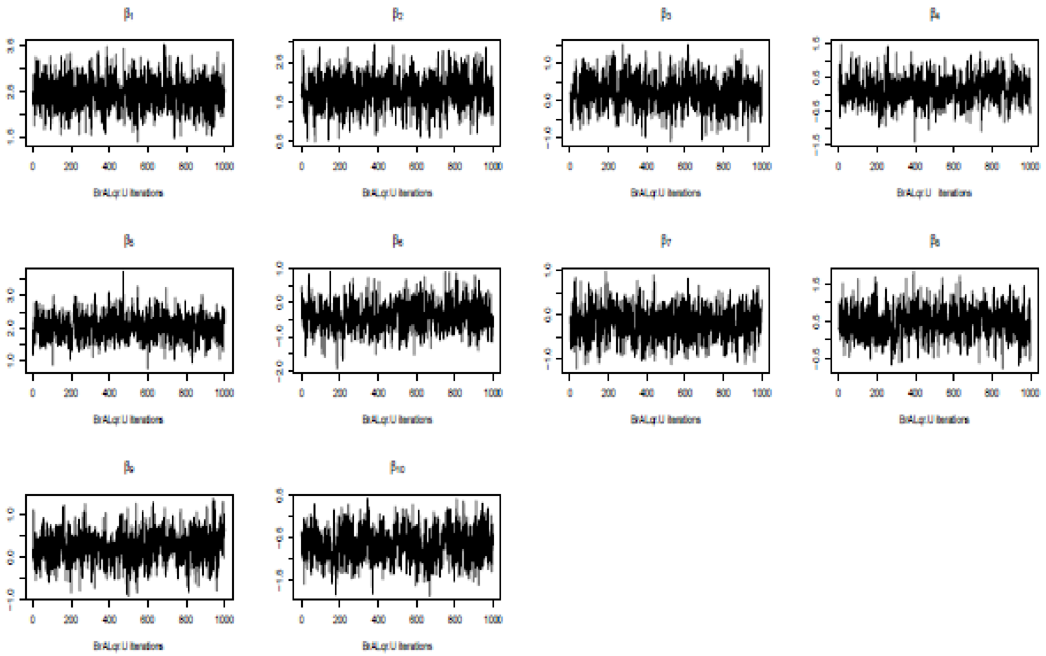
$$\mathrm{MMAE} = \mathrm{Median}\ (\mathrm{mean} \left|x_i^T \beta^{predicted} - x_i^T \beta^{true}\right|)$$

where me is the median which is taken over 100 simulations. The results of the simulations are listed in Tables 1 and 2. We can see that our proposed approaches (BrALqr.U and BrALqr.N) perform well compared with the other existing approaches. For all the simulated cases, convergence of the corresponding MCMC Gibbs sampler was evaluated by trace plots and histograms of the simulated samples. Trace plot is a convergence diagnoses technique, commonly is using to indicate if the generated samples from MCMC for the posterior distribution of parameters convergence to stationary distribution. Moreover, the histograms are used for checking the distribution class of the interested variable.

Table 1: MMADs and SD for Simulation study 1. In the parentheses are standard deviations of the MMADs.

q	Method	Error Distribution			
		normal	normal mixture	Laplace	Laplace mixture
$q = 0.1$	BrALqr.U	1.7297 (0.2322)	**1.0895** (0.0880)	2.1400 (0.5579)	1.4016 (0.1422)
	BrALqr.N	1.7333 (0.4328)	1.1115 (0.2039)	3.3173 (0.6734)	1.4115 (0.0854)
	BrLqr.N	2.3296 (0.0248)	1.3035 (0.0773)	3.0689 (0.5632)	1.4477 (0.1217)
	BLqr	**1.7020** (0.3332)	1.1053 (0.0940)	2.2803 (0.0019)	**1.0215** (0.1571)
	BBqr	2.3571 (0.1040)	1.1701 (0.0016)	2.9227 (0.4265)	1.3839 (0.0044)
	lasso	1.8434 (0.2696)	1.1544 (0.0592)	**2.0156** (0.0954)	1.7939 (0.1828)
	qr	2.1835 (0.4175)	1.4124 (0.1953)	3.3177 (0.6585)	1.5017 (0.0834)
	qrL_1	2.2706 (0.3197)	2.5132 (0.0147)	3.6636 (0.0382)	3.4927 (0.1289)
$q = 0.5$	BrALqr.U	2.0867 (0.0963)	**1.0111** (0.0269)	**2.0318** (0.3623)	**1.0891** (0.0167)
	BrALqr.N	2.0482 (0.2565)	1.1074 (0.0253)	2.0923 (0.1293)	1.0935 (0.0541)
	BrLqr.N	2.6332 (0.1559)	1.0268 (0.0184)	2.6763 (0.4507)	1.2551 (0.1187)
	BLqr	1.9731 (0.3635)	0.9695 (0.0736)	2.1828 (0.0589)	1.1251 (0.0644)
	BBq	2.3483 (0.0716)	1.0429 (0.0693)	2.4132 (0.0733)	1.3746 (0.0374)
	lasso	**1.7680** (0.1724)	1.1377 (0.0421)	2.0342 (0.1479)	1.5184 (0.2037)
	qr	2.9489 (0.2614)	1.1088 (0.0247)	2.5924 (0.1265)	1.4641 (0.0547)
	qrL_1	1.8050 (0.0589)	1.7951 (0.1302)	2.0867 (0.3955)	1.2363 (0.1083)
$q = 0.9$	BrALqr.U	**1.5136** (0.2996)	1.8480 (0.0702)	2.2124 (0.4044)	**2.0800** (0.4193)
	BrALqr.N	1.5939 (0.1432)	1.7914 (0.1961)	2.0410 (0.3639)	2.1818 (0.5647)
	BrLqr.N	1.8423 (0.0336)	1.9018 (0.0184)	3.2503 (0.7134)	2.6721 (0.3137)
	BLqr	1.9275 (0.8329)	**1.4396** (0.0445)	**2.0000** (0.1885)	2.3599 (0.1620)
	BBqr	1.7361 (0.0940)	2.0129 (0.0015)	2.6501 (0.4572)	2.6081 (0.5594)
	lasso	1.5275 (0.5322)	1.4406 (0.0499)	2.4353 (0.3578)	2.3113 (0.1602)
	qr	2.1953 (0.1431)	1.7860 (0.2017)	2.8979 (0.3307)	3.0812 (0.5532)
	qrL_1	2.8491 (0.0103)	2.9340 (0.3537)	3.3749 (1.1403)	3.1683 (0.1564)

In Table 1, we can see that the proposed method Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms (BrALqr.U) performs better than the other approaches in 6 out 12 cases.



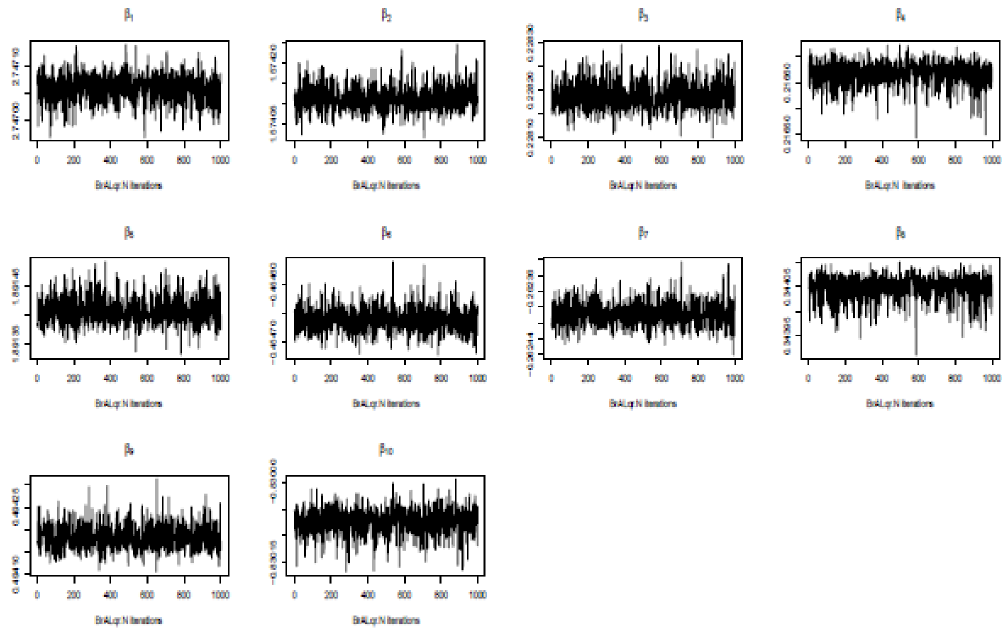


Figure 1: Trace plots based on posterior samples for Simulation 1 when the error is normal and q = 0.1 using BrALqr.U and BrALqr.N methods.

The above figure (1) shows that the trace plots explains no flat bits and that MCMC algorithm suffer no slow mixing which indicates that the proposed methods have good mixing properties. Figure (2) illustrated the distributions of the parameter estimates $\beta_1 - \beta_{10}$ through the histograms and it is clearly that the distribution of the parameters follows the normal distribution.

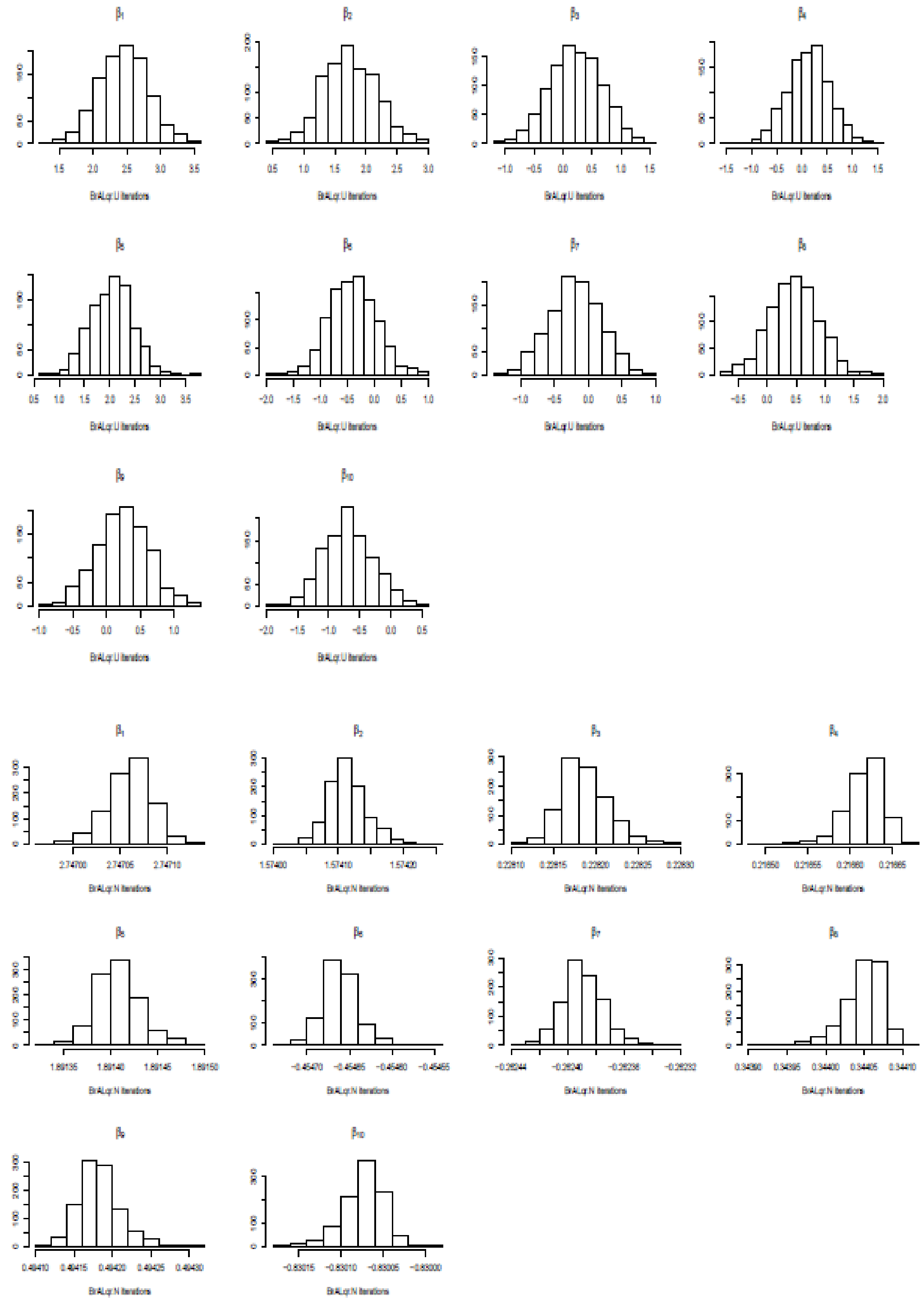


Figure 2: Histograms based on posterior samples for Simulation 1 when the error is normal and q = 0.1 using BrALqr.U and BrALqr.N methods.

Table 2: MMADs and SD for Simulation study 2. In the parentheses are standard deviations of the MMADs.

q	Method	Error Distribution			
		normal	normal mixture	Laplace	Laplace mixture
$q = 0.1$	BrALqr.U	2.0818 (0.3723)	**1.5580** (0.2062)	1.7507 (0.2971)	2.0058 (0.4423)
	BrALqr.N	2.0738 (0.4447)	1.5880 (0.2810)	**1.7484** (0.1945)	**1.7003** (0.5117)
	BrLqr.N	2.1323 (0.1594)	1.6956 (0.2005)	2.1629 (0.2323)	1.7168 (0.3034)
	BLqr	2.0717 (0.4157)	1.5740 (0.0835)	2.3009 (0.2567)	1.8355 (0.4048)
	BBqr	2.0631 (0.3254)	1.6113 (0.1636)	2.0708 (0.2617)	1.8383 (0.4022)
	lasso	**1.8016** (0.1148)	1.7843 (0.2305)	2.4886 (0.3036)	2.0975 (0.2582)
	qr	2.5745 (0.4226)	1.9882 (0.3010)	1.7488 (0.1738)	1.7005 (0.5160)
	qrL_1	2.4674 (0.3754)	2.6825 (0.4064)	3.6040 (0.3187)	3.0462 (0.6319)
$q = 0.5$	BrALqr.U	2.0042 (0.3671)	**1.0055** (0.3932)	2.2152 (0.5126)	1.5674 (0.3289)
	BrALqr.N	2.0477 (0.2571)	1.0128 (0.3399)	**2.0239** (0.6915)	1.9802 (0.3909)
	BrLqr.N	2.2545 (0.1108)	1.1098 (0.3049)	2.1546 (0.6179)	1.8590 (0.2465)
	BLqr	2.0817 (0.2201)	1.0978 (0.3816)	2.4019 (0.4739)	**1.4716** (0.1926)
	BBqr	2.1253 (0.3323)	1.1140 (0.1913)	2.3812 (0.6012)	1.5086 (0.2593)
	lasso	**1.9356** (0.3544)	1.1554 (0.5953)	3.2013 (0.5498)	1.4905 (0.4641)
	qr	2.4495 (0.2422)	1.2121 (0.3238)	2.1268 (0.6837)	1.9805 (0.3886)
	qrL_1	2.0528 (0.4799)	1.9311 (0.2435)	2.0842 (0.3803)	1.6359 (0.4531)
$q = 0.9$	BrALqr.U	**1.9335** (0.3287)	**1.5152** (0.2278)	1.9291 (0.3366)	1.4842 (0.4030)
	BrALqr.N	1.9498 (0.3505)	1.5768 (0.3275)	**1.9005** (0.4232)	**1.4130** (0.4131)
	BrLqr.N	2.3590 (0.2980)	1.6746 (0.2173)	2.0143 (0.2567)	1.5034 (0.4568)
	BLqr	1.9922 (0.2251)	1.5984 (0.2727)	2.2615 (0.2451)	1.9088 (0.2813)
	BBqr	1.9645 (0.2164)	1.7517 (0.1312)	1.9330 (0.4355)	1.8351 (0.3376)
	lasso	2.3203 (0.2788)	1.5935 (0.4409)	2.4973 (0.2618)	1.9109 (0.2504)
	qr	2.1299 (0.3565)	1.8765 (0.3217)	2.1206 (0.4194)	1.4169 (0.4196)
	qrL_1	2.1939 (0.3385)	2.1006 (0.2268)	3.4474 (0.2902)	2.5425 (0.3549)

In Table 2, we can see that the proposed method Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of normals (BrALqr.N) performs better than the other approaches in 5 out 12 cases and the proposed method BrALqr.U performs 2 better than the other approaches in 4 out 12 cases.

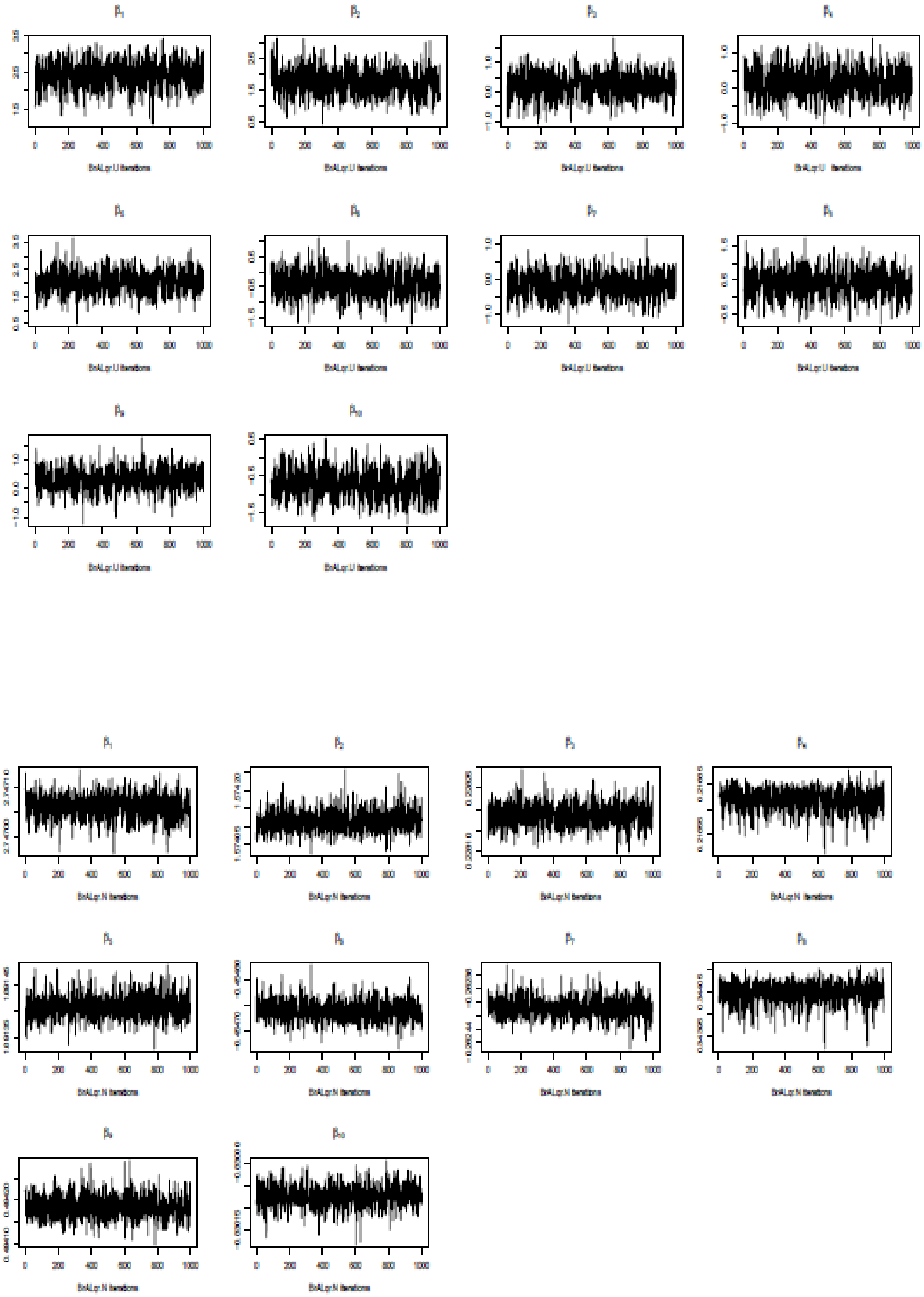


Figure 3: Trace plots based on posterior samples for Simulation 1 when the error is normal and q = 0.5 using BrALqr.U and BrALqr.N methods.

The above figure (3) shows that the trace plots explains no flat bits and that MCMC algorithm suffer no slow mixing which indicates that the proposed methods have good mixing properties. Figure (4) illustrated the distributions of the parameter estimates $\beta_1 - \beta_{10}$ through the histograms and it is clearly that the distribution of the parameters follows the normal distribution.

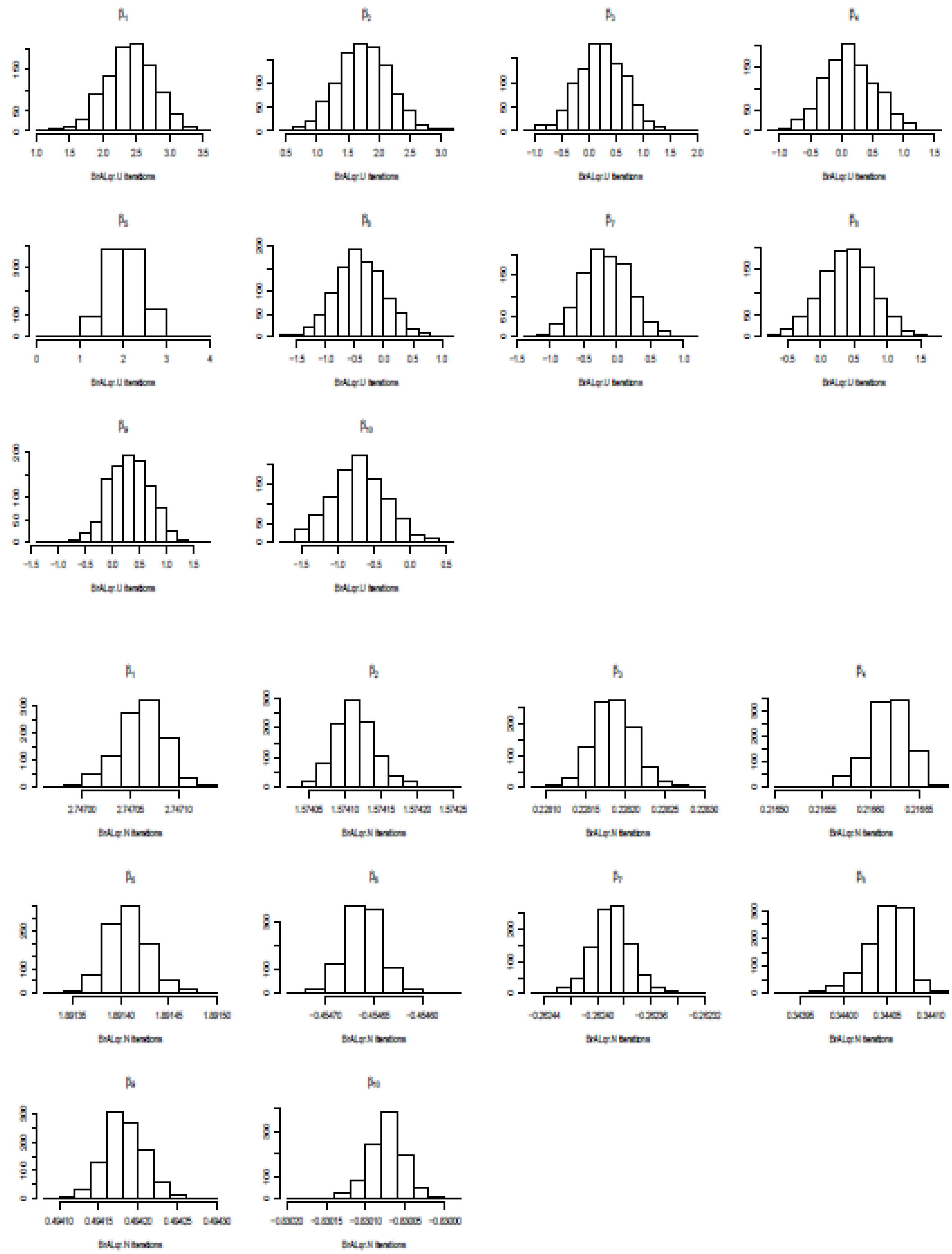


Figure 4: Histograms based on posterior samples for Simulation 1 when the error is normal and q = 0.5 using BrALqr.U and BrALqr.N methods.

Conclusions:

The lacks in the least squares methods motivates the authors to create more reliable estimation methods that are known as the regularization methods (Ridge and lasso, …etc.). The reciprocal adaptive lasso is the latest version of the regularization methods, we introduced Bayesian reciprocal adaptive lasso in quantile regression and this work is the first as application study that discussed the employing of this type of regularization methods in quantile regression. New hierarchical prior models have discussed based on two scale mixture representation for the prior distribution of the interested parameter; the first scale mixture is of normls and the second one is the scale mixture of uniforms. Furthermore, new full conditional posterior densities have developed based on the proposed hierarchical prior models, as well as the fast implement Gibbs sampler algorithm have used for the computations. We focused on the comparison idea between the two proposed models and few estimation methods to check the quality of the parameter estimates through conducted four simulation scenarios and in one real data analysis. The criterion that is named median mean absolute deviation and its standard deviation has used to assess the quality of the parameter estimation methods in simulation results. Results of the simulation scenarios show that the proposed methods are comparable to the other methods.

References:

Alhamzawi, R., & Mallick, H. (2020). Bayesian reciprocal LASSO quantile regression. Communications in Statistics-Simulation and Computation, 1-16.

Alhusseini, F.H.H. (2017) " BAYESIAN QUANTILE REGRESSION WITH SCALEMIXTURE OF UNIFORM PRIOR DISTRIBUTIONS " International Journal of Pure and Applied Mathematics pp 77-91.

Almusaedi, M. and Flaih,A. (2021). Simulation Study for Penalized Bayesian Elastic Net Quantile Regression. Al-Qadisiyah Journal of Pure Science. Vol.26, 3,pp.7-22.

Almusaedi, M. and Flaih,A. (2020). Bayesian Regularized Quantile Regression Analysis Based on Asymmetric Laplace Distribution. Journal of Applied Mathematics and Physics 08(01):70-84.

Armagan, A., D. B. Dunson, and J. Lee (2013). Generalized double pareto shrinkage. Statistica Sinica 23 (1), 119.

Chatterjee, S. & Hadi, A.S (2013). Regression Analysis by example. Wiley series in probability and statistics, Fifth edition.

Koenker, R. and J. A. F. Machado (1999). Goodness of fit and related inference processes for quantile regression. Journal of the American Statistical Association 94, 1296-1310.

Koenker, R. and G. J. Bassett (1978). Regression quantiles. Econometrica 46, 33-50.

Koenker, R. and V.Dorey (1987). Algorithm AS 229: Computing regression quantile. Journal of the Royal Statistical Society: Series C (Applied Statistics) 36, 383-393.

Kozumi, H. and Kobayashi, G. (2011). "Gibbs sampling methods for Bayesian quantile regression." Journal of statistical computation and simulation, 81(11): 1565–1578. 3.

Li, Y. and J. Zhu .(2008). l_1 -norm quantile regressions. Journal of Computational and Graphical Statistics 17, 163-185.

Leng, (2010), Bayesian adaptive Lasso,arXiv:1009.2300vl.

Li, Q., R. Xi, N. Lin, et al. (2010). Bayesian regularized quantile regression. Bayesian Analysis 5 (3), 533-556.

Mallick, H., R. Alhamzawi, and V. Svetnik (2020). The reciprocal Bayesian lasso. arXiv preprint arXiv:2001.08327.

Marasinghe,D. (2014). Quantile regression for climate data. Thesis of Master of Science,Mathematical Science. Clemson University.

Song, Q. (2018). An overview of reciprocal L1-regularization for high dimensional regression data. Wiley Interdisciplinary Reviews: Computational Statistics, 10(1):e1416.

Song, Q. and Liang, F. (2015). High-dimensional variable selection with reciprocal L1-regularization. Journal of the American Statistical Association 110 (512), 1607-1620.

Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society: Series B (Methodological), 58(1), 267-288.

Yu, K. and Moyeed, R. (2001). Bayesian quantile regression. Statistics & Probability Letters, 54:437-447.

Yu, K., & Stander, J. (2007). Bayesian analysis of a Tobit quantile regression model. Journal of Econometrics, 137(1), 260-276.

Zou, H. (2006). The adaptive lasso and its oracle properties.Journal of the American statistical association, 101(476),1418-1429.