العلاقة بين انواع دوال الهوية

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The relationship between types of identifications

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الخلاصة :
في هذا البحث تستخدم تعاريف المجموعات المفتوحة من انماط
($\alpha -$ open , pre $-$ open , b $-$ open , $\beta -$ open)
لتحديد تعاريف جديده لدوال الهوية في الفضاءات التبولوجية, اسميناها
$\alpha -$ identification, pre $-$ identification, b $-$ identification , $\beta -$ identification
وناقشنا العلاقة فيما بينهم . وايضا "بعض صفات تلك الدوال دُرست وبُرهنت .
الدالة المفتاحية :
$\alpha -$ identification, pre $-$ identification, b $-$ identification , $\beta -$ identification

Abstract
In this paper , used the definitions of ($\alpha -$ open , pre $-$ open , b $-$ open , $\beta -$ open) sets in order to limit the identifications in topological space namely ($\alpha -$ identification, pre $-$ identification , b $-$ identification , $\beta -$ identification) functions and we discuss the relationship between them , as well as several properties of these functions are proved.
Keyword :
$\alpha -$ identification, pre $-$ identification, b $-$ identification and $\beta -$ identification

Introduction and Preliminaries:
The concept of continuous($\alpha -$continuous, pre $-$continuous , b $-$continuous, $\beta -$continuous) function, irresolute($\alpha -$ irresolute , pre $-$ irresolute , b $-$ irresoljute , $\beta -$ irresolute)function and contra $-$ continouous(contra $-$ $\alpha -$ continouous, contra pre $-$ continouous , contra $-$ b $-$ continouous , contra $-$ $\beta -$ continouous) have been introduced and investigated by Mashhour [12 ,13],Andrjevic [3] ,El-Monsef [5],(Maheshwair and Thakur) [10], (Jafaris and Noiri) [7, 8] and Calda [4] respectively. By using" semi-, ($\alpha -$, pre $-$, $\beta -$, b$-$) open sets " have been introduced and investigated by Levine [9],Njasted [18], Mashhour [12,13], Andrjevic [3], El-Monsef [5] respectively.

AL-kutabi [1] in 1996 , introduces and studies some week identifications , the notion of semi-identification, Mazl [14] introduces the notion of b- identification. In this work , we study the concepts of types of identifications and discuss the relation between them .Also, we investigate it's relationship with other types of identifications.

" Throughout this paper $\mathcal{H}$, $\mathcal{M}$ *and* $\aleph$,will denote topological spaces for a subset $\mathcal{A}$ of space ($\mathcal{H}$,$\mathfrak{I}$), int($\mathcal{A}$) , cl($\mathcal{A}$) , denoted the interior and closure of a set $\mathcal{A}$, respectively ", and we indicate them by the following symbols : gof $= \mathcal{W}$, $\mathfrak{f}^{-1} = \mathfrak{H}$, g^{-1}=$\mathfrak{h}$, $f(\mathfrak{f}^{-1}) = \mathcal{F}$.

" A subset $\mathcal{A}$ of a space $\mathcal{H}$ is said to be:

1. $\alpha -$open set [18](for short $\mathfrak{D}-$) if $\mathcal{A} \subseteq \text{int}\Big(\text{cl}\big(\text{int}(\mathcal{A})\big)\Big)$. So $\mathcal{A}^c$ called $\alpha -$ closed (for short $\mathfrak{D} =$).
2. pre $-$open set [12] (for short $\mathfrak{p}-$) if $\mathcal{A} \subseteq \text{int}\big(\text{cl}(\mathcal{A})\big)$. So $\mathcal{A}^c$ called pre $-$ closed (for short $\mathfrak{p} =$).
3. $\beta -$Open set [5] (for short $\mathfrak{B}-$) if $\mathcal{A} \subseteq \text{cl}\Big(\text{int}\big(\text{cl}(\mathcal{A})\big)\Big)$. So $\mathcal{A}^c$ called $\beta -$ closed (for short $\mathfrak{B} =$).
4. b $-$open set [3] (for short $\mathfrak{b} -$) if $\mathcal{A} \subseteq \Big(\text{cl}\big(\text{int}(\mathcal{A})\big) \cup \text{int}\big(\text{cl}(\mathcal{A})\big)\Big)$. So $\mathcal{A}^c$ called b $-$ closed (for short $\mathfrak{b} =$)."

The family of all ($\mathfrak{D}-$, $\mathfrak{p}-$, $\mathfrak{B}-$, $\mathfrak{b}-$)sets is denoted by $\mathfrak{D}O(\mathcal{H})$, $\mathfrak{p}O(\mathcal{H})$, $\mathfrak{B}O(\mathcal{H})$, $\mathfrak{b}O(\mathcal{H})$.

Remark : the diagram below shows the relationship between open sets .

$$\text{open} \longrightarrow \mathfrak{D}- \longrightarrow \mathfrak{p}- \longrightarrow \mathfrak{b}- \longrightarrow \mathfrak{B}-$$

figure (1)

" The converse of these implications are not true in general".

Example 1 :

Let $\mathcal{H} = \{d, k, p, O, C\}$ on $\mathfrak{I} = \{\mathcal{H}, \varphi, \{p, O\}, \{d, k\}, \{d, k, p, O\}\}$.

Then

- A subset $\{d\}$ of $\mathcal{H}$ is $\mathfrak{p}-$ but it does not $\mathfrak{D}-$.
- A subset $\{d, k, C\}$ of $\mathcal{H}$ is $\mathfrak{b}-$ but it does not $\mathfrak{p}-$.
- A subset $\{p, C\}$ of $\mathcal{H}$ is $\mathfrak{B}-$but it does not $\mathfrak{b}-$.

"The following definitions and results were introduced and studied ".

Definition 2: "Let a function of a space $\mathcal{H}$ into a space $\mathcal{M}$ then:

1- $\mathfrak{f}$ is called open (closed) function if the image of each open (closed) set in $\mathcal{H}$ is open(closed) set in $\mathcal{M}$ [6].

2- $\mathfrak{f}$ is called $\mathfrak{D}-$ ($\mathfrak{D}=$) function if the image of each $\alpha-$ open ($\mathfrak{D}=$) set in $\mathcal{H}$ is $\mathfrak{D}-$ ($\mathfrak{D}=$) set in $\mathcal{M}$ [13].

3- $\mathfrak{f}$ is called $\mathfrak{p}-$ ($\mathfrak{p}=$) function if the image of each $\mathfrak{p}-$ ($\mathfrak{p}=$) set in $\mathcal{H}$ is $\mathfrak{p}-$ ($\mathfrak{p}=$) set in $\mathcal{M}$ [12].

4- $\mathfrak{f}$ is called $\mathfrak{b}-$ ($\mathfrak{b}=$) function if the image of each $\mathfrak{b}-$ ($\mathfrak{b}=$) set in $\mathcal{H}$ is $\mathfrak{b}-$ ($\mathfrak{b}=$) set in $\mathcal{M}$ [3].

5- $\mathfrak{f}$ is called $\mathfrak{B}-$ ($\mathfrak{B}=$) function if the image of each $\mathfrak{B}-$ ($\mathfrak{B}=$) set in $\mathcal{H}$ is $\mathfrak{B}-$($\mathfrak{B}=$) set in $\mathcal{M}$ [5]. "

Remark : the diagram below holds for a functions .

$$\text{open fun.} \longrightarrow \mathfrak{D}-\text{fun.} \longrightarrow \mathfrak{p}-\text{fun.} \longrightarrow \mathfrak{b}-\text{fun.} \longrightarrow \mathfrak{B}-\text{fun.}$$

figure (2)

"Now by [3,5,12,13]and the following examples illustrate that The converse of these implication are not true in general" .

Definition 3 : A function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is called:

1- Acontinuous function if $\mathfrak{H}$ of any open set in $\mathcal{M}$ is a open set in $\mathcal{H}$ [6].

2- $\alpha-$continuous function if $\mathfrak{H}$ of any open set in $\mathcal{M}$ is $\mathfrak{D}-$set in $\mathcal{H}$ [13].

3- pre $-$continuous function if $\mathfrak{H}$ of any open set in $\mathcal{M}$ is $\mathfrak{p}-$ set in $\mathcal{H}$ [12].

4- b $-$continuous function if $\mathfrak{H}$ of any open set in $\mathcal{M}$ is $\mathfrak{b}-$ in $\mathcal{H}$ [2].

5- $\beta-$continuous function if $\mathfrak{H}$ of any open set in $\mathcal{M}$ is $\mathfrak{B}-$set in $\mathcal{H}$ [5].

Remark : Mubarki in 2013 presented the following diagram that illustrates the relationship between the types of continuous functions . [15]

$$\text{cont.} \longrightarrow \alpha-\text{cont.} \longrightarrow \text{pre}-\text{cont.} \longrightarrow \text{b}-\text{cont.} \longrightarrow \beta-\text{cont.}$$

figure (3)

"The converse of these implications are not true in general and the following examples" .

Example. 4:

Let $\mathcal{H} = \{d, k, p, O, C\}$ on $\mathfrak{I} = \{\mathcal{H}, \varphi, \{p, \text{d}\}, \{d, k\}, \{d, k, p, \text{d}\}\}$

1-Then , $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{H}$ defined by $\mathfrak{f}(d) = k$, $\mathfrak{f}(k) = d$, $\mathfrak{f}(p) = p$, $\mathfrak{f}(O) = k$, $\mathfrak{f}(C) = C$, is pre $-$continuous function but it is not $\alpha-$ cont.

2- Then , f:$\mathcal{H} \longrightarrow \mathcal{H}$ defined by$\mathfrak{f}(d) = d$, $\mathfrak{f}(k) = k$, $\mathfrak{f}(p) = p$, $\mathfrak{f}(O) = O$, $\mathfrak{f}(C) = k$, is b $-$cont. but it is not pre $-$ cont.

3- Then, $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{H}$ defined by $\mathfrak{f}(d)=p, \mathfrak{f}(k)=\mathcal{C}, \mathfrak{f}(p)=d, \mathfrak{f}(\mathcal{O})=\mathcal{O}, \mathfrak{f}(\mathcal{C})=k$, is β −cont. but it is not b − cont.

Definition 5 :

A mapping $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ is called irresolute function[10] (resp.α − irresolute [10], pre − irresolute[13], b − irresolute [3]β − irresolute[5]) if $\mathfrak{H}$ (u) is open($\mathfrak{D}$ −, $\mathfrak{p}$ −, $\mathfrak{b}$− , $\mathfrak{B}$−)in $\mathcal{H}$ for each open ($\mathfrak{D}$ − , $\mathfrak{p}$ − , $\mathfrak{b}$−, $\mathfrak{B}$−) in $\mathcal{M}$.

"Diagram (4)" :

irresol. → α − irresol.→ pre − irresol. → b − irresol. → β − irresol.

generally speaking ,the opposite of the implication s is not necessarily true , as follows instance .

Example 6 :

Let $\mathcal{H}=\{d,k,p,\mathcal{O},\mathcal{C}\}$ on $\mathfrak{I}=\{\mathcal{H},\varphi,\{p,\mathcal{O}\},\{d,k\},\{d,k,p,\mathcal{O}\}\}$

1- Then , the $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{H}$ defined by $\mathfrak{f}(d)=d, \mathfrak{f}(k)=p, \mathfrak{f}(p)=k, \mathfrak{f}(\mathcal{O})=\mathcal{O}, \mathfrak{f}(\mathcal{C})=\mathcal{C}$,is pre − irresol.and not α − irresol .

2- Then , the $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{H}$ defined by $\mathfrak{f}(d)=d, \mathfrak{f}(k)=k, \mathfrak{f}(p)=\mathcal{C}, \mathfrak{f}(\mathcal{O})=\mathcal{O}, \mathfrak{f}(\mathcal{C})=p$, is b − irresol. and not pre − irresol .

3- Then the $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{H}$ defined by $\mathfrak{f}(d)=p, \mathfrak{f}(k)=k, \mathfrak{f}(p)=d, \mathfrak{f}(\mathcal{O})=\mathcal{C}, \mathfrak{f}(\mathcal{C})=\mathcal{O}$, is β − irresol. and not b − irresol .

Definition 7 :

A function $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ is called contra − continouous (resp.contra α − continouous , contra pre − continouous [6,7], contra b − continouous [2]contra β − continouous[4]) , if $\mathfrak{H}$ (u) is closed ($\mathfrak{D}$ = , $\mathfrak{p}$ = , $\mathfrak{b}$ = , $\mathfrak{B}$ =) in $\mathcal{H}$,for each open set u of $\mathcal{M}$.

."Diagram (5)" :

contra − cont. ⟶ contra α − cont. ⟶ contra pre – cont. ⟶ contra b − cot. ⟶ contra β − cot.

The examples show that the reversal of the chart is incorrect .

Example 8 :

Let $\mathcal{H}=\{d,k,p,\mathcal{O},\mathcal{C}\}$ on $\mathfrak{I}=\{\mathcal{H},\varphi,\{p,\mathcal{O}\},\{d,\mathrm{b}\},\{d,k,p,\mathcal{O}\}\}$

1-Then , $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{H}$ defined by $\mathfrak{f}(d)=\mathcal{C}, \mathfrak{f}(k)=k, \mathfrak{f}(p)=d, \mathfrak{f}(\mathcal{O})=\mathcal{O}, \mathfrak{f}(\mathcal{C})=p$. Iscontra pre – cont.but it is not contra α − cont.

2- Then , $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{H}$ defined by $\mathfrak{f}(d)=p, \mathfrak{f}(k)=\mathcal{O}, \mathfrak{f}(p)=d, \mathfrak{f}(\mathcal{O})=k, \mathfrak{f}(\mathcal{C})=\mathcal{C}$. Is contra b − cont .but not contra pre – cont.

3-Then , $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{H}$ defined by $\mathfrak{f}(d)=d, \mathfrak{f}(k)=\mathcal{O}, \mathfrak{f}(p)=p, \mathfrak{f}(\mathcal{O})=k, \mathfrak{f}(\mathcal{C})=\mathcal{C}$. Is contra β − cont.but not contra b − cont.

A Study of some new types of identifications:

In this section, we introduce new definitions of (α − identification, pre – identification, b − identification, β − identification) functions by using ($\mathfrak{D}$ −, $\mathfrak{p}$ −, $\mathfrak{b}$− , $\mathfrak{B}$−) sets and study the relations between them .

Definition 9 : " A function $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ is called α − identification Iff $\mathfrak{f}$ is onto and one of the following condition satisfies "

--- U is $\mathfrak{D}$ − in $\mathcal{M}$ iff $\mathfrak{H}$(u) is $\mathfrak{D}$ − in $\mathcal{H}$.

--- U is $\mathfrak{D}$ =in $\mathcal{M}$ iff $\mathfrak{H}$ (u) is $\mathfrak{D}$ = in $\mathcal{H}$.

For example : Let $\mathcal{H}=\{d,k,p,\mathcal{O}\}$ and $\mathcal{M}=\{1,2,3\}$ be equipped with the topologies $\mathfrak{I}_{\mathcal{H}}=\{\mathcal{H},\varphi,\{k,p\},\{d,k\},\{k\}\}$, $\mathfrak{I}_{\mathcal{M}}=\{\varphi,\mathcal{M},\{1,2\},\{2,3\},\{2\}\}$

If $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ defined by $\mathfrak{f}(d)=1, \mathfrak{f}(k)=2, \mathfrak{f}(p)=3, \mathfrak{f}(\mathcal{O})=3$.

we get $\mathfrak{f}$ is α − identification.

Proposition 10 :

Every $\alpha-$ irresolute and $\mathfrak{D}-(\mathfrak{D}=)$ onto functions is $\alpha-$ identification.

Proof : A $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$, $\mathfrak{f}(U)$ is $\mathfrak{D}-$ since $\mathfrak{f}$ is onto and $\mathfrak{D}-$,
so $(\mathcal{F}(U))=U$ is $\mathfrak{D}-$. and $U\subseteq\mathcal{M}$.
is $\alpha-$ irresolute hanc $\mathfrak{H}(U)$ is $\mathfrak{D}-$ in $\mathcal{H}$, so $\mathfrak{f}$ is $\alpha-$ identification. $\mathfrak{f}$

While if :

If $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ is onto , $\mathfrak{D}=$ and $\alpha-$ irresolute ,
Hence $\mathfrak{H}$ (U) is $\mathfrak{D}-$, implies $(\mathfrak{H}(U))^c=\mathfrak{H}(U^c)$ is $\mathfrak{D}=$, which $(\mathcal{F}(U^c))=(U^c)$ is $\mathfrak{D}=$,
Since $\mathfrak{f}$ is $\alpha-$ irresolute by def. so $\mathfrak{f}$ is $\alpha-$ identification. $\mathfrak{p}-$in $\mathcal{H}$".

Definition11 :

" A function $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ is called pre $-$ identification if $\mathfrak{f}$ is onto and U is $\mathfrak{p}-$ in $\mathcal{M}$ iff $\mathfrak{f}^{-1}(u)$ is $\mathfrak{p}-$ in $\mathcal{H}$ ".

Remark : from figure (1) we get every $\alpha-$ identification is pre – identification but the opposite is not true .

As follows instance .

$\mathcal{H}=\{d,k,p,O,C\}$, $\mathcal{M}=\{d,k,p,O\}$ be equipped with topologies $\mathfrak{I}_x=\{\mathcal{H},\varphi,\{p,O\},\{d,k\},\{d,k,p,O\}\}$ and $\mathfrak{I}_{\mathcal{M}}=\{\varphi,Y,\{d,k\},\{k,p\},\{k\}\}$.

If $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ defined by $\mathfrak{f}(d)=d$, $\mathfrak{f}(k)=k$, $\mathfrak{f}(p)=p$,
$\mathfrak{f}(O)=O$, $\mathfrak{f}(C)=p$, we get $\mathfrak{f}$ is pre $-$ identification.

Lemma 12 : A onto function $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ is called pre – identification , U is $\mathfrak{p}=$ in $\mathcal{M}$ iff $\mathfrak{H}$ (U) is $\mathfrak{p}=$in $\mathcal{H}$.

Proposition 13 :

Every pre $-$ irresolute and $\mathfrak{p}-(\mathfrak{p}=)$ ontofunctions is pre $-$ identification

Proof :

from " figure (1 ,4)" every $\mathfrak{D}-$ function is $\mathfrak{p}-$function and $\alpha-$ irresolute is pre – irresolute , by Proposition 10 , we get every $\mathfrak{f}$ $\alpha-$ irresolute is pre $-$ irresolute.

Definition 14:

" A function $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ is called b $-$ identification if $\mathfrak{f}$ is onto and one of the following condition satisfies "

1) U is $\mathfrak{b}-$ in $\mathcal{M}$ iff $\mathfrak{H}$ (u) is $\mathfrak{b}-$ in $\mathcal{H}$.
2) U is $\mathfrak{b}=$ in $\mathcal{M}$ iff $\mathfrak{H}$ (u) is $\mathfrak{b}=$ in $\mathcal{H}$. [3]

"from figure (1) every $\mathfrak{p}-$ is $\mathfrak{b}-$ then for each pre – identification is b $-$ identification."

We note from an example 1 :

be defined by $\mathfrak{f}(d)=d$, $\mathfrak{f}(k)=k$, $\mathfrak{f}(p)=C$, $\mathfrak{f}(O)=O$, $\mathfrak{f}(C)=p$, A $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{H}$
then $\mathfrak{f}$ is b $-$ identification but not pre $-$ identification, since $\mathfrak{H}\{d,k,p\}=\{d,k,C\}\notin PO(\mathcal{H})$.

Proposition 15 :

If $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$ is onto , $\mathfrak{b}-(\mathfrak{b}=)$ and b $-$ irresolute then $\mathfrak{f}$ is b $-$ identification. [3].

Proposition 16 :

The composition of two, $\alpha-$identification(pre $-$ identification, b $-$ identification) functions is $\alpha-$ identification(pre $-$ identification, b $-$ identification).

Proof :

Suppose that $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$, $\mathcal{G}:\mathcal{M}\longrightarrow\aleph$ are $\alpha-$ identifications
"Whenever The compo. of two onto functions is onto ".

Now ,if U be any $\mathfrak{D}-$in $\aleph$, by hypo. $\mathcal{G}$, $\mathfrak{f}$ are $\alpha-$ identifications then $\mathfrak{h}$ (U) is $\mathfrak{D}-$ in $\mathcal{M}$ and we have $\mathfrak{H}\left(\mathfrak{h}(U)\right)=(\mathcal{W})^{-1}(U)$ is $\mathfrak{D}-$ in $\mathcal{H}$. implies U is $\mathfrak{D}-$ in $\mathcal{H}$, thus $\mathcal{W}$ is $\alpha-$ identification.

Similarly ,to prove $\mathcal{W}$ is (pre − identification, b − identification).

Proposition 17 :

A $\mathfrak{f}:\mathcal{H} \longrightarrow \mathcal{M}$ and $\mathcal{G}:\mathcal{M} \longrightarrow \aleph$ are functions and $\mathfrak{f}$ is α − identification (pre − identification, b − identification)

then the following statement are valid :

1- If $\mathcal{W}$ is α − cont. (pre − cont., b − cont.)then $\mathcal{G}$ is α − cont. (pre − cont., b − cont.).
2- If $\mathcal{W}$ is α − irresolute (pre − irresolute , b − irresolute .)then $\mathcal{G}$ is α − irresolute . (pre − irresolute , b − irresolute).
3- If $\mathcal{W}$ is contra α − cont. (contra pre − cont., contra b − cont.) then $\mathcal{G}$ is contra α − cont. (contra pre − cont.,contra b − cont.).

Proof :

1) Let $\mathcal{W}:\mathcal{H} \longrightarrow \aleph$ is α − cont. , Assume that k any an open set in $\aleph$, Let V = $\mathfrak{h}$ (k) and U= $\mathfrak{H}$ (V) , whenever $\mathcal{W}^{-1}$ (k) = $\mathfrak{H}$ ($\mathfrak{h}$ (k)) =U is $\mathfrak{D}$ − in $\mathcal{H}$, then $\mathcal{W}^{-1}$ (k) $\mathfrak{D}$ − in $\mathcal{H}$,but $\mathfrak{f}$ is α − identif.
then V is $\mathfrak{D}$ − in $\mathcal{M}$ .So $\mathfrak{h}$(k) $\mathfrak{D}$ − in $\mathcal{M}$, so $\mathcal{G}$ is α − cont.
2) Assume that k any an $\mathfrak{D}$ − set in $\aleph$, Let V = $\mathfrak{h}$ (k) and U= $\mathfrak{H}$ (V) , we have $\mathcal{W}^{-1}$ (k) = $\mathfrak{H}$ ($\mathfrak{h}$ (k)) =U ,that is, U is$\mathfrak{D}$ − in $\mathcal{H}$, we get $\mathcal{W}^{-1}$ (k) $\mathfrak{D}$ − in $\mathcal{H}$,but $\mathfrak{f}$ is α − identif .
, then V is $\mathfrak{D}$ − in $\mathcal{M}$. whenever $\mathfrak{h}$ (k) $\mathfrak{D}$ −in $\mathcal{M}$.
So $\mathcal{G}$ is α − irresolute .
3) Assume that k any an $\mathfrak{D}$ − set in Z , Let V = $\mathfrak{h}$ (k) and U= $\mathfrak{H}$(V), we have $\mathcal{W}^{-1}$ (k) = $\mathfrak{H}$($\mathfrak{h}$ (k)) =U is $\mathfrak{D}$ − in $\mathcal{H}$, then $\mathcal{W}^{-1}$ (k) $\mathfrak{D}$ −in $\mathcal{H}$,but $\mathfrak{f}$ is α − identif. , then V = $\mathfrak{h}$ (k) is $\mathfrak{D}$ = in $\mathcal{M}$, thus $\mathcal{G}$ is contra α − cont.

Definition 18 :

" A function $\mathfrak{f}:\mathcal{H} \longrightarrow \mathcal{M}$ is called β − identification if $\mathfrak{f}$ is onto and U is $\mathfrak{B}$ − in $\mathcal{M}$ iff $\mathfrak{H}$ (u) is $\mathfrak{B}$ − in $\mathcal{H}$".

" from figure (1) we get every b − identification is β – identification but the converse is not true". From instance 1: let $\mathfrak{f}:\mathcal{H} \longrightarrow \mathcal{H}$ be defined by $\mathfrak{f}(d) = d$, $\mathfrak{f}(k) = k$, $\mathfrak{f}(p) = \mathcal{C}$, $\mathfrak{f}(\mathcal{O}) = \mathcal{O}$,, $\mathfrak{f}(\mathcal{C}) = p$

then $\mathfrak{f}$ is b − identification but not pre − identification, since $\mathfrak{H}\{d,k,p\} = \{d,k,\mathcal{C}\} \notin$ $\mathfrak{p}O(\mathcal{H})$.

Proposition 19:

A onto function $\mathfrak{f}:\mathcal{H} \longrightarrow \mathcal{M}$ is called β – identification if U is $\mathfrak{B}$ =in $\mathcal{M}$ iff $\mathfrak{H}$ (U) is $\mathfrak{B}$ = in $\mathcal{H}$.

Proof : If U subset of $\mathcal{M}$, $\mathfrak{B}$ = then U^c is $\mathfrak{B}$ − in $\mathcal{M}$, since $\mathfrak{f}$ is β − identification, so $\mathfrak{H}$ (U) is $\mathfrak{B}$ = in $\mathcal{H}$,(by def. $\mathfrak{f}$ is onto , $(\mathfrak{H}(U))^c = \mathfrak{H}(U^c)$ is $\mathfrak{B}$ − in $\mathcal{H}$. Similarly ,if $\mathfrak{H}$ (U) is $\mathfrak{B}$ = , in $\mathcal{H}$, we get $\mathfrak{H}(U)^c = \mathfrak{H}(U^c)$ is $\mathfrak{B}$ − in $\mathcal{H}$ and $\mathfrak{f}$ is β − identif. , we get U is $\mathfrak{B}$ = in $\mathcal{M}$.

Assume that U be $\mathfrak{B}$ − in Y then U^c is $\mathfrak{B}$ = in $\mathcal{M}$, whenever $(\mathfrak{H}(U))^c = \mathfrak{H}(U^c)$ is $\mathfrak{B}$ = in $\mathcal{H}$, so $\mathfrak{H}$ (U) is $\mathfrak{B}$ − in $\mathcal{H}$. Similarly,
if $\mathfrak{H}$ (U) is $\mathfrak{B}$ = in $\mathcal{H}$, we get $(\mathfrak{H}(U))^c = \mathfrak{H}(U^c)$ is $\mathfrak{B}$ = in $\mathcal{H}$,
and then $U^c$is $\mathfrak{B}$ = , so U is $\mathfrak{B}$ − .

proposition 20 :

If $\mathfrak{f}:\mathcal{H} \longrightarrow \mathcal{M}$ is onto ,$\mathfrak{B}$ − ($\mathfrak{B}$ =) and β − irresolute then $\mathfrak{f}$ is β − identification.

Proof :
Assume that U is $\mathfrak{B}=$ in $\mathcal{H}$, $U\subseteq \mathcal{M}$, such that $\mathfrak{H}$ (U) is $\mathfrak{B}=$ in $\mathcal{H}$ .whenever $(\mathcal{F}(U))=$ U ,we get U is $\mathfrak{B}=$ in $\mathcal{H}$ (Since $\mathfrak{H}$ (U) is $\mathfrak{B}=$in $\mathcal{H}$, ,and $\mathfrak{f}$ is $\mathfrak{B}=$ in $\mathcal{H}$).
,so U^c is $\mathfrak{B}-$ in $\mathcal{H}$, and since $\mathfrak{f}$ is $\beta-$ irresolute then $\mathfrak{H}$ (U)is $\mathfrak{B}-$ in $\mathcal{H}$, whenever $\mathfrak{f}$ is onto $(\mathfrak{H}(U))^c=\mathfrak{H}(U^c)$ imples $\mathfrak{H}(U)$ is $\mathfrak{B}-$ in $\mathcal{H}$, thus ,by Proposition 19 , then $\mathfrak{f}$ is $\beta-$ identification.

Theorem 21 : The below stated expressions are hold .
1- every identification is $\alpha-$ identification.
2- every $\alpha-$ identification is pre $-$ identification.
3- every pre $-$ identification is b $-$ identification.
4- every b $-$ identification is $\beta-$ identification.

Proof : obvious.

Remark : " the above examples show that the inverse theorem is not necessarily true ."

Proposition 22 :
"The composition of two $\beta-$ identification functions is $\beta-$ identification".

Proof:
Let $\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$, $\mathcal{G}:\mathcal{M}\longrightarrow\aleph$ are $\beta-$ identifications
"Whenever The compo. of two onto functions is onto" .
,If U be any $\mathfrak{B}-$ in $\aleph$, by hypo. $\mathcal{G}$, $\mathfrak{f}$ are $\beta-$ identifications then $\mathfrak{h}$ (U) is $\mathfrak{B}-$ in $\mathcal{M}$ and we have $\mathfrak{H}(\mathfrak{h}(U))=(\mathcal{W})^{-1}(U)$ is $\mathfrak{B}-$ in $\mathcal{H}$, implies U is $\mathfrak{B}-$ in $\mathcal{H}$, thus $\mathcal{W}$ is $\beta-$ identification.

Proposition23 :
$\mathfrak{f}:\mathcal{H}\longrightarrow\mathcal{M}$, $\mathcal{G}:\mathcal{M}\longrightarrow\aleph$ be functions and $\mathfrak{f}$ is $\beta-$ identificationthen the following statement are valid :

1- If $\mathcal{W}$ is $\beta-$ cont. then $\mathcal{G}$ is $\beta-$ cont.
2- If $\mathcal{W}$ is $\beta-$ irresolute then $\mathcal{G}$ is $\beta-$ irresolute .
3- If $\mathcal{W}$ is contra $\beta-$ cont. then $\mathcal{G}$ is contra $\beta-$ cont.

Proof :

1) Let $\mathcal{W}$ f : $\mathcal{H}\longrightarrow\aleph$ is $\beta-$ cont. , Assume that k any an open set in $\aleph$, Let V = $\mathfrak{h}$ (k) and U= $\mathfrak{H}$(V) , we have W^{-1} (k) = $\mathfrak{H}(\mathfrak{h}$ (k)) =U is $\mathfrak{B}-$ in $\mathcal{H}$, then $\mathcal{W}^{-1}$ (k) $\mathfrak{B}-$ in $\mathcal{H}$,but $\mathfrak{f}$ is$\beta-$ identification , then V is $\mathfrak{B}-$ in $\mathcal{M}$ .So $\mathfrak{h}$ (k) $\mathfrak{B}-$ in $\mathcal{M}$, thus $\mathcal{G}$ is $\beta-$ cont.

2) Assume that k any an $\mathfrak{B}-$ set in $\aleph$, Let V = $\mathfrak{h}$ (k) and U= $\mathfrak{H}$ (V) , we have $\mathcal{W}^{-1}$ (k) = $\mathfrak{H}(\mathfrak{h}$ (k)) =U ,that is, U $\mathfrak{B}-$ in $\mathcal{H}$, we get $\mathcal{W}^{-1}$ (k) $\mathfrak{B}-$ in $\mathcal{H}$,but $\mathfrak{f}$ is $\beta-$ identif. ,
then V is $\mathfrak{B}-$ in $\mathcal{M}$, thus $\mathcal{G}$ is $\beta-$ irresolute .

3) Assume that k any an $\mathfrak{B}-$ set in $\aleph$, Let V = $\mathfrak{h}$ (k) and U= $\mathfrak{H}$(V), we have $\mathcal{W}^{-1}$ (k) = $\mathfrak{H}(\mathfrak{h}$ (k)) =U is $\mathfrak{B}-$ in $\mathcal{H}$. So $\mathcal{W}^{-1}$ (k) $\mathfrak{B}-$ in $\mathcal{H}$,but $\mathfrak{f}$ is $\beta-$ identifi.,
, then V = $\mathfrak{h}$.

Remark : from the above discussion and known results we have the following implications .

identification $\longrightarrow$ $\alpha-$ identification $\longrightarrow$ pre $-$ identification $\longrightarrow$ b $-$ identification $\longrightarrow$ $\beta-$ identification

figure (6)

Definition 24 : " A space $(\mathcal{H},\mathfrak{I})$ is said to be $\alpha-\mathfrak{I}_1$ (pre-$\mathfrak{I}_1$,$b-\mathfrak{I}_1$, $\beta-\mathfrak{I}_1$) [8,11,16, 18] iff for each a pair of distinct points $x, y \in \mathcal{H}$, each belongs to an $\mathfrak{D}-($ $\mathfrak{p}-$,$\mathfrak{b}-$, $\mathfrak{B}-$)sets which does not contain the other .

Theorem 25 : A function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is $\alpha-$ identification and $\mathcal{M}$ is $\alpha-\mathfrak{I}_1$,then $\mathcal{H}$ is $\alpha-\mathfrak{I}_1$.

Proof : let $x, y \in \mathcal{H}$, $x \neq y$,since $\mathcal{M}$ is $\alpha-\mathfrak{I}_1$,there exist $\mathfrak{D}-$ s ets M_1 and M_2 ,Of $\mathcal{M}$ such that $\mathfrak{f}(x) \in M_1$ and $\mathfrak{f}(y) \in M_2$,$\mathfrak{f}(y) \notin M_1$ and $\mathfrak{f}(x) \notin M_2$.

Since function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is $\alpha-$ identification , we have

$$x \in \mathfrak{H}(M_1), y \in \mathfrak{H}(M_2) \text{ and } x \notin \mathfrak{H}(M_2), y \notin \mathfrak{H}(M_1)$$

hence then $\mathcal{H}$ is $\alpha-\mathfrak{I}_1$.

Theorem 26 : A function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is pre $-$ identification and $\mathcal{M}$ is pre -$\mathfrak{I}_1$,then $\mathcal{H}$ is pre $-\mathfrak{I}_1$.

Proof : let $x, y \in \mathcal{H}$, $x \neq y$,since $\mathcal{M}$ is pre $-\mathfrak{I}_1$,there exist $\mathfrak{p}-$ sets M_1 and M_2 ,0f $\mathcal{M}$ such that $\mathfrak{f}(x) \in M_1$ and $\mathfrak{f}(y) \in M_2$,$\mathfrak{f}(y) \notin M_1$ and $\mathfrak{f}(x) \notin M_2$ .Since function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is pre $-$ identification , we have $x \in \mathfrak{H}(M_1), y \in \mathfrak{H}(M_2)$and $x \notin \mathfrak{H}(M_2), y \notin \mathfrak{H}(M_1)$ hence then $\mathcal{H}$ is pre $-\mathfrak{I}_1$.

Theorem 27: A function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is $b-$ identification and$\mathcal{M}$ is $b-\mathfrak{I}_1$, then $\mathcal{H}$ is $b-\mathfrak{I}_1$.

Proof : let$x, y \in \mathcal{H}$, $x \neq y$,since $\mathcal{M}$ is $b-\mathfrak{I}_1$,there exist $\mathfrak{b}-$ sets M_1 and M_2 ,of $\mathcal{M}$ such that $\mathfrak{f}(x) \in M_1$ and $\mathfrak{f}(y) \in M_2$,$\mathfrak{f}(y) \notin M_1$ and $\mathfrak{f}(x) \notin M_2$ .Since function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is $b-$ identification ,

we have $x \in \mathfrak{H}(M_1), y \in \mathfrak{H}(M_2)$and $x \notin \mathfrak{H}(M_2), y \notin \mathfrak{H}(M_1)$

hence then $\mathcal{H}$ is $b-\mathfrak{I}_1$.

Theorem 28 : A function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is $\beta-$ identification and $\mathcal{M}$ is $\beta-\mathfrak{I}_1$ then $\mathcal{H}$ is $\beta-\mathfrak{I}_1$.

Proof : let $x, y \in \mathcal{H}$, $x \neq y$,since $\mathcal{M}$ is $\beta-\mathfrak{I}_1$,there exist $\mathfrak{B}-$ sets M_1 and M_2 ,of $\mathcal{M}$ such that $\mathfrak{f}(x) \in M_1$ and $\mathfrak{f}(y) \in M_2$,$\mathfrak{f}(y) \notin M_1$ and $\mathfrak{f}(x) \notin M_2$ .Since function $\mathfrak{f}: \mathcal{H} \longrightarrow \mathcal{M}$ is $\beta-$ identification , we have $x \in \mathfrak{H}(M_1)$, $y \in \mathfrak{H}(M_2)$and $x \notin \mathfrak{H}(M_2), y \notin \mathfrak{H}(M_1)$

hence then $\mathcal{H}$ is $\beta-\mathfrak{I}_1$.

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