

Some Results on Reduced Rings

*Mehsin Jabel Atteya**

*Dalal Ibraheem Rissan**

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Abstract:

The main purpose of this paper is to study some results concerning reduced ring with another concepts as semiprime ring ,prime ring,essential ideal ,derivations and homomorphism ,we give some results a bout that.

Key words: Reduced Ring ,Semiprime Ring ,Prime Ring ,Essential Ideal ,Derivations and Homomorphism.

1- Introduction

Many authors study behaviour and interrelations between various types of rings with reduced ring, they gave many results a bout that .By using the concept of annihilator and reduced ring, Fraser and Nicholson [1] showed that a ring R is a reduced p.p.-ring if and only if is a (left and right) p.p.-ring in which every idempotent is central ,where R is left p.p.-ring ,in brevity ,an l.p.p.-ring, if every principal left ideal of R ,regarded as a left R -module ,is projective. Dually ,we may define the right p.p.-rings (r.p.p.-rings),we call a ring R a p.p.-ring if R is both on l.p.p.-ring and r.p.p.-ring.Reduced rings with the maximum condition on annihilator were first studied by Cornish and Stewart [2] ,Xiaojiang and Shum[3] proved that ,let R be a p.p.-ring and $E(R)$ the set of all idempotents of R ,then R is reduced ,Muhittin and Nazim [4] proved that let R be a reduced ring,then R is right nonsingular ,we say R is a right nonsingular ring if $Z(R_R)=0$,let M be a right module over a ring R , an element $m \in M$ is said to be a singular element of M if the right ideal $r_R(m)$ is essential in R_R ,the set of all singular elements of M is denote by $Z(M)$,we say that M_R is a singular (resp. nonsingular)module if $Z(M)=M$ (resp. $Z(M)=0$)Kosan [5] proved that ,let $R[x]$ be a right IN-ring,then R is a right IN- ring in R is a reduced ring

,where a ring R is called a right Ikeda – Nakayama (for short IN-ring) if the left annihilator of the intersection of any two right ideals is the sum of the left annihilators .The objective of this paper is to study behaviour of reduced ring with types of noncommutative ring (prime and semiprime ring) and additive mapping (a derivations)we give some results a bout that.

2- Preliminaries

Throughout R will represent an associative ring with center $Z(R)$, R is said reduced if there are no nilpotent elements not equal to zero , is said to be prime if $xRy=0$ for $x,y \in R$ implies $x=0$ or $y=0$,and semiprime if $xRx=0$ with $x \in R$ implies $x=0$. An ideal I is said to essential ideal if whenever $I \cap J=0$, where J is an ideal of R implies that $J=0$,and nilpotent proved that $I^n=0$, n is nilpotency index of I a positive integer. If I is a non-empty subset of R ,then the centralizer of I in R ,denoted by $C_R(I)$,is defined by: $C_R(I)=\{a \in R/ax=xa \text{ for all } x \in I\}$.If $a \in C_R(I)$ we say that a centralizes I . An additive mapping $d:R \rightarrow R$ is called a derivation if $d(xy)=d(x)y+xd(y)$ holds for all $x,y \in R$,and an inner derivation if there exists $a \in R$ such that $d(x)=[a,x]$ for all $x \in R$.Also is called skew-centralizing on subset I of R (resp. skew-commuting on subset I of R)if $d(x)x+xd(x) \in Z(R)$ holds for all

* Department of Mathematics- College of Education – Al- Mustansiriyah University.

$x \in I$ (resp. $d(x)x + xd(x) = 0$) holds for all $x \in I$, and d acts as a homomorphism on I if $d(xy) = d(x)d(y)$ holds for all $x, y \in I$.

We write $[x, y]$ for $xy - yx$ and make extensive use of basic commutator identities $[xy, z] = x[y, z] + [x, z]y$ and $[x, yz] = y[x, z] + [x, y]z$.

To achieve our purposes, we mention the following results.

Proposition 1 [6]

The following properties of a reduced ring R are equivalent

- (a) $xR \cap yR = 0$ implies $xy = 0$ for all $x, y \in R$.
- (b) $I \cap J = 0$ implies $IJ = 0$ for all right ideals I, J of R .
- (c) Q_m is strongly regular, Q_m is the maximal ring of quotients.

Proposition 2 [7 : Proposition 1.1.6]

The following statements are equivalent, for any ring R

- (a) R is prime.
- (b) If I and J are ideals of R and $IJ = 0$, then either $I = 0$ or $J = 0$.

Lemma 3

Every reduced ring R has no divisors of zero.

Proof: Let $x \in R$, suppose that $x^n = 0$ for all $x \in R$. for some n positive integer.

Since R is reduced ring, then $x = 0$. Replacing x by xy in the relation $x^n = 0$, we obtain $(xy)^n = 0$ for all $x, y \in R$. Since R is reduced ring then $xy = 0$ for all $x, y \in R$. But $x = 0$, therefore, R has no zero divisors.

Remark 4 [6]

If Q_m is a strongly regular ring, then R must be reduced.

3-Reduced Ring as Prime or Semiprime Rings.

In this section we will discuss interrelations between prime and semiprime rings with reduced rings.

Theorem 3.1

Let R be a reduced ring. If R satisfies one of the following conditions

- (i) $xR \cap yR = 0$ for all $a, b \in R$.
- (ii) $I \cap J = 0$ for all a non-zero right ideals I, J of R . Then R is semiprime ring.

Proof: (i) For all $x, y \in R$, we have $xR \cap yR = 0$. Since R is reduced ring, then by Proposition 1(a), we obtain $xy = 0$. By Lemma 3, we get either $x = 0$ or $y = 0$. Replacing y by rx , we get $xrx = 0$ for all $x, r \in R$.

Then, we have $xRx = 0$, with $x \in R, x = 0$, therefore, R is semiprime ring. Similarly for y .

(ii) For all a non-zero right ideals I, J of R , we have $I \cap J = 0$. Since R is reduced ring, by Proposition 1(b), we obtain $IJ = 0$.

Let $x \in I$ and $y \in J$, then $xy = 0$ for all $x \in I$. By same method in part (i) we get, R is semiprime ring.

Theorem 3.2

Let R be a ring and I is a right nilpotent ideal with nilpotency index 2 and J is a right ideal of R such that $I \cap J = 0$. Then R is reduced ring.

Proof: We have $I \cap J = 0$, let $x \in I$ and $I \cap J$ implies $x \in J$, we obtain $I \subset J$. Similarly, when $y \in J$ and $I \cap J$, we obtain $J \subset I$, then $I = J$. Left-multiplying this relation by I , we get

Thus, we have $I \cap J = IJ = 0$. Since $I \cap J = M$, M is ideal of R .

Let $x \in I$ and $y \in J$, then $xy = 0$ for all $x \in I, y \in J$. But $xy \in M$, then $xy = 0 = z$ for all $z \in M$. Now, it is easy we obtain R is semiprime ring.

Theorem 3.3

Let R be a ring and I is a right essential ideal of R , then R is reduced ring.

Proof: We have I is a right essential ideal of R , then there exists J is a right ideal of R such that $I \cap J = 0$ implies $J = 0$. Left-multiplying the relation $J = 0$ by I

,we obtain $I \cap J = 0$.Then by Proposition 1(b and c) and Remark 4,R is reduced ring.

Corollary3.4

Let R be a reduced ring and I is a right essential ideal of R ,then R is prime ring.

Proof:Since I is a right essential ideal of R ,then there exists J is a right ideal of R such that $I \cap J = 0$, by Proposition 1(b) ,we get $IJ = 0$. But I is right essential ideal ,then we obtain $J = 0$.By Proposition 2(a and b) ,we get R is prime ring.

By same method in Corollary 3.4 and Theorem 3.1(ii),we can prove the following theorem .

Theorem 3.5

Let R be a reduced ring and I is a right essential ideal of R ,then R is semiprime ring.

Theorem3.6

Let R be a prime ring , I and J are right ideals of R such that $IJ = 0$. Then R is reduced ring .

Proof: We have $IJ = 0$ for all right ideals of R .

By Proposition 2(b) ,we get,either $I = 0$ or $J = 0$.

When $I = 0$,by intersection J with I ,we obtain $I \cap J = 0$ for all right ideals. And we have $IJ = 0$.By Proposition 1(b and c)and Remark 4 , we obtain,R is reduced ring. Similary when $J = 0$.

Proposition 3.7

If R is reduced ring and I is one-sided ideal of R ,x is nilpotent element of R , $x \in I$ then R is semiprime ring.

Proof: Since x is nilpotent element and $x \in I$ i.e.

$x^n = 0$ (for some n positive integer). Since R is reduced ring ,then $x = 0$. Left -multiplying by xr ,we obtain $xRx = 0$,with $x \in R$.It is easy we obtain ,R is semiprime ring.

4-Reduced Ring with Derivations.

In this section we obtain necessary and sufficient conditions for a derivation d of reduced ring with $\text{char}.R \neq 2$ to become skew-centralizing and skew-commuting .Also we study effect of d acts as skew -centralizing and skew-commuting on subset of reduced rings with $\text{char}.R \neq 2$.

Theorem4.1

Let R be a reduced ring with $\text{char}.R \neq 2$, and I is a subset of R .

If R admits a derivation d to satisfy (a)d acts as a skew-commuting on I ,then $d(I) = 0$.

(b)d acts as a skew-centralizing on I ,then $d(I)$ centralizes I.

Proof: (a) Since d is skew-commuting ,then

$$d(x)x + xd(x) = 0 \text{ for all } x \in I \quad (1)$$

Left -multiplying (1) by x,we obtain

$$xd(x)x + x^2d(x) = 0 \text{ for all } x \in I. \quad (2)$$

From (1) ,we get $d(x^2) = 0$ for all $x \in I$ (3)

In (3) replacing x by x+y,we obtain $d(x^2) + d(xy) + d(yx) + d(y^2) = 0$ for all $x, y \in I$.

According to (3) ,a bove equation become

$$d(xy) + d(yx) = 0 \text{ for all } x, y \in I. \text{Then}$$

$$d(x)y + xd(y) + d(y)x + yd(x) = 0 \text{ for all } x, y \in I .$$

Replacing y by x^2 and according to(3),

$$\text{gives } d(x)x^2 + x^2d(x) = 0 \text{ for all } x \in I \quad (4)$$

Then

$$x^2d(x) = -d(x)x^2 \text{ for all } x \in I \quad (5)$$

By substituting (4) in (2) ,we get

$$xd(x)x - d(x)x^2 = 0 \text{ for all } x \in I . \text{Then}$$

$$[x, d(x)]x = 0 \text{ for all } x \in I. \text{Then by}$$

Lemma 3,we obtain either $x = 0$

or $[x, d(x)] = 0$ for all $x \in I$.

If $x = 0$ for all $x \in I$.Then $d(x) = 0$ for all $x \in I$, i.e. $d(I) = 0$.

We have ,when $[x, d(x)] = 0$ for all $x \in I$.Then

$$d(x)x - xd(x) = 0 \text{ for all } x \in I. \quad (6)$$

From (6) and(1) ,we obtain

$2d(x)x=0$ for all $x \in I$. Since $\text{char } R \neq 2$, then

$d(x)x=0$ for all $x \in I$. Then by Lemma 3, we get

$d(x)=0$ for all $x \in I$. Then $d(I)=0$.

(b) We will now discuss, when d acts as a skew-centralizing on I . Then we have $d(x)x+xd(x) \in Z(R)$ for all $x \in I$.

$d(x^2) \in Z(R)$ for all $x \in I$. i.e.

$$[d(x^2), r]=0 \text{ for all } x \in I, r \in R. \quad (7)$$

Also, by replacing r by x in (7), we obtain

$$(d(x))x+xd(x) = x(d(x)x+xd(x)) \text{ for all } x \in I. \quad (8)$$

Then $d(x)x^2+xd(x)x = xd(x)x+x^2d(x)$ for all $x \in I$. Then

$d(x)x^2-x^2d(x)=0$ for all $x \in I$. Then

$$[d(x), x^2]=0 \text{ for all } x \in I. \quad (9)$$

In (7), replacing x by $x+y$, we obtain

$$[d(x^2)+d(xy) + d(yx)+d(y^2), r]=0 \text{ for all } x, y \in I, r \in R.$$

According to (7), we obtain

$$[d(x)y+xd(y)+d(y)x+yd(x), r]=0 \text{ for all } x, y \in I, r \in R.$$

Replacing y by x^2 , we obtain

$$[d(x)x^2+xd(x^2)+d(x^2)x+x^2d(x), r]=0 \text{ for all } x \in I, r \in R.$$

According to (7) and (9), we get

$$[x^2d(x)+xd(x^2)+xd(x^2)+x^2d(x), r]=0 \text{ for all } x \in I, r \in R.$$

$$2[x^2d(x)+xd(x^2), r]=0 \text{ for all } x \in I, r \in R.$$

Since $\text{char } R \neq 2$, then we obtain

$$[x^2d(x)+xd(x^2), r]=0 \text{ for all } x \in I, r \in R.$$

$$[x(xd(x)+d(x^2)), r]=0 \text{ for all } x \in I, r \in R.$$

$$x[xd(x)+d(x^2), r]+[x, r](xd(x)+d(x^2))=0 \text{ for all } x \in I, r \in R.$$

According to (7), a above equation become

$$x[xd(x), r]+[x, r](xd(x)+d(x^2))=0 \text{ for all } x \in I, r \in R.$$

Replacing r by x , we obtain $x[xd(x), x]=0$ for all $x \in I$. Then

$x^2[d(x), x]=0$ for all $x \in I$. Then by Lemma 3, we get either $x^2=0$ or

$[d(x), x]=0$ for all $x \in I$. If $x^2=0$. Since R is reduced ring, then $x=0$, for all $x \in I$.

Right multiplying by $d(x)$, we obtain $xd(x)=0$ for all $x \in I$. (10)

Again left – multiplying by $d(x)$, we obtain

$$d(x)x=0 \text{ for all } x \in I \quad (11)$$

Subtracting (10) and (11), we get $[d(x), x]=0$ for all $x \in I$. Thus

$d(I)$ centralizes of I . We get same result when $[d(x), x]=0$ for all $x \in I$.

Theorem 4.2

Let R be a reduced ring with $\text{char } R \neq 2$ and I a subset of R , then a derivation d is skew-centralizing and skew-commuting on I . If R admits d to satisfy

- (a) d acts as a homomorphism on I .
- (b) d acts as an anti-homomorphism on I .

Proof: (a) d acts as a homomorphism on I . We have d is a derivation, then $d(xy)=d(x)y+xd(y)$ for all $x, y \in I$. Then $[d(xy), r]=[d(x)y, r]+[xd(y), r]$ for all $x, y \in I, r \in R$. Since d acts as a homomorphism, then $[d(x)d(y), r]=[d(x)y, r]+[xd(y), r]$ for all $x, y \in I, r \in R$. Replacing r by $d(y)$, we obtain

$$[d(x), d(y)]d(y)=[d(x)y, d(y)]+[xd(y), d(y)] \text{ for all } x, y \in I.$$

$$[d(x), d(y)]d(y)=d(x)[y, d(y)]+[d(x), d(y)][y, d(y)] \text{ for all } x, y \in I.$$

Replacing y by x , we obtain $d(x)[x, d(x)]+[x, d(x)]d(x)=0$ for all $x \in I$.

Then $[d(x)^2, x]=0$ for all $x \in I$. Then $d(x)^2 \in Z(R)$ for all $x \in I$. Since d acts as a homomorphism, then $d(x^2) \in Z(R)$ for all $x \in I$, i.e. $d(x)x+xd(x) \in Z(R)$ for all $x \in I$, then d is skew-centralizing on I .

We will, now discuss when d is skew-commuting on I .

By same method in first part, we obtain $d(x)^2 \in Z(R)$ for all $x \in I$.

Also, we have $d(xy) = d(x)y+xd(y)$ for all $x, y \in I$. Replacing x by x^2 , we obtain

$$d(x^2y) = d(x^2)y+x^2d(y) \text{ for all } x, y \in I.$$

Since d acts as a homomorphism, then $d(x^2)d(y)=d(x^2)y+x^2d(y)$ for all $x, y \in I$. Then

$d(x^2)(d(y)-y) = x^2 d(y)$ for all $x, y \in I$. (12)

Then $[d(x^2)(d(y)-y), r] = [x^2 d(y), r]$ for all $x, y \in I, r \in R$.

$$d(x^2)[d(y)-y, r] + [d(x^2), r](d(y)-y) - x^2[d(y), r] - [x^2, r]d(y) = 0$$

for all $x, y \in I, r \in R$. Since d acts as a homomorphism and $d(x)^2 \in Z(R)$ for all $x \in I$,

above equation become $d(x)^2[d(y)-y, r] - x^2[d(y), r] - [x^2, r]d(y) = 0$

for all $x, y \in I, r \in R$. Replacing r by y , we obtain

$$d(x)^2[d(y)-y, y] - x^2[d(y), y] - [x^2, y]d(y) = 0$$

for all $x, y \in I$. Then

$$d(x)^2[d(y), y] - x^2[d(y), y] - [x^2, y]d(y) = 0$$

for all $x, y \in I$. Then

$$(d(x)^2 - x^2)[d(y), y] - [x^2, y]d(y) = 0 \text{ for all } x, y \in I.$$

Replacing y by x , we obtain

$$(d(x)^2 - x^2)[d(x), x] = 0 \text{ for all } x \in I.$$

Since R is reduced ring, then by Lemma3, we obtain either $d(x)^2 - x^2 = 0$ or $[d(x), x] = 0$

for all $x \in I$. We have when $d(x)^2 - x^2 = 0$ for all $x \in I$. Then $d(x)^2 = x^2$ for all $x \in I$.

Substituting the relation $d(x)^2 = x^2$, in (12), we obtain

$$d(x)^2(d(y)-y) = d(x)^2 d(y) \text{ for all } x, y \in I.$$

Thus $-d(x)^2 y = 0$ for all $x, y \in I$.

Since R is reduced ring then by Lemma3, we have either $d(x)^2 = 0$ for all $x \in I$ or $y = 0$ for all $y \in I$. If $d(x)^2 = 0$ for all $x \in I$.

Since d acts as a homomorphism, then $d(x^2) = 0$ for all $x \in I$. Then

$$d(x)x + xd(x) = 0 \text{ for all } x \in I.$$

Then d is skew-commuting on I .

If $y = 0$ for all $y \in I$. Replacing y by x , we obtain $x = 0$ for all $x \in I$.

Left-multiplying by $d(x)$, we obtain

$$d(x)x = 0 \text{ for all } x \in I. \tag{13}$$

Again right-multiplying by $d(x)$, we obtain

$$xd(x) = 0 \text{ for all } x \in I. \tag{14}$$

From (13) and (14), we get

$$d(x)x + xd(x) = 0 \text{ for all } x \in I.$$

Then d is skew-commuting on I . We will now discuss when $[d(x), x] = 0$ for all $x \in I$. Since d is a derivation, we have

$$d(x^2) = d(x)x + xd(x) \text{ for all } x \in I. \tag{15}$$

Also, we have

$$d(x)x - xd(x) = 0 \text{ for all } x \in I. \tag{16}$$

From (15) and (16), we obtain $d(x^2) = 2d(x)x$ for all $x \in I$. Since d acts as a homomorphism, we have $d(x)^2 = 2d(x)x$ for all $x \in I$. Then

$d(x)(d(x)-2x) = 0$ for all $x \in I$. Since R is reduced ring, then by Lemma3, we obtain either $d(x) = 0$ or $d(x)-2x = 0$ for all $x \in I$.

If $d(x) = 0$ for all $x \in I$. Then it is easy to get d is skew-commuting on I .

When $d(x) = 2x$ for all $x \in I$. (17)

Substituting (17) in (15), we obtain

$$d(x^2) = 4x^2 \text{ for all } x \in I. \tag{18}$$

Substituting (18) in (12), we obtain

$$4d(x^2)(d(y)-y) = d(x^2)d(y) \text{ for all } x, y \in I.$$

Then

$$4d(x^2)d(y) - 4d(x^2)y - d(x^2)d(y) = 0 \text{ for all } x, y \in I.$$

Then since d acts as a homomorphism, we obtain

$$4d(x^2y) - 4d(x^2)y - d(x^2)y = 0 \text{ for all } x, y \in I.$$

Then

$$3d(x^2y) - 4d(x^2)y = 0 \text{ for all } x, y \in I.$$

Replacing y by x^2 , we obtain

$$3d(x^4) - 4d(x^2)x^2 = 0 \text{ for all } x \in I.$$

According to (18), we obtain $3d(x^4) - d(x^2)d(x^2) = 0$ for all $x \in I$. Since d acts as a homomorphism, we obtain

$$3d(x^4) - d(x^4) = 0 \text{ for all } x \in I.$$

Then $2d(x^4) = 0$ for all $x \in I$. Since $\text{char } R \neq 2$, then

$$d(x^4) = 0 \text{ for all } x \in I.$$

Since d acts as a homomorphism, we obtain

$$d(x^2)^2 = 0 \text{ for all } x \in I.$$

Since R is reduced ring, then

$$d(x^2) = 0 \text{ for all } x \in I.$$

Then

$$d(x)x + xd(x) = 0 \text{ for all } x \in I.$$

Thus, we obtain d is skew-commuting on I . The proof of (b) is similar.

Then by Theorem 4.1 and Theorem 4.2, we obtain the following corollary.

Corollary 4.3

Let R be a reduced ring with $\text{char } R \neq 2$ and I a subset of R then a derivation $d(I) = 0$ and $d(I)$ centralizes I .

If R admits d to satisfy

(a) d acts as a homomorphism on I .

(b) d acts as an anti-homomorphism on I .

Remark 4.4

In Theorem 4.1 and Theorem 4.2, we can not exclude the condition $\text{char.}R \neq 2$, as it is shown in the following example.

Example 4.5

Let R be the ring of all 2×2 matrices over a field F with $\text{char.}R=2$, let $a = \begin{pmatrix} o & o \\ o & 1 \end{pmatrix}$, $I = \{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} \mid a, b \in F \}$ be a subset of R .

Let d be the inner derivation given by:

$$d(x) = x \begin{pmatrix} o & o \\ o & 1 \end{pmatrix} - \begin{pmatrix} o & o \\ o & 1 \end{pmatrix} x, \text{ when } x \in I$$

, then $x = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$ therefore

$$d(x) = \begin{pmatrix} o & b \\ -b & o \end{pmatrix}. \text{ Then}$$

$$\begin{aligned} d(x) \\ x + xd(x) &= \begin{pmatrix} o & b \\ -b & o \end{pmatrix} \begin{pmatrix} a & b \\ b & a \end{pmatrix} + \begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} o & b \\ -b & o \end{pmatrix} \\ &= \begin{pmatrix} b^2 & ba \\ -ba & -b^2 \end{pmatrix} + \begin{pmatrix} -b^2 & ab \\ -ba & b^2 \end{pmatrix} \\ &= \begin{pmatrix} o & 2ba \\ -2ba & o \end{pmatrix}. \text{ Since char. } R=2, \text{ then} \\ &= \begin{pmatrix} o & o \\ o & o \end{pmatrix}. \text{ Then } d \text{ is skew-centralizing} \end{aligned}$$

and skew-commuting on I , i.e. $d(I) = o$ and $d(I)$ centralizes I .

Also when we have d acts as homomorphism (resp. acts as an anti-homomorphism).

$$\begin{aligned} d(x)d(x) &= d \left(\begin{pmatrix} a & b \\ b & a \end{pmatrix} \right) d \left(\begin{pmatrix} a & b \\ b & a \end{pmatrix} \right) \\ &= d \left(\begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} a & b \\ b & a \end{pmatrix} \right) \end{aligned}$$

$$\begin{aligned} &= d \begin{pmatrix} a^2 + b^2 & ab + ba \\ ba + ab & b^2 + a^2 \end{pmatrix} \\ &= \begin{pmatrix} o & 2ab \\ -2ab & o \end{pmatrix}. \text{ Since char. } R=2, \text{ we} \\ &\text{obtain} \\ &= \begin{pmatrix} o & o \\ o & o \end{pmatrix}. \text{ Thus } d \text{ is skew-centralizing} \\ &\text{and skew-commuting on } I. \end{aligned}$$

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بعض النتائج حول الحلقات الاختزالية

دلال ابراهيم رسن*

محسن جبل عطية*

*الجامعة المستنصرية/ كلية التربية

الملخص :

الغرض الرئيسي من هذا البحث هو دراسة بعض النتائج بخصوص الحلقات الاختزالية مع مفاهيم اخرى كالحلقات شبه الاولية والحلقات الاولية والمثالي الاساسي والاشتقاقات والتشاكلات وقد اعطينا بعض النتائج حول ذلك.