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## Future of Mathematical Modelling: A Review of COVID-19 Infected Cases Using S-I-R Model

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### Abstract:

The spread of novel coronavirus disease (COVID-19) has resulted in chaos around the globe. The infected cases are still increasing, with many countries still showing a trend of growing daily cases. To forecast the trend of active cases, a mathematical model, namely the SIR model was used, to visualize the spread of COVID-19. For this article, the forecast of the spread of the virus in Malaysia has been made, assuming that all Malaysian will eventually be susceptible. With no vaccine and antiviral drug currently developed, the visualization of how the peak of infection (namely flattening the curve) can be reduced to minimize the effect of COVID-19 disease. For Malaysians, let's ensure to follow the rules and obey the SOP to lower the  $R_0$  value from time to time, hoping that the virus will vanish one day.

**Key words:** COVID-19, Mathematical modelling, SIR epidemic model.

### Introduction:

In Wuhan, Hubei Province, China, a novel strain of Coronavirus (SARS-CoV-2) was detected in December 2019, causing a severe and possibly fatal respiratory syndrome, i.e., COVID-19. Since then, the pandemic announced on 11th March by the World Health Organization (WHO) has spread worldwide. Currently, social distancing, self-quarantine, and wearing a face mask have arisen as the most adopted method for preventing and managing the pandemic in the absence of a proper medication or vaccination.

Mathematical modelling is a field utilized to visualize a problem into a mathematical equation, which gives an overview, answer, and forecast to the problem. Researchers produce the mathematical equations that can tell the nature of the problem and prepare the solution to prevent it from becoming a disaster (1). To demonstrate the possible outcome of a pandemic and warn public health interventions, mathematical models may project how infectious diseases spread. Moreover, to determine parameters for different infectious diseases, models employ simple assumptions or gathered statistics along with

mathematics to measure the results of various interventions (2). Modeling can determine which intervention(s) may prevent, test, or forecast potential development trends (3).

In the context of COVID-19, a mathematical model has become one of the main components in strategizing a preventive measure to minimize the spread's effect (4). Through the forecast, the government and Ministry of Health (MOH) may take proper action to prevent the spread of the virus (5), for example, initiating lockdown and movement control order (MCO) (MOH statement on 18th March 2020), social distancing, mandate the usage of mask and hand sanitizer in public and so on. The result obtained can also help the government prepare for action, mainly to prevent overwhelming in hospital beds due to a higher number of COVID-19 patients.

Numerous studies were performed in studying attributes related to the COVID-19 pandemic using the SIR model. For example, (6) used the procedure to fit a set of SIR and SIRD models, with time-dependent contact rate, to

COVID-19 data for a group of most affected countries. The author found that SIR and SIRD models with constant transmission coefficients cannot fit COVID-19 data for most countries (mainly because social distancing, lockdown, etc., make those time-dependent). The authors also state that any time-dependent contact rate decaying with time can help fit SIR and SIRD models for most countries.

Moreover, (7) contributes to understanding the COVID-19 contagion in Italy, who developed a modified SIR model for the contagion, and used official data of the pandemic up to 30th March 2020 for identifying the parameters of this model. The non-standard part of the approach resides because they considered model parameters, the initial number of susceptible individuals, and the proportionality factor relating the detected number of positives with the actual (and unknown) number of infected individuals. Identifying the contagion, recovery, and death rates, as well as the mentioned parameters, amounts to a non-convex identification problem that was solved employing a two-dimensional grid search in the outer loop, with a standard weighted least-squares optimization problem as the inner step. Additionally, (8) reported new analytical results and numerical routines suitable for the SIR model's parametric estimation. The manuscript introduces iterative algorithms approximating the incidence variable, which allows for analysis of the model parameters from the numbers of observed cases. The numerical approach is exemplified with data from the European Centre

for Disease Prevention and Control (ECDC) for several European countries from Jan 2020 to Jun 2020.

With limited medical facilities and a high transmission rate, the study of COVID-19 progression and its subsequent trajectory must be analyzed in Malaysia. Consolidation of these additional parameters is helpful to provide a broader picture of COVID-19 dissemination in Malaysia. Forecasts about where and when the disease will occur may be of great usefulness for public decision-makers, as they give them the time to intervene in the local public health systems.

**SIR Model:**

SIR model (Susceptible – Infected – Recovered) is an epidemic model used to explain a virus's spread (9). The construct of the simple SIR model was first articulated by (10). There are three components in this model, Susceptible – people that might be infected, Infected – people that got infected, dan Recovered – patients who recovered. Several literatures have also categorized the death of a patient as R (11). The SIR model may provide us with observations and forecasts that the recorded data alone will not transmit the virus into populations. Our study highlights the value of modeling the distribution of COVID-19 via the SIR model as it will aid by making useful forecasts to determine the effects of the disease. From these three components, these assumptions can be made, thus produced the related differential equations (12), as shown in Fig 1.



Figure 1. The dynamics of SIR model

**1<sup>ST</sup> ASSUMPTION**

There are  $n$  number of people in the community that might be susceptible to the disease. The number might reduce from time to time as the people might get infected. The differential equation for  $S(t)$  is given by

$$\frac{dS}{dt} = -aS(t)I(t), \quad (1)$$

where  $a$  denotes the rate of infectivity.

**2<sup>nd</sup> ASSUMPTION**

Given that infected people will infect a susceptible person,  $I(t)$ , the number of infected people will increase. However, the amount will eventually decrease once they recover or die, denoted

as  $R(t)$ . Thus, the differential equation produced is given by

$$\frac{dI}{dt} = aI(t)S(t) - bI(t). \quad (2)$$

**3<sup>rd</sup> ASSUMPTION**

In this case, consider that  $I(t)$  will eventually recover from the disease. Thus, the differential equation is given by

$$\frac{dR}{dt} = bI(t). \quad (3)$$

Under the condition that  $S + I + R = I$ , these are the three components in the SIR model.

**Parameter Setting:**

Several parameters need to be set. First of all, it is considered that there are no import cases

due to MCO. Secondly, the recovery rate is given by parameter  $b = \frac{1}{\lambda}$  (11), where  $\lambda$  is the incubation period of the infected patient (i.e., the duration of the infected patient). For this case,  $\lambda = 14$  days is assumed as this value is considered the number of days required to undergo a mandatory quarantine for COVID-19 patients (13). The  $a$  value is obtained using the following formula (11), given by

$$R_0 = \frac{aS_0}{b}, \quad (4)$$

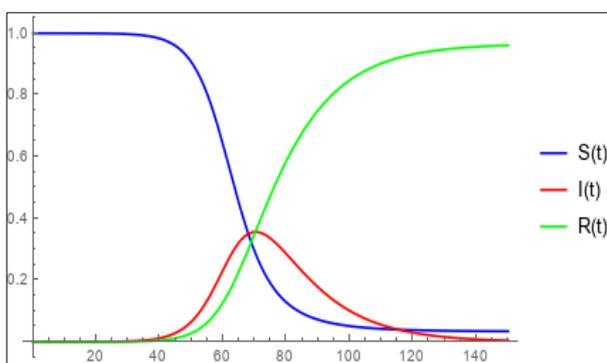
where  $S_0$  is the assumed susceptible person infected in Malaysia divided by the number of the total population in Malaysia. Here, the cases after MCO's execution are only considered from 18<sup>th</sup> March 2020 onwards.

**Data Analysis:**

For the first case, it is considered that all Malaysians are susceptible to the infection, and there are already around 0.001% (estimating around 300 cases as of 18<sup>th</sup> March 2020) who are infected at  $t = 0$ . Note that the value of  $R_0 = 3.5$  (Ministry of Health Malaysia. Press Statement on 18<sup>th</sup> May 2020). Then, from Equation (4), the equation for  $a$  is given by

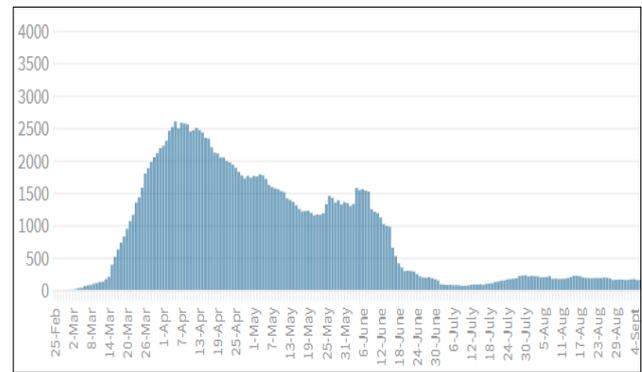
$$a = \frac{S_0}{bR_0}. \quad (5)$$

From Equation (5), the value of  $a \approx 0.25$  is obtained, where the results were analyzed using Wolfram Mathematica. Figure 2 shows that the infection reaches its peak at  $t = 70$  days with  $I[70]$  is 0.356282 (around 36% of the total population), whereas the infection becomes plateau after  $t = 140$  days.



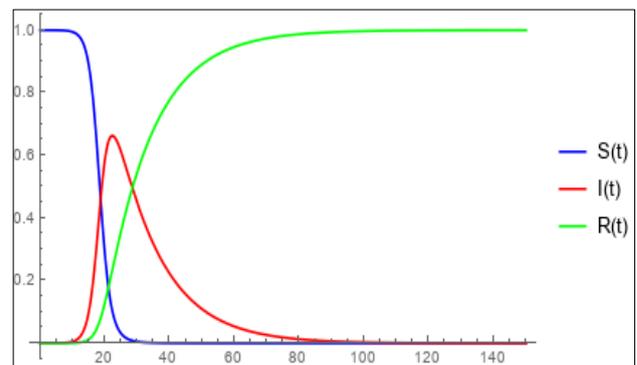
**Figure 2. The graph for Susceptible  $S(t)$ , Infected  $I(t)$ , Removed  $R(t)$ , based on the values  $R_0 = 3.5$ ,  $a = 0.25$ , and incubation period of 14 days.**

Data included in this analysis are those from 18<sup>th</sup> March 2020 onwards. The graph of Fig. 2 is then compared with the official data released by MOH (Fig. 3).



**Figure 3. Daily active cases in Malaysia from 25<sup>th</sup> February until 4<sup>th</sup> September 2020 (image retrieved from <https://ukkdosm.github.io/COVID-19>).**

Upon comparing Fig. 2 with Fig. 3, the peak occurs on 7<sup>th</sup> April 2020 (data from MOH), 20 days after MCO is implemented. This shows a clear difference in terms of the cases reaching their peak. It might indicate that our assumptions are wrong or data are incorrect. Several pieces of literature were studied to investigate parameters to obtain a more favorable result. From (14), the value of the parameter  $a$  given is 0.6931163, which results in a more favorable output.



**Figure 4. Simulation of the SIR graph based on  $a = 0.6931163$ (11) and the incubation period of 14 days.**

Figure 4 shows that the infection reaches its peak at  $t = 22$  days with  $I(22)$  with  $a = 0.661743$  (around 66.17% of the total population), whereas the infection becomes plateau after  $t = 90$  days. It shows that the active case peak occurs at  $t = 22$  days, closer to MOH data (20 days). This also implies that the  $R_0$  value at  $t = 0$  is closer to 9.7. Suppose that MCO implementation on 18<sup>th</sup> March 2020 could reduce the amount of initial susceptible persons to 20%. There is a significant drop in the peak of active cases, as shown in Fig. 5 and 6.

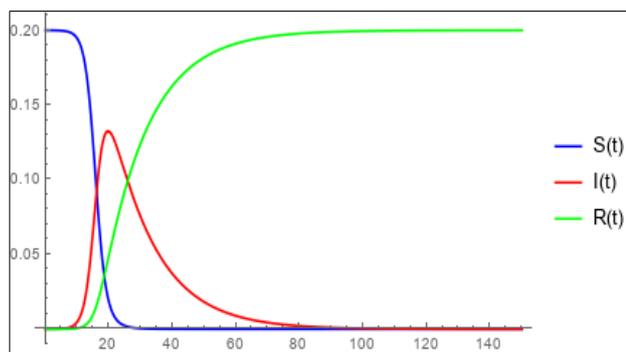


Figure 5. SIR graph based on the value of  $S_0 = 0.2$ ,  $R_0 = 9.7$ ,  $a = 3.462$ .

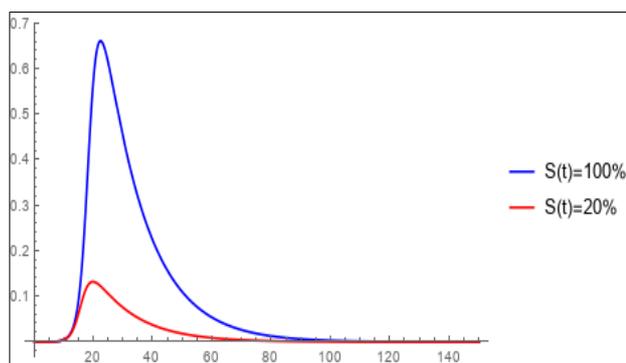


Figure 6. Comparison for active case when  $R_0 = 9.7$  with  $S(0) = 100\%$  (blue graph) and  $S(0)=20\%$  (red graph) with the incubation period of 14 days.

Figure 6 shows that the reduction of susceptible person  $S(0)$  may result in lower active cases in Malaysia. This is possible with the implementation of MCO and EMCO and abiding by the Malaysian government's SOP, such as wearing a facemask, practicing social distancing, frequently washing hands, practicing good respiratory hygiene, etc. By comparing the reported data with the data from our modeling, we infer that if adequate controls and firm policies are enforced to monitor the infection rates early after the spread of the disease, the spread of COVID-19 could be monitored in all populations considered.

### Discussion:

According to the SIR model, there are significant differences between the expected number of infected people and how the  $S(0)$  parameter is defined. In addition, this paper did not consider those Malaysians returning from overseas (import cases). Active case detection also becomes a problem with the limited testing kit at the early stage and how contact tracing may affect the data. There is also a possibility of unreported cases, as stated by (15). During an epidemic situation, restricting human mobility is to reduce the spread of infectious disease. Minimizing the spread of the

contagious disease will help the health care community by reducing or eliminating the surge in people coming to treatment facilities. Hospitals' surge capability is always small, and due to insufficient available resources, a higher than the average number of patients may result in poor treatment. Consequently, measures such as the lockdown of a region or a whole province as implemented by Malaysia during Movement Control Order (MCO) would result in "flattening the epidemic curve" to some degree.

However, this model can forecast the time,  $t$ , taken for active cases to reach their peak, given that no source of infection from outside occurs (import case). Since COVID-19 is a new virus spreading worldwide, there are still many parameters yet remain unknown. More research still needs to be conducted. However, the visualization for the spread of COVID-19 can be made via a mathematical model. First, let us discuss the  $R_0$  parameter, known as an indicator of the infection rate or disease spread (16). Moreover, the reproduction number,  $R_0$ , denotes the average number of new cases of a disease that arise from a single case (17). From equations (2) and (3), the equation for  $R_0$  given by,

$$R_0 = \frac{aS_0}{b}. \quad (5)$$

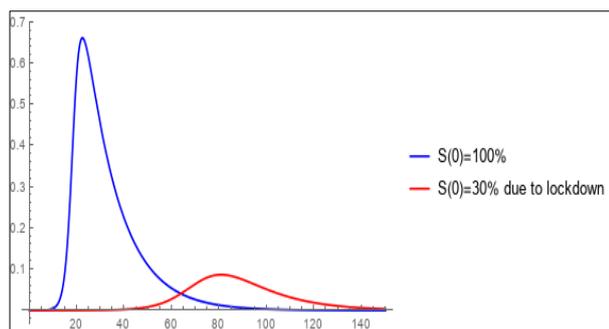
What  $R_0$  tell?

- If  $R_0 < 1$ , for every 'one' case, it will result in less than 'one' new positive case in the community. In this case, the virus will die and eventually stop the spread of the virus.
- If  $R_0 = 1$ , every existing case will result in 'one' new case.
- If  $R_0 > 1$ , every 'one' case will result in more than 'one' new positive case.

For  $R_0 > 1$ , there is a potential for the virus to keep spreading, resulting in a pandemic causing more severe trouble in the future (17).

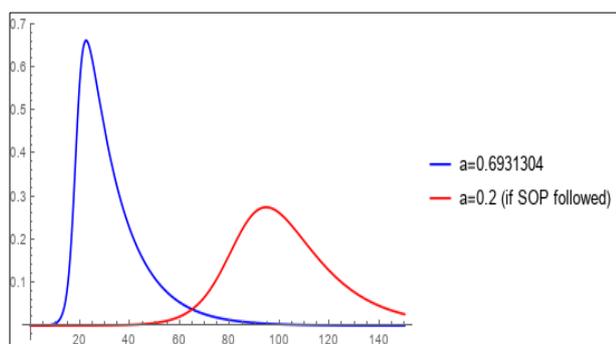
From the formula above, the government can strategize on how to lower the  $R_0$  value. This measure is essential to ensure our hospital capacity is enough to treat COVID-19 patients, thus flattening the curve. First of all, assume that the parameter  $b$  remains unchanged since there is no antiviral drug specifically for the virus. In other words, all treatment protocols are based on the symptoms. Therefore, only two parameters can be used to lower down the value of  $R_0$ . First is the value of  $S_0$ , which signifies that the number of early susceptible persons can be reduced by administering a vaccine or initiating lockdown. Since the vaccine is still being developed, the only possible way is to initiate lockdown on the affected areas. From Fig. 7, the lockdown (MCO implementation by the

government) was able to reduce the peak of infection significantly.



**Figure 7. The difference when the lockdown is not initiated (blue line) and lockdown (red line) is initiated.**

The second measure that can be taken is by lowering the infectivity rate,  $a$ . This can be done by following the SOP outlined by MOH, such as wearing a face mask in public, using hand sanitizer, avoid going out if symptoms are shown, and social distancing. By abiding to the SOP, the infectivity rate  $a$  can be lowered, as shown in Fig. 8.



**Figure 8. Graph for  $a=0.693134$  and  $a=0.2$  with  $S(0)$  at 100%.**

SIR model was constructed based on three compartments, depending on our assumption. It is difficult to predict the exact figure due to a lack of information and how it is handled in real life (18). SIR model can not predict a sudden increase in cases due to import cases (Malaysian coming from abroad and illegal immigrants). Proper quarantine measures are needed to prevent the virus spread to the community, as this is what happened with the Sivagangga cluster. The failure of those who return from abroad to quarantine themselves increased inactive cases in Malaysia, specifically in Kedah.

### Conclusion:

COVID-19 hit the world hard, including Malaysians. Much research is still ongoing to study the effect and spread of COVID-19 along with producing the vaccine and cure. On the bright side,

mathematical modeling can be used to forecast the spread of COVID-19. However, it is not perfect due to the unknown nature of the virus. Some researchers incorporated the parameter Exposure (SEIR) in the modeling, and also others consider the spread as a form of waves and more. With several assumptions, a forecast for active cases can be produced and thus prompt better implementation of the proper action to stop the spread of COVID-19. For Malaysians, make sure to follow the rules and obey the SOP to ensure  $R_0$  is lowered from time to time, hoping that the virus will vanish one day.

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### Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for republication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University Sains Islam Malaysia.

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## مستقبل النمذجة الرياضية: مراجعة للحالات المصابة بفيروس COVID-19 باستخدام نموذج S-I-R

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### الخلاصة:

أدى انتشار مرض فيروس كورونا الجديد (كوفيد-19) إلى حدوث فوضى في جميع أنحاء العالم. ولا تزال الحالات المصابة تتزايد، حيث لا يزال في العديد من البلدان ظهور متزايد في الحالات اليومية. للتنبؤ بظهور الحالات النشطة، تم استخدام نموذج رياضي، وهو نموذج SIR، لتصور انتشار COVID-19. في هذا المقال، تم التنبؤ بانتشار الفيروس في ماليزيا، على افتراض أن جميع الماليزيين سيكونون في النهاية عرضة للإصابة به. مع عدم تطوير لقاح وعقار مضاد للفيروسات حاليًا، يمكن تقليل تصور كيفية ذروة العدوى (أي تسطيح المنحنى) لتقليل تأثير مرض COVID-19 وبالنسبة للماليزيين، علنا ان نؤكد على اتباع القواعد والامتثال لإجراءات السلامة الموحدة لخفض قيمة  $R_0$  من وقت لآخر، على أمل أن يختفي الفيروس يومًا ما.

الكلمات المفتاحية: كوفيد 19 ، النمذجة الرياضية، نموذج وباء SIR .