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Discussing Fuzzy Reliability Estimators of Function of Mixed Probability Distribution by Simulation

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Abstract:

This paper deals with constructing mixed probability distribution from exponential with scale parameter (β) and also Gamma distribution with (2,β), and the mixed proportions are $(\frac{\alpha}{\alpha+1},\frac{1}{\alpha+1})$ .first of all, the probability density function (p.d.f) and also cumulative distribution function (c.d.f) and also the reliability function are obtained. The parameters of mixed distribution, (α,β) are estimated by three different methods, which are maximum likelihood, and Moments method,as well proposed method (Differential Least Square Method)(DLSM).The comparison is done using simulation procedure, and all the results are explained in tables.

Key words: Maximum Likelihood Estimator(MLE), Mean square error(MSE), Mixed Probability Distribution (MPD), Moment estimator(MOM), Proposed Method (DLSM).

Introduction:

The compound and mixture distributions provide a mathematical approach for doing statistical modeling different verity random phenomena because the mixture distribution are flexible models for and lyzing random durations, in the heterogeneous population. Also, the mixture distributions have vital role in practical applications for research that deal with economics, medicine, agriculture, life testing, and reliability for classical reliability theory. There are several methods and models in which the parameters assume precise, but in real world application, due to vague and randomness affect the life times distribution, also when the parameters of life time distribution are fuzzy, then there is difficulty for handling reliability and hazard functions Many researches work on fuzzy reliability and introduce development for this field, as this in dictated. In (2017), (1)two researchers (M.A. Hussian) and (Esssam A A. min) discussed fuzzy exponential distribution and discussed how to compute reliability in case of stress- strength model and ranked set sampling.Also in (2013) (2), two researchers (Elbatal) and (M.elgarhy) introduced Transmuted Quasi Lindley Distribution and worked on deriving (r_{th}) moment and moment generating function.

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In (2010) (3), (Mohamoudi and Zakerzaadeh) worked on generalized Poisson-Lindley Distribution and introduced different methods of estimation and comparing results by MSE, while in 2016 (4) (Nedjar and Zeghdoudi) worked on deriving gamma Lindley Distribution and studied its properties by Simulation. In (1970) (5) (Sankaran, M.) studied discrete Poisson- Lindley Distribution, and discussed its estimat. Also in (2013) (6) (Shanker and Mishra) introduced a research about quasi lindley distribution to Journal of Mathematics and Computer Science.

A Generalization of Lindley Distribution was introduced by (Zakerzadeh and Dolati) (2009) (7). Also (Zeghdoudi and Nedjar) in 2016 (8) introduced properties and application on Poisson Gamma Lindley Distribution. In May –Jun. (2015) (9) (Dutta and Borah) introduced a study about Poisson –Quasi Lindley Distribution, and derived its mathematical properties of coefficient of skewness and kurtosis coefficient of variation. In (2017) (10), (Rama Shanker and etl.) discuss three parameters Lindley,and studied their different estimators of parameter and reliability function, all mathematical and statistical properties were discussed. The aim of this research is to build mixed failure to time model from exponential and Gamma distribution where this model is necessary when the observation of time to failure cannot be represented by single probability distribution, which is recognized by scatter diagram, so two types of distributions need to be mixed which are exponential and Gamma using certain proportion as weights function, and the sum of this weight equals one.

Theoretical Parts:

Let$x$be random variable distribution exponential with scale parameter(β) is defined in equation(1) as:

$h_1(x) = \beta\, e^{-\beta x} x>0\ ,\beta > 0$ … (1)

Also there is another random variable $(y = x)$ distributionas Gamma (2,β) as exponential in equation (2):

$h_2(x) = \beta^2 x\ \ e^{-\beta x} x>0, \beta > 0$ ….(2)

Then Mixing p.d.f of (1) and (2) by applying proportion ($\frac{\alpha}{\alpha+1}, \frac{1}{\alpha+1}$) leads to:

$$f(x,\alpha,\beta) = \frac{\alpha}{\alpha+1} h_1(x) + \frac{1}{\alpha+1} h_2(x) \quad \text{….}(3)$$

$$f(x,\alpha,\beta) = \frac{\alpha\beta}{\alpha+1} e^{-\beta x} + \frac{1}{\alpha+1}\beta^2\ x\ \ e^{-\beta x}$$

$$f(x,\alpha,\beta) = \frac{\beta}{\alpha+1} e^{-\beta x}(\alpha+\beta x) \quad \alpha > -1 \ \text{….}(4) \quad \beta > 0 \quad x > 0$$

This p.d.f$f\,(x,\alpha,\beta)$ represent Quasi Lindy distribution with two parameters
$(\beta{:}\ scale\ parameter)$
And
(α: is (mixed proportion) parameter)

The cumulative distribution function (C.D.F) correspoinal to p.d.f in equation (4) is:

$$F_x(x,\alpha,\beta) = pr\ (X \le x)$$

$$= \int_0^x f(t,\alpha,\beta)dt$$

$$= 1 - \frac{(1+\alpha+\beta x)e^{-\beta x}}{(\alpha+1)} \quad \text{….}(5)$$

While Reliability Function is:

$$R_x(x) = \frac{(1+\alpha+\beta x)}{(\alpha+1)} e^{-\beta x} \quad \text{….}(6)$$

and hazard rate function:

$$h(x) = \frac{f(x)}{R(x)}$$

$$= \frac{\frac{\beta e^{-\beta x}(\alpha+\beta x)}{(\alpha+1)}}{\frac{(1+\alpha+\beta x)}{(\alpha+1)} e^{-\beta x}}$$

$$h(x) = \frac{\beta(\alpha+\beta x)}{(1+\alpha+\beta x)} \quad \text{….}(7)$$

and the fuzzy hazard Rate function

$$\tilde{h}(x) = \frac{\hat{\beta}\left(\hat{\alpha} + \hat{\beta}\hat{k}_i x_i\right)}{\left(1 + \ \hat{\alpha} + \hat{\beta}\hat{k}_i x_i\right)} \quad \text{….}(8) \quad ,\hat{k}_i\ is\ vague\ factor$$

Estimation Method:

In this section, some methods are presented for estimating (α,β) by different ways like moments, maximum likelihood, and differential least – squares method
(D L S M) (proposed method).

The First Method: (Moment Method):

First of all, the formula of the moments about origin is derived from

$$\mu r' = E(x^r) = \int_0^\infty x^r\ f(x,\alpha,\beta)\, dx$$

After some steps we find

$$\mu r' = \frac{\Gamma(r+1)(\alpha+r+1)}{\beta^r(\alpha+1)} \quad \text{….}(9)$$

Then from

$$\acute{\mu}_1 = \frac{\sum xi}{n} = \frac{\Gamma(2)(\alpha+2)}{\beta(\alpha+1)} = \bar{x}$$

And from $\acute{\mu}_1 = \frac{\sum xi^2}{n}$

$$E(x^2) = \frac{2\ (\alpha+3)}{\beta^2(\alpha+1)}$$

Then, from equation of variance which is

$$\sigma^2 = E(x^2) - (Ex)^2$$

$$= \frac{\alpha^2+4\alpha+2}{\beta^2(\alpha+1)^2}$$

the Coefficient of Variable (C.V) is:

$$(C.V) = \frac{\sigma}{\acute{\mu}_1}$$

$$= \frac{\sqrt{\alpha^2+4a+2}}{2\alpha^2+2} \quad \text{….}(10)$$

where the $(C.V)$ is function of α, so can equate $(C.V)$ from sample with equation (10), and solving resulted equation, $\hat{\alpha}_{\,mom}$ is obtained then gives $\hat{\beta}_{mom}$

$$\hat{\beta}_{mom} = \sqrt{\frac{2n\ (\hat{\alpha}+3)}{\sum_{i=1}^n xi^2(\hat{\alpha}+1)}} \quad \text{….}(11)$$

The Second Method: Maximum Likelihood Estimator(MLE):

The estimation on by this method depends on maximizing

$$L = \prod_{i=1}^n f(xi\,,\alpha\,,\beta)$$

$$= \beta^n(\alpha+1)^{-n} e^{-\beta \sum_{i=1}^n xi} \prod_{i=1}^n (\alpha+\beta xi)$$

$$\log L = n \log\beta - n\log(\alpha+1) - \beta\sum_{i=1}^n xi + \sum_{i=1}^n \log(\alpha+\beta xi)$$

$$\frac{\partial \log L}{\partial\beta} = \frac{n}{\beta} - \sum_{i=1}^n xi + \sum_{i=1}^n \frac{xi}{(\alpha+\beta xi)}$$

$$\frac{\partial \log L}{\partial\alpha} = \frac{-n}{(\alpha+1)} + \sum_{i=1}^n \frac{1}{(\alpha+\beta xi)}$$

$$from\ \frac{\partial \log L}{\partial\beta} = 0 \quad \rightarrow$$

$$\frac{n}{\hat{\beta}} = \sum_{i=1}^{n} xi - \sum_{i=1}^{n} \frac{xi}{(\hat{\alpha} + \hat{\beta} xi)}$$

$$\hat{\beta}_{MLE} = \frac{n}{\sum_{i=1}^{n} xi - \sum_{i=1}^{n} \frac{xi}{(\hat{\alpha}+\hat{\beta}xi)}}$$

$$and\ from\ \frac{\partial \log L}{\partial \alpha} = 0$$

$$\sum_{i=1}^{n} (\hat{\alpha} + \hat{\beta}\, xi)^{-1} = \frac{n}{\hat{\alpha} + 1}$$

$$\hat{\alpha} + 1 = \frac{n}{\sum_{i=1}^{n} (\hat{\alpha} + \hat{\beta} xi)^{-1}}$$

from equation $\hat{\beta}_{MLE}$, $\hat{\beta}_{MLE}$ is found to be an implicit function from $(\hat{\alpha}, \hat{\beta})$so iterative procedure can be used according to the generated values of (Xi, using assumed α, β) to find $\left|\hat{\beta}_{i+1\,MLE} - \hat{\beta}_{i\,MLE}\right|$ <to lerance also $\hat{\alpha}$ is found using same procedur.

$$\hat{\alpha}_{MLE} = \frac{n}{\sum_{i=1}^{n} (\hat{\alpha}+\hat{\beta}xi)^{-1}} - 1 \ldots (12)$$

The Third method: (Differential Least Square Method (DLSM):

It is assumed here that data sets, are generated from Quasi Lindlly with tow parameters (α *and* β), since β is a scale parameter it can be estimated by (DLSM), assuming that (α *is known*). Also the two parameters (α *and* β) are estimated Moments, and Maximum likelihood, as explained previously.

$$f(x) = \left(\frac{\beta}{\alpha + 1}\right) e^{-\beta x} (\alpha + \beta x)$$

$$= \left(\frac{1}{\alpha + 1}\right) e^{-\beta x} (\alpha\beta + \beta^2 x)$$

$$\frac{df}{dx} = \left(\frac{1}{\alpha + 1}\right) \left[e^{-\beta x} (\beta^2) + (\alpha\beta + \beta^2 x)\ e^{-\beta x} (-\beta) \right]$$

$$= \left(\frac{1}{\alpha + 1}\right) \beta^2 \left[e^{-\beta x} - (\alpha + \beta x)\ e^{-\beta x} \right]$$

$$\frac{d^2 f}{dx^2} = \frac{\beta^2}{(\alpha + 1)} \left[-\beta\, e^{-\beta x} - (\alpha + \beta x)\ e^{-\beta x} (-\beta) + e^{-\beta x} (\beta) \right]$$

$$= \frac{\beta^2}{(\alpha + 1)} \left[-\boldsymbol{\beta}\, e^{-\beta x} + \beta\ e^{-\beta x} (\alpha + \beta x) - \beta\ e^{-\beta x} \right]$$

$$= \frac{\beta^2}{(\alpha + 1)} \left[\beta\ e^{-\beta x} (\alpha + \beta x) - 2\beta\ e^{-\beta x} \right]$$

$$\therefore\ f(x) = \left(\frac{\beta}{\alpha + 1}\right) e^{-\beta x} (\alpha + \beta x)$$

There for

$$\frac{d^2 f}{dx^2} = \left[\beta^2 f(x) - 2\ \frac{\beta^3}{\alpha + 1}\ e^{-\beta x} \right]$$

The scale parameter β can be obtained by Minimizing

$$T = \sum_{i=1}^{n} \left[\beta^2 f(xi) - \frac{2\beta^3}{\alpha + 1}\ e^{-\beta xi} - \hat{f}(xi) \right]^2 \ldots (13)$$

Using instruction (f: solve)in math lab

Simulation Procedures:

The following three steps are concluded:

1-Generate values of random ~ (Quasi Lindely) with (Parameters $(\alpha + \beta)$)using rejection and accept method.

2- Generate random variable Zi∼ uniform (0,1)

3- Generate two random variable $Vi \sim \exp\beta$), Wi $\sim Gamma\ (2, \beta)$

If $ui \leq p = \frac{\alpha i}{\alpha+1}\ then\ xi = vi$

other wise $xi = wi$, $x > 0$

$\beta > 0$

$\alpha > -1$

And by inverse transformation from

$$F_x(xi, \alpha, \beta) = 1 - \frac{(1 + \alpha + \beta xi) e^{-\beta xi}}{(\alpha + 1)}$$

$$ui = 1 - \frac{(1 + \alpha + \beta xi) e^{-\beta xi}}{(\alpha + 1)}$$

$$\frac{(1 + \alpha + \beta xi)}{(\alpha + 1)} = (1 - ui)$$

$$1 + \frac{\beta}{(\alpha + 1)} xi = (1 - ui)$$

$$ui = -\frac{\beta}{(\alpha + 1)} xi$$

$\log\ ui = (-\frac{\beta}{\alpha+1}) xi$, Θ=$- \frac{\beta}{\alpha+1} \alpha > -1$

$\log\ ui = \Theta xi$, 0< $xi < \infty$

$$xi = \frac{ui}{\Theta}$$

According to the given values of $(\beta\ , \alpha)$ and$(0 \leq ui \leq 1)$, values of (Xi) are generated These observation are generated according to different sample size n=(25, 50, 80).

There are different values for $(\alpha = 0.8\ ,1.2\ )$, $(\beta = 1.5\ ,2.5\ )$ and $(\ \tilde{k} = 0.3\ ,0.7\ )$

And each experiment is repeated $R = 500$

All the results of comparisons of fuzzy reliability functions, are explained in different Tables(1,2,3,4,5,6,7,8) and according to different sets of initial values. The comparison is done using statistical Measures Mean Square Error.

The values of variable(xi), are generate by simulation, using different sets of sample size(n= 25, 50, 80) and also different sets of initial values of parameters which are $(\alpha = 0.8\ ,1.2\ )$, $(\beta = 1.5\ ,2.5\ )$ and vague factor $(\ \tilde{k} = 0.3\ ,0.7\ )$.

Table 1. Estimator Fuzzy Reliability when $\beta = 1.5$, $\alpha = 0.8$, $\tilde{k} = 0.3$

n	t_i	Rreal	R_{mle}	R_{mom}	R_{pro}	mse-mle	mse-mom	mse-pro	Best
	0.0272	0.946049	0.97691	0.785567	0.999292	0.059432	0.736766	0.056627	mse-pro
	0.1176	0.825691	0.935831	0.780076	0.984853	0.053082	0.050132	0.383169	mse-mom
	0.2636	0.801620	0.909449	0.780059	0.920505	3.15E-02	0.323819	0.035277	mse-mle
	0.2763	0.801321	0.844168	0.777832	0.912604	0.025516	0.320932	0.033574	mse-mle
25	0.3065	0.786306	0.839904	0.764597	0.892580	7.08E-03	0.299041	0.023326	mse-mle
	0.3156	0.777789	0.835302	0.750762	0.886178	0.002859	0.279858	0.018034	mse-mle
	0.4304	0.775390	0.828518	0.747624	0.793320	0.002068	0.259567	1.60E-02	mse-mle
	0.4870	0.758288	0.827583	0.747442	0.740868	2.06E-03	0.239961	0.015138	mse-mle
	0.5508	0.698358	0.799396	0.709178	0.678283	1.68E-03	0.175245	0.013039	mse-mle
	0.5654	0.682541	0.792465	0.589471	0.663601	0.157895	8.46E-04	7.19E-03	mse-mom
	0.1130	0.957881	0.993002	0.745025	0.97914	0.014515	0.778453	0.026833	mse-mle
	0.1170	0.925756	0.960244	0.721437	0.977799	0.01448	0.658603	0.022971	mse-mle
	0.1180	0.897648	0.949353	0.720171	0.977548	1.45E-02	0.529383	0.018778	mse-mle
	0.1400	0.896738	0.937497	0.713703	0.969649	0.523588	0.010281	0.01031	mse-mom
50	0.1429	0.876332	0.932636	0.696337	0.968338	5.25E-03	0.50743	0.010234	mse-mle
	0.1554	0.847696	0.920616	0.686673	0.963216	0.004862	0.430279	0.006763	mse-mle
	0.1660	0.843932	0.907975	0.682311	0.958572	0.00246	0.407352	6.64E-03	mse-mle
	0.1860	0.840514	0.905161	0.664759	0.949236	5.40E-04	0.404829	0.005127	mse-mle
	0.1944	0.826236	0.894905	0.614678	0.945096	1.82E-04	0.379749	0.000138	mse-pro
	0.2149	0.799407	0.877087	0.577119	0.934418	7.50E-05	0.320517	5.55E-06	mse-mle
	0.0090	0.969321	0.982877	0.689771	0.999148	0.011761	0.831525	0.013222	mse-mle
	0.0312	0.954974	0.977239	0.689001	0.994266	0.005293	0.742213	0.009864	mse-pro
	0.0477	0.947730	0.975136	0.674799	0.988969	0.00925	0.72747	3.24E-03	mse-mle
	0.0494	0.944398	0.942233	0.671358	0.988384	0.001855	0.717598	0.009228	mse-pro
	0.0539	0.931762	0.941943	0.646874	0.986719	1.79E-03	0.005465	0.643143	mse-mom
	0.0555	0.921143	0.930798	0.621636	0.986123	0.001686	0.641488	0.0043	mse-pro
80	0.1425	0.902972	0.928522	0.603719	0.941921	0.000498	0.541825	7.89E-04	mse-mle
	0.1682	0.894948	0.911721	0.600688	0.925659	2.28E-04	0.000487	0.537192	mse-mom
	0.2005	0.892319	0.911695	0.597813	0.903666	.000386	0.520319	6.47E-050	mse-mle
	0.2318	0.877676	0.851887	0.558438	0.88105	6.13E-06	0.491288	4.78E-05	mse-mle

Table 2. Estimator Fuzzy Reliability when $\beta = 1.5$, $\alpha = 1.2$, $\tilde{k} = 0.3$

n	t_i	Rreal	R_{mle}	R_{mom}	R_{pro}	mse-mle	mse-mom	mse-pro	Best
	0.0356	0.946049	0.957776	0.835522	0.999168	0.050506	0.684143	0.037605	mse-pro
	0.1540	0.825691	0.890355	0.830157	0.982229	0.035319	0.329527	0.050077	mse-mle
	0.3450	0.80162	0.868772	0.826366	0.907276	2.86E-02	0.275702	0.033925	mse-mle
	0.3620	0.801321	0.829565	0.824991	0.898134	0.032620	0.27198	0.018972	mse-pro
25	0.4016	0.786306	0.828043	0.810744	0.875028	0.250062	4.14E-03	0.028663	mse-mom
	0.4136	0.777789	0.82297	0.797465	0.86766	0.001703	0.232625	0.028066	mse-mle
	0.5640	0.77539	0.818081	0.792971	0.761861	2.56E-02	0.214717	0.001535	mse-pro
	0.6382	0.758288	0.816586	0.791864	0.703033	9.11E-04	0.196189	0.015045	mse-mle
	0.7217	0.698358	0.799875	0.754682	0.633793	0.135772	8.80E-04	0.013628	mse-mom
	0.7409	0.682541	0.799746	0.621953	0.617708	8.04E-04	0.120655	1.25E-02	mse-mle
	0.1490	0.957881	0.98909	0.791153	0.977218	0.013709	0.731406	0.025744	mse-mle
	0.1540	0.925756	0.949153	0.765381	0.975755	0.601741	0.011774	0.022356	mse-mom
	0.1550	0.897648	0.944543	0.762237	0.975482	1.16E-02	0.481905	0.018216	mse-mle
	0.1830	0.896738	0.926988	0.756952	0.966867	0.008248	0.472976	0.009398	mse-pro
50	0.1873	0.876332	0.923504	0.740323	0.965439	4.66E-03	0.446839	0.009255	mse-pro
	0.2036	0.847696	0.917901	0.723021	0.959857	0.004264	0.375446	0.006331	mse-pro
	0.2175	0.843932	0.904422	0.721824	0.9548	0.357113	0.002148	6.16E-03	mse-mom
	0.2437	0.840514	0.902403	0.709172	0.944638	3.64E-04	0.353119	0.004720	mse-mle
	0.2547	0.826236	0.894512	0.650829	0.940135	3.29E-04	0.328024	8.08E-05	mse-pro
	0.2816	0.799407	0.881307	0.605311	0.928529	7.94E-06	0.270491	7.69E-06	mse-pro
	0.0118	0.969321	0.978848	0.728729	0.999138	0.011727	0.790279	0.008291	mse-pro
	0.0409	0.954974	0.973983	0.726382	0.994202	0.694981	0.004866	0.009851	mse-mom
	0.0626	0.94773	0.97067	0.713714	0.988846	2.69E-03	0.678182	0.009248	mse-mle
	0.0647	0.944398	0.941513	0.710561	0.988254	0.009203	0.667729	0.001903	mse-pro
	0.0706	0.931762	0.939375	0.678789	0.986571	1.75E-03	0.597277	0.005652	mse-mle
80	0.0727	0.921143	0.931259	0.658526	0.985968	0.584735	0.001331	0.004281	mse-mom
	0.1867	0.902972	0.928057	0.634207	0.941288	0.000416	0.491057	8.48E-04	mse-mle
	0.2204	0.894948	0.916283	0.631981	0.924855	3.22E-04	0.484445	0.000523	mse-pro
	0.2628	0.892319	0.91542	0.628945	0.902638	0.4724	6.02E-05	3.81E-04	mse-mom
	0.3037	0.877676	0.872582	0.582181	0.879797	2.44E-05	0.439766	9.67E-06	mse-pro

Table 3. Estimator Fuzzy Reliability when $\beta = 2.5$, $\alpha = 0.8, \tilde{k} = 0.3$

n	t_i	Rreal	R_{mle}	R_{mom}	R_{pro}	mse-mle	mse-mom	mse-pro	Best
	0.0356	0.946049	0.957776	0.735522	0.999168	0.037605	0.684143	0.050506	mse-mle
	0.1540	0.825691	0.890355	0.730157	0.982229	0.035319	0.329527	0.050077	mse-mle
	0.3450	0.801620	0.868772	0.726366	0.907276	2.86E-02	0.275702	0.033925	mse-pro
	0.3620	0.801321	0.829565	0.724991	0.898134	0.018972	0.271980	0.032620	mse-mle
25	0.4016	0.786306	0.828043	0.710744	0.875028	4.14E-03	0.250062	0.028663	mse-pro
	0.4136	0.777789	0.82297	0.697465	0.86766	0.232625	0.001703	0.028066	mse-mom
	0.5640	0.775390	0.818081	0.692971	0.761861	0.001535	0.214717	2.56E-02	mse-mle
	0.6382	0.758288	0.816586	0.691864	0.703033	0.196189	9.11E-04	0.015045	mse-mom
	0.7217	0.698358	0.799875	0.654682	0.633793	8.80E-04	0.135772	1.36E-02	mse-mle
	0.7409	0.682541	0.799746	0.521953	0.617708	8.04E-04	1.25E-02	0.120655	mse-pro
	0.1490	0.957881	0.98909	0.691153	0.977218	0.013709	0.731406	0.025744	mse-mle
	0.1540	0.925756	0.949153	0.665381	0.975755	0.011774	0.601741	0.022356	mse-mle
	0.1550	0.897648	0.944543	0.662237	0.975482	1.16E-02	0.481905	0.018216	mse-pro
	0.1830	0.896738	0.926988	0.656952	0.966867	0.008248	0.472976	0.009398	mse-mle
	0.1873	0.876332	0.923504	0.640323	0.965439	4.66E-03	0.446839	0.009255	mse-mle
50	0.2036	0.847696	0.917901	0.623021	0.959857	0.375446	0.004264	0.006331	mse-mom
	0.2175	0.843932	0.904422	0.621824	0.9548	0.002148	0.357113	6.16E-03	mse-mle
	0.2437	0.840514	0.902403	0.609172	0.944638	0.353119	3.64E-04	0.00472	mse-mom
	0.2547	0.826236	0.894512	0.550829	0.940135	3.29E-04	0.328024	8.08E-05	mse-pro
	0.2816	0.799407	0.881307	0.505311	0.928529	7.94E-06	0.270491	7.69E-06	mse-pro
	0.0118	0.969321	0.978848	0.628729	0.999138	0.008291	0.790279	0.011727	mse-mle
	0.0409	0.954974	0.973983	0.626382	0.994202	0.009851	0.694981	0.004866	mse-pro
	0.0626	0.94773	0.97067	0.613714	0.988846	0.678182	2.69E-03	0.009248	mse-mom
	0.0647	0.944398	0.941513	0.610561	0.988254	0.001903	0.667729	0.009203	mse-mle
	0.0706	0.931762	0.939375	0.578789	0.986571	1.75E-03	0.597277	0.005652	mse-pro
80	0.0727	0.921143	0.931259	0.558526	0.985968	0.584735	0.001331	0.004281	mse-mom
	0.1867	0.902972	0.928057	0.534207	0.941288	0.000416	0.491057	8.48E-04	mse-mle
	0.2204	0.894948	0.916283	0.531981	0.924855	3.22E-04	0.484445	0.000523	mse-mle
	0.2628	0.892319	0.91542	0.528945	0.902638	0.4724	6.02E-05	3.81E-04	mse-mom
	0.3037	0.877676	0.872582	0.482181	0.879797	2.44E-05	0.439766	9.67E-06	mse-mle

Table 4. Estimator Fuzzy Reliability when $\beta = 2.5$, $\alpha = 1.2, \tilde{k} = 0.3$

n	t_i	Rreal	R_{mle}	R_{mom}	R_{pro}	mse-mle	mse-mom	mse-pro	Best
	0.1350	0.946049	0.957776	0.806106	0.999071	0.037605	0.850763	0.050033	mse-mle
	0.3260	0.825691	0.890355	0.792447	0.981117	0.035319	0.435708	0.049150	mse-mle
	0.5280	0.801620	0.868772	0.736442	0.904239	2.86E-02	0.372494	0.035261	mse-pro
	0.5440	0.801321	0.829565	0.721052	0.894975	0.018972	0.371400	0.032219	mse-mle
	0.5785	0.786306	0.828043	0.720283	0.871627	0.336924	4.14E-03	0.028932	mse-mom
25	0.5887	0.777789	0.822970	0.709760	0.86420	0.001703	0.318107	0.027504	mse-mle
	0.7092	0.775390	0.818081	0.695313	0.75827	0.312801	0.001535	2.64E-02	mse-mom
	0.7638	0.758288	0.816586	0.694722	0.699792	9.11E-04	0.275060	0.015851	mse-mle
	0.8223	0.698358	0.799874	0.665831	0.631228	8.80E-04	1.28E-02	0.166598	mse-pro
	0.8353	0.682541	0.799746	0.524129	0.615333	8.04E-04	0.143179	1.18E-02	mse-mle
	0.3190	0.957881	0.98909	0.696275	0.975735	0.013709	0.890331	0.025061	mse-mle
	0.3250	0.925756	0.949153	0.668325	0.974208	0.774819	0.011774	0.021896	mse-mom
	0.3260	0.897648	0.944543	0.651369	0.973923	1.16E-02	0.661552	0.017798	mse-mle
	0.3610	0.896738	0.926988	0.649965	0.964966	0.008248	0.648514	0.008903	mse-mle
	0.3660	0.876332	0.923504	0.641219	0.963486	0.008743	0.598803	4.66E-03	mse-pro
50	0.3848	0.847696	0.917901	0.603245	0.957713	0.004264	0.500005	0.006098	mse-mle
	0.4004	0.843932	0.904422	0.593858	0.952497	0.485212	0.002148	5.80E-03	mse-mom
	0.4287	0.840514	0.902403	0.584925	0.942052	3.64E-04	0.476957	0.004455	mse-mle
	0.4402	0.826236	0.894512	0.54555	0.937436	3.29E-04	0.436495	5.02E-05	mse-pro
	0.4675	0.799407	0.881307	0.514413	0.925572	7.94E-06	0.368791	3.38E-08	mse-pro
	0.0698	0.969321	0.978848	0.610637	0.999004	0.927656	0.008291	0.011459	mse-mom
	0.1470	0.954974	0.973983	0.599582	0.993565	0.004866	0.867785	0.009725	mse-mle
	0.1900	0.94773	0.97067	0.588242	0.987795	0.009222	0.851801	2.69E-03	mse-pro
	0.1930	0.944398	0.941513	0.578586	0.987161	0.001903	0.839705	0.008995	mse-mle
	0.2039	0.931762	0.939375	0.555417	0.985364	0.772646	1.75E-03	0.006338	mse-mom
80	0.2074	0.921143	0.931259	0.551491	0.984723	0.001331	0.756505	0.004124	mse-mle
	0.3654	0.902972	0.928057	0.528129	0.938092	1.10E-03	0.68055	0.000416	mse-pro
	0.4036	0.894948	0.916283	0.524952	0.921216	3.22E-04	0.652755	0.000703	mse-mle
	0.4485	0.892319	0.91542	0.52473	0.89854	0.633093	6.02E-05	3.41E-04	mse-mom
	0.4892	0.877676	0.872582	0.506231	0.875361	2.44E-05	0.590866	3.98E-05	mse-mle

Table 5. Estimator Fuzzy Reliability when $\beta = 1.5$, $\alpha = 0.8, \tilde{k} = 0.7$

n	t_i	Rreal	R_{mle}	R_{mom}	R_{pro}	mse-mle	mse-mom	mse-pro	Best
	0.0272	0.946049	0.976910	0.785567	0.999292	0.059432	0.736766	0.056627	mse-pro
	0.1180	0.825691	0.935831	0.780076	0.984853	0.053082	0.383169	0.050132	mse-pro
	0.2640	0.80162	0.909449	0.780059	0.920505	3.15E-02	0.323819	0.035277	mse-mle
	0.2760	0.801321	0.844168	0.777832	0.912604	0.320932	0.025516	0.033574	mse-mom
25	0.3065	0.786306	0.839904	0.764597	0.89258	7.08E-03	0.299041	0.023326	mse-mle
	0.3156	0.777789	0.835302	0.750762	0.886178	0.279858	0.002859	0.018034	mse-mom
	0.4304	0.77539	0.828518	0.747624	0.79332	1.60E-02	0.259567	0.002068	mse-pro
	0.4870	0.758288	0.827583	0.747442	0.740868	2.06E-03	0.239961	0.015138	mse-mle
	0.5508	0.698358	0.799396	0.709178	0.678283	0.175245	1.68E-03	1.30E-02	mse-mom
	0.5654	0.682541	0.792465	0.589471	0.663601	8.46E-04	0.157895	7.19E-03	mse-mle
	0.1130	0.957881	0.993002	0.745025	0.97914	0.026833	0.778453	0.014515	mse-pro
	0.1170	0.925756	0.960244	0.721437	0.977799	0.01448	0.658603	0.022971	mse-mle
	0.1180	0.897648	0.949353	0.720171	0.977548	0.529383	1.45E-02	0.018778	mse-mom
	0.1400	0.896738	0.937497	0.713703	0.969649	0.01031	0.523588	0.010281	mse-pro
	0.1429	0.876332	0.932636	0.696337	0.968338	0.50743	5.25E-03	0.010234	mse-mom
50	0.1554	0.847696	0.920616	0.686673	0.963216	0.004862	0.430279	0.006763	mse-mle
	0.1660	0.843932	0.907975	0.682311	0.958572	6.64E-03	0.407352	0.00246	mse-pro
	0.1860	0.840514	0.905161	0.664759	0.949236	5.40E-04	0.404829	0.005127	mse-mle
	0.1944	0.826236	0.894905	0.614678	0.945096	1.82E-04	0.379749	1.38E-04	mse-pro
	0.2149	0.799407	0.877087	0.677119	0.934418	5.55E-06	0.320517	7.50E-05	mse-mle
	0.0090	0.969321	0.982877	0.689771	0.999148	0.013222	0.831525	0.011761	mse-pro
	0.0312	0.954974	0.977239	0.689001	0.994266	0.005293	0.742213	0.009864	mse-mle
	0.0477	0.94773	0.975136	0.674799	0.988969	0.72747	3.24E-03	0.00925	mse-mom
	0.0494	0.944398	0.942233	0.671358	0.988384	0.001855	0.717598	0.009228	mse-mle
	0.0539	0.931762	0.941943	0.646874	0.986719	0.643143	1.79E-03	0.005465	mse-mom
80	0.0555	0.921143	0.930798	0.621636	0.986123	0.004300	0.641488	0.001686	mse-pro
	0.1425	0.902972	0.928522	0.603719	0.941921	0.000498	0.541825	7.89E-04	mse-mle
	0.1682	0.894948	0.911721	0.600688	0.925659	0.537192	2.28E-04	0.000487	mse-mom
	0.2005	0.892319	0.911695	0.597813	0.903666	6.47E-05	0.520319	3.86E-04	mse-mle
	0.2318	0.877676	0.851887	0.558438	0.88105	4.78E-05	0.491288	6.13E-06	mse-pro

Table 6. Estimator Fuzzy Reliability when $\beta = 1.5$, $\alpha = 1.2, \tilde{k} = 0.7$

n	t_i	**Rreal**	R_{mle}	R_{mom}	R_{pro}	**mse-mle**	**mse-mom**	**mse-pro**	**Best**
	0.1150	0.946049	0.97691	0.821601	0.999202	0.059432	0.878107	0.055044	mse-pro
	0.2770	0.825691	0.935831	0.809373	0.983756	0.053082	0.522471	0.050092	mse-pro
	0.4490	0.80162	0.909449	0.766512	0.917157	0.452422	3.15E-02	0.033783	mse-mom
	0.4620	0.801321	0.844168	0.756158	0.909079	0.450393	0.025516	0.033174	mse-mom
	0.4919	0.786306	0.839903	0.754626	0.888667	7.08E-03	0.418610	0.024847	mse-mle
25	0.5006	0.777789	0.835301	0.743143	0.882158	0.396370	0.002859	0.016999	mse-mom
	0.6030	0.77539	0.828518	0.73283	0.788421	0.002068	0.389922	1.63E-02	mse-mle
	0.6494	0.758288	0.827582	0.732153	0.735894	0.353631	2.06E-03	0.015073	mse-mom
	0.6992	0.698358	0.799395	0.703182	0.673509	1.68E-03	0.241447	1.41E-02	mse-mle
	0.7102	0.682541	0.792465	0.609653	0.658913	0.214262	8.46E-04	8.06E-03	mse-mom
	0.2710	0.957881	0.993002	0.732692	0.977578	0.014515	0.908204	0.026079	mse-mle
	0.2770	0.925756	0.960244	0.709352	0.976165	0.820878	0.01448	0.022479	mse-mom
	0.2780	0.897648	0.949353	0.697224	0.975901	1.45E-02	0.72235	0.01833	mse-pro
	0.3070	0.896738	0.937498	0.695965	0.967613	0.010281	0.706002	0.009739	mse-pro
	0.3112	0.876332	0.932636	0.685431	0.966242	0.673818	5.25E-03	0.009637	mse-mom
50	0.3272	0.847696	0.920616	0.657775	0.960897	0.581912	0.004862	0.006389	mse-mom
	0.3405	0.843932	0.907975	0.656931	0.956064	0.00246	0.560133	6.36E-03	mse-mle
	0.3645	0.840514	0.905161	0.648705	0.946381	0.555923	5.40E-04	0.004831	mse-mom
	0.3743	0.826236	0.894905	0.619845	0.942099	1.82E-04	0.517014	9.47E-05	mse-pro
	0.3975	0.799407	0.877087	0.605087	0.931087	0.449981	5.55E-06	2.84E-05	mse-mom
	0.0594	0.969321	0.982877	0.667049	0.999006	0.013222	0.936323	0.011466	mse-pro
	0.1250	0.954974	0.977239	0.662175	0.99358	0.89027	0.005293	0.009728	mse-mom
	0.1610	0.94773	0.975136	0.650775	0.987823	3.24E-03	0.879166	0.009223	mse-mle
	0.1650	0.944398	0.942233	0.644231	0.98719	0.869739	0.001855	0.009001	mse-mom
	0.1734	0.931762	0.941943	0.632608	0.985398	1.79E-03	0.811282	0.006296	mse-mle
	0.1764	0.921143	0.930798	0.623691	0.984757	0.805816	0.001686	0.004129	mse-mom
80	0.3107	0.902972	0.928522	0.612401	0.93823	0.73893	0.000498	1.09E-03	mse-mom
	0.3432	0.894948	0.911721	0.611899	0.92139	2.28E-04	0.714307	0.000694	mse-mle
	0.3813	0.892319	0.911695	0.61038	0.898761	0.691735	6.47E-05	3.42E-04	mse-mom
	0.4160	0.877676	0.851887	0.601786	0.875629	4.78E-05	0.658569	3.80E-05	mse-pro

Table 7. Estimator Fuzzy Reliability when $\beta = 2.5$, $\alpha = 0.8$, $\tilde{k} = 0.7$

n	t_i	Rreal	R_{mle}	R_{mom}	R_{pro}	mse-mle	mse-mom	mse-pro	Best
	0.1350	0.946049	0.957776	0.906106	0.999071	0.037605	0.850763	0.050033	mse-mle
	0.3260	0.825691	0.890355	0.892447	0.981117	0.04915	0.435708	0.035319	mse-pro
	0.5280	0.801620	0.868772	0.836442	0.904239	2.86E-02	0.372494	0.035261	mse-mle
	0.5440	0.801321	0.829565	0.821052	0.894975	0.371400	0.018972	0.032219	mse-mom
	0.5785	0.786306	0.828043	0.820283	0.871627	0.028932	0.336924	4.14E-03	mse-pro
25	0.5887	0.777789	0.82297	0.809760	0.864200	0.001703	0.318107	0.027504	mse-mle
	0.7092	0.775390	0.818081	0.795313	0.758270	0.312801	0.001535	2.64E-02	mse-mom
	0.7638	0.758288	0.816586	0.794722	0.699792	9.11E-04	0.275060	0.015851	mse-mle
	0.8223	0.698358	0.799874	0.765831	0.631228	0.166598	8.80E-04	1.28E-02	mse-mom
	0.8353	0.682541	0.799746	0.624129	0.615333	8.04E-04	0.143179	1.18E-02	mse-mle
	0.3190	0.957881	0.989090	0.796275	0.975735	0.013709	0.890331	0.025061	mse-mle
	0.3250	0.925756	0.949153	0.768325	0.974208	0.011774	0.774819	0.021896	mse-mle
	0.3260	0.897648	0.944543	0.751369	0.973923	0.017798	0.661552	1.16E-02	mse-pro
	0.3610	0.896738	0.926988	0.749965	0.964966	0.648514	0.008248	0.008903	mse-mom
50	0.3660	0.876332	0.923504	0.741219	0.963486	4.66E-03	0.598803	0.008743	mse-mle
	0.3848	0.847696	0.917901	0.703245	0.957713	0.500005	0.004264	0.006098	mse-mom
	0.4004	0.843932	0.904422	0.693858	0.952497	0.002148	0.485212	5.80E-03	mse-mle
	0.4287	0.840514	0.902403	0.684925	0.942052	0.476957	3.64E-04	0.004455	mse-mom
	0.4402	0.826236	0.894512	0.645550	0.937436	3.29E-04	0.436495	5.02E-05	mse-pro
	0.4675	0.799407	0.881307	0.614413	0.925572	7.94E-06	0.368791	3.38E-08	mse-pro
	0.0698	0.969321	0.978848	0.710637	0.999004	0.008291	0.927656	0.011459	mse-mle
	0.1470	0.954974	0.973983	0.699582	0.993565	0.009725	0.867785	0.004866	mse-pro
	0.1900	0.94773	0.97067	0.688242	0.987795	2.69E-03	0.851801	0.009222	mse-mle
	0.1930	0.944398	0.941513	0.678586	0.987161	0.839705	0.001903	0.008995	mse-mom
	0.2039	0.931762	0.939375	0.655417	0.985364	0.772646	1.75E-03	0.006338	mse-mom
80	0.2074	0.921143	0.931259	0.651491	0.984723	0.001331	0.756505	0.004124	mse-mle
	0.3654	0.902972	0.928057	0.628129	0.938092	0.000416	0.68055	1.10E-03	mse-pro
	0.4036	0.894948	0.916283	0.624952	0.921216	0.652755	3.22E-04	0.000703	mse-mom
	0.4485	0.892319	0.91542	0.62473	0.89854	6.02E-05	0.633093	3.41E-04	mse-mle
	0.4892	0.877676	0.872582	0.606231	0.875361	0.590866	2.44E-05	3.98E-05	mse-mom

Table 8. Estimator Fuzzy Reliability when $\beta = 2.5$, $\alpha = 1.2$, $\tilde{k} = 0.7$

n	t_i	Rreal	R_{mle}	R_{mom}	R_{pro}	mse-mle	mse-mom	mse-pro	Best
	0.0042	0.906624	0.929847	0.961187	0.999964	0.025341	0.514816	0.047484	mse-mle
	0.0908	0.877562	0.897507	0.902607	0.988491	0.017071	0.445863	0.043087	mse-mle
	0.1440	0.869927	0.896769	0.894314	0.972842	0.415733	1.52E-02	0.041727	mse-mom
	0.1760	0.845484	0.846783	0.890705	0.960825	0.00765	0.348554	0.025077	mse-mle
	0.2410	0.835614	0.842028	0.884944	0.930093	6.61E-03	0.324132	0.020458	mse-mle
25	0.3542	0.816326	0.819209	0.870124	0.861179	0.005568	0.312786	0.018831	mse-mle
	0.4273	0.802467	0.818442	0.858315	0.808464	0.003662	0.286087	1.29E-02	mse-mle
	0.5302	0.795691	0.805012	0.843318	0.726896	1.27E-03	0.268752	0.004775	mse-mle
	0.6221	0.770583	0.783094	0.818483	0.649989	6.69E-04	0.228415	2.07E-03	mse-mle
	0.6364	0.665621	0.765686	0.799474	0.637910	5.46E-04	0.093831	2.44E-04	mse-pro
	0.0839	0.915373	0.966542	0.831095	0.988329	0.00637	0.560693	0.009473	mse-mle
	0.1910	0.90848	0.95935	0.82625	0.951291	0.00608	0.541813	0.008002	mse-mle
	0.2470	0.905329	0.950874	0.825965	0.924439	5.42E-03	0.517887	0.006300	mse-mle
	0.2710	0.90515	0.940235	0.820685	0.91184	0.002779	0.511727	0.006212	mse-mle
	0.2772	0.90158	0.934378	0.814938	0.908171	2.23E-03	0.511298	0.001833	mse-pro
50	0.3061	0.893846	0.925875	0.803091	0.891616	0.001369	0.49060	0.000834	mse-pro
	0.3147	0.892156	0.923792	0.797576	0.886499	0.000661	5.23E-04	0.483280	mse-mom
	0.4240	0.891002	0.883722	0.794433	0.815698	5.81E-04	0.473820	0.000387	mse-pro
	0.4651	0.866194	0.880717	0.774808	0.78682	3.87E-04	8.08E-06	0.416680	mse-mom
	0.4658	0.865144	0.846335	0.772187	0.786329	2.05E-04	0.415138	4.97E-06	mse-pro
	0.0633	0.969863	0.98566	0.807808	0.987001	0.002965	0.777798	0.007177	mse-mle
	0.0658	0.960493	0.970566	0.801268	0.986193	0.004045	0.728935	0.001814	mse-pro
	0.0811	0.951746	0.959723	0.793186	0.980951	1.68E-03	0.001198	0.695013	mse-mom
	0.1200	0.949914	0.95831	0.789574	0.965462	0.001632	0.654703	0.000963	mse-pro
	0.1647	0.932088	0.941532	0.776106	0.943798	1.36E-03	0.619643	0.000898	mse-mle
80	0.1704	0.931744	0.920717	0.758109	0.940865	0.000948	0.000703	0.599185	mse-mom
	0.1968	0.927555	0.915751	0.748493	0.926563	0.000716	0.571203	1.17E-04	mse-pro
	0.2003	0.913774	0.909943	0.722708	0.924601	5.52E-04	3.24E-05	0.545688	mse-mom
	0.2052	0.906256	0.903502	0.713612	0.921867	3.05E-05	0.52409	3.37E-04	mse-mle
	0.2053	0.90121	0.883938	0.694959	0.921786	2.22E-04	1.94E-05	0.514347	mse-mom

Conclusion:

From Table (1), the results of simulation .indicate that the best first fuzzy estimators of reliability function when (n= 25) and (n= 50) and (n=80), is Maximum Likelihood Estimator (MLE) with percentage (63.33%), and second moment($\hat{\hat{R}}_{mom}$ with percentage 20.000%), and finally ($\hat{\hat{R}}_{proposed}$ is best with percentage 16.666%). From these results, it is concluded that $\hat{\hat{R}}_{ML}$ is first best and $\hat{\hat{R}}_{mom}$ is the second best and finally $\hat{\hat{R}}_{proposed}$ is the third best estimator.

Andfrom Table (2), the results indicate that the first best one estimator is the proposed since it has the best percentage(40%), and second one is MLE with percentage (36.666%), and third one is moment estimators since it has percentage (23.333%).

Also from Table (3), the results of simulation indicate that, the first best fuzzy reliability estimator is (MLE) since it has the percentage (50%) or$\left(\frac{15}{30}\right)$%, and the second best estimator for fuzzy reliability is proposed with percentage (26.666%) or $\left(\frac{8}{30}\right)$%, while third one is moment estimator, has the percentage (23.333%) or $\left(\frac{7}{30}\right)$% .

The next tables summarize the results of simulation as:

Table 9. Summary of percentage of preference from each tables (1, 2, … . , 8) for three different estimators

Tables	% MLE	% MOM	% Proposed
Table (1)	63.333	20	16.66
Table (2)	36.666	23.34	40
Table(3)	50.000	23.333	26.666
Table (4)	53.333	23.33	23.33
Table(5)	40	26.66	33.33
Table(6)	26.666	50	23.333
Table(7)	43.333	33.333	23.333
Table(8)	50.000	23.333	26.333

From the summary of results in Table (9) , it is concluded that MLE is best with percentage(50%) , and then proposed is best with (25%) and also MOM is best with (25%) .

Conflicts of Interest: None.

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مناقشة عدة مقدرات للمعولية الضبابية لتوزيع احتمالي مختلط بواسطة المحاكاة

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الخلاصة:
يتناول هذا البحث تقدير المعولية الضبابية لتوزيع احتمالي مختلط يتكون من مزج التوزيع الاسي ذو المعلمة (β) ،مع توزيع كاما ذي المعلمتين (2,β)، وان معلمات المزج هي $(\frac{\alpha}{\alpha+1},\frac{1}{\alpha+1})$ ،وقد تم اشتقاق الدالة الاحتمالية والتراكمية وكذلك دالة المعولية ، وتم تقدير المعلمات بطريقة الإمكان الأعظم وطريقة العزوم وطريقة المربعات الصغرى ، وبعد اشتقاق المقدرات بالطرائق الثلاث، أجريت تجارب محاكاة لمقارنة النتائج وإيجاد مقدر المعولية الضبابي الذي يحقق اصغر متوسط مربعات خطأ.

الكلمات المفتاحية:مقدرات الإمكان الأعظم ،متوسط مربعات الخطأ، توزيع احتمالي مختلط ،مقدرات العزوم ،مقدرات مقترحة.