Open Access **Baghdad Science Journal** **P-ISSN: 2078-8665**
Published Online First: November 2021 **E-ISSN: 2411-7986**

DOI: http://dx.doi.org/10.21123/bsj.2022.19.3.0551

Variable Selection Using aModified Gibbs Sampler Algorithm with Application on Rock Strength Dataset

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Received 11/12/2020, Accepted31/1/2021, Published Online First 20/11/2021

Abstract:

Variable selection is an essential and necessary task in the statistical modeling field. Several studies have triedto develop and standardize the process of variable selection, but it isdifficultto do so. The first question a researcher needs to ask himself/herself what are the most significant variables that should be used to describe a given dataset's response. In thispaper, a new method for variable selection using Gibbs sampler techniqueshas beendeveloped.First, the model is defined, and the posterior distributions for all the parameters are derived.The new variable selection methodis tested usingfour simulation datasets. The new approachiscompared with some existingtechniques: Ordinary Least Squared (OLS), Least Absolute Shrinkage and Selection Operator (Lasso), and Tikhonov Regularization (Ridge). The simulation studiesshow that the performance of our method is better than the othersaccording to the error and the time complexity. Thesemethodsare applied to a real dataset, which is called Rock StrengthDataset.The new approach implemented using the Gibbs sampler is more powerful and effective than other approaches.All the statistical computations conducted for this paper are done using R version 4.0.3 on a single processor computer.

Keywords: Bayesian, Gibbs, Lasso,Markov chain Monte Carlo,Posterior, Ridge, Variable selection.

Introduction:

The relationship between a set of variables, $x_1, x_2, \dots, x_p$, and a response, y, can be expressed by the following linear regression model,

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2)\ \forall i = 1, \dots, n. \tag{1}$$

The system of n equations (Eq.1) can be expressed in matrix notation as follows,

$$y = X\beta + \epsilon, \qquad \epsilon \sim MVN(0_n, \Sigma_{n\times n}) \tag{2}$$

where y is a $n \times 1$ vector, X is a $n \times (p+1)$ matrix, β is a $(p+1) \times 1$ vector, ϵ is a $n \times 1$ vector, 0 is a $n \times 1$ vector and Σ is a $n \times n$ matrix.Generally, selectingcertain explanatory variables that can be used to describe the response variable is called feature selection (shrinkage). The feature selection is used to i) remove the unimportant variables which do not add any information; ii) reduce the computation time by shrinking the data size; iii) avoid the overfitting. To decide which variables are irrelevant is hard for high dimensional datasets. On the other hand, it is difficult to build and interpret a model that uses all the explanatory variables. In this case,variables selection techniques can play an important role. The set of coefficients, β, can express whether the explanatory variablesare important for the model or not. When the value of a coefficient is zero or very close to zero, then its corresponding variable is not significant to be chosen in the model.

Variable selection can be made using several traditionalapproaches. For example, Chi-square [1], ANOVA[2], and Pearson correlations can compute the variables' impact. Depending onthe coefficient values, itcan be determined whether the variable is important or not. Moreover, forward and backward selection methods are used to select the best subsets of variables by following some steps[3].These methods are slow with large datasets[4]. In this paper, variables are selected based onthe influence of their coefficientson the model.

In general, the set of parameters, $\{\beta_i : i = 1, \dots, n\}$,can be estimated from $n$of the observations using the Ordinary Least Squares (OLS) criterion.

The set of estimated parameters is denoted by $\hat{\beta}_{OLS}$ and defined as follows,

$$\hat{\beta}_{OLS} = argmin_{\beta\in\mathbb{R}^p}\left||Y - X\beta|\right|^2 = argmin_{\beta\in\mathbb{R}^p}(Y - X\beta)^T(Y - X\beta) \quad (3)$$

From Eq.3, $\hat{\beta}_{OLS}$ is the value of β that givesas the minimum squared norm of error between the observed value and estimated value. The first step to derive $\hat{\beta}_{OLS}$, let $h = (Y - X\beta)^T(Y - X\beta)$, and by expanding h yields:

$$h = Y^TY - \beta^TX^TY - Y^TX\beta + \beta^TX^TX\beta = Y^TY - 2\beta^TX^TY + \beta^TX^TX\beta$$

Taking the derivative to β,

$$\frac{\partial h}{\partial \beta} = -2X^TY + 2X^TX\beta. \text{If} \frac{\partial h}{\partial \beta} = 0, \text{then } X^TX\beta = X^TY$$

Therefore,

$$\hat{\beta}_{OLS} = (X^TX)^{-1}X^TY \quad (4)$$

OLS will have unbiased results if X and $y$have an approximately linear relationship.It also has a low variance if $n \gg p$. However, in real life, in many datasets such as health, business, and economy datasets, the number of explanatory variables can be much larger than the number of samples, $p \gg n$. Hence,the OLS solution is not unique. A datasetmay be high variability in the estimators, which causes poor predictive and overfitting. For these types of datasets,researchers usually use Ridge and Lasso modelsto select variables.

This paper is organized as follows: Section 2 provides a background for Ridge and Lasso models. In section 3, Bayesian inference is discussed. Markov chain Monte Carlo and Gibbs sampler are discussed in sections 4 and 5; respectively. In section 6, a new variable selection method is applied to a simulation dataset. Real data analysis is introduced in section 7. Section 8 presentsresults and discussion of the variable selection for a real dataset between the new method and some commonly used methods. In the end, the conclusionis given in section 9.

Ridge and Lasso

This section reviews two variable selection (shrinkage) methods named Ridge[5]and Lasso. Shrinkagemethods can be used under some constraints depending onthe size of thedataset.The regression model can be fitted using all the p variables, but the shrinkage technique improves the accuracy and stability by reducing the number of variables. TheRidge and Lasso models aim to estimate the coefficient of some variables as0 or close to zero so that those variables can be excluded from the model.$\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_p)$ that minimizes the Residual Sum Squares (*RSS*) is the solution to the OLS fitting procedure; i.e.,

$$RSS = \sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{n}\beta_j x_{ij}\right)^2$$

Similarly, Ridge regression seeks the vector $\hat{\beta}^{ridge}$ that minimizes the penalized RSS, $RSS + \lambda\sum_{j=1}^{p}\beta_j^2$,i.e.,

$$\hat{\beta}^{ridge} = minimize\left(\sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{n}\beta_j x_{ij}\right)^2 + \lambda\sum_{j=1}^{p}\beta_j^2\right), \text{subject to } \sum_{j=1}^{p}\beta_j^2 \le t$$

where the value of$t$ is the upper bound for the sum of the coefficients. The complexity parameter,λ, is greater than or equals to 0. If $\lambda = 0$, then $\hat{\beta}^{ridge} = \hat{\beta}$, as $\lambda \to \infty$, $\hat{\beta}^{ridge} \to 0_p$. And $0 < \lambda < \infty$ balances linear regression model fitting and shrinkage of the coefficients. The shrinkage penalty is small when$\beta_1, \beta_2, \dots,$ andβ_p are close to zero[6].

Unlike OLS, ridge solutions are not unique. As a result, before the estimation, the inputs should be standardized. First,$\beta_0$is estimated separately as $\bar{y} = \sum_{i=1}^{n}\frac{y_i}{n}$, and the remaining parameterscan be estimated by using the data matrix X as follows,

$\hat{\beta}^{ridge} = (X^TX + \lambda I)^{-1}X^Ty$, where I is the $p \times p$ identity matrix.

To constrain the size of OLS estimatesdifferent kinds of penalization can be considered. For example, L_1 norm can be used as penalty encompasses, so

$$\hat{\beta}^{lasso} = minimize\left(\sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{n}\beta_j x_{ij}\right)^2 + \lambda\sum_{j=1}^{p}|\beta_j|\right), \quad \lambda \ge 0 \quad (5)$$

Lasso propertyallowsexcluding variables by setting their coefficients to be zero[7]. The reduced model becomes more efficient, especially when the number of variables is much larger than the number of samples, $p \gg n$.

Bayesian Inference of a Multivariate Linear Regression

Eq.2 is used to apply Gibbs sampler in the Multivariate Linear Regression (MLR). From Eq.2,it can be concluded that $y \sim MVNn(X\beta; \sigma^2 In)$.Therefore, the likelihood function,denoted by$L(y)$, can be expressed as follows,

$$L(y) = \prod_{i=1}^{n} f(y_i)$$

$$= \frac{1}{\left(\sqrt{\sigma^2}\right)^n} e^{\left(\frac{-1}{2}(y-X\beta)^T(\sigma^2 I_n)^{-1}(y-X\beta)\right)}$$

$$= \frac{1}{(\sigma^2)^{\frac{n}{2}}} e^{\left(\frac{-(y-X\beta)^T(y-X\beta)}{2\sigma^2}\right)} (6)$$

The prior for $\beta$is chosen to be Multivariate Normal Distribution with mean 0 and covariance matrix$c_0 I_{p+1}$; i. e., $\beta \sim MVN(0, c_0 I_{p+1})$.$c_0$ is usually chosen to be a large positive value that leads to a large variance[8]. The prior for $\sigma^2$is chosen to be inverse Gamma; i. e., $\sigma^2 \sim IG(a_0, b_0)$.$a_0$ and $b_0$are the initial valuesthat can be any positive numbers. The conditional posterior distribution for β can be written as follows,

$$\pi(\beta|\sigma^2, \quad D) \quad \propto L(y) \times \prod(\sigma^2|\beta, D)$$

$$\propto \text{EXP}\left(-\frac{1}{2}\left[\frac{(y-X\beta)^T(y-X\beta)}{\sigma^2} + \frac{\beta^T\beta}{c_0}\right]\right)$$

$$\propto \text{EXP}\left(-\frac{1}{2}\left[\frac{y^Ty - y^TX\beta - \beta^TX^Ty + \beta^TX^TX\beta}{\sigma^2} + \frac{\beta^T\beta}{c_0}\right]\right)$$

$$\propto \text{EXP}\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}(-y^TX\beta - \beta^TXy + \beta^TX^TX\beta) + \frac{\beta^T\beta}{c_0}\right]\right)$$

$$\propto \text{EXP}\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}(-2\beta^TX^Ty + \beta^TX^TX\beta) + \frac{\beta^T\beta}{c_0}\right]\right)$$

$$\propto \text{EXP}\left(-\frac{1}{2}\left[\frac{-2\beta^TX^Ty + \beta^TX^TX\beta}{\sigma^2} + \frac{\beta^T\beta}{c_0}\right]\right)$$

$$\propto \text{EXP}\left(-\frac{1}{2}\left[-2\beta^T\frac{X^Ty}{\sigma^2} + \beta^T\left(\frac{X^TX}{\sigma^2}\right)\beta + \beta^T\left(\frac{I}{c_0}\right)\beta\right]\right)$$

$$\propto \text{EXP}\left(-\tfrac{1}{2}\left[-2\beta^T\tfrac{X^Ty}{\sigma^2} + \beta^T\left[\tfrac{X^TX}{\sigma^2} + \tfrac{I}{c_0}\right]\beta\right]\right).$$

Consider$b = \frac{X^Ty}{\sigma^2}$ and $A = \frac{X^TX}{\sigma^2} + \frac{I}{c_0}$, so the posterior distribution for β can be written as follows,

$$\pi(\beta|\sigma^2, \quad D) \quad \propto f(y) \times \prod(\sigma^2)$$

$$\propto \text{EXP}\left(\left(-\frac{1}{2}[\beta^TA\beta - 2\beta^Tb]\right)\right)$$

$$\propto \text{EXP}\left(-\frac{1}{2}(\beta - p)^T\,\phi(\beta - p)\right)$$

$$\propto \text{EXP}\left(-\frac{1}{2}(\beta - A^{-1}b)^T\,A(\beta - A^{-1}b)\right) \quad (7)$$

Eq.7 is a MVN density with $\mu = A^{-1}b$ and $\Sigma = A^{-1}$.Hence, the full conditional posterior for β is

$$\beta \sim MVN(A^{-1}b, A^{-1}), \text{ where } A = \frac{X^TX}{\sigma^2} + \frac{I}{c_0}$$

$$\text{and } b = \frac{X^Ty}{\sigma^2}$$

The full conditional posterior distribution for σ^2 is

$$\pi(\sigma^2|\beta, D)$$

$$\propto (\sigma^2)^{-\frac{n}{2}}\,\text{EXP}\left(-\frac{1}{2}\frac{E}{\sigma^2}\right)(\sigma^2)^{-(a_0+1)}\text{EXP}\left(-\frac{b_0}{\sigma^2}\right),$$

where $E = (y - X\beta)^T\,(y - X\beta)$

$$\propto (\sigma^2)^{-\frac{n}{2}-(a_0+1)}\,\text{EXP}\left(-\frac{1}{\sigma^2}\left(\frac{E}{2} + b_0\right)\right)$$

$$\propto (\sigma^2)^{-\left(\frac{n}{2}+a_0+1\right)}\,\text{EXP}\left(-\frac{1}{\sigma^2}\left(\frac{E}{2} + b_0\right)\right)$$

The above function isthe density function of the inverse gamma distribution with a shape equal to $\frac{n}{2} + a_0$ and rate equal to $\frac{E}{2} + b_0$. i.e.,

$$\sigma^2 \sim IG\left(\frac{n}{2} + a_0, \frac{E_1}{2} + b_0\right) \quad (8)$$

So far, the posterior distributions for β(Eq.7) and σ^2(Eq.8) have been derived. Hence, the estimated values for β and σ^2 can be found by calculating theirsample means. This can be done using Markov chain Monte Carlo (MCMC)without calculating the marginal likelihood for β and σ^2.In the following section, a brief discussion of MCMC is given, and thena particular case from MCMC (Gibbs sampler) is explained in detail.

Markov Chain Monte Carlo

MCMC is an essential technique, and it is used frequently in many statistical applications. In many cases, it is challenging to sample from a target posterior density. ThenMCMC is used[9]. There are three popular MCMC sampling techniques, such as Metropolis-Hastings, slice sampling[10], and Gibbs sampling[11].MCMC methods are derived from a Monte Carlo (MC)[12]. A chain is used to approximate samples of desired distribution in MCMC, and the approximation is generallyimproved after several stepshave been done[13].

Gibbs Sampler

In Bayesian inference, Gibbs sampling is commonly used bystatistical inference without calculating the marginal likelihood function. A high dimensional problem can be broken down into numbers of lowdimensional problemswhen Gibbs

sampler is used.The vector of parametersshould besplit into several blocks, and theneach block can besampled from its conditional distribution given other blocks.That means Gibbs sampling generated posterior samples by sweeping through each block variables[14]. Gibbs sampling issimilarto the other MCMC algorithms that generate a chain of samples where each of them is correlated with its nearby samples. Therefore, if the independent samples are desired, the samples should be thinned to get an independent sample set[7]. Suppose there are$n$ parameters$\theta_1, \theta_2, \dots, \theta_n$,Gibbs sampling canestimate the parameters by updating them one by one. Evaluating the joint posterior$f(\theta_1, \theta_2, \dots, \theta_n|Data)$ is the first step, and it can be done by multiplying the likelihood with the prior $f(\theta_1), f(\theta_2), \dots, f(\theta_n)$. For instance,the conditional posterior forθ_1, $f(\theta_1|\theta_2, \theta_3, \dots, \theta_n)$, can be found by assuming$\theta_2, \dots, \theta_n$are fixed atcurrent values. This process should be repeated for all the parameters $\theta_2, \theta_3, \dots, \theta_n$. Algorithm1 summarizesGibbs sampler steps[15].

Algorithm:Gibbs Sampler

INPUT: $\boldsymbol{\theta^{(i-1)}} = (\boldsymbol{\theta_1}^{(i-1)}, \boldsymbol{\theta_2}^{(i-1)}, \dots, \boldsymbol{\theta_k}^{(i-1)})$
OUTPUT: $\boldsymbol{\theta^{(i)}} = (\boldsymbol{\theta_1}^{(i)}, \boldsymbol{\theta_2}^{(i)}, \dots, \boldsymbol{\theta_k}^{(i)})$

for $\boldsymbol{j = 1, 2, \dots, N}$ **do**

$\boldsymbol{\theta_1}^{(j)} \sim \boldsymbol{f(\theta_1|\theta_2}^{(j-1)}, \boldsymbol{\theta_2}^{(j-1)}, \dots, \boldsymbol{\theta_k}^{(j-1)})$

$\boldsymbol{\theta_2}^{(j)} \sim \boldsymbol{f(\theta_2|\theta_1}^{(j)}, \boldsymbol{\theta_3}^{(j-1)}, \dots, \boldsymbol{\theta_k}^{(j-1)})$

$\boldsymbol{\theta_3}^{(j)} \sim \boldsymbol{f(\theta_3|\theta_1}^{(j)}, \boldsymbol{\theta_2}^{(j)}, \dots, \boldsymbol{\theta_k}^{(j-1)})$

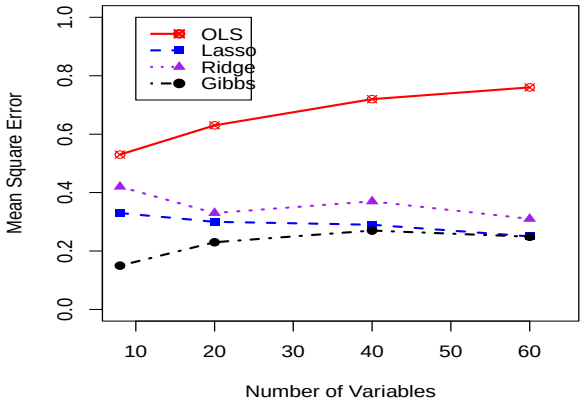
$\vdots$

$\boldsymbol{\theta_k}^{(j)} \sim \boldsymbol{f(\theta_k|\theta_2}^{(j)}, \boldsymbol{\theta_2}^{(j)}, \dots, \boldsymbol{\theta_{k-1}}^{(j)})$

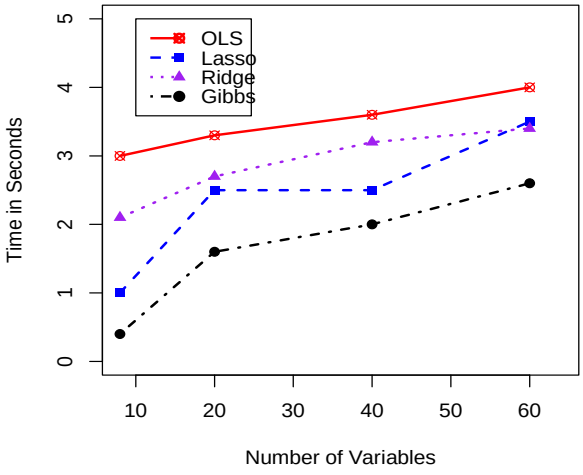
end for

Simulation Studies

Four datasetswithfour differentcovariates (8, 20, 40, 60)were generated with 1000cases.The covariates were simulated independently from a normal distribution, and then Eq.1was used to find the responses for each dataset. The smallest dataset, 8 covariates, is discussed in detail, and the results of other datasets are summarized. Mean Squared Error (MSE) and time-complexity are represented in Fig.1. Figure 1a shows that Gibbs gives the lowest MSE in all four simulated datasets. Moreover, time consumption is checked forall datasets. Fig.1bshows that Gibbs uses less time compared to the other methods.



a. Comparing the MSE



b. Comparing the consuming time

Figure 1. Comparison among 4 methods (OLS, Lasso, Ridge, and Gibbs) in the 4 simulation datasets.

The true parametersfor the first simulation datasetare: $\beta_0 = 1.1$, $\beta_1 = -2.2$, $\beta_2 = 4.3$, $\beta_3 = 1.2$, $\beta_4 = -2.2$, $\beta_5 = 6.7$, $\beta_6 = -1.3$, $\beta_7 = 3.3$ and $\beta_8 = 3.1$. The posterior distribution that has been derived in Eq.7 and 8have beenrun with 10000 iterations. Simulated samples arethinned at every 5^{th}sample to reduce the correlation between the samples.Both Gibbs sampler and Lasso methodsare used to identifythe most important variables from the 8 variables.Parameters aresummarized from their corresponding posterior means, and some of themarevery good estimatorsof the corresponding true value. In Fig.2, The samples are plotted as histograms, and the true valuesare marked with the blue lines. The distributions for some of the posterior samples are approximately normal. The true values of the parameters are close to the estimated parameters.Thecovariates

associated withβ_0, β_2, β_4, $\beta_6$and $\beta_7$were selected as the most significant covariates because they were close to the true model coefficients, as showninTable 1.However, in Lasso and Ridge methods,all the covariateswere selected as important variables.Computationally, selecting all the variables as important variables is inefficient because both the error and time willincrease for the large datasets.

Moreover, in Table1, the parameters' actual values are compared with their corresponding posterior sample means.The 95percentcredible intervals (CI) are calculated for all the parameters. It is clear that the values that are not considered a significant lie in large CI; On the other hand, all significant parametersare centered innarrow CI. This indicates that the estimation is reasonable and practical.

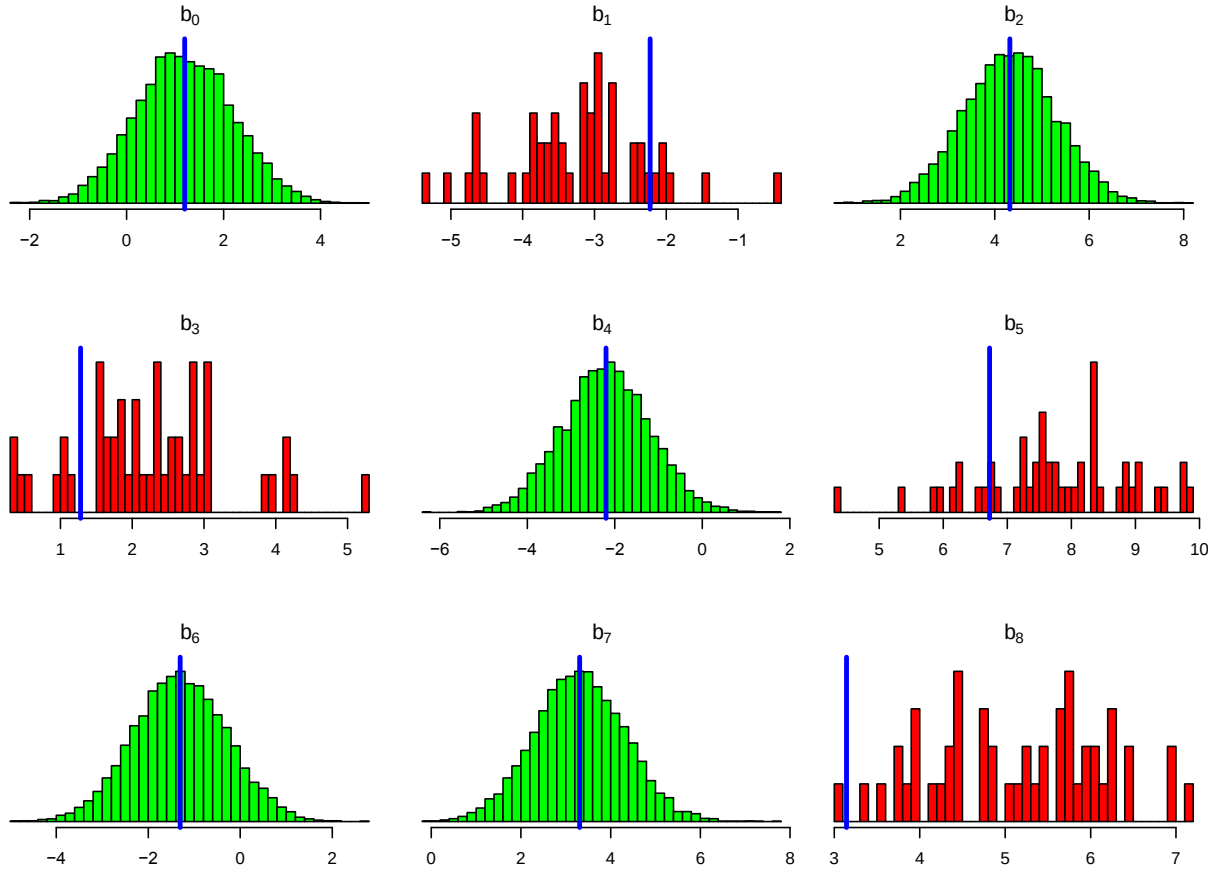


Figure 2. Posterior histograms for $\beta_0, \ldots, \beta_8$, blue lines denote the simulation's actual values.

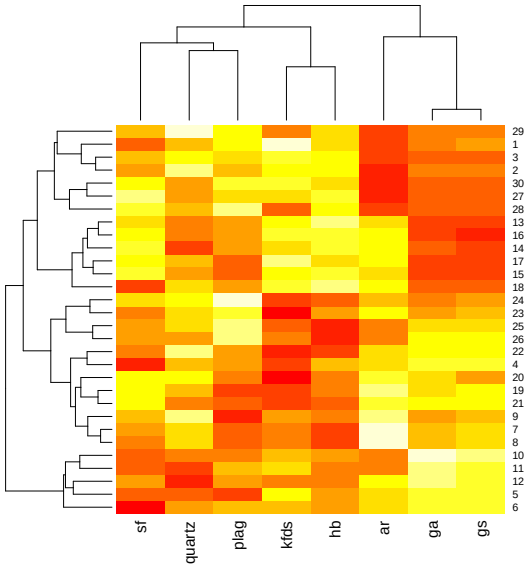
Table 1. True values, predicted values, and 95% credible intervals.

Parameter	True Value	Posterior Means	95% CI	Selected Variables (YES/NO)		
				Gibbs	Lasso	Ridge
β_0	1.1	1.003	(−0.534 , 2.398)	YES	YES	YES
β_1	−2.2	−3.321	(−5.822 , −1.272)	NO	YES	YES
β_2	4.3	4.221	(3.482 , 5.448)	YES	YES	YES
β_3	1.2	2.322	(0.238 , 4.230)	NO	YES	YES
β_4	−2.2	−2.331	(−3238 , 0.382)	YES	YES	YES
β_5	6.7	8.263	(5.239 , 9.384)	NO	YES	YES
β_6	−1.3	−1.294	(−3.484 , 1.823)	YES	YES	YES
β_7	3.3	3.309	(1.349 , 5.282)	YES	YES	YES
β_8	3.1	4.872	(3.392 , 7.849)	NO	YES	YES

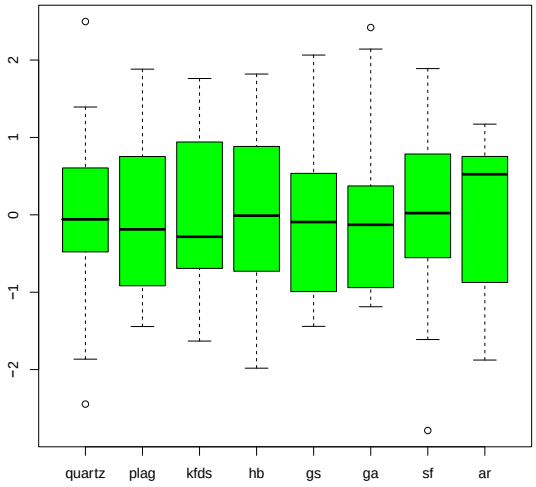
Real data Analysis

In this section, the Rock Strength Dataset (RSD) is analyzed. RSD contains information regarding the relationship between 8 predictors, which are %Quartz (**quatz**), %Plagaoclase(**plag**), %K. feldspar (**kfds**),%Hornblende (**hb**), Grain size (**gs**), Grain area (**ga**), Shape Factor (**sf**) ,Aspect Ratio (**ar**), and the response, Uniaxial Compressive Strength (**UCS**), for 30 rock specimens.The dataset is collected from the**UCI Machine Learning Repository.**

Figure 3a shows the heatmap for the8 predictors. The heatmap did not give us sufficient information about the data. As can be seen,the level of correlation is represented across all samples. The orange colorrepresents the high correlation, and the low correlation is marked with yellow color. The dataset was normalized, and then predictors' boxplots are plotted.InFig.3b, some outliers in the dataset are realized.So,they areremoved before running Gibbsand Lasso variables selection methods.The correlation matrix for the 8 predictors in the real data set (**RSD**) is given in Fig.4. Most of the covariates are approximately normally distributed. ***gs***and ***ga***havea strongpositive correlation, while ***plag***and ***kfds***have a very low correlation.



a. Heatmap for SRD variables



b. Boxplot for SRD variables

Figure 3.Heatmap and boxplot for the 8 predictors in SRD.

Baghdad Science Journal
P-ISSN: 2078-8665
E-ISSN: 2411-7986

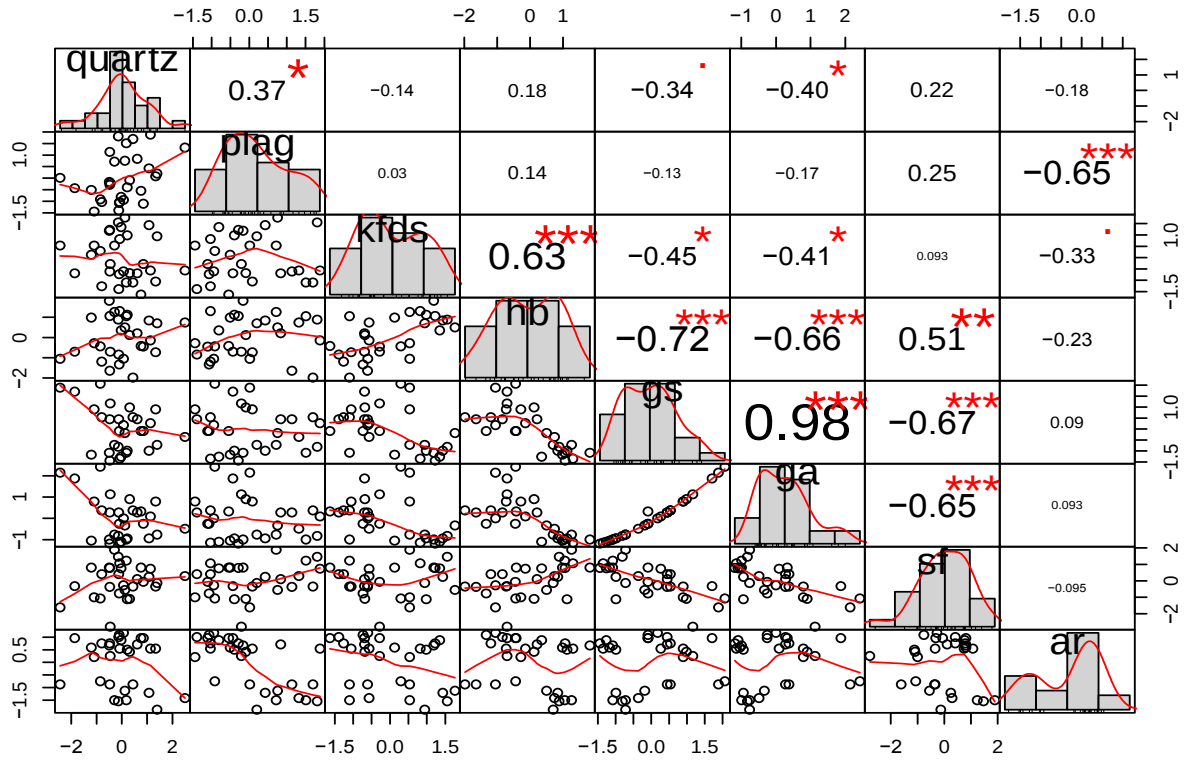


Figure 4.correlation matrix for the 8 predictors in SRD and their distributions.

Result and Discussion of the Variable Selection in RSD

Gibbs sampler has been used to select the essential variables in RSD. If all the variables are used, then the general multivariate model should be

$$y = \beta_0 + \beta_1 \boldsymbol{quartz} + \beta_2 \boldsymbol{plag} + \beta_3 \boldsymbol{kfds} + \beta_4 \boldsymbol{hb} + \beta_5 \boldsymbol{gs} + \beta_6 \boldsymbol{ga} + \beta_7 \boldsymbol{sf} + \beta_8 \boldsymbol{ar} + \epsilon$$

where $\epsilon \sim MNN(O_n, I_n)$

In the beginning, the dataset isnormalized. Gibbs sampler is run for1000 iterations to create the posterior samples of the parameters. The prediction performanceischecked by using leave-one-out-cross validation (LOOCV)(10). In LOOCV,one of the observations left out,and the model's coefficients are estimatedwith the rest of the observations. Since the real data has only 30 observations, the procedure isrepeated 30 times. It was found that 5 out of8 posterior means lie inside the 95percent credible intervals. These variables were selected as important predictors. Therefore, using the Gibbs method, the new model becomes

$$y = \beta_0 + \beta_1 \boldsymbol{quartz} + \beta_3 \boldsymbol{kfds} + \beta_5 \boldsymbol{gs} + \beta_7 \boldsymbol{sf} + \epsilon \text{where } \epsilon \sim MNN(O_n, I_n)$$

OLS, Ridge, Lasso, and Gibbs selection methods were applied on the RSD, and the outputs are shown in Table2. Gibbs methodgives the smallest MSE, and takes less time compared to the other methods. Table 3. shows that the posterior means for the parameter arerepresented with their 95percent CI. Gibbs selects 4 variables as essential variables: **quartz, Kfds, gs, and sf**, while Lasso selects**6**variables, and Ridge selects all the **8** variables.

Table 2. Comparison between OLS, Ridge, Lasso, and Gibbs selection methods

Parameter	OLS	Ridge	Lasso	Gibbs
MSE	2.311	1.934	0.773	**0.460**
Time	≈8 seconds	≈6 seconds	≈3 seconds	**<1 second**

Table 3.Posterior means, 95% CI, and the selected variables.

Parameter	Posterior Means	95% CI	Selected Variables (YES/NO)		
			Gibbs	Lasso	Ridge
β_0	104.943	(103.533 , 106.232)	**YES**	**YES**	**YES**
β_1 **quartz**	2.351	(0.375 , 4.234)	**YES**	**YES**	**YES**
β_2 **plag**	−3.685	(−3.561 , 13.873)	**NO**	**NO**	**YES**
β_3 **kfds**	−0.396	(−2.832 , 2.134)	**YES**	**YES**	**YES**
β_4 **hb**	8.566	(−4.054 , 7.184)	**NO**	**YES**	**YES**
β_5 **gs**	−3.124	(−4.334 , 1.334)	**YES**	**YES**	**YES**
β_6 **ga**	−5.604	(−3.442 , 12.497)	**NO**	**YES**	**YES**
β_7 **sf**	2.761	(0.476 , 4.512)	**YES**	**YES**	**YES**
β_8 **ar**	9.268	(−2.442 , 7.583)	**NO**	**NO**	**YES**

Conclusions:

A new variable selection approachusing the Gibbs samplerhas been discussed in this article.The posterior distributions for $\beta$and σ^2have been derived, andthe Gibbs sampler algorithmis used to sample from the corresponding distributions. The simulation datasets show that the Gibbs sampler is better than other existing methods (Lasso and Ridge, OLS). In both simulations and real datasets,the variable selection using Gibbs is faster and gives less error. Asshown in SRD, the new method performs better than the other strategies by selecting only 50 percent of the variables; In contrast, Lasso and Ridge have selected 75 percent and 100 percent of the variables; respectively, with less accuracy and more time-consuming.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

Authors' contributions statement:

Ghadeer J. M. Mahdi contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript. Othman M. Salih performed the computationsand verified the analytical methods. All authors discussed the results and contributed to the final manuscript**.**

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اختيار المتغيرات باستخدام خوارزمية Gibbsالمطورة وتطبيقها على بيانات Rock Strength

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الخلاصة:

اختيار المتغيرات مهمة ضرورية ومطلوبة في مجال النمذجة الإحصائية. حاولت العديد من الدراسات تطوير وتوحيد طرق اختيار المتغيرات، ولكن من الصعب القيام بذلك. السؤال الأول الذي يحتاج الباحث أن يسأل نفسه عنه هو ما هو أهم المتغيرات التي يجب استخدامها لوصف الاستجابة لمجموعة بيانات معينة. في هذا العمل، تمت مناقشة طريقة جديدة في الاستدلال بايزي لأختيار المتغيرات باستخدام تقنيات عينات Gibbs. بعد تحديد النموذج، تم اشتقاق التوزيعات الخلفية لجميع المعلمات. تم اختبار طريقة الاختيار للمتغير الجديد باستخدام 4 مجاميع من البيانات. تمت مقارنة الطريقة الجديدة مع بعض الطرق المعروفة التي هي قليل مربعات الخطأ (OLS)، عامل انكماش مطلق واختيار (Lasso)، وتسوية تيكونوف (Ridge). أظهرت دراسات المحاكاة أن أداء طريقتنا أفضل من الأخرى حسب الخطأ ووقت الاستهلاك. تم تطبيق الطرق على مجموعة بيانات Rock Strength، وكانت الطريقة الجديدة التي تم تقديمها أكثر كفاءة ودقة.

الكلمات المفتاحية: اختيار المتغيرات، طريقة المربعات الصغرى، طريقة الانكماش، خوارزمية Gibbs، نظرية بايز.