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Bayesian and Non - Bayesian Inference for Shape Parameter and Reliability Function of Basic Gompertz Distribution

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Abstract:

In this paper, some estimators of the unknown shape parameter and reliability function of Basic Gompertz distribution (BGD) have been obtained, such as MLE, UMVUE, and MINMSE, in addition to estimating Bayesian estimators under Scale invariant squared error loss function assuming informative prior represented by Gamma distribution and non-informative prior by using Jefferys prior. Using Monte Carlo simulation method, these estimators of the shape parameter and R(t), have been compared based on mean squared errors and integrated mean squared, respectively

Key words: Basic Gompertz distribution, MinMSE estimator, MLE, Scale invariant squared error loss function**,** UMVUE**.**

Introduction:

The Gompertz distribution (GD) was originally introduced by Gompertz in1825 (1).This distribution is used in model survival times, modeling human mortality and actuarial tables. It has many real life applications, especially in medical and actuarial studies. Due to its complicated form, it has not received enough attention in the past. However, recently, this distribution has received considerable attention from actuaries and demographers. The probability density function of the (GD) is given by (2):

$$f(t\,;\varphi) = \varphi \exp\left[ct + \frac{\varphi}{c}\,(1 - e^{ct})\right] \;;t \geq 0\,, c, \varphi > 0$$

Where φ is the shape parameter and c is the scale parameter of the Gompertz distribution. In the current paper, it will be assumed c=1 which is a special case of Gompertz distribution known as Basic Gompertz distribution with the following probability density function (3):

$$f(t\,;\varphi) = \varphi e^{t+\varphi(1-e^{t})}\;; t \geq 0, \quad \varphi > 0 \qquad \ldots(1)$$

The corresponding cumulative distribution function F(t) is given by

$$F(t) = 1 - \exp[\varphi\,(1 - e^{t})] \qquad ; \quad t \geq 0 \qquad \ldots(2)$$

Accordingly, R(t) is given by

$$R(t) = \overline{F(t)} = \exp[\,\varphi(1 - e^{t})] \quad ; \quad t \geq 0$$

Non-Bayes Estimators of the Shape Parameter

Maximum likelihood Estimator (MLE)

Assume that, $\underline{t} = t_1, t_2,\ldots, t_n$ be the set of n random lifetimes from the Basic Gompertz distribution defined by equation (1), the likelihood function for the sample observation will be as follows (4):

$$L\,(t_1, t_2, \ldots, t_n;\, \varphi) = \pi_{i=1}^{n} f(t_i\,; \varphi)$$

$$= \varphi^n \exp[\sum_{i=1}^{n} t_i + \varphi \sum_{i=1}^{n}(1 - e^{t_i})] \qquad \ldots(3)$$

By letting, $\frac{\partial}{\partial\varphi}\ln L\,(t_1, t_2, \ldots, t_n;\, \varphi) = 0$, the MLE of φ becomes

$$\hat{\varphi}_{ML} = \frac{-n}{T} \qquad \ldots(4)$$

Where $T = \sum_{i=1}^{n}(1 - e^{t_i})$

Based on the invariant property of the MLE, the MLE for R(t) will be as follows

$$R(t) = \exp[\hat{\varphi}_{MLE}\,(1 - e^{t})]$$

Uniformly Minimum Variance Unbiased Estimator (UMVUE)

The probability density function of the BGD (1) belongs to the exponential family. Therefore, $T = \sum_{i=1}^{n}(1 - e^{t_i})$ is a (C.S.S) for φ. Then, depending on the theorem of Lehmann-Scheffe (5), the UMVUE of φ, denoted by $\hat{\varphi}_U$ will be as follows

$\hat{\varphi}_U = \frac{n-1}{\sum_{i=1}^{n}(e^{t_i}-1)}$ ... (5)

Hence,

The UMVUE of R(t) is approximated as

$\hat{R}(t)_U = \exp[\hat{\varphi}_U(1-e^t)]$ ... (6)

Minimum Mean Squared Error Estimators Method (MinMSE)

The Minimum Mean Squared Error (Min MSE) estimator can be found in the class of estimators of the form $\left(Y = \frac{k}{W}\right)$ where, k is a constant, $W = \sum_{i=1}^{n}(e^{t_i}-1)$. Therefore,

$MSE(\hat{\varphi}_{MinMSE}) = E\left[\frac{k}{W} - \varphi\right]^2$

$= k^2 E\left(\frac{1}{W^2}\right) - 2\varphi k E\left(\frac{1}{W}\right) + \varphi^2$ ... (7)

To minimize (MSE) for $(\hat{\varphi}_{MinMSE})$, partial derivative will be taken, with respect to (k) and then equating it to zero, as follows

$\frac{\partial}{\partial c} MSE(\hat{\varphi}_{MinMSE}) = 2kE\ \frac{1}{(W^2)} - 2\varphi E\frac{1}{(W)} = 0$

$k = \frac{\varphi E\left(\frac{1}{W}\right)}{E\left(\frac{1}{W^2}\right)}$ ... (8)

To obtaining $E\left(\frac{1}{W}\right)$ and $E\left(\frac{1}{W^2}\right)$, assuming that,

$Y = e^t - 1$

This implies that,

$t = \ln(y+1)$, then

$\frac{dt}{dy} = y + 1$

According to the transformation technique,

$g(y) = f(t)\left|\frac{dt}{dy}\right|$

$= \varphi e^{-\varphi y}$, y > 0

Therefore,

$Y \sim Exponential(\varphi)$ and, $W = \sum_{i=1}^{n} Y_i \sim \Gamma(n, \varphi)$ with the following p.d.f.

$g_1(w) = \frac{\varphi^n}{\Gamma n} w^{n-1} e^{-\varphi w}$, w > 0

Let $Z = \frac{1}{W}$

Hence, $Z \sim$ Inverted Gamma (n, φ)

Since, $E\left(\frac{1}{W}\right) = E(Z) = \frac{\varphi}{n-1}$

$E\left(\frac{1}{W^2}\right) = E(Z^2) = \frac{\varphi^2}{(n-1)(n-2)}$

After substitution into (8), yields

$k = \frac{\varphi^2/n-1}{\varphi^2/((n-1)(n-2))}$

Since k = n-2, the MinMSE estimator for (φ) became as follows

$\hat{\varphi}_{MinMSE} = \frac{(n-2)}{\sum_{i=1}^{n}(e^{t_i}-1)}$

The MinMSE of R(t) is approximated as

$\hat{R}(t)_{MinMSE} = \exp[\hat{\varphi}_{MinMSE}(1-e^t)]$; $t \geq 0$

Bayesian estimation

Posterior Density Functions Using Gamma Distribution

In this subsection, Gamma distribution has been considered as a prior distribution of the shape parameter φ which is defined as follows (7):

$g_1(\varphi) = \frac{\delta^\gamma}{\Gamma(\gamma)} \varphi^{\gamma-1} e^{-\delta\varphi}$; $\varphi > 0, \gamma, \delta > 0$ ... (9)

In general, the posterior p.d.f. of the shape parameter φ can be expressed as

$$\pi(\varphi|\underline{t}) = \frac{L(t_1, t_2, \ldots, t_n; \varphi)\, g(\varphi)}{\int_{\forall\varphi} L(t_1, t_2, \ldots, t_n; \varphi)\, g(\varphi) d\varphi} \quad \ldots (10)$$

Now, combining equation (3) with equation (9) in equation (10), results in

$$\pi_1(\varphi|\underline{t}) = \frac{\varphi^{n+\gamma-1} e^{-\varphi(\delta-T)}}{\int_0^\infty \varphi^{n+\gamma-1} e^{-\varphi(\delta-T)} d\varphi}$$

After simplification, gives

$$\pi_1(\varphi|\underline{t}) = \frac{(\delta-T)^{n+\gamma}\, \varphi^{n+\gamma-1}\, e^{-\varphi(\delta-T)}}{\Gamma(n+\gamma)}$$

The posterior p.d.f. of the parameter φ is Gamma distribution, i.e.,

$\varphi|\underline{t} \sim \Gamma(n+\gamma, (\delta-T))$, with,

$E(\varphi|\underline{t}) = \frac{n+\gamma}{\delta-T}$, $Var(\varphi|\underline{t}\sim\varphi) = \frac{n+\gamma}{(\delta-T)^2}$

Posterior Density Functions Using Jeffreys Prior

Supposing φ to have non-informative prior presented after using Jeffreys prior information $g_2(\varphi)$ which is signified by (8):

$g_2(\varphi) \propto \sqrt{I(\varphi)}$

Where $I(\varphi)$ stands for Fisher information designated as follows (9):

$$I(\varphi) = -nE\left(\frac{\partial^2 \ln f(t,\varphi)}{\partial\varphi^2}\right)$$

Hence,

$$g_2(\varphi) = k\sqrt{-nE\left(\frac{\partial^2 \ln f(t;\varphi)}{\partial\varphi^2}\right)} \quad \ldots (11)$$

Where, k is a constant

$\ln f(t;\varphi) = \ln\varphi + t + \varphi(1-e^t)$

$\frac{\partial^2 \ln f}{\partial\varphi^2} = -\frac{1}{\varphi^2}$

Thus,

$E\left(\frac{\partial^2 \ln f(t;\varphi)}{\partial\varphi^2}\right) = -\frac{1}{\varphi^2}$

After substitution into equation (11) yields,

$g_2(\varphi) = \frac{k}{\varphi}\sqrt{n}$, $\varphi > 0$

After substituting in equation (10), the posterior density function based on Jeffreys prior $\pi_2(\varphi|t_1, \ldots, t_n)$ is

$$\pi_2(\varphi|t_1, \ldots, t_n) = \frac{\varphi^{n-1} e^{-\varphi\sum_{i=1}^{n}(e^{t_i}-1)}}{\int_0^\infty \varphi^{n-1} e^{-\varphi\sum_{i=1}^{n}(e^{t_i}-1)} d\varphi}$$

$$= \frac{P^n \varphi^{n-1} e^{-\varphi P}}{\Gamma(n)}$$

Where $P = \sum_{i=1}^{n}(e^{t_i} - 1)$
$(\varphi|t_1, \dots, t_n) \sim \Gamma(n, P)$, with:

Bayes Estimator under Scale Invariant Squared Error Loss Function

The Scale invariant squared error loss function (SISELF) is a continuous and non-negative symmetric loss function. It has been discussed by De Groot (1970) (10), and is defined as:

$$L(\hat{\varphi},\ \varphi) = \left(\frac{\varphi - \hat{\varphi}}{\varphi}\right)^2$$

Based on (SISELF), risk function $R(\hat{\varphi}, \varphi)$ can be derived as

$$R(\hat{\varphi}, \varphi) = E(L\,(\hat{\varphi}, \varphi))$$
$$= \int_0^\infty L\,(\hat{\varphi}, \varphi)\pi(\varphi|t_1 \dots\dots\dots .. t_n)d\varphi$$

By letting $\frac{\partial R_{SI}(\hat{\varphi},\varphi)}{\partial \hat{\varphi}} = 0$, gives,

$$2\,\hat{\varphi}\, E\left(\frac{1}{\varphi^2}\right) - 2\,E\left(\frac{1}{\varphi}\right) = 0$$

Therefore, Bayesian estimator under (SISELF), that minimizes the risk function, as follows

$$\hat{\varphi}_B = \frac{E(\frac{1}{\varphi})}{E(\frac{1}{\varphi^2})} \quad \dots (12)$$

Bayes Estimation under (SISELF) with Gamma Prior

Bayes estimator relative to (SISELF) based on Gamma prior, can be derived as follows

$$E\left(\frac{1}{\varphi}|\underline{t}\right) = \int_0^\infty \frac{1}{\varphi}\pi(\varphi|\underline{t})d\varphi$$

$$= \frac{(\delta-T)\,\Gamma(n+\gamma-1)}{\Gamma(n+\gamma)} \int_0^\infty \frac{(\delta-T)^{n+\gamma-1}\,\varphi^{n+\gamma-2}e^{-\varphi(\delta-T)}d\varphi}{\Gamma(n+\gamma-1)}$$
$$= \frac{(\delta - T)}{n+\gamma-1}$$

$$E\left(\frac{1}{\varphi^2}|\underline{t}\right) = \int_0^\infty \frac{1}{\varphi^2}\pi(\varphi|\underline{t})d\varphi$$

$$= \frac{(\delta-T)^2\Gamma(\nu)}{\Gamma(n+\gamma)}\int_0^\infty \frac{(\delta-T)^\nu \varphi^{\nu-1}e^{-\varphi(\delta-T)}d\varphi}{\Gamma(\nu)}$$

Where, $\nu = n + \gamma - 2$
Thus,

$$E\left(\frac{1}{\varphi^2}|\underline{t}\right) = \frac{(\delta-T)^2}{(n+\gamma-1)(n+\gamma-2)}$$

The posterior density is then realized as the Gamma distribution density, i.e.

$E(\varphi|\underline{t}) = \frac{n}{P}$, $Var(\varphi|\underline{t}\sim\varphi) = \frac{n}{P^2}$

After substituting into (12), yields

$$\hat{\varphi}_{BG} = \frac{\frac{\delta - T}{n+\gamma-1}}{\frac{(\delta-T)^2}{n+\gamma-1(n+\gamma-2)}}$$

Thus, Bayesian estimation for the shape parameter of BGD under (SISELF) with Gamma prior, denoted by $\hat{\varphi}_{BG}$ is

$$\hat{\varphi}_{BG} = \frac{n+\gamma-2}{(\delta-T)} \quad \dots (13)$$

Now, Bayesian estimation for R(t) by using (SISELF) can be obtained using equation (12) as follows

$$\hat{R}(t) = \frac{E\left[\frac{1}{R(t)}\right]}{E\left[\frac{1}{(R(t))^2}\right]} \quad \dots (14)$$

$$E\left(\frac{1}{R(t)}\right) = \int_0^\infty e^{-\varphi(1-e^t)} \frac{(\delta-T)^{n+\gamma}\varphi^{n+\gamma-1}e^{-\varphi(\delta-T)}d\varphi}{\Gamma(n+\gamma)}$$

$$= \frac{(\delta-T)^{n+\gamma}}{(\xi)^{n+\gamma}} \int_0^\infty \frac{(\xi)^{n+\gamma}\varphi^{n+\gamma-1}e^{-\varphi(\xi)}d\varphi}{\Gamma(n+\gamma)}$$

Where, $\xi = (\delta - T) + (1 - e^t)$
Hence,

$$E\left(\frac{1}{R(t)}\right) = \frac{(\delta-T)^{n+\gamma}}{((\delta-T)+(1-e^t))^{n+\gamma}}$$

Now,

$$E\left[\frac{1}{(R(t))^2}\right] = \int_0^\infty \frac{1}{(R(t))^2}\pi(\varphi|\underline{t})d\varphi$$

$$= \int_0^\infty e^{-2\varphi(1-e^t)}\frac{(\delta-T)^{n+\gamma}\,\varphi^{n+\gamma-1}e^{-\varphi(\delta-T)}}{\Gamma(n+\gamma)}d\varphi$$

$$= \frac{(\delta-T)^{n+\gamma}}{[\Theta]^{n+\gamma}}\int_0^\infty \frac{(\Theta)^{n+\gamma}\,\varphi^{n+\gamma-1}e^{-\varphi(\Theta)}}{\Gamma(n+\gamma)}d\varphi$$

Where,
$\Theta = (\delta - T) + 2(1 - e^t)$.
Hence,

$$E\left[\frac{1}{(R(t))^2}\right] = \frac{(\delta-T)^{n+\gamma}}{[(\delta-T)+2(1-e^t)]^{n+\gamma}}$$

After substituting into(14), the Bayes estimator for R(t) under (SISELF) with Gamma prior is given by

$$\hat{R}(t)_{BG} = \left[\frac{(\delta-T)+2(1-e^{t})}{(\delta-T)+(1-e^{t})}\right]^{n+\gamma} \quad \dots (15)$$

Bayes Estimation under (SISELF) with Jefferys Prior

The Bayes estimator for the shape parameter φ under Jefferys prior can be obtained using equation (12) as follows:

$$\mathbf{E}\left(\frac{1}{\varphi}|\underline{t}\right) = \int_0^\infty \frac{1}{\varphi}\pi(\varphi|\underline{t})d\varphi$$

$$=\int_0^\infty \frac{1}{\varphi}\frac{P^n}{\Gamma(n)}\ \varphi^{n-1}e^{-\varphi P}d\ \varphi$$

$$=\frac{P\,\Gamma(n-1)}{\Gamma(n)}\ \int_0^\infty \frac{P^{n-1}\ \varphi^{n-2}e^{-\varphi P}d\varphi}{\Gamma(n-1)}$$

$$\mathbf{E}\left(\frac{1}{\varphi}|\underline{t}\right) = \frac{P}{n-1}$$

$$E\left(\frac{1}{\varphi^2}|\underline{t}\right) = \int_0^\infty \frac{1}{\varphi^2}\pi\left(\varphi|\underline{t}\right)d\varphi$$

$$=\frac{P^2\Gamma(n-2)}{\Gamma(n)}\int_0^\infty \frac{P^{n-2}\varphi^{n-3}e^{-\varphi P}d\varphi}{\Gamma(n-2)}$$

$$=\frac{P^2}{(n-1)(n-2)}$$

After substituting into (12), Bayesian estimation for the shape parameter of BGD under (SISELF) with Jefferys prior, denoted by $\hat{\varphi}_{BJ}$ is

$$\hat{\varphi}_{BJ} = \frac{n-2}{P} \quad \dots (16)$$

This is equivalent to $\hat{\varphi}_{MinMSE}$.

Now, Bayesian estimation for R(t) under (SISELF) can be obtained using equation (14), as follows

$$E\left(\frac{1}{R(t)}\right) = \int_0^\infty \frac{1}{R(t)}\pi\left(\varphi|\underline{t}\right)d\varphi$$

$$= \int_0^\infty e^{-\varphi(1-e^{t})}\frac{P^n\varphi^{n-1}e^{-\varphi p}d\varphi}{\Gamma(n)}$$

$$= \frac{P^n}{(P+(1-e^{t}))^n}\int_0^\infty \frac{\left(P+(1-e^{t})\right)^n\varphi^{n-1}e^{-\varphi(P+(1-e^{t}))}d\varphi}{\Gamma(n)}$$

$$= \frac{P^n}{\left(P+(1-e^{t})\right)^n}$$

$$E\left[\frac{1}{(R(t))^2}\right] = \int_0^\infty e^{-2\varphi(1-e^{t})}\frac{P^n\varphi^{n-1}e^{-\varphi P}}{\Gamma(n)}d\varphi$$

$$= \frac{P^n}{[\varpi]^n}\int_0^\infty \frac{(\varpi)^n\varphi^{n-1}e^{-\varphi(\varpi)}}{\Gamma(n)}d\varphi$$

Where,

$$\varpi = P + 2(1 - e^{t})$$

$$= \frac{P^n}{(P+2(1-e^{t}))^n}$$

After substituting into(14), Bayesian estimator for R(t) using (SISELF) with based on Jefferys prior, denoted by $\hat{R}(t)_{BJ}$ is

$$\hat{R}(t)_{BJ} = \left[\frac{P+2(1-e^{t})}{P+(1-e^{t})}\right]^n \quad \dots (17)$$

Simulation study

To compare the behavior of the different estimators of φ and R(t) Monte-Carlo simulation has been employed. The process has been repeated 5000 times (L=5000) with different sample sizes (n= 15, 50, and100). The values of φ have chosen as φ= 0.5 and 3.

Two values of the parameters of Gamma prior are chosen as γ=0.8 and 3, δ=0.5 and 3.

All estimators for φ derived previously are evaluated based on their mean squared errors (MSE's), where,

$$MSE(\varphi) = \frac{\sum_{i=0}^{L}(\hat{\varphi}_i-\varphi)^2}{L}\ \ ;\ \ i = 1,2,3,\dots,L$$

The integrated mean squared error (IMSE) has been employed to compare the behavior of the Bayesian estimation for R(t). IMSE is an important global measure and it more accurate than MSE, where,

$$IMSE(\hat{R}(t)) = \frac{1}{L}\ \sum_{i=1}^{L}\ [\frac{1}{n_t}\sum_{j=1}^{n_t}\ (\hat{R}_i\ (t_j) - R(t_j)\ )]^2$$

$$= \frac{1}{n_t}\ \sum_{j=1}^{n_t} MSE(\hat{R}_i(t_j))$$

Where i=1, 2,…, L, n_t the random limits of t_i.

In this paper, we chose t = 0.1, 0.2, 0.3, 0.4, 0.5.

Results and Discussion

Estimating the shape parameter

The results of estimating the shape parameter of Basic Gompertz distribution φ including expected values (EXP) and mean squared errors (MSE's) are tabulated in Tables 1-4.

The discussion of the results can be summarized as follows:

- The comparison between MSE's of the three non-Bayesian estimators (MLE, UMVUE and MinMSE) for the shape parameter φ showed that (theoretically and empirically), the performance of (MinMSE**)** is the best estimator, followed by UMVUE. In other words, MSE(MinMSE) ≤ MSE(UMVUE) ≤ MSE(MLE)
- The simulation experiments results show a convergence between the expected values (EXP) to the true values of the parameter φ with an increase in the sample sizes.
- The Bayes estimator under Scale invariant squared error loss function with Gamma prior is the best estimator for the shape parameter in comparing to others, such that the value of the shape parameter of Gamma prior γ should be less than (1) for all cases and the value of the scale parameter of Gamma prior should be chosen greater than 1 if the shape parameter of Basic Gompertz distribution is less than 1 and vice versa.

- The MinMSE estimator for the shape parameter and Bayes estimator with Jefferys prior are equivalent.
- Generally, the MSE's of all estimators of the shape parameter φ increase with the increase of the shape parameter value.

Table 1. The expected values (EXP) and MSE's for different estimators of the shape parameter of Basic Gompertz distribution φ when φ =0.5.

n	Criteria	MLE	UMVU	MINMSE	Jeffreys prior	Gamma prior			
						γ = 0.8		γ = 3	
						δ = 0.5	δ = 3	δ = 0.5	δ =3
15	EXP	0.535349	0.499659	0.463970	0.463970	0.483241	0.441950	0.560279	0.512405
	MSE	0.023567	0.019441	0.018061	0.018061	0.017675	**0.015239**	0.027016	0.016110
50	EXP	0.510212	0.500008	0.489804	0.489804	0.495385	0.482878	0.517718	0.504647
	MSE	0.005678	0.005353	0.005240	0.005240	0.005219	**0.004974**	0.005991	0.005134
100	EXP	0.505357	0.500303	0.495250	0.495250	0.498021	0.491761	0.509110	0.502712
	MSE	0.002650	0.002569	0.002540	0.002540	0.002537	**0.002474**	0.002730	0.002522

Table 2. The expected values (EXP) and MSE's for different estimators of the shape parameter of Basic Gompertz distribution φ when φ =3.

n	Criteria	MLE	UMVU	MINMSE	Jeffreys prior	Gamma prior			
						γ = 0.8		γ = 3	
						δ = 0.5	δ = 3	δ = 0.5	δ =3
15	EXP	3.212099	2.997957	2.783821	2.783821	2.651703	1.768695	3.074444	2.050661
	MSE	0.848427	0.699889	0.650205	0.650205	**0.548626**	1.596538	0.579961	1.009360
50	EXP	3.061275	3.000048	2.938821	2.938821	2.897267	2.517173	3.027889	2.630656
	MSE	0.204416	0.192715	0.188672	0.188672	**0.179087**	0.328218	0.184850	0.240282
100	EXP	3.032140	3.001823	2.971498	2.971498	2.950575	2.743830	3.016275	2.804920
	MSE	0.095417	0.092509	0.091458	0.091458	**0.089096**	0.130291	0.090820	0.105633

Estimating the Reliability function

The discussion of the results IMSE's of all estimators of R(t) were tabulated in Tables (5-8) and the dissection can be expressed by the following important point:

- According to the results for Non-Bayesian methods, when the shape parameter less than 1, the performance of MinMSE (approximated) is the best estimator for R(t), followed by UMVUE (approximated). In other words, $\text{MSE}(\text{MinMSE}) \leq \text{MSE}(\text{UMVUE}) \leq \text{MSE}(\text{MLE})$ (see Table 5). While the UMVUE(approximated) is the best estimator for R(t), followed by MLE when the value of the shape parameter φ greater than 1(see Table 6).
- The Bayes estimator under Scale invariant squared error loss function with Gamma prior is the best estimator for R(t) in comparison to others, such that the value of the shape parameter of Gamma prior γ should be greater than (1) for all cases while the value of the scale parameter of Gamma prior should be greater than 1 if the shape parameter of Basic Gompertz distribution is less than 1 and vice versa (see Tables 7-8).
- The IMSE's of all estimators of R(t) increase with the increase of the shape parameter value.

Table 3. IMSE's of different estimators of the reliability function of Basic Gompertz distribution when φ =0.5

n	MLE	UMVUE (approximated)	MINMSE (approximated)	Jeffreys prior	Gamma prior			
					γ = 0.8		γ = 3	
					δ = 0.5	δ = 3	δ = 0.5	δ =3
15	0.001459	0.001248	**0.001216**	0.001643	0.001414	0.001085	0.002465	**0.000777**
50	0.000384	0.000366	**0.000363**	0.000398	0.000383	0.000344	0.000479	**0.000312**
100	0.000186	0.000182	**0.000181**	0.000190	0.000186	0.000176	0.000210	**0.000168**

Table 4. IMSE's of different estimators of the reliability function of Basic Gompertz distribution when φ =3

n	MLE	UMVUE (approximated)	MINMSE (approximated)	Jeffreys prior	Gamma prior			
					γ = 0.8		γ = 3	
					δ = 0.5	δ = 3	δ = 0.5	δ =3
15	0.006947	**0.006914**	0.007888	0.009866	0.006196	0.074618	**0.005891**	0.058172
50	0.002111	**0.002108**	0.002190	0.002375	0.002032	0.015987	**0.001996**	0.013066
100	0.001066	**0.001066**	0.001086	0.001132	0.001045	0.005355	**0.001036**	0.004508

Conclusion:

The simulation study has shown that:

1. In general, Bayesian estimation of the shape Parameter and the Reliability function of Basic Gompertz distribution under Scale invariant squared error loss function with Gamma prior is the best compared

1. AIP Conf. Proc. 2015; (1643): (125-134).

Hogg R V with the corresponding estimates.

2. To increase the accuracy of Bayesian estimation of R(t) under Scale invariant squared error loss function using Gamma prior, the value of scale parameter (δ) of Gamma prior should be chosen to be inversely proportional to the value of the shape parameter of Basic Gompertz distribution (φ).

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- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Mustansiriyah University

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الاستدلال البيزي وغير البيزي لمعلمة الشكل ودالة المعولية لتوزيع Basic Gompertz

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الخلاصة:

في هذا البحث، تم الحصول على بعض المقدرات لمعلمة الشكل غير المعروفة ودالة المعولية لتوزيع Basic Gompertz ، مثل مقدّر الإمكان الأعظم والمقدر المنتظم غير المتحيز ذى اقل تباين ومقدرمتوسط مربعات الخطأ الأنى والمقدرات البيزية تحت دالة خسارة الخطأ التربيعية الثابتة باستخدام دوال اسبقية معلوماتية تمثلت بتوزيع كاما وغير معلوماتية باستخدام دالة اسبقية جيفري. تم إجراء محاكاة مونت كارلو لمقارنة أداء جميع تقديرات معلمة الشكل ودالة المعولية ، استنادًا إلى متوسط مربعات الخطأ ومتوسط مربعات الخطأ التكاملي، على التوالي.

الكلمات المفتاحية: توزيع Basic Gompertz، مقدرمتوسط مربعات الخطأ الأنى، دالة خسارة الخطأ التربيعية الثابتة، المقدر المنتظم غير المتحيز ذى اقل تباين.