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An Asymptotic Analysis of the Gradient Remediability Problem for Disturbed Distributed Linear Systems

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Abstract:

The goal of this work is demonstrating, through the gradient observation of a disturbed distributed parameter systems of type linear ($DDPL$-systems), the possibility for reducing the effect of any disturbances (pollution, radiation, infection, *etc.*) asymptotically, by a suitable choice of related actuators of these systems. Thus, a class of asymptotically gradient remediable system (AGR-system) was developed based on finite time gradient remediable system (GR-system). Furthermore, definitions and some properties of this concept AGR-system and asymptotically gradient controllable system (AGC-controllable) were stated and studied. More precisely, asymptotically gradient efficient actuators ensuring the weak asymptotically gradient compensation system ($WAGC$-system) of known or unknown disturbances are examined. Consequently, under convenient hypothesis, the existence and the uniqueness of the control of type optimal, guaranteeing the asymptotically gradient compensation system (AGC-system), are shown and proven. Finally, an approach that leads to a Mathematical approximation algorithm is explored.

Keywords: Asymptotic analysis, Controllability, Disturbance, Optimal control, Remediability.

Introduction:

Driven by environmental, pollution[1], radiation and infection problems [2-3], the authors have studied the problem with regard to the gradient observation of a class of $DDPL$-systems considering the possibility of lessening or compensating asymptotically the effect of any disturbances. Thus, the study constitutes a development to the case of asymptotic type for the previous investigates to the remediability linear parabolic problem of different systems, introduced in the finite time case [4-7] and asymptotic case [4, 8, 9].

One can note that studying compensation problem with respect to the gradient observation and the so-called gradient remediability, is of considerable interest [10]. Thus, it was shown that there exists a system that is not remediable, however may be gradient remediable.

Gradient remediability concept in usual and regional case is considered and studied for DPL-systems [10-12]. Regarding the asymptotic case aspect [13], the great importance of the asymptotic analysis in systems theory [14-15], takes into consideration the problem of AGC-systems and studies a prospective extension of the development methods, in addition to analyzing the results in finite time. Hereafter, through likeness the relationship among the remediability and controllability of the gradient case has been inspected and studied in a considerable time.

Also, the link among remediability and controllability in asymptotic gradient case has been studied and analyzed.

This paper is structured as follows:

Section 2, is devoted to the introduction of the gradient remediability concepts of type exact and weak under convenient hypothesis.

Section 3 relates to the asymptotic form in various cases in connection with suitable actuators and sensors. Also, an asymptotically gradient efficient actuators enable the guaranteeing an asymptotic gradient compensation of weak type is presented.

In section 4, weakly and exactly a asymptotically gradient controllable system

($WEAGC$-system) are defined and characterized. Then, the link between $WEAGC$-system and weakly and exactly asymptotically gradient remediable system ($WEAGR$-system) are studied and analyzed, and it is shown that AGR-system is dependent on the appropriate sensors with corresponding actuators.

While, in section 5, the AGR-problem through the energy of type minimum is examined.

In the last section, the control of optimal type, is used to obtain a mathematical algorithm approach.

Formulation of the Considered Problem:

Assume that Ω stands as an open and bounded set in IR^n, with a boundary of smooth type $\partial\Omega$. Considering a class of $DDPL$-system defined by the form:

$$(S)\begin{cases} \dot{y}(t) = \mathcal{A}\, y(t) + B\, u(t) + f(t) \ ; 0 < t < T \\ y(0) = y_0 \end{cases}$$

where $\mathcal{A}$ generates a strongly continuous semi-group $\left(S(t)\right)_{t\geq 0}$; $B \in \mathcal{L}(U,\mathcal{X}), u \in L^2(0,T;U)$, U is a space of Hilbert type is denoted the input space and $\mathcal{X} = H_0^1(\Omega)$, the space of state.

The system (S) admits a unique solution $y \in C\left(0,T;H_0^1(\Omega)\right) \cap C^1\left(0,T;L^2(\Omega)\right)$ given by [13]:

$$y(t) = S(t)y_0 + \int_0^t S(t-s)\, Bu(s)\, ds + \int_0^t S(t-s)\, f(s) ds$$

The system (S) is augmented by the following output (gradient observation) equation:

$$(\mathcal{O})\ z_{u,f}(t) = C\nabla y(t)\ ; 0 < t < T$$

where $C \in \mathcal{L}\left(\left(L^2(\Omega)\right)^n, Y\right), Y$ is a Hilbert space (gradient observation space) and ∇ is the operator defined by:

$$\nabla: H_0^1(\Omega) \to \left(L^2(\Omega)\right)^n$$

$$y \to \nabla y = \left(\frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \dots, \frac{\partial y}{\partial x_n}\right)$$

while ∇^* its adjoint operator. Then, the gradient observation at the final time T is given by:

$$z_{u,f}(T) = C\nabla S(T)y_0 + C\nabla H_T \mathrm{u} + C\nabla F_T f$$

where H_T and F_T are operators formulated by

$$H_T: L^2(0,T;U) \longrightarrow \mathcal{X}$$

$$u \longrightarrow H_T\, u = \int_0^T S(T-s)\, Bu(s) ds$$

and

$$F_T: L^2(0,T;\mathcal{X}) \longrightarrow \mathcal{X}$$

$$f \longrightarrow F_T f = \int_0^T S(T-s) f(s) ds$$

In the autonomous case, that is to say, deprived of disturbance ($f = 0$) and control ($u = 0$) the observation of gradient, $z_{0,0}(.) = C\nabla S(.)y_0$, is then normal. But if the system is disturbed by a term f, the gradient observation becomes

$$z_{0,f}(T) = C\nabla S(T)y_0 + C\nabla F_T f$$

Generally $z_{0,f}(.) \neq C\nabla S(.)y_0$. Then a control term Bu is introduced in order to reduce, in finite time, the effect of this disturbance according to the gradient observation, such that: For any $f \in L^2(0,T;\mathcal{X})$, there exists $u \in L^2(0,T;U)$ satisfying

$$C\nabla H_T\, u + C\nabla F_T f = 0$$

The next **definition 1** characterizes the gradient remediable notion of type exactly and weakly in finite time as follows:

Definition 1 [10]

1. System (S) augmented by $(\mathcal{O})$, (or $(S) + (\mathcal{O})$) is called exactly gradient remediable (EGR-system) on $[0,T]$, if for every $f \in L^2(0,T;X)$, there exists a control $u \in L^2(0,T;U)$ such that $C\nabla H_T u + C\nabla F_T f = 0$.
2. $(S) + (\mathcal{O})$ is called weakly gradient remediable (WGR-system) on $[0,T]$, if for every $f \in L^2(0,T;\mathcal{X})$ and for every $\varepsilon > 0$, there exists a control $u \in L^2(0,T;U)$ such that $\|C\nabla H_T u + C\nabla F_T f\|_Y < \varepsilon$.

Remark 1

The finite time gradient compensation problem is equivalent to:

For any $f \in L^2(0,T;\mathcal{X})$, does there exists a control $u \in L^2(0,T;U)$ such that

$$\int_0^T C\nabla S(T-s)Bu(s)ds + \int_0^T C\nabla S(T-s)f(s)ds = 0$$

or equivalently

$$\int_0^T C\nabla S(t)Bv(t)dt + \int_0^T C\nabla S(t)g(t)dt = 0 \tag{1}$$

where $g(t) = f(T-t)$ and $v(t) = u(T-t)$.

Consequently, the finite time gradient remediability of $(S) + (E)$ can be also formulated as follows:

For any $g \in L^2(0,T;\mathcal{X})$, there exists a control $v \in L^2(0,T;U)$ satisfying Eq.1.

The characterizations consequences on the $WEGR$-systems and in limited time have been established by Rekkab and Benhadid, and they have shown that the remediability concept of type gradient is a weaker than controllability of type gradient [10].

Asymptotic Gradient Compensation Problem: Formalism statement:

An asymptotic analysis of the problem is given by considering the system:

$$(S_\infty)\begin{cases}\dot{y}(t) = \mathcal{A}y(t) + B\,u(t) + f(t)\ ; t > 0\\ y(0) = y_0\end{cases}$$

augmented by the output (gradient observation) equation:

$$(\mathcal{O}_\infty)\ z_{u,f}(t) = C\,\nabla y(t)\ ; t > 0$$

with $f \in L^2(0, +\infty; \mathcal{X})$ and $u \in L^2(0, +\infty; U)$.
Let us consider the following operators

$$H_\infty: L^2(0, +\infty; U) \to \mathcal{X}$$
$$u \to H_\infty u = \int_0^{+\infty} S(s)Bu(s)ds$$

and

$$F_\infty: L^2(0, +\infty; \mathcal{X}) \to \mathcal{X}$$
$$f \to F_\infty f = \int_0^{+\infty} S(s)f(s)ds$$

The asymptotic gradient remediability problem was studied to consist an investigation regarding the output operator C, the existence of an input one B confirming the gradient compensation asymptotically of any disturbance, that is : For any $f \in L^2(0, +\infty; \mathcal{X})$, there exists $u \in L^2(0, +\infty; U)$ such that

$$C\nabla H_\infty u + C\nabla F_\infty f = 0 \qquad 2$$

Note that the operators H_∞ and F_∞ are not generally well defined. They are, if and only if the following condition is verified [14]:
$\exists k \in L^2(0, +\infty; \mathbb{R}^+)$ such that

$$\|S(t)\| \le k(t);\ \forall t \ge 0 \qquad 3$$

Remark 2

- If $\left(S(t)\right)_{t\ge0}$ is exponentially stable, that is to say, if $\exists\beta > 0$ *and* $\exists\alpha > 0$ such that
$$\|S(t)\| \le \beta e^{-\alpha t};\ \forall t \ge 0$$

then Eq.3, is satisfied with $k(t) = \beta e^{-\alpha t} \in L^2(0, +\infty; \mathbb{R}^+)$, consequently H_∞ and F_∞ are well defined. This hypothesis concern the choice of the dynamics $\mathcal{A}$ of the system through the semi-group $\left(S(t)\right)_{t\ge0}$ and also the input operator B.

- Actually, this work is concerned with the operators K_C^∞ and R_C^∞ which are defined by
$$K_C^\infty: L^2(0, +\infty; U) \to Y$$
$$u \to K_C^\infty\, u = \int_0^{+\infty} C\nabla S(t)B\, u\,(t)dt$$

and

$$R_C^\infty: L^2(0, +\infty; X) \to Y$$
$$f \to R_C^\infty f = \int_0^{+\infty} C\nabla S(t)f(t)dt$$

Then some weaker hypotheses are needed than Eq.3. Certainly, it is supposed that $\exists\ k \in L^2(0, +\infty;\ \mathbb{R}^+)$ satisfied

$$\|C\nabla S(t)\ \| \le k\ (t);\ \forall t \ge 0 \qquad 4$$

In this case, K_C^∞ and R_C^∞ are well defined and Eq.2 becomes:

$$K_C^\infty u + R_C^\infty f = 0$$

Under hypothesis Eq.4, therefore, the $WEAGR$-system can be expressed in the next manner**:**

Definition 2

(i) $(S_\infty) + (\mathcal{O}_\infty)$ is called $EAGR$-system, if $\forall\ f \in L^2(0, +\infty; \mathcal{X})$, there exists a control $u \in L^2(0, +\infty; U)$ such that $K_C^\infty u + R_C^\infty f = 0$.

(ii) $(S_\infty) + (\mathcal{O}_\infty)$ is called $WAGR$-system, if $\forall\ f \in L^2(0, +\infty; \mathcal{X})$ and every $\varepsilon > 0$ there exists a control $u \in L^2(0, +\infty; U)$ such that $\|K_C^\infty u + R_C^\infty f\|_{IR^q} < \varepsilon$.

Let us note that for $T > 0$; $f \in L^2(0, +\infty; \mathcal{X})$ and $u \in L^2(0, +\infty; U)$ and under hypothesis Eq.4, it follows that:

$$K_C^\infty u + R_C^\infty f = \int_0^T C\nabla\ S\ (t)\ B\ u(t)dt + \int_0^T C\nabla S\ (t)f\ (t)dt + \int_T^{+\infty} C\nabla\ S\ (t)\ B\ u(t)dt + \int_T^{+\infty} C\nabla\ S\ (t)\ f(t)dt$$

$$= \int_0^T C\nabla S(t)Bu(t)dt + \int_0^T C\nabla S(t)f(t)dt + [\varepsilon_1(T) + \varepsilon_2(T)]$$

where $\varepsilon_1(T) = \int_T^{+\infty} C\nabla S(t)Bu(t)dt$ and

$\varepsilon_2(T) = \int_T^{+\infty} C\nabla S(t)f(t)dt$, with
$\varepsilon_1(T) + \varepsilon_2(T) \to 0$ when $T \to +\infty$, then for any $f \in L^2(0,+\infty;X)$ and $u \in L^2(0,+\infty;U)$, it follows that

$$\lim_{T\to+\infty}\left(\int_0^T C\nabla S(t)Bu(t)dt + \int_0^T C\nabla S(t)f(t)dt\right) = K_C^{\infty}u + R_C^{\infty}f$$

Characterization:

For the following results, let B^* and C^* be the adjoint operators of B and C respectively and $(S^*(t))_{t\geq 0}$ is considered for the semigroup of $(S(t))_{t\geq 0}$ of type adjoint. Let also $\mathcal{X}', U'$ and Y' be the dual space of $\mathcal{X}, U$ and Y. Under hypothesis Eq.4, the following general characterization results are obtained:

Proposition 1

The following properties are equivalent

(i) $(S_\infty) + (E_\infty)$ is $EAGR$-system.
(ii) $Im(R_C^{\infty}) = Im(K_C^{\infty})$.
(iii) $\exists \gamma > 0$ such that $\forall \theta \in Y'$, it follows that

$$\|S^*(.)\nabla^* C^*\theta\|_{L^2(0,+\infty;X')} \leq \gamma \|B^* S^*(.)\nabla^* C^*\theta\|_{L^2(0,+\infty;U')}$$

Proof

$(i) \Leftrightarrow (ii)$ Derives from Definition 1. Indeed, it is assumed that $(S_\infty) + (E_\infty)$ is $EAGR$-system.

Let $y \in Im(R_C^{\infty})$, then there exists $f \in L^2(0,+\infty;\mathcal{X})$ such that $y = R_C^{\infty}f$.

From the property of exact asymptotic gradient remediability for the considered system, there exists $u \in L^2(0,+\infty;U)$ such that $K_C^{\infty}u + R_C^{\infty}f = 0 \Longrightarrow R_C^{\infty}f = -K_C^{\infty}u$.

By the linearity of the operator K_C^{∞}, it follows that $y = R_C^{\infty}f = K_C^{\infty}(-u)$, then $y \in Im(K_C^{\infty})$.

The other inclusion is obtained as the previous one. Then, it follows that $Im(R_C^{\infty}) = Im(K_C^{\infty})$.

- Conversely, it is assumed that $Im(R_C^{\infty}) = Im(K_C^{\infty})$ and one can show that $(S_\infty) + (E_\infty)$ is $EAGR$-system.

Let $f \in L^2(0,+\infty;X)$, then $R_C^{\infty}f \in Im(R_C^{\infty})$. Since $Im(R_C^{\infty}) \subset Im(K_C^{\infty})$, it follows that $R_C^{\infty}f \in Im(K_C^{\infty})$ then there exists $u \in L^2(0,+\infty;U)$ such that $R_C^{\infty}f = K_C^{\infty}u$, this gives $R_C^{\infty}f - K_C^{\infty}u = 0$ and by putting $u_1 = -u \in L^2(0,+\infty;U)$. Thus $R_C^{\infty}f + K_C^{\infty}u_1 = 0$ where $(S) + (E)$ is $EAGR$-system.

$(ii) \Leftrightarrow (iii)$ Derives from the fact that the adjoint operators $(R_C^{\infty})^*$ and $(K_C^{\infty})^*$ of (R_C^{∞}) and (K_C^{∞}) respectively, are defined by

$$(R_C^{\infty})^*: Y' \to L^2(0,+\infty;X')$$
$$\theta \to (R_C^{\infty})^*\theta = S^*(.)\nabla^* C^*\theta$$

and

$$(K_C^{\infty})^*: Y' \to L^2(0,+\infty;U')$$
$$\theta \to (K_C^{\infty})^*\theta = B^*(R_C^{\infty})^*\theta = B^*S^*(.)\nabla^* C^*\theta$$

Set $P = (R_C^{\infty})^*$, $Q = (K_C^{\infty})^*$ and use the following lemma.

Lemma 1 [14]

Let $\mathcal{X}, Y, Z$ be spaces of Banach reflexive type, $\mathcal{P} \in \mathfrak{L}(\mathcal{X}, Z)$ and $Q \in \mathfrak{L}(Y, Z)$. There is an equivalence between:

$$Im(\mathcal{P}) \subset Im(Q)$$

and

$$\exists \gamma > 0 \text{ such that } \|\mathcal{P}^* z^*\|_{\mathcal{X}'} \leq \gamma \|Q^* z^*\|_{Y'}, \quad \forall z^* \in Z'.$$

□

The following **proposition 2** is proved with regard of the weak asymptotic gradient remediability characterization.

Proposition 2

There is equivalence between

(i) $(S_\infty) + (\mathcal{O}_\infty)$ is $WAGR$-system.
(ii) $Im(R_C^{\infty}) \subset \overline{Im(K_C^{\infty})}$.
(iii) $Ker\ (B^*(R_C^{\infty})^*) = Ker\ ((R_C^{\infty})^*)$.

Proof

$(i) \Leftrightarrow (ii)$ Derives from Definition 1. Indeed, it is assumed that $(S_\infty) + (\mathcal{O}_\infty)$ is $WAGR$-system.

Let $f \in L^2(0,+\infty;\mathcal{X})$, then $\forall \varepsilon > 0$, $\exists u \in L^2(0,+\infty;U)$ such that $\|K_C^{\infty}u + R_C^{\infty}f\|_Y < \varepsilon$, that is to say $\|R_C^{\infty}f - K_C^{\infty}(-u)\|_Y < \varepsilon$.

Set $u = -u \in L^2(0,+\infty;U)$, then $\forall \varepsilon > 0$, $\exists u_1 \in L^2(0,+\infty;U)$ such that $\|R_C^{\infty}f - K_C^{\infty}u_1\|_Y < \varepsilon$, this gives $R_C^{\infty}f \in \overline{Im(K_C^{\infty})}$, where $Im(R_C^{\infty}) \subset \overline{Im(K_C^{\infty})}$.

Conversely, assume that $Im(R_C^{\infty}) \subset \overline{Im(K_C^{\infty})}$ and let $f \in L^2(0,+\infty;\mathcal{X})$, then $R_C^{\infty}f \in \overline{Im(K_C^{\infty})}$, then $\forall \varepsilon > 0$, $\exists u_1 \in L^2(0,+\infty;U)$ such that $\|R_C^{\infty}f - K_C^{\infty}u\|_Y < \varepsilon$.

Put $u_1 = -u \in L^2(0,+\infty;U)$, then $\forall \varepsilon > 0$, $\exists u \in L^2(0,+\infty;U)$ such that $\|R_C^{\infty}f + K_C^{\infty}u\|_Y < \varepsilon$ where $(S_\infty) + (E_\infty)$ is $WAGR$-system.

$(ii) \Leftrightarrow (iii)$ by considering orthogonal. Indeed, it is assumed that $(S_\infty) + (\mathcal{O}_\infty)$ is $WAGR$-system. So, one can show that $\ker(B^*(R_C^{\infty})^*) = Ker\ ((R_C^{\infty})^*)$.

Let $\theta \in IR^q$ such that $B^*(R_C^{\infty})^*\theta = 0$.

In addition, $(K_C^{\infty})^* = B^*(R_C^{\infty})^*$, this gives $\theta \in \ker((K_C^{\infty})^*)$. Thus $\overline{Im(K_C^{\infty})} = (Ker\ ((K_C^{\infty})^*))^{\perp}$.

By Proposition 3.5, if follows that $Im(R_C^{\infty}) \subset \overline{Im(K_C^{\infty})}$. Then, $Im(R_C^{\infty}) \subset (Ker\ ((K_C^{\infty})^*))^{\perp}$

$\Longrightarrow \forall f \in L^2(0,+\infty;X)$; $R_C^{\infty}f \in (Ker\ ((K_C^{\infty})^*))^{\perp}$

$\Longrightarrow \langle R_C^{\infty}f, \theta\rangle = 0$, because $\theta \in Ker((K_C^{\infty})^*)$

$\Longrightarrow \theta \in \left(Im(R_C^{\infty})\right)^{\perp} = Ker((R_C^{\infty})^*)$,

and then $Ker(B^*(R_C^{\infty})^*) \subset Ker((R_C^{\infty})^*)$ where $\ker(B^*(R_C^{\infty})^*) = \ker((R_C^{\infty})^*)$.

Conversely, assume that $Ker(B^*(R_C^\infty)^*) = Ker((R_C^\infty)^*)$ and one can show that $Im(R_C^\infty) \subset \overline{Im(K_C^\infty)}$.
Let $f \in L^2(0,+\infty; X)$ such that $f \in Im(R_C^\infty)$, it follows that $\overline{Im\ (K_C^\infty)} = (Ker\ ((K_C^\infty)^*))^\perp$.
For every $\theta \in IR^q$ such that $(K_C^\infty)^*\theta = 0$ that is, $B^*(R_C^\infty)^*\theta = 0$, it follows that $(R_C^\infty)^*\theta = 0$ because $Ker(B^*(R_C^\infty)^*) = Ker\ ((R_C^\infty)^*)$, then $\langle R_C^\infty f, \theta\rangle = 0$. □

Asymptotic Gradient Remediability via Actuators and Sensors:

In connection with the system (S_∞) is motivated by $(\Omega_k, g_k)_{1\le k\le p}$, actuators suite of type zone with $g_i \in L^2\ (\Omega_k)$ and $,\Omega_k = Supp\ (g_k) \subset \Omega, \forall\ k = 1, \dots, p$, with control space $U = \mathbb{R}^p$ and B is specified by

$$B: \mathbb{R}^p \longrightarrow \mathcal{X}$$
$$u(t) \longrightarrow Bu(t) = \sum_{k=1}^p \chi_{\Omega_k} g_k u_k(t)$$

and where $u = (u_1, \dots, u_p) \in L^2(0,+\infty; \mathbb{R}^p)$. Its adjoint is given by

$$B^*z = \left(\langle g_1, z_1\rangle_{L^2(\Omega_i)}, \dots, \langle g_p, z_p\rangle_{L^2(\Omega_p)}\right) \in \mathbb{R}^p \quad 5$$

then, the following result is obtained:

Corollary 1

$(S_\infty) + (\mathcal{O}_\infty)$ is $EAGR$-systems $\Leftrightarrow\ \exists\ \gamma > 0$ satisfied the next inequality

$$\int_0^{+\infty} \|S^*(t)\nabla^*C^*\theta\|_{\mathcal{X}'}^2 dt \le \gamma \int_0^{+\infty} \sum_{k=1}^p (\langle g_k, S^*(t)\nabla^*C^*\theta\rangle)^2\, dt$$

for every $\theta \in Y$.

Proof

Since Proposition 1, $(S_\infty) + (\mathcal{O}_\infty)$ is $EAGR$-systems $\Leftrightarrow\ \exists\ \gamma > 0$ with

$$\|S^*(.)\nabla^*C^*\theta\|_{L^2(0,+\infty;X')} \le \gamma\|B^*S^*(.)\nabla^*C^*\theta\|_{L^2(0,+\infty;U')}$$

for every $\theta \in Y$
By using Eq.5, the formula of the operator B^*, yields that
$\int_0^{+\infty}\|S^*(t)\nabla^*C^*\theta\|_{\mathcal{X}'}^2 dt \le \gamma \int_0^{+\infty} \sum_{k=1}^p (\langle g_k, S^*(t)\nabla^*C^*\theta\rangle)^2\, dt$
□

By supposing the output function (S_∞) is specified via suite of sensor of type zones $(D_l, h_l)_{1\le l\le q}$, $h_i \in L^2(D_l)$, represent the distribution zone sensor, $D_l = Supp\ h_l \subset \Omega$, intended for $l = 1, \dots, q$ as well as $D_l \cap D_j = \emptyset$ for $l \ne j$, $Y = \mathbb{R}^q$ and the operator C is formed by

$$C : \left(L^2(\Omega)\right)^n \to \mathbb{R}^q$$
$$y(t) \mapsto Cy(t) = \left(\sum_{l=1}^n \langle h_1, y_l(t)\rangle_{D_1}, \dots, \sum_{l=1}^n \langle h_q, y_l(t)\rangle_{D_q}\right)$$

its adjoint is given by C^* with for $\theta = (\theta_1, \dots, \theta_q) \in \mathbb{R}^q$

$$C^*\theta = \left(\sum_{i=1}^q \chi_{D_i}\theta_i h_i, \dots, \sum_{i=1}^q \chi_{D_i}\theta_i h_i\right) \in \left(L^2(\Omega)\right)^n \quad 6$$

Without loss of generality, consider the system (S_∞) with a dynamics $\mathcal{A}$ having the form

$$\mathcal{A}\, y = \sum_{m=1}^{+\infty} \lambda_m \sum_{j=1}^{r_m} \langle y, \varphi_{mj}\rangle_{L^2(\Omega)}\varphi_{mj}, \forall y \in \mathcal{D}(\mathcal{A})$$

where $\lambda_1, \lambda_2, \dots$ are real parameters such that $\lambda_1 > \lambda_2 > \lambda_3 > \dots$, $(\varphi_{mj})_{\substack{1\le j\le r_m \\ m\ge 1}}$ is an orthogonal basis in $H_0^1(\Omega)$ of eigenvectors for $\mathcal{A}$ which is orthonormal in $L^2(\Omega)$, related to eigenvalues λ_n with a multiplicity r_n. It is well known that $\mathcal{A}$ produces a semi − group $(S(t))_{t\ge 0}$ of type strongly continuous given by [14, 15]:

$$S(t)y = \sum_{m\ge 1} e^{\lambda_m t} \sum_{j=1}^{r_m} \langle y, \varphi_{mj}\rangle_{L^2(\Omega)}\varphi_{mj}$$

Obviously, if $\sup_{m\ge 1} \lambda_m = \lambda_1 < 0$, $(S(t))_{t\ge 0}$ is exponentially stable.
□

The following characterization results have obtained

Corollary 2

$(S_\infty) + (E_\infty)$ is $EAGR$-systems $\Leftrightarrow\ \exists\ \gamma > 0$ satisfied the next inequality

$$\sum_{m\ge 1} \frac{-1}{2\lambda_m} \sum_{j=1}^{r_m} \langle C^*\theta, \nabla\varphi_{mj}\rangle^2_{(L^2(\Omega))^n} \le \gamma \sum_{k=1}^p \sum_{m\ge 1} \frac{-1}{2\lambda_m} \sum_{j=1}^{r_m} \langle C^*\theta, \nabla\varphi_{mj}\rangle^2_{(L^2(\Omega))^n} \langle g_k, \varphi_{mj}\rangle^2_{\Omega_k}$$

for every $\theta = (\theta_1, \dots, \theta_q) \in \mathbb{R}^q$.

Proof

Since Corollary 1, $(S_\infty) + (E_\infty)$ is $EAGR$-systems $\Leftrightarrow\ \exists\ \gamma > 0$ satisfied that $\forall\ \theta = (\theta_1, \dots, \theta_q) \in \mathbb{R}^q$, yields that

$$\int_0^{+\infty} \|S^*(t)\nabla^*C^*\theta\|_{\mathcal{X}'}^2 dt \le \gamma \int_0^{+\infty} \sum_{k=1}^p \left(\langle g_k, S^*(t)\nabla^*C^*\theta\rangle_{L^2(\Omega)}\right)^2 dt$$

Since

$$S(t)y = \sum_{m\geq 1} e^{\lambda_m t} \sum_{j=1}^{r_m} \langle y, \varphi_{mj}\rangle_{L^2(\Omega)} \varphi_{mj}$$

it follows that

$$\int_0^{+\infty} \|S^*(t)\nabla^* C^*\theta\|^2_{\mathcal{X}'} dt$$

$$\leq \int_0^{+\infty} \|S^*(t)\nabla^* C^*\theta\|^2_{L^2(\Omega)} dt$$

$$= \int_0^{+\infty} \sum_{m\geq 1} e^{2\lambda_m t} \sum_{j=1}^{r_m} \left(\langle \nabla^* C^*\theta, \varphi_{mj}\rangle\right)^2 dt$$

$$= \sum_{m\geq 1} \frac{-1}{2\lambda_m} \sum_{j=1}^{r_m} \left(\langle C^*\theta, \nabla\varphi_{mj}\rangle\right)^2$$

and

$$\int_0^{+\infty} \sum_{k=1}^{p} (\langle g_k, S^*(t)\nabla^* C^*\theta\rangle)^2 dt =$$

$$\sum_{k=1}^{p} \int_0^{+\infty} \sum_{m\geq 1} e^{2\lambda_m t} \sum_{j=1}^{r_m} \langle \nabla^* C^*\theta, \varphi_{mj}\rangle^2_{\Omega} \langle g_k, \varphi_{mj}\rangle^2_{\Omega_k} dt$$

$$= \sum_{k=1}^{p} \sum_{m\geq 1} \frac{-1}{2\lambda_m} \sum_{j=1}^{r_m} \langle C^*\theta, \nabla\varphi_{mj}\rangle^2_{\left(L^2(\Omega)\right)^n} \langle g_k, \varphi_{mj}\rangle^2_{\Omega_k}$$

□

By using Eq.6, the formula of the operator C^*, the following Corollary is obtained:

Corollary 3

$(S_\infty) + (E_\infty)$ is $EAGR$-systems $\Leftrightarrow$ $\exists\, \gamma > 0$ satisfied the next inequality

$$\sum_{m\geq 1} \frac{-1}{2\,\lambda_m} \sum_{j=1}^{r_m} \sum_{l=1}^{n} \sum_{i=1}^{q} \langle \theta_i h_i, \frac{\partial\varphi_{mj}}{\partial x_l}\rangle^2_{L^2(D_i)}$$

$$\leq \gamma \sum_{k=1}^{p} \sum_{m\geq 1} \frac{-1}{2\,\lambda_m} \sum_{j=1}^{r_m} \langle g_k, \varphi_{mj}\rangle^2_{L^2(\Omega_k)} \sum_{l=1}^{n} \sum_{i=1}^{q} \langle \theta_i h_i, \frac{\partial\varphi_{mj}}{\partial x_l}\rangle^2_{L^2(D_i)}, \qquad \forall\theta = \left(\theta_1, \dots, \theta_q\right) \in \mathbb{R}^q.$$

Proof

Since Corollary 2, $(S_\infty) + (E_\infty)$ is $EAGR$-systems $\Leftrightarrow$ $\exists\, \gamma > 0$ satisfied that $\forall\, \theta = (\theta_1, \dots, \theta_q) \in \mathbb{R}^q$, then

$$\sum_{m\geq 1} \frac{-1}{2\lambda_m} \sum_{j=1}^{r_m} \langle C^*\theta, \nabla\varphi_{mj}\rangle^2_{\left(L^2(\Omega)\right)^n} \leq$$

$$\gamma \sum_{k=1}^{p} \sum_{m\geq 1} \frac{-1}{2\lambda_m} \sum_{j=1}^{r_n} \langle C^*\theta, \nabla\varphi_{mj}\rangle^2_{\left(L^2(\Omega)\right)^n} \langle g_k, \varphi_{mj}\rangle^2_{\Omega_k}$$

Using the formula of the operator C^*, in Eq.6, yields that

$$\langle C^*\theta, \nabla\varphi_{mj}\rangle_{\left(L^2(\Omega)\right)^n}$$

$$= \left\langle \begin{pmatrix} \sum_{i=1}^{q} \chi_{D_i}\theta_i h_i \\ \sum_{i=1}^{q} \chi_{D_i}\theta_i h_i \\ \vdots \\ \sum_{i=1}^{q} \chi_{D_i}\theta_i h_i \end{pmatrix}, \begin{pmatrix} \frac{\partial\varphi_{mj}}{\partial x_1} \\ \frac{\partial\varphi_{mj}}{\partial x_2} \\ \vdots \\ \frac{\partial\varphi_{mj}}{\partial x_n} \end{pmatrix} \right\rangle_{\left(L^2(\Omega)\right)^n}$$

$$= \sum_{l=1}^{n} \langle \sum_{i=1}^{q} \chi_{D_i}\theta_i h_i, \frac{\partial\varphi_{mj}}{\partial x_l}\rangle_{L^2(\Omega)}$$

$$= \sum_{l=1}^{n} \sum_{i=1}^{q} \langle \theta_i h_i, \frac{\partial\varphi_{mj}}{\partial x_l}\rangle_{L^2(D_i)}$$

□

Asymptotic Gradient Efficient Actuators and Sensors:

The notion of asymptotic gradient efficient actuator have been presented analogy to the concept of gradient efficient actuator in finite time given as follows:

Definition 3 [10]

The suite $(\Omega_k, g_k)_{1\leq k\leq p}$, is called asymptotic gradient efficient actuators ($AG\mathbb{E}$-actuators) if, $(S_\infty) + (E_\infty)$ is $WAGR$-systems.

Proposition 3

The suite $(\Omega_k, g_k)_{1\leq k\leq p}$, $AG\mathbb{E}$-actuators if and only if

$$\bigcap_{m\geq 1} Ker\ (M_m f_m) = Ker\ (B^*(R_C^\infty)^*)$$

anywhere, for $m \geq 1$, M_m is the matrix of order $(p \times r_m)$ defined by

$$M_m = \left(\langle g_k, \varphi_{mj}\rangle_{L^2(\Omega_k)}\right)_{kj}, 1 \leq k \leq p \ and$$
$$1 \leq j \leq r_m$$

and

$$f_m : \theta \in \mathbb{R}^q \longrightarrow f_m(\theta)$$
$$= \left(\langle \nabla^* C^* \theta, \varphi_{m1}\rangle, \langle \nabla^* C^* \theta, \varphi_{m2}\rangle, \dots, \langle \nabla^* C^* \theta, \varphi_{mr_m}\rangle\right) \in \mathbb{R}^{r_m}$$

Proof

Since Proposition 3, $(S_\infty)+(\mathcal{O}_\infty)$ is $WAGR$-systems if and only if

$$Ker\ (B^*(R_C^\infty)^*) = Ker\ ((R_C^\infty)^*)$$

Let $\theta \in \mathbb{R}^q$, then

$$B^*(R_C^\infty)^*\ \theta = B^* S^*(.)\nabla^* C^*\ \theta = \begin{pmatrix} \langle g_1, S^*(.)\nabla^* C^*\ \theta\rangle_{L^2(\Omega_1)} \\ \langle g_2, S^*(.)\nabla^* C^*\ \theta\rangle_{L^2(\Omega_2)} \\ \vdots \\ \langle g_p, S^*(.)\nabla^* C^*\ \theta\rangle_{L^2(\Omega_p)} \end{pmatrix} =$$
$$\begin{pmatrix} \sum_{m\geq 1} e^{\lambda_m(.)} \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)} \langle g_1, \varphi_{mj}\rangle_{L^2(\Omega_1)} \\ \sum_{m\geq 1} e^{\lambda_m(.)} \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)} \langle g_2, \varphi_{mj}\rangle_{L^2(\Omega_2)} \\ \vdots \\ \sum_{m\geq 1} e^{\lambda_m(.)} \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)} \langle g_p, \varphi_{mj}\rangle_{L^2(\Omega_p)} \end{pmatrix}$$

and then, for $m \geq 1$,

$$M_m f_m(\theta) = \begin{pmatrix} \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)} \langle g_1, \varphi_{mj}\rangle_{L^2(\Omega_1)} \\ \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)} \langle g_2, \varphi_{mj}\rangle_{L^2(\Omega_2)} \\ \vdots \\ \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)} \langle g_p, \varphi_{mj}\rangle_{L^2(\Omega_p)} \end{pmatrix}$$

Assume that $\theta \in \bigcap_{m\geq 1} Ker\ (M_m f_m)$, this gives

$$\theta \in Ker\ (M_m f_m)\,, \forall\, m \geq 1 \Longrightarrow$$
$$\sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)} \langle g_k, \varphi_{mj}\rangle_{L^2(\Omega_k)} = 0,$$
$$\forall\, k \in \{1,\ 2,\ \dots,\ p\},\ \forall\, m\ \geq\ 1 \Longrightarrow$$
$$\sum_{m\geq 1} e^{\lambda_m(.)} \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)} \langle g_k, \varphi_{mj}\rangle_{\Omega_k} = 0, \forall\, k \in \{1,\ 2,\ \dots,\ p\}, \forall\, m\ \geq\ 1 \Longrightarrow$$
$$B^*\ (R_C^\infty)^*\ \theta = 0 \Longrightarrow \theta \in Ker\ \ (B^*\ (R_C^\infty)^*).$$

Where

$$\bigcap_{m\geq 1} Ker\ (M_m f_m) \subset Ker\ \ (B^*\ (R_C^\infty)^*)$$

that is

$$\bigcap_{m\geq 1} Ker\ (M_m f_m) = Ker\ \ (B^*\ (R_C^\infty)^*)$$

□

Proposition 4

The suite $(\Omega_k, g_k)_{1\leq k\leq p}$, is $AG\mathbb{E}$-actuators if and only if

$$Ker\ (\nabla^* C^*) = \bigcap_{m\geq 1} Ker\ \ (M_m f_m)$$

Proof

Suppose that the suite $(\Omega_k, g_k)_{1\leq k\leq p}$, is $AG\mathbb{E}$-actuators to prove

$$Ker\ (\nabla^* C^*) = \bigcap_{m\geq 1} Ker\ (M_m f_m)$$

Since Proposition 2 and Proposition 3, $(S_\infty)+(E_\infty)$ is $WAGR$-system if and only if

$$\bigcap_{m\geq 1} Ker\ (M_m f_m) = Ker\ \ (B^*(R_C^\infty)^*)$$
$$= Ker\ \ ((R_C^\infty)^*)$$

it follows that for every $\theta \in \mathbb{R}^q$,

$$(R_C^\infty)^*\ \theta = S^*(.)\nabla^* C^* \theta$$
$$= \sum_{m\geq 1} e^{\lambda_m(.)} \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)}\ \varphi_{mj}$$

Assuming that $\theta \in Ker\ ((R_C^\infty)^*)$, then $(R_C^\infty)^*\ \theta = 0$.

Then.

$$(R_C^\infty)^*\ \theta = S^*(.)\nabla^* C^* \theta$$
$$= \sum_{m\geq 1} e^{\lambda_m(.)} \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)}\ \varphi_{mj} = 0$$
$$\Longrightarrow \forall\ m \geq\ 1, \qquad \sum_{j=1}^{r_m} \langle \nabla^* C^*\ \theta, \varphi_{mj}\rangle_{L^2(\Omega)}\ \varphi_{mj} = 0$$
$$\Longrightarrow \nabla^* C^*\ \theta = 0 \Longrightarrow \theta \in Ker\ \ (\nabla^* C^*\ \theta).$$

Hence,

$$Ker\ ((R_C^\infty)^*) \subset Ker\ \ (\nabla^* C^*\ \theta)$$

On the other hand, if it is assumed that $\theta \in Ker\ \ (\nabla^* C^*)$, then $\nabla^* C^*\ \theta = 0$ that is to say,

$$(R_C^\infty)^*\ \theta = S^*(.)\nabla^* C^* \theta = 0$$
$$\Longrightarrow \theta \in Ker\ \ ((R_C^\infty)^*).$$

That is to say, $Ker\ (\nabla^* C^*) \subset Ker\ \ ((R_C^\infty)^*)$, then $Ker\ (\nabla^* C^*) = \ker((R_C^\infty)^*)$.

□

By analogy with the finite time case and under a condition given by: If there exists $m_0 \geq 1$ such that

$$rank\ G_{m_0}^{tr} = q \qquad 7$$

where, for $m \geq 1$, G_m is the matrix of order $(q \times r_m)$ defined by

$$G_m = \left(\sum_{l=1}^{n} \langle h_i, \frac{\partial \varphi_{mj}}{\partial x_l}\rangle_{L^2(D_i)}\right)_{ij},$$
$$1 \leq i \leq q \quad and \quad 1 \leq j \leq r_m$$

and G_m^{tr} is the transposal matrix of G_m, the two following corollary's are obtained, where the demonstrations are similar to the finite time case given in [12].

Corollary 4

The suite $(\Omega_k, g_k)_{1\le k\le p}$, is $AG\mathbb{E}-$ actuators if and only if

$$\bigcap_{m\geq 1} Ker\ (M_m G_m^{tr}) = \{0\}$$

Corollary 5

If

$$rank\ \left(M_{m_0} G_{m_0}^{tr}\right) = q \quad \text{or} \quad rank\ M_{m_0} = r_{m_0}$$

Then, the suite $(\Omega_k, g_k)_{1\le k\le p}$, is $AG\mathbb{E}$-actuators

Asymptotic Gradient Remediability and Asymptotic Gradient Controllability:

The case of asymptotic relation is difficult and requires more conditions.

Asymptotic Gradient Controllability:

Assuming the system that is described by the following equation:

$$(S_0)\begin{cases} \dot{y}\ (t) = \mathcal{A}y(t) + B\ u(t)\ ;t > 0 \\ y\ (0) = y \end{cases}$$

and A is supposed generates a strongly continuous semi-group $\left(S(t)\right)_{t\geq 0}$ such that $\exists k \in L^2(0,+\infty;\mathbb{R}^+)$ such that

$$\|\nabla S(t)\| \leq k(t);\ \forall t \geq 0 \qquad 8$$

Next, some sufficient conditions to characterize the AGC-system are given in the following results.

Definition 4

System (S_0) is called

- $EAG\mathcal{C}$-system if for every $y \in \mathcal{E} = \left(L^2(\Omega)\right)^n$, there exists $u \in L^2(0,+\infty;U)$ such that $\nabla y_0 + \nabla H_\infty\ u = y$, or equivalently $Im\nabla H_\infty = \left(L^2(\Omega)\right)^n$.
- $WAG\mathcal{C}$-system if for every $y_d \in \mathcal{E} = \left(L^2(\Omega)\right)^n$, and every $\varepsilon > 0$, there exists $u \in L^2(0,+\infty;U)$ such that $\|\nabla y_0 + \nabla H_\infty\ u - y_d\| < \varepsilon$, or equivalently $\overline{Im\nabla H_\infty} = \left(L^2(\Omega)\right)^n$.

Let $\mathcal{E}', U'$ be the dual spaces of $\mathcal{E}$ and U respectively, then using Lemma 1, it is easy to show the following results the following proposition 5 characterizes the $EAGR$-systems, and $WAGR$-systems.

Proposition 5

The system (S_0) is

(i) $EAGR$-systems if and only if $\exists\gamma > 0$ such that $\forall z^* \in \mathcal{E}'$,

$$\|z^*\|_{\mathcal{E}'} \leq \gamma\|(\nabla H_\infty)^* z^*\|_{L^2(0,+\infty;U')}$$

Or equivalently $\exists\gamma > 0$ such that $\forall z^* \in \mathcal{E}'$,

$$\|z^*\|_{\mathcal{E}'} \leq \gamma\|B^* S^*(.)\nabla^* z^*\|_{L^2(0,+\infty;U')}$$

(ii) $WAGR$-systems if and only if : $Ker\ [(\nabla H_\infty)^*] = \{0\}$

The following results in proposition 6 demonstrate that the asymptotic controllability concept of type gradient is strongest than the asymptotic remediability of type gradient in various situations.

Proposition 6

If (S_0) is $EAG\mathcal{C}$-system (resp. $WAG\mathcal{C}$-system), then, $(S_\infty) + (E_\infty)$, it is $EAGR$-system (resp. $WAGR$-system).

Proof

- By hypothesis Eq.8, if follows that, for $\theta \in Y'$,

$$\|S^*(.)\nabla^* C^*\theta\|_{L^2(0,+\infty;X')} = \left(\int_0^{+\infty}\|S^*(t)\nabla^* C^*\theta\|_{X'}^2 dt\right)^{\frac{1}{2}} \leq \left(\int_0^{+\infty}\|S^*(t)\nabla^*\|^2\|C^*\theta\|_{\mathcal{E}'}^2 dt\right)^{\frac{1}{2}} = \left(\int_0^{+\infty}\|(\nabla S(t))^*\|^2\|C^*\theta\|_{\mathcal{E}'}^2 dt\right)^{\frac{1}{2}} \leq k\|C^*\theta\|_{\mathcal{E}'};\ \text{with } k > 0.$$

from Proposition 5, and since (S_0) is $EAG\mathcal{C}$-system, $\exists\ \gamma > 0$, with

$$\|C^*\theta\|_{\mathcal{E}'} \leq \gamma\|B^*S^*(.)\nabla^* C^*\theta\|_{L^2(0,+\infty;U')}$$

then,

$$\|S^*(.)\nabla^* C^*\theta\|_{L^2(0,+\infty;X')} \leq M\|B^*S^*(.)\nabla^* C^*\theta\|_{L^2(0,+\infty;U')} \text{ with } M = k\gamma > 0.$$

By using the equivalence of part (i) and part (ii) in proposition 1, $(S_\infty) + (E_\infty)$ is $EAGR$-systems.

- From Proposition 5, $(S_\infty) + (E_\infty)$ is $WAG\mathcal{C}$-system and remains equivalent to, $Ker\ (B^*(R_C^\infty)^*) = Ker\ ((R_C^\infty)^*)$, that is to say $Ker\ (B^*(R_C^\infty)^*) \subset Ker\ ((R_C^\infty)^*)$.

This is equivalent to $Ker\ ((K_C^\infty)^*) \subset Ker\ ((R_C^\infty)^*)$, because $(K_C^\infty)^* = B^*(R_C^\infty)^*$.

For $\theta \in Ker\ ((K_C^\infty)^*)$, it follows that

$$(K_C^\infty)^*\theta = B^*S^*(.)\nabla^* C^*\theta = (\nabla H_\infty)^* C^*\theta = 0,$$

then $C^*\theta = 0$ because $Ker[(\nabla H_\infty)^*] = \{0\}$, and then $\theta \in Ker\ (C^*) \subset Ker\ ((R_C^\infty)^*)$.

□

Remark 3

The opposite of Proposition 6 is not correct; this case may be exemplified via the following.

Example 1

Reflect the subsequent one dimensional system of type diffusion.

$$(S_1)\begin{cases}\frac{\partial y}{\partial t}(x,t)=\Delta y(x,t)+f(x,t)+\sum_{k=1}^{p} g_k(x)u_k(t) \ ; in\ \Omega\times]0,+\infty[\\ y(x,0)=y_0(x)\ ;\ in\ \Omega \\ y(x,t)=0\ ;\ in\ \partial\Omega\times]0,+\infty[\end{cases}$$

boosted via observation function allows by q sensors of type zone

$$(\mathcal{O}_1)\ z_{u,f}(t)=C\nabla y(t) = \left(\sum_{l=1}^{n}\langle h_1,\frac{\partial y}{\partial x_l}(t)\rangle_{D_1},\ldots,\sum_{l=1}^{n}\langle h_q,\frac{\partial y}{\partial x_l}(t)\rangle_{D_q}\right)$$

So, $\Omega=[0,1]$ gives the corresponding operator Δ of type Laplace that confesses an appropriate basis of eigenfunctions via next form

$$\varphi_m(x)=\sqrt{2}\ \sin(m\pi x)\ \ ; m\geq 1$$

The correspondent eigenvalues are specified through $\lambda_m=-m^2\pi^2\ ;\ m\geq 1$. The operator Δ generates a self adjoint strongly continuous semi group $(S(t))_{t\geq 0}$ defined by

$$S(t)y=\sum_{m\geq 1} e^{-m^2\pi^2 t}\langle y,\varphi_m\rangle\varphi_m$$

is exponentially stable [14] with the transformations

$$H_\infty u = \sum_{k=1}^{p}\sum_{m=1}^{+\infty}\int_0^{+\infty} e^{-m^2\pi^2 t}u_k(t)dt\ \langle g_k,\varphi_m\rangle\varphi_m$$

and

$$F_\infty f=\sum_{m=1}^{+\infty}\int_0^{+\infty} e^{-m^2\pi^2 t}\ \langle f(.,t),\varphi_m\rangle\varphi_m dt$$

are well defined and since Corollary 3, $(S_1)+(\mathcal{O}_1)$ is $EAGR$-systems if and only if $\exists\,\gamma>0$ such that

$$\sum_{m\geq 1}\frac{1}{2m^2\pi^2}\sum_{i=1}^{q}\langle\theta_i h_i,\frac{\partial\varphi_m}{\partial x}\rangle^2_{L^2(D_i)} \leq \gamma\sum_{k=1}^{p}\sum_{m\geq 1}\frac{1}{2m^2\pi^2}\langle g_k,\varphi_m\rangle^2_{L^2(\Omega_k)}\sum_{i=1}^{q}\langle\theta_i h_i,\frac{\partial\varphi_m}{\partial x}\rangle^2_{L^2(D_i)}$$

for every $\theta=(\theta_1,\ldots,\theta_q)\in\mathbb{R}^q$

If a unique actuator (sensor) represents the input (output) of system $(S_1)+(\mathcal{O}_1)$[13-14], then the last inequality becomes as follows:

$$\sum_{m\geq 1}\frac{1}{2m^2\pi^2}\langle\theta h,\frac{\partial\varphi_m}{\partial x}\rangle^2_{L^2(D)} \leq\gamma\sum_{m\geq 1}\frac{1}{2m^2\pi^2}\langle g,\varphi_m\rangle^2_{L^2(\Omega)}\langle\theta h,\frac{\partial\varphi_m}{\partial x}\rangle^2_{L^2(D)}\ ;\ \forall\theta\in\mathbb{R}$$

Or equivalently,

$$\sum_{m\geq 1}\frac{1}{2m^2\pi^2}\langle h,\frac{\partial\varphi_m}{\partial x}\rangle^2_{L^2(D)} \leq\gamma\sum_{m\geq 1}\frac{1}{2m^2\pi^2}\langle g,\varphi_m\rangle^2_{L^2(\Omega)}\langle h,\frac{\partial\varphi_m}{\partial x}\rangle^2_{L^2(D)}$$

for $g=\varphi_{m_0}$ with $m_0\geq 1$, it is obtained that

$$\frac{1}{2{m_0}^2\pi^2}\langle h,\frac{\partial\varphi_{m_0}}{\partial x}\rangle^2_{L^2(D)}\leq\gamma\frac{1}{2{m_0}^2\pi^2}\langle h,\frac{\partial\varphi_{m_0}}{\partial x}\rangle^2_{L^2(D)}$$

this is verified for $\gamma\geq 1$. But the considered system (S_1) is not $EAGC$-system because it is not $WAGC$-system. Indeed,

let $y\in L^2(\Omega)$

$$(\nabla H_\infty)^* y=(H_\infty)^*\nabla^* y=B^*S^*(.)\nabla^* y =\sum_{m\geq 1} e^{-m^2\pi^2(.)}\langle y,\nabla\varphi_m\rangle B^*\varphi_m =\sum_{m\geq 1} e^{-m^2\pi^2(.)}\langle y,\nabla\varphi_m\rangle\langle g,\varphi_m\rangle$$

for $g=\varphi_{m_0}$ with $m_0\geq 1$, it follows that

$$(\nabla H_\infty)^* y=e^{-{m_0}^2\pi^2(.)}\langle y,\nabla\varphi_{m_0}\rangle =m_0\pi e^{-{m_0}^2\pi^2(.)}\sqrt{2}\int_0^1 y(x)\cos(m_0\pi x)\,dx$$

Putting $y(x)=\sin(m_0\pi x)$, yields that

$$(\nabla H_\infty)^* y =e^{-{m_0}^2\pi^2(.)}\sqrt{2}\int_0^1 m_0\pi\sin(m_0\pi x)\cos(m_0\pi x)\,dx =\frac{\sqrt{2}}{2}e^{-{m_0}^2\pi^2(.)}[\sin^2(m_0\pi x)]_0^1=0$$

then $Ker\,[(\nabla H_\infty)^*]\neq\{0\}$ and by proposition 5, the result is proven.

□

Asymptotic Gradient Remediability with Minimum Energy:

Under the condition Eq.7, and the hypothesis of $WAGR$-system, then in the present section the problem of $WAGR$-system with Minimal Energy is studied. Thus, through $f\in L^2(0,+\infty;X)$, there exists a control of type optimal $u\in L^2(0,+\infty;\mathbb{R}^p)$ ensuring, asymptotically, the gradient remediability of the disturbance f such that $K_C^\infty u+R_C^\infty f=0$, are studied. That is the set defined by

$$D=\{u\in L^2(0,+\infty;\mathbb{R}^p): K_C^\infty u+R_C^\infty f=0\} \quad 9$$

is non empty. Next, the following function is considered

$$J(u)=\|K_C^\infty u+R_C^\infty f\|^2_{\mathbb{R}^q}+\|u\|^2_{L^2(0,+\infty;\mathbb{R}^p)}$$

The considered problem becomes : $\min_{u\in D} J(u)$.

For its resolution, one can use a modification of (**H. U. M**) [14].

For $\theta\in\mathbb{R}^q$, it is noted that

$$\|\theta\|_*=\left(\int_0^{+\infty}\|B^*S^*(t)\nabla^*C^*\theta\|^2_{\mathbb{R}^p}dt\right)^{\frac{1}{2}}$$

The conforming inner product is specified by

$$\langle\theta,\sigma\rangle_*=\int_0^{+\infty}\langle B^*S^*(t)\nabla^*C^*\theta,B^*S^*(t)\nabla^*C^*\sigma\rangle dt$$

and the operator $\Lambda_C^\infty: \mathbb{R}^q \longrightarrow \mathbb{R}^q$ defined by

$$\Lambda_C^\infty \theta = K_C^\infty (K_C^\infty)^* \theta$$

Then, the following proposition have obtained.

Proposition 7

If the condition Eq.7, is verified, then $\|.\|_*$ is a norm on $\mathbb{R}^q$ if and only if $(S_\infty)+(E_\infty)$ is $WAGR$-system and the operator Λ_C^∞ is invertible.

Proof

Since,

$$\|\theta\|_* = \left(\int_0^{+\infty} \|B^*S^*(t)\nabla^*C^*\theta\|_{\mathbb{R}^p}^2 dt\right)^{\frac{1}{2}} = 0$$

$$\Rightarrow \|B^*S^*(.)\nabla^*C^*\theta\|_{L^2(0,+\infty;\mathbb{R}^q)}^2 = 0$$

$$\Rightarrow B^*S^*(.)\nabla^*C^*\theta = 0$$

$$\Rightarrow \theta \in Ker\,(B^*S^*(.)\nabla^*C^*) = Ker\ B^*(R_C^\infty)^*$$

However, from Proposition 3, it follows that $\bigcap_{m\geq 1} Ker\,(M_m f_m) = Ker\,(B^*(R_C^\infty)^*)$ and also $\bigcap_{m\geq 1} Ker\,(M_m f_m) = \bigcap_{m\geq 1} Ker\,(M_m G_m^{tr})$. Indeed, let $\theta \in \mathbb{R}^q$, then

$\theta \in \bigcap_{m\geq 1} Ker\,(M_m G_m^{tr}) \Leftrightarrow (M_m G_m^{tr})\theta = 0, \forall\, m \geq 1.$

$\Leftrightarrow$

$\sum_{i=1}^{q} \sum_{j=1}^{r_m} \langle g_k, \varphi_{mj}\rangle \langle h_i, \sum_{l=1}^{n} \frac{\partial \varphi_{mj}}{\partial x_l}\rangle = 0, \forall\, m \geq 1, \forall k = 1, \ldots, p.$

$\Leftrightarrow$

$\sum_{j=1}^{r_m} \langle g_k, \varphi_{mj}\rangle \langle \nabla^*C^*\theta, \varphi_{mj}\rangle = 0,\ \forall\, m \geq 1, \forall\, k = 1, \ldots, p.$

$\Leftrightarrow (M_m f_m)\theta = 0, \forall m \geq 1.$

$\Leftrightarrow \theta \in \bigcap_{m\geq 1} Ker\,(M_m f_m).$

Where $Ker\,(B^*(R_C^\infty)^*) = \bigcap_{m\geq 1} Ker\,(M_m G_m^{tr})$ this gives $\theta \in \bigcap_{m\geq 1} Ker\,(M_m G_m^{tr})$ and since the Corollary 4, the result is obtained.

Alternatively the Λ_C^∞ is an operator of type symmetric. Actually,

$$\langle \Lambda_C^\infty \theta, \sigma\rangle_{\mathbb{R}^q} = \langle K_C^\infty (K_C^\infty)^*\theta, \sigma\rangle_{\mathbb{R}^q} = \langle \theta, K_C^\infty (K_C^\infty)^*\sigma\rangle_{\mathbb{R}^q} = \langle \theta, \Lambda_C^\infty \sigma\rangle_{\mathbb{R}^q}$$

and positive definite. Indeed,

$$\langle \Lambda_C^\infty \theta, \theta\rangle_{\mathbb{R}^q} = \langle K_C^\infty (K_C^\infty)^*\theta, \theta\rangle_{\mathbb{R}^q} = \langle (K_C^\infty)^*\theta, (K_C^\infty)^*\theta\rangle_{L^2(0,+\infty;\,\mathbb{R}^q)}$$

Finally, the mapping Λ_C^∞ has an inverse operator.

□

Now, the next consequence demonstrates, the existence of an optimal control in which guaranteed the AGR-system.

Proposition 8

For $f \in L^2(0,+\infty;\ X)$, there exists a unique $\theta_f \in \mathbb{R}^q$ such that

$$\Lambda_C^\infty \theta_f = -R_C^\infty f$$

and the control u_{θ_f} defined by *:*

$$u_{\theta_f} = B^*S^*(.)\nabla^*C^*\theta_f = (K_C^\infty)^*\theta_f$$

verifies $K_C^\infty u_{\theta_f} + R_C^\infty f = 0.$

Moreover, it is optimal and

$$\left\|u_{\theta_f}\right\|_{L^2(0,+\infty;\,\mathbb{R}^p)} = \left\|\theta_f\right\|_*.$$

Proof

By utilizing Proposition 7, the mapping Λ_C^∞ has inverse, now, $f \in L^2(0,+\infty;\ X)$, then there exists a unique $\theta_f \in \mathbb{R}^q$ such that $\Lambda_C^\infty \theta_f = -R_C^\infty f$ and by putting $u_{\theta_f} = (K_C^\infty)^*\theta_f$, yields that

$\Lambda_C^\infty \theta_f = K_C^\infty(K_C^{\infty *})\theta_f = \int_0^{+\infty} C\nabla S(t)BB^*S^*(t)\nabla^*C^*\theta_f dt = K_C^\infty u_{\theta_f} = -R_C^\infty f \Rightarrow K_C^\infty u_{\theta_f} + R_C^\infty f = 0.$

The set D defined by Eq.9, is closed, convex and not empty.

For $u \in D$, $J(u) = \|u\|_{L^2\,(0,+\infty;\mathbb{R}^p)}^2$. So, J is convex mapping of type strictly in D, and hence ensures a unique minimum at $u^* \in D$, characterized by $\langle u^*, v - u^*\rangle_{L^2(0,+\infty;\mathbb{R}^p)} \geq 0;\ \forall v \in D$.

For $v \in D$,

$$\begin{aligned}\langle u_{\theta_f}, v - u_{\theta_f}\rangle_{L^2(0,+\infty;\mathbb{R}^p)} &= \langle (K_C^\infty)^*\theta_f, v - (K_C^\infty)^*\theta_f\rangle_{L^2(0,+\infty;\mathbb{R}^p)} \\ &= \langle \theta_f, K_C^\infty v - \Lambda_C^\infty \theta_f\rangle_{L^2(0,+\infty;\mathbb{R}^p)} \\ &= 0\end{aligned}$$

Since u^* is unique, then $u^* = u_{\theta_f}$ and u_{θ_f} is optimal with

$$\left\|u_{\theta_f}\right\|_{L^2(0,+\infty;\,\mathbb{R}^p)}^2 = \left\|B^*S^*(.)\nabla^*C^*\theta_f\right\|_{L^2(0,+\infty;\,\mathbb{R}^p)}^2 = \left\|\theta_f\right\|_*^2$$

□

Mathemetical Approximations

The current part of this paper, presents important approximations augmented with an approximation approach for AGR-system. First we give an approximation of θ_f as a solution of a finite dimension linear system $A\theta_f = b$ and then the optimal control u_{θ_f}, with a comparison between the corresponding observation noted $z_{u_{\theta_f},f}$, and the normal case.

The Approximations Approach:

- *<u>System coefficients components</u>*:

For $i, j \geq 1$, consider $a_{ij} = \langle \Lambda_C^\infty e_i, e_j\rangle_{\mathbb{R}^q}$ such that $(e_i)_{1\leq i\leq q}$ is the canonical basis of $\mathbb{R}^q$, it follows that

$$\Lambda_C^\infty e_i = \int_0^{+\infty} C\nabla S(t)BB^*S^*(t)\nabla^*C^*e_i\, dt$$

and since N and M represent the number of eigenfunctions of the dynamic operator A. Thus, sufficiently large because the space have an infinite dimension.

Then, M, N be sufficiently large:

$$a_{ij} \cong \sum_{m=1}^{M}\sum_{l=1}^{r_m}\sum_{m'=1}^{N}\sum_{h=1}^{r_{m'}}\sum_{r=1}^{p}\left(\frac{-1}{\lambda_m+\lambda_{m'}}\right)\langle g_r,\varphi_{ml}\rangle_{\Omega_r}\langle g_r,\varphi_{m'h}\rangle_{\Omega_r}\sum_{k'=1}^{n}\langle\frac{\partial\varphi_{m'h}}{\partial x_{k'}},h_i\rangle_{\mathrm{D}_i}\sum_{k=1}^{n}\langle\frac{\partial\varphi_{ml}}{\partial x_k},h_j\rangle_{\mathrm{D}_j} \quad 10$$

and $b_j = -\langle R_C^{\infty} f, e_j\rangle_{\mathbb{R}^q}$.

Because N represent the number of eigenvectors $(\varphi_{mj})_{\substack{1\le j\le r_m\\ m\ge 1}}$ and really it is infinite.

For the applications it is considered sufficiently large and then

$$b_j \cong -\sum_{m'=1}^{N}\sum_{h=1}^{r_{m\prime}}\sum_{k=1}^{n}\langle\frac{\partial\varphi_{m\prime l}}{\partial x_k},h_j\rangle_{L^2(\mathrm{D}_j)}\int_0^{+\infty}e^{\lambda_m t}\langle f(t),\varphi_{m'h}\rangle_{L^2(\Omega)}\,dt \quad 11$$

- *The optimal control*:

In this part, an approximation of the optimal control u_{θ_f} is given, which is defined by:

$$u_{\theta_f}(s) = B^*S^*(t)\nabla^*C^*\theta_f$$

Its function coordinates $u_{j,\theta_f}(.)$ are given, for a large integer N, by

$$u_{j,\theta_f}(.) = \langle g_j, S^*(t)\nabla^*C^*\theta_f\rangle_{L^2(\mathrm{D}_j)}$$

$$\cong \sum_{m'=1}^{N}\sum_{h=1}^{r_{m'}}\sum_{k=1}^{n}\sum_{i=1}^{q}\theta_{i,f}e^{\lambda_{m\prime}(.)}\langle g_j,\varphi_{m'h}\rangle_{L^2(\Omega_j)}\langle\frac{\partial\varphi_{m'h}}{\partial x_k},h_i\rangle_{L^2(\mathrm{D}_i)} \quad 12$$

- *Cost:*

The minimum energy (cost), for N sufficiently large, is defined by

$$\left\|u_{\theta_f}\right\|_{L^2(0,+\infty;\,\mathbb{R}^p)} = \left(\int_0^{+\infty}\left\|B^*S^*(t)\nabla^*C^*\theta_f\right\|_{\mathbb{R}^p}^2 dt\right)^{\frac{1}{2}}$$

$$\cong\left(\sum_{j=1}^{p}\int_0^{+\infty}\left(\sum_{m'=1}^{N}\sum_{h=1}^{r_{m'}}\sum_{k=1}^{n}\sum_{i=1}^{q}\theta_{i,f}e^{\lambda_{m\prime}t}\langle g_j,\varphi_{m'h}\rangle_{L^2(\Omega_j)}\langle\frac{\partial\varphi_{m'h}}{\partial x_k},h_i\rangle_{L^2(\mathrm{D}_i)}\right)^2 dt\right)^{\frac{1}{2}}$$

- *The related observation*:

The measurement information related to a given control is described by

$$z_{u_{\theta_f},f}(t) = C\nabla S(t)y^0 + C\nabla\int_0^t S(\tau)Bu_{\theta_f}(\tau)\,d\tau + C\nabla\int_0^t S(\tau)f(\tau)\,d\tau \quad 13$$

Its coordinates $\left(z_{j,u_{\theta_f},f}(.)\right)_{1\le j\le q}$ are achieved for a specific integer N, given by:

$$\begin{aligned} z_{j,u_{\theta_f},f}(t) \cong{}& \sum_{m'=1}^{N}\sum_{h=1}^{r_{m'}}\sum_{k=1}^{n}e^{\lambda_{m'}t}\langle y_0,\varphi_{m'h}\rangle_{L^2(\Omega)}\langle\frac{\partial\varphi_{m'h}}{\partial x_k},h_j\rangle_{L^2(\mathrm{D}_j)}\\ &+\sum_{m'=1}^{N}\sum_{h=1}^{r_{m'}}\sum_{k=1}^{n}\langle g_i,\varphi_{m'h}\rangle_{L^2(\Omega_i)}\langle\frac{\partial\varphi_{m'h}}{\partial x_k},h_j\rangle_{L^2(\mathrm{D}_j)}\int_0^t e^{\lambda_{m'}\tau}u_{j,\theta_f}(\tau)\,d\tau\\ &+\sum_{m'=1}^{N}\sum_{h=1}^{r_{m'}}\sum_{k=1}^{n}\langle\frac{\partial\varphi_{m'h}}{\partial x_k},h_j\rangle_{L^2(\mathrm{D}_j)}\int_0^t e^{\lambda_{m\prime}\tau}\langle f(\tau),\varphi_{m'h}\rangle_{L^2(\Omega)}\,d\tau \end{aligned} \quad 14$$

The Mathematical Approach:

Remember the problem considered above:

$$(\mathbb{P})\begin{cases}\text{Calculate } u^* \in L^2(0,+\infty;U)\text{, with}\\ K_C^\infty u^* + R_C^\infty f = 0\end{cases}$$

So, depending on the above result., and employment the preceding consequences in this investigation, one can improve an algorithm which permits to define controls suite which tends to u^* of $(\mathbb{P})$. The measurement information is specified via Eq.13 and Eq.14.

Algorithm

First Step: Data: domain Ω, initial state y^0, disturbance function f, sensors (D,h), gradient of efficient actuators (σ,g) and precision threshold ε.

Second Step: Select a truncation low of order $M = N$.

Third Step: Calculate $z_{0,0}$: output with $f = 0$ and $u = 0$.

Fourth Step: Calculate $z_{0,f}$: output with $f \neq 0$ and $u = 0$.

Fiveth Step: Resolve a finite system $A\theta = b$ such that the parameters are represented by Eq.10 and Eq.11.

Sixth Step:. Calculate u given by Eq.12.

Seventh Step:. Compute $z_{u,f}$: output where $f \neq 0$ and $u \neq 0$.

Eighth Step: If $\left\|z_{u,f} - z_{0,\ 0}\right\|_{L^2(\Omega)} \leq \varepsilon$, then stop. Otherwise,

Ninth Step: $M \leftarrow M+1$ and $N \leftarrow N+1$ and return to third step.

Ten Step: Control u of type optimal links to u^* the solution of $(\mathbb{P})$.

Conclusion:

In this paper, the problem of AGCanalysis has been presented. Certainly, it is based on suitable hypothesis and an appropriate choice of operators and spaces. Furthermore, $WEAGR$-system and $AG\mathbb{E}$-actuators have been presented firstly. Also the problem of $WEAGC$-system has been examined under a suitable hypothesis with appropriate choice of spaces and operators. More precisely, the relationship between $WEAGC$-system and AGR-system has been demonstrated in different important results. Indeed, in the asymptotic case, it has been proved that the controllability concept of gradient type remains stronger than the remediability concept of gradient type, that is to say, AGR-system can be asymptotically gradient remediable but, it is not AGC-system.

Thus, through the choice of sensors and hypothesis of $WAGR$-system, the problem of $EAGR$-system with minimum energy has been studied. Moreover, the issue of how to discover an optimal control has been examined in a way compensateing for the influence of the disturbances about the observation of gradient via the use of **H U M** modified.

Regarding the digital processing, some mathematical approximations are proposed, using a multi-step algorithm.

Later, the obtained outcomes have been introduced for class $DDPL$-systems and may be interesting to expand this work to regional or regional bounded case with other classes under the suitable different select of spaces, for example, the possibility to replace the observability concept in this paper by an asymptotic observer.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Mentouri.

Authors' contributions statement:

S. R. and S. B. conceived of the presented idea and developed the theory. R. AL. verified the analytical methods and contributed to the analysis of the results and to the writing of the manuscript. All authors discussed the results and contributed to the final manuscript.

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تحليل مقارب لمسائل قابلية معالجة التدرج للأنظمة الخطية التوزيعية المضطربة

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الخلاصة:

الهدف من هذا العمل، برهان إمكانية التقليل من تأثير أي اضطرابات (تلوث، إشعاع، عدوى، الخ) بشكل تقاربي، من خلال مراقبة تدرج نوع من الأنظمة الخطية المضطربة ذات المعاملات التوزيعية (انظمة-$DDPL$)، بواسطة اختيار مناسب للمحفزات ذات العلاقة بتلك الانظمة . وهكذا، تم تطوير منظومة قابلية معالجة التدرج (منظومة- AGR) , بالاعتماد على منظومة قابلية معالجة التدرج في زمن محدود (منظومة-GR). وعلاوة على ذلك، درست وقدمت تعاريف وبعض خصائص مفاهيم تتعلق بمنظومة - AGR ومنظومة قابلية السيطرة في التدرج المقارب (قابلية السيطرة-AGC). بشكل ادق، فحصت المحفزات الفعالة المتدرجة بشكل مقارب والتي تضمن تعويض التدرج الضعيف بشكل مقارب للخلل المعروف أو غير المعروف (منظومة-$WAGC$). وبالتالي، في ظل فرضية ملائمة، أثبتت وبرهنت وجود ووحدانية مسيطر امثل يضمن منظومة تعويض تدرج مقارب (منظومة-AGC). أخيرا، تم أيضا اكتشاف تقريبا يؤدي إلى خوارزمية تقريبية رياضية.

الكلمات المفتاحية: تحليل مقارب، قابلية التحكم، اضطراب، التحكم الأمثل، قابلية المعالجة.