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Further Results on (a, d) -total Edge Irregularity Strength of Graphs

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Abstract

Consider a simple graph $G = (V, E)$ on l vertices and m edges together with a total h – labeling $\rho: V(G) \cup E(G) \rightarrow \{1,2,3, \dots, h\}$. Then ρ is called (a, d)–total edge irregular labeling if there exists a one-to-one correspondence, say $\psi: E(G) \rightarrow \{a, a + d, a + 2d, \dots + a + (m - 1)d\}$ defined by $\psi(uv) = \rho(u) + \rho(v) + \rho(uv)$ for all $uv \in E(G)$, where $a \geq 3, d \geq 2$. Also, the value $\psi(uv)$ is said to be the edge weight of uv. The (a, d) −total edge irregularity strength of the graph G is indicated by $(a, d) - tes(G)$ and is the least h for which G admits (a, d) – edge irregular h-labeling. In this article, $(a, d) - tes(G)$ for some common graph families are examined. In addition, an open problem (3,2)– $tes(K\_(m, n)), m, n > 2$ is solved affirmatively.

Keywords: (a, d) – Irregular labeling, Edge irregular labeling, Irregular labeling, Irregularity strength, Total edge irregular labeling.

Introduction

This paper considers finite simple undirected graphs. Under certain conditions, graph labeling refers to the assignment of numbers to graph elements such as vertices, edges, or both. These specifications are described by means of some evaluating function values (weights). Chartrand et al.[1] initially proposed irregular graph labeling as follows: Assume $G = (V, E)$ to be a connected graph of order n ≥ 3 with an edge h-labeling $\alpha : E \rightarrow \{1, 2, \dots, h\}$ and weight of a vertex v to be defined as $w(v) = \sum_{v \in e} \alpha(e)$. When all the vertex weights are distinct, this is referred to as irregular labeling. The least positive integer h such that G admits an irregular graph labeling is known as irregularity strength $s(G)$. Irregular labeling of a graph was modified as the total vertex irregular labeling and the total edge irregular labeling by Baca et al.[2] Let $G = (V, E)$ be a connected graph of order $n \geq 3$. Let $\beta : V \cup E \rightarrow \{1, 2, \dots, k\}$ be a function and let the weight of the edge $e = uv$ be defined by $\omega(e) = \beta(u) + \beta(v) + \beta(e)$. Then β is called a total edge irregular labeling if all the edge weights are distinct. The total edge irregularity strength $tes(G)$ is the smallest positive integer k such that there is a total edge irregular labeling $\beta: V \cup E \rightarrow \{1, 2, \dots, k\}$. For a polar grid graph, Salama[3] computed the total edge irregularity strength. Susanti Y, et al.[4] presented the exact value of total edge irregularity strength of staircase graphs, double staircase graphs, and mirror-staircase graphs. Ratnasari L, et al.[5] determined the exact value of the total edge irregularity strength of ladder-related families of graphs. Muthu Guru Packiam[6] introduced the concept of (a, d) −total edge irregularity strength of the graph G as follows: Consider a simple graph $G = (V, E)$ on l vertices and m edges

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together with a total $h-$ labeling $\rho: V(G) \cup E(G) \rightarrow \{1,2,3,\dots,h\}$. Then ρ is called (a,d)–total edge irregular labeling if there exists a one-to-one correspondence, say $\psi: E(G) \rightarrow \{a, a+d, a+2d, \dots + a+(m-1)d\}$ defined by $\psi(uv) = \rho(u) + \rho(v) + \rho(uv)$ for all $uv \in E(G)$, where $a \geq 3, d \geq 2$. Also, the value $\psi(uv)$ is said to be the edge weight of uv. The (a,d) −total edge irregularity strength of the graph G is indicated by $(a,d) - tes(G)$ and is the least h for which G admits (a,d) – edge irregular h −labeling. Additionally, they provided upper and lower bounds for the parameter and established the evaluations of $(a,d) - tes(G)$ for some families of graphs. The purpose of this paper is to focus on the study of (a,d) −total edge irregular graph labeling. P_n indicates a path containing n vertices[7,8], whereas C_n represents a cycle with n vertices[9,10,11]. The slanting ladder SLn is a graph constructed by connecting each u_i with v_{i+1}, $1 \leq i \leq n-1$ in two paths $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$. A triangular ladder $TL_n, n \geq 2$ is constructed by adding the edges $u_i v_{i+1}$, $1 \leq i \leq n-1$ to ladder. The vertices of TL_n are u_i and v_i. u_i and v_i are the two paths in the graph TL_n where $i = \{1,2,\dots,n\}$. A complete bipartite graph is a graph with bipartition (U,V) in which every vertex of U is connected with every vertex of V. If $|U| = a$ and $|V| = b$, then the complete bipartite graph is denoted by $K_{a,b}$. If either $|U| = 1$ or $|V| = 1$, then $K_{1,b}$ or $K_{a,1}$ is known as star graph.

Proposition 1:[4] Suppose G be a graph having p vertices and q edges. For integers $a \geq 3$ and $d \geq 2$, $\left\lceil \frac{a+(q-1)d}{3} \right\rceil \leq (a,d) - tes(G) \leq a - 2 + (q-1)d$.

Proposition 2:[4] If P_n is a path of order n, then $(3,2)$– $tes(P_n) = \left\lceil \frac{2n-1}{3} \right\rceil$.

Proposition 3:[4] Let $K_{1,n}$ be a star graph. Then $(3,2) - tes(K_{1,n}) = n,\ n \geq 2$.

Results and Discussion

Theorem1: If SL_n is a slanting ladder with $2n$ vertices, then $(3,2)$– $tes(SL_n) = 2n-1$ for any $n \geq 2$.

Proof: Let the vertex set of $SL_n = V_1 \cup V_2$ where $V_1 = \{u_1, u_2, \dots, u_n\}$, and $V_2 = \{v_1, v_2, \dots, v_n\}$, and $E(SL_n) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} / 1 \leq i \leq n-1\}$.

Define total labeling $\rho: V(SL_n) \cup E(SL_n) \rightarrow \{1,2,\dots,2n-1\}$ as follows:

$\rho(u_1) = 1,\ \rho(u_2) = 1,\ \rho(u_i) = 2i-1,\ 3 \leq i \leq n,$

$\rho(v_1) = 3,\ \rho(v_2) = 3,\ \rho(v_i) = 2i-1,\ 3 \leq i \leq n,$

$\rho(u_1 u_2) = 1,\quad \rho(u_2 u_3) = 3, \rho(u_3 u_4) = 3, \rho(u_i u_{i+1}) = 2i-3,\ 4 \leq i \leq n-1,$

$\rho(v_1 v_2) = 1,\quad \rho(v_i v_{i+1}) = 2i+1,\ 2 \leq i \leq n-1,$

$\rho(u_1 v_2) = 1,\quad \rho(u_2 v_3) = 5, \rho(u_i v_{i+1}) = 2i-1,\ 3 \leq i \leq n-1,$

induced edge weight function $\psi: E(SL_n) \rightarrow \{3,5,7,\dots,6n-5\}$ is given by

$\psi(u_i u_{i+1}) = 6i-3,\ 1 \leq i \leq n-1,$

$\psi(v_i v_{i+1}) = 6i+1,\ 1 \leq i \leq n-1,$

$\psi(u_i v_{i+1}) = 6i-1,\ 1 \leq i \leq n-1.$

therefore, the induced edge weights of SL_n generates an arithmetic progression and it differs by 2 and hence $(3,2)$– $tes(SL_n) \leq 2n-1$.

Proposition1 shows that $(3,2)$– $tes(SL_n) \geq 2n-1$, this concludes the proof. ■

Example 1: $(3,2)$ total edge irregular labeling of SL_{10} is given in Fig 1.

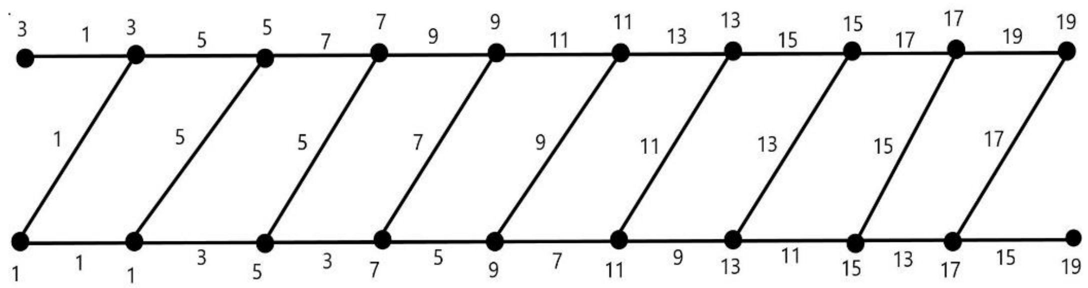

Figure 1. $(3,2)$– $tes(SL_{10}) = 19$.

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Theorem 2: If C_n is a cycle having n vertices, then $(3,2)-\ tes(C_n) = \left\lceil\frac{2n+1}{3}\right\rceil, n \geq 6$.

Proof: Let C_n be a cycle with n vertices.

Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $E(C_n) = \{e_i = v_i v_{i+1} / 1 \leq i \leq n\}$.

Define total labeling $\rho: V(C_n) \cup E(C_n) \rightarrow \left\{1, 2, \dots, \left\lceil\frac{2n+1}{3}\right\rceil\right\}$ as follows.

Case i If $n \equiv 0\ (mod\ 3)$

$\rho(v_1) = 1, \quad \rho(v_2) = 1, \quad \rho(v_n) = 3,$

$\rho(v_{3i}) = 4i - 1, \quad 1 \leq i \leq \left\lfloor\frac{n}{6}\right\rfloor,$

$\rho(v_{3i+1}) = 4i + 1, \quad 1 \leq i \leq \left\lfloor\frac{n}{6}\right\rfloor,$

$\rho(v_{3i+2}) = 4i + 1, \quad 1 \leq i \leq \left\lceil\frac{n}{6}\right\rceil - 1,$

$\rho\left(v_{n-3i)}\right) = 4i + 3, \quad 0 \leq i \leq \left\lceil\frac{n}{6}\right\rceil - 1,$

$\rho\left(v_{n-(3i+1)}\right) = 4i + 3, \quad 0 \leq i \leq \left\lceil\frac{n}{6}\right\rceil - 1,$

$\rho\left(v_{n-(3i+2)}\right) = 4i + 5, \quad 0 \leq i \leq \left\lfloor\frac{n}{6}\right\rfloor - 1,$

$\rho(e_n) = 1,$

$\rho(e_{3i+1}) = 4i + 1, \quad 0 \leq i \leq \left\lfloor\frac{n}{6}\right\rfloor,$

$\rho(e_{3i+2}) = 4i + 3, \quad 0 \leq i \leq \left\lceil\frac{n}{6}\right\rceil - 1,$

$\rho(e_{3i}) = 4i - 1, \quad 1 \leq i \leq \left\lfloor\frac{n}{6}\right\rfloor,$

$\rho\left(e_{n-(3i-2)}\right) = 4i - 1, \quad 1 \leq i \leq \left\lfloor\frac{n}{6}\right\rfloor,$

$\rho\left(e_{n-(3i-1)}\right) = 4i + 1, \quad 1 \leq i \leq \left\lceil\frac{n}{6}\right\rceil - 1,$

$\rho\left(e_{n-3i)}\right) = 4i + 1, \quad 1 \leq i \leq \left\lceil\frac{n}{6}\right\rceil - 1.$

Case ii If $n \equiv 1\ (mod\ 3)$

$\rho(v_1) = 1, \quad \rho(v_2) = 1,$

$\rho(v_{3i}) = 4i - 1, \quad 1 \leq i \leq \left\lceil\frac{n-1}{6}\right\rceil,$

$\rho(v_{3i+1}) = 4i + 1, \quad 1 \leq i \leq \left\lfloor\frac{n-1}{6}\right\rfloor,$

$\rho(v_{3i+2}) = 4i + 1, \quad 1 \leq i \leq \left\lceil\frac{n-1}{6}\right\rceil - 1,$

$\rho\left(v_{n-3i)}\right) = 4i + 3, \quad 0 \leq i \leq \left\lceil\frac{n-1}{6}\right\rceil - 1,$

$\rho\left(v_{n-(3i+1)}\right) = 4i + 3, \quad 0 \leq i \leq \left\lfloor\frac{n-1}{6}\right\rfloor - 1,$

$\rho\left(v_{n-(3i+2)}\right) = 4i + 5, \quad 0 \leq i \leq \left\lfloor\frac{n-1}{6}\right\rfloor - 1,$

$\rho(e_n) = 1,$

$\rho(e_{3i+1}) = 4i + 1, \quad 0 \leq i \leq \left\lfloor\frac{n-1}{6}\right\rfloor,$

$\rho(e_{3i+2}) = 4i + 3, \quad 0 \leq i \leq \left\lceil\frac{n-1}{6}\right\rceil - 1,$

$\rho(e_{3i}) = 4i - 1, \quad 1 \leq i \leq \left\lceil\frac{n-1}{6}\right\rceil,$

$\rho\left(e_{n-(3i-2)}\right) = 4i - 1, \quad 1 \leq i \leq \left\lceil\frac{n-1}{6}\right\rceil,$

$\rho\left(e_{n-(3i-1)}\right) = 4i + 1, \quad 1 \leq i \leq \left\lceil\frac{n-1}{6}\right\rceil,$

$\rho\left(e_{n-3i)}\right) = 4i + 1, \quad 1 \leq i \leq \left\lfloor\frac{n-1}{6}\right\rfloor.$

Case iii If $n \equiv 2\ (mod\ 3)$

$\rho(v_1) = 1, \quad \rho(v_2) = 1,$

If n is odd, $\left(v_{\left\lfloor\frac{n}{2}\right\rfloor+1}\right) = \frac{2n+2}{3}$,

If n is even, $\left(v_{\left(\frac{n}{2}\right)+1}\right) = \frac{2n+2}{3}$,

$\rho(v_{3i}) = 4i - 1, \quad 1 \leq i \leq \left\lfloor\frac{n-2}{6}\right\rfloor,$

$\rho(v_{3i+1}) = 4i + 1, \quad 1 \leq i \leq \left\lfloor\frac{n-2}{6}\right\rfloor,$

$\rho(v_{3i+2}) = 4i + 1, \quad 1 \leq i \leq \left\lfloor\frac{n-2}{6}\right\rfloor - 1,$

$\rho\left(v_{n-3i)}\right) = 4i + 3, \quad 0 \leq i \leq \left\lceil\frac{n-2}{6}\right\rceil - 1,$

$\rho\left(v_{n-(3i+1)}\right) = 4i + 3, \quad 0 \leq i \leq \left\lceil\frac{n-2}{6}\right\rceil - 1,$

$\rho\left(v_{n-(3i+2)}\right) = 4i + 5, \quad 0 \leq i \leq \left\lfloor\frac{n-2}{6}\right\rfloor - 1,$

$\rho(e_n) = 1,$

If n is odd, $\left(e_{\frac{n-1}{2}}\right) = \frac{2n-4}{3}, \rho\left(e_{\frac{n+1}{2}}\right) = \frac{2n+2}{3}$,

If n is even, $\left(e_{\frac{n}{2}}\right) = \frac{2n-4}{3}, \rho\left(e_{\frac{n}{2}+1}\right) = \frac{2n+2}{3}$,

$\rho(e_{3i+1}) = 4i + 1, \quad 0 \leq i \leq \left\lceil\frac{n-2}{6}\right\rceil - 1,$

$\rho(e_{3i+2}) = 4i + 3, \quad 0 \leq i \leq \left\lfloor\frac{n-2}{6}\right\rfloor - 1,$

$\rho(e_{3i}) = 4i - 1, \quad 1 \leq i \leq \left\lfloor\frac{n-2}{6}\right\rfloor,$

$\rho\left(e_{n-(3i-2)}\right) = 4i - 1, \quad 1 \leq i \leq \left\lceil\frac{n-2}{6}\right\rceil,$

$\rho\left(e_{n-(3i-1)}\right) = 4i + 1, \quad 1 \leq i \leq \left\lfloor\frac{n-2}{6}\right\rfloor,$

$\rho\left(e_{n-3i)}\right) = 4i + 1, \quad 1 \leq i \leq \left\lceil\frac{n-2}{6}\right\rceil - 1.$

induced edge weight function $\psi: E(C_n) \rightarrow \{3, 5, 7, \dots, 2n+1\}$ is given as

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$\psi(e_i) = 4i - 1, \quad 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil, ,$
$\psi(e_n) = 5,$
$\psi(e_{n-i}) = 4i + 5, \quad 1 \leq i \leq \left\lceil \frac{n+1}{2} \right\rceil - 2.$
therefore, the induced edge weights of C_n , generates an arithmetic progression and it differs by 2 and hence$(3,2)-$ $tes(C_n) \leq \left\lceil \frac{2n+1}{3} \right\rceil$.

Proposition 1 shows that $(3,2)-$ $tes(C_n) \geq \left\lceil \frac{2n+1}{3} \right\rceil$, this concludes the proof. ■

Example 2:$(3,2)$ total edge irregular labeling of C_{10} is given in Fig 2.

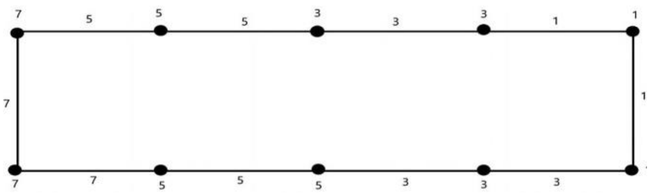

Figure 2. $\boldsymbol{(3,2)-}$ $\boldsymbol{tes(C_{10}) = 7}$**.**

Remark 1:

For $n = 3,4,5,$ $(3,2)-$ $tes(C_n) \leq \left\lceil \frac{2n+1}{3} \right\rceil$, total edge irregular labeling for C_n is shown as follows in Figs 3-5.

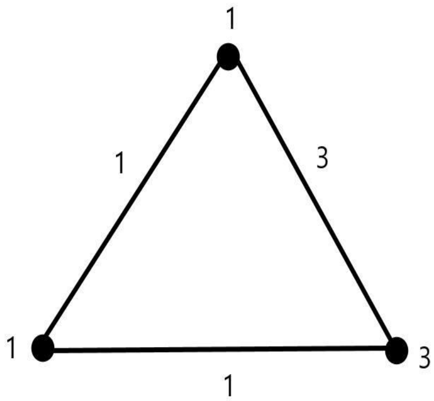

Figure 3. $\boldsymbol{(3,2)-}$ $\boldsymbol{tes(C_3) = 3}$**.**

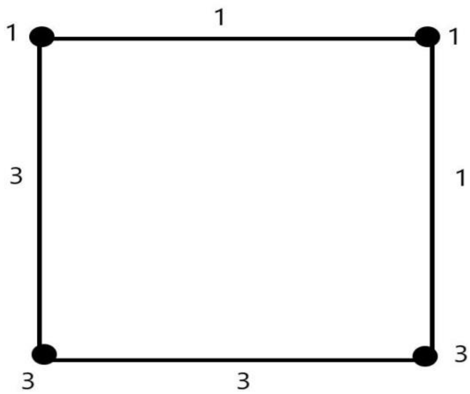

Figure 4. $\boldsymbol{(3,2)-}$ $\boldsymbol{tes(C_4) = 3}$**.**

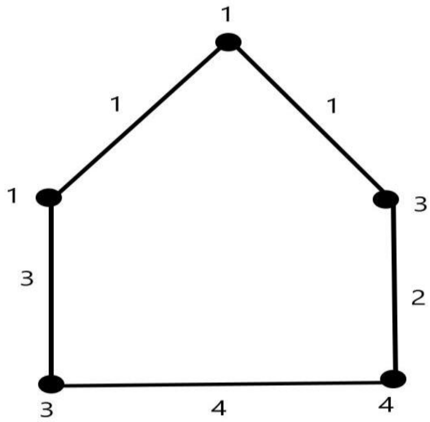

Figure 5. $\boldsymbol{(3,2)-}$ $\boldsymbol{tes(C_5) = 4}$**.**

Theorem 3: $(3,2)-$ $tes(TL_n) = \left\lceil \frac{8n}{3} \right\rceil - 2$ for $n \geq 2$.

Proof: Let the vertex set of $TL_n = V_1 \cup V_2$ where $V_1 = \{u_1, u_2, \ldots, u_n\}$, and $V_2 = \{v_1, v_2, \ldots, v_n\}$ and $E(TL_n) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_i / 1 \leq i \leq n\} \cup \{u_i v_{i+1} / 1 \leq i \leq n-1\}$.

Define total labeling $\rho: V(TL_n) \cup E(TL_n) \to \left\{1, 2, \ldots, \left\lceil \frac{8n}{3} \right\rceil - 2\right\}$ as follows:

<u>Case i</u> If $n \equiv 0 \ (mod\ 3)$

$\rho(u_{3i-2}) = 8i - 7, \quad 1 \leq i \leq \frac{n}{3},$
$\rho(u_{3i-1}) = 8i - 3, \quad 1 \leq i \leq \frac{n}{3},$
$\rho(u_{3i}) = 8i - 1, \quad 1 \leq i \leq \frac{n}{3},$
$\rho(v_{3i-2}) = 8i - 7, \quad 1 \leq i \leq \frac{n}{3},$
$\rho(v_{3i-1}) = 8i - 5, \quad 1 \leq i \leq \frac{n}{3},$
$\rho(v_{3i}) = 8i - 3, \quad 1 \leq i \leq \frac{n}{3},$

$\rho(u_{3i-2}u_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n}{3},$

$\rho(u_{3i-1}u_{3i}) = 8i-3, \quad 1 \le i \le \frac{n}{3},$

$\rho(u_{3i}u_{3i+1}) = 8i+1, \quad 1 \le i \le \frac{n-3}{3},$

$\rho(v_{3i-2}v_{3i-1}) = 8i-7, \quad 1 \le i \le \frac{n}{3},$

$\rho(v_{3i-1}v_{3i}) = 8i-3, \quad 1 \le i \le \frac{n}{3},$

$\rho(v_{3i}v_{3i+1}) = 8i-1, \quad 1 \le i \le \frac{n-3}{3},$

$\rho(u_{3i-2}v_{3i-2}) = 8i-7, \quad 1 \le i \le \frac{n}{3},$

$\rho(u_{3i-1}v_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n}{3},$

$\rho(u_{3i}v_{3i}) = 8i-1, \quad 1 \le i \le \frac{n}{3},$

$\rho(u_{3i-2}v_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n}{3},$

$\rho(u_{3i-1}v_{3i}) = 8i-3, \quad 1 \le i \le \frac{n}{3},$

$\rho(u_{3i}v_{3i+1}) = 8i-1, \quad 1 \le i \le \frac{n-3}{3}.$

<u>Case ii</u> If $n \equiv 1\ (mod\ 3)$

$\rho(u_{3i-2}) = 8i-7, \quad 1 \le i \le \frac{n+2}{3},$

$\rho(u_{3i-1}) = 8i-3, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i}) = 8i-1, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(v_{3i-2}) = 8i-7, \quad 1 \le i \le \frac{n+2}{3},$

$\rho(v_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(v_{3i}) = 8i-3, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i-2}u_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i-1}u_{3i}) = 8i-3, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i}u_{3i+1}) = 8i+1, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(v_{3i-2}v_{3i-1}) = 8i-7, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(v_{3i-1}v_{3i}) = 8i-3, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(v_{3i}v_{3i+1}) = 8i-1, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i-2}v_{3i-2}) = 8i-7, \quad 1 \le i \le \frac{n+2}{3},$

$\rho(u_{3i-1}v_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i}v_{3i}) = 8i-1, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i-2}v_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i-1}v_{3i}) = 8i-3, \quad 1 \le i \le \frac{n-1}{3},$

$\rho(u_{3i}v_{3i+1}) = 8i-1, \quad 1 \le i \le \frac{n-1}{3}.$

<u>Case iii</u> If $n \equiv 2\ (mod\ 3)$

$\rho(u_n) = \frac{8(n-2)}{3} + 4, \rho(v_n) = \frac{8(n-2)}{3} + 3,$

$\rho(u_n v_n) = \frac{8(n-2)}{3} + 4, \ \rho(u_{n-1}u_n) = \frac{8(n-2)}{3} + 3$

$\rho(u_{3i-2}) = 8i-7, \quad 1 \le i \le \frac{n+1}{3},$

$\rho(u_{3i-1}) = 8i-3, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(u_{3i}) = 8i-1, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(v_{3i-2}) = 8i-7, \quad 1 \le i \le \frac{n+1}{3},$

$\rho(v_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(v_{3i}) = 8i-3, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(u_{3i-2}u_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(u_{3i-1}u_{3i}) = 8i-3, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(u_{3i}u_{3i+1}) = 8i+1, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(v_{3i-2}v_{3i-1}) = 8i-7, \quad 1 \le i \le \frac{n+1}{3},$

$\rho(v_{3i-1}v_{3i}) = 8i-3, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(v_{3i}v_{3i+1}) = 8i-1, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(u_{3i-2}v_{3i-2}) = 8i-7, \quad 1 \le i \le \frac{n+1}{3},$

$\rho(u_{3i-1}v_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(u_{3i}v_{3i}) = 8i-1, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(u_{3i-2}v_{3i-1}) = 8i-5, \quad 1 \le i \le \frac{n+1}{3},$

$\rho(u_{3i-1}v_{3i}) = 8i-3, \quad 1 \le i \le \frac{n-2}{3},$

$\rho(u_{3i}v_{3i+1}) = 8i-1, \quad 1 \le i \le \frac{n-2}{3},$

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induced edge weight function $\psi: E(TL_n) \to \{3, 5, 7, \ldots, 8n-5\}$ is given by
$\psi(u_i u_{i+1}) = 8i + 1, \quad 1 \le i \le n-1,$
$\psi(v_i v_{i+1}) = 8i - 3, \quad 1 \le i \le n-1,$
$\psi(u_i v_i) = 8i - 5, \quad 1 \le i \le n,$
$\psi(u_i v_{i+1}) = 8i - 1, \quad 1 \le i \le n-1.$
therefore, the induced edge weights of TL_n generates an arithmetic progression and it differs by 2 and hence $(3,2)-\ tes(TL_n) \le \left\lceil \frac{8n}{3} \right\rceil - 2.$

Proposition 1 shows that $(3,2)-\ tes(TL_n) \ge \left\lceil \frac{8n}{3} \right\rceil - 2$, this concludes the proof. ■

Example 3: $(3,2)$ total edge irregular labeling of TL_{10} is given in Fig 6.

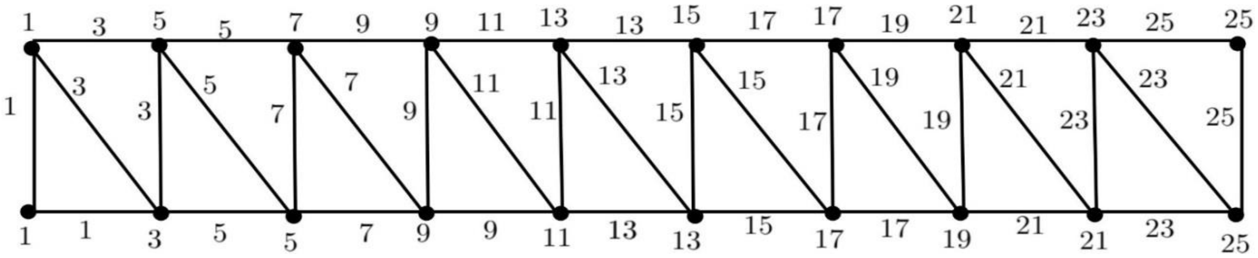

Figure 6. $(3,2)-\ tes(TL_{10}) = 25$.

Theorem 4: If $K_{m,n}$ is a complete bipartite graph on $(m+n)$ vertices, then

$$(3,2)-\ tes(K_{m,n}) = \begin{cases} n(m-1)+1, & 3 \le m \le n \\ \left\lfloor \frac{4n}{3} \right\rfloor + 1, & m = 2, \text{for any } n \ge 2. \end{cases}$$

Proof: Let $K_{m,n}$ be a complete bipartite graph on $(m+n)$ vertices with the vertex set, $V(K_{m,n}) = V_1 \cup V_2$ where $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$ and $E(K_{m,n}) = \{u_i v_j / 1 \le i \le m, 1 \le j \le n\}$.

Case i Suppose $3 \le m \le n$.

Define the total labeling $\rho: V(K_{m,n}) \cup E(K_{m,n}) \to \{1, 2, \ldots, n(m-1)+1\}$ as
$\rho(u_1) = 1,$
$\rho(u_i) = ni - n + 1, \quad 2 \le i \le m,$
$\rho(v_j) = 2j - 1, \quad 1 \le j \le n,$
$\rho(u_1 v_j) = 1, \quad 1 \le j \le n,$
For $2 \le i \le m,$
$\rho(u_i v_j) = ni - n + 1, \quad 1 \le j \le n.$
induced edge weight function $\psi: E(K_{m,n}) \to \{3, 5, 7, \ldots, 2mn+1\}$ is given as
$\psi(u_i v_j) = 2(i-1)n + 2j + 1, \ 1 \le i \le m, \ 1 \le j \le n.$

From the above labeling, it is easy to verify that $(3,2)-\ tes(K_{m,n}) \le n(m-1)+1$. Further, it is not possible to obtain $(3,2)-$ total edge irregular labeling by assigning labels fewer than $mn - n + 1$, hence $(3,2)-\ tes(K_{m,n}) = n(m-1)+1$.

Case ii Suppose $m = 2$, for any $n > 2$

Then $V(K_{2,n}) = V_1 \cup V_2$ where $V_1 = \{u_1, u_2\}$, $V_2 = \{v_1, v_2, \ldots, v_n\}$ and
$E(K_{2,n}) = \{u_i v_j / 1 \le i \le 2, 1 \le j \le n\}$.
Define the total labeling $\rho: V(K_{2,n}) \cup E(K_{2,n}) \to \left\{1, 2, \ldots, \left\lfloor \frac{4n}{3} \right\rfloor + 1\right\}$ as follows:

Subcase i If $n \equiv 1 \ (mod\ 3)$
$\rho(u_1) = 1, \quad \rho(u_2) = \frac{4n+2}{3},$

$$\rho(v_j) = \begin{cases} 2j - 1, & 1 \le j \le \frac{n+5}{3} \\ \frac{2n+10}{3} + \left(j - \left(\frac{n+8}{3}\right)\right), & \frac{n+8}{3} \le j \le n \end{cases}$$

$$\rho(u_1 v_j) = \begin{cases} 1, & 1 \le j \le \frac{n+5}{3} \\ 2 + \left(j - \left(\frac{n+8}{3}\right)\right), & \frac{n+8}{3} \le j \le n \end{cases}$$

$$\rho(u_2 v_j) = \begin{cases} \frac{2n+4}{3}, & 1 \le j \le \frac{n+5}{3} \\ \frac{2n+7}{3} + \left(j - \left(\frac{n+8}{3}\right)\right), & \frac{n+8}{3} \le j \le n \end{cases}$$

Subcase ii If $n \equiv 2 \ (mod\ 3)$
$\rho(u_1) = 1, \quad \rho(u_2) = \frac{4n+1}{3},$

$$\rho(v_j) = \begin{cases} 2j-1, & 1 \le j \le \frac{n+4}{3} \\ \frac{2n+8}{3} + \left(j - \left(\frac{n+7}{3}\right)\right), & \frac{n+7}{3} \le j \le n \end{cases}$$

$$\rho(u_1 v_j) = \begin{cases} 1, & 1 \le j \le \frac{n+4}{3} \\ 2 + \left(j - \left(\frac{n+7}{3}\right)\right), & \frac{n+7}{3} \le j \le n \end{cases}$$

$$\rho(u_2 v_j) = \begin{cases} \frac{2n+5}{3}, & 1 \le j \le \frac{n+4}{3} \\ \frac{2n+8}{3} + \left(j - \left(\frac{n+7}{3}\right)\right), & \frac{n+7}{3} \le j \le n \end{cases}$$

Subcase iii If $n \equiv 0 \ (mod\ 3)$

$\rho(u_1) = 1, \quad \rho(u_2) = \frac{4n}{3},$

$$\rho(v_j) = \begin{cases} 2j-1, & 1 \le j \le \frac{n+6}{3} \\ \frac{2n+12}{3} + \left(j - \left(\frac{n+9}{3}\right)\right), & \frac{n+9}{3} \le j \le n \end{cases}$$

$$\rho(u_1 v_j) = \begin{cases} 1, & 1 \le j \le \frac{n+6}{3} \\ 2 + \left(j - \left(\frac{n+9}{3}\right)\right), & \frac{n+9}{3} \le j \le n \end{cases}$$

$$\rho(u_2 v_j) = \begin{cases} \frac{2n+6}{3}, & 1 \le j \le \frac{n+6}{3} \\ \frac{2n+9}{3} + \left(j - \left(\frac{n+9}{3}\right)\right), & \frac{n+9}{3} \le j \le n. \end{cases}$$

induced edge weight function $\psi: E(K_{2,n}) \to \{3, 5, 7, \dots, 4n+1\}$ is given by

$\psi(u_1 v_j) = 2j + 1, \quad 1 \le j \le n,$

$\psi(u_2 v_j) = 2n + 2j + 1, \quad 1 \le j \le n.$

therefore, the induced edge weights of $K_{2,n}(n \ge 3)$ generates an arithmetic progression and it differs by 2 and hence $(3,2)-\ tes(K_{2,n}) \le \left\lceil \frac{4n}{3} \right\rceil + 1$.

Proposition 1 shows that $(3,2)-\ tes(K_{2,n}) \ge \left\lceil \frac{4n}{3} \right\rceil + 1$, this concludes the proof. ■

Remark 2:

If $m = n = 1$, then $K_{1,1} \cong P_2$ and hence it follows from Proposition 2.

If $m = 1 < n$, then $K_{1,n}$ and by proposition 3, the result follows.

■

In the case of the complete bipartite graph $K_{m,n}$, the upper bound of $(3,2)$–total edge irregular strength given in Proposition 1 is reformed as follows.

Corollary 1:

If $K_{m,n}$ is a complete bipartite graph on $(m+n)$ vertices, then

$\left\lceil \frac{a+(q-1)d}{3} \right\rceil \le (3,2)-\ tes(K_{m,n}) \le n(m-1)+1.$

■

Example 4: $(3,2)$ total edge irregular labeling of $K_{4,6}$, $K_{2,6}$, $K_{5,6}$ and $K_{3,5}$ are given in Figs7, 8, 9, and 10.

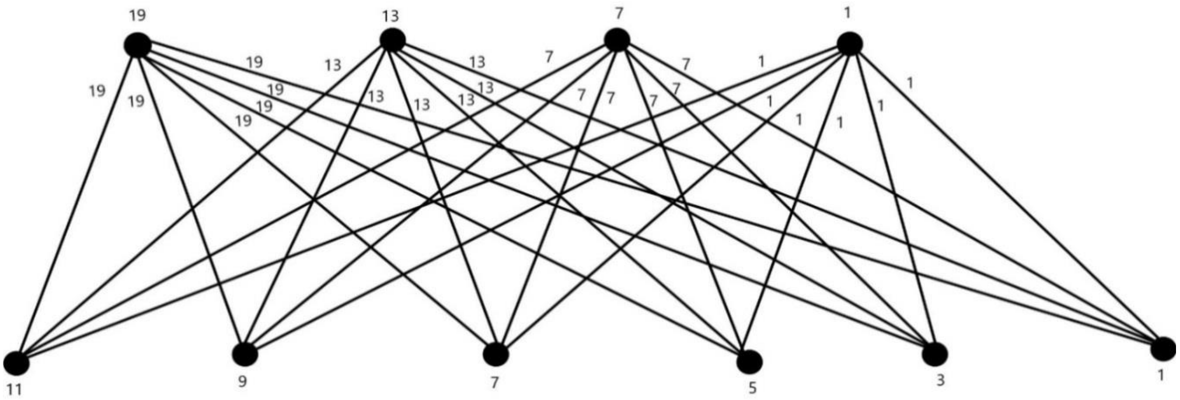

Figure 7. $\mathbf{(3,2)-\ tes(K_{4,6}) = 19}$**.**

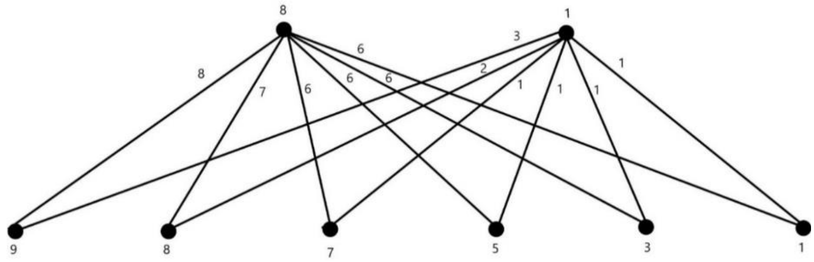

Figure 8. $(3,2)-\ tes(K_{2,6}) = 9$.

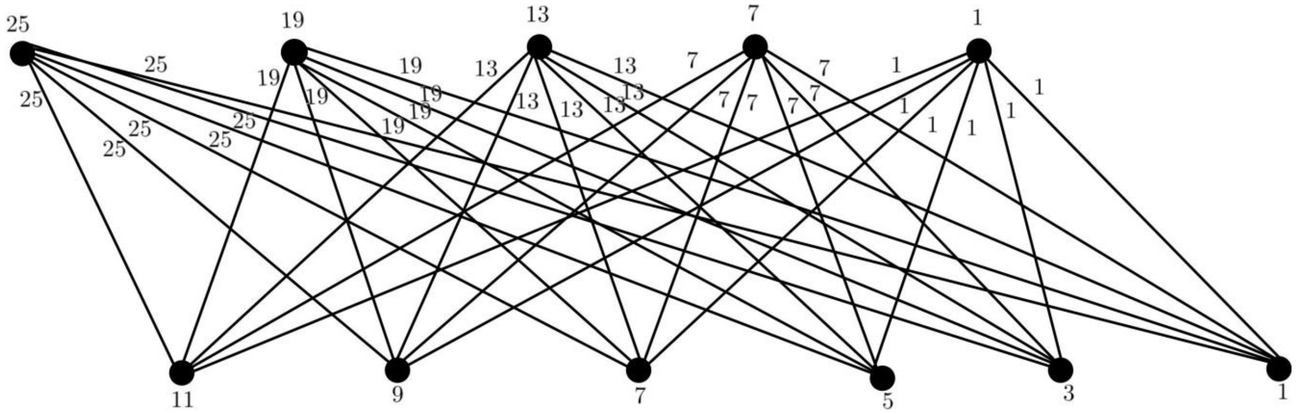

Figure 9. $(3,2)-\ tes(K_{5,6}) = 25$.

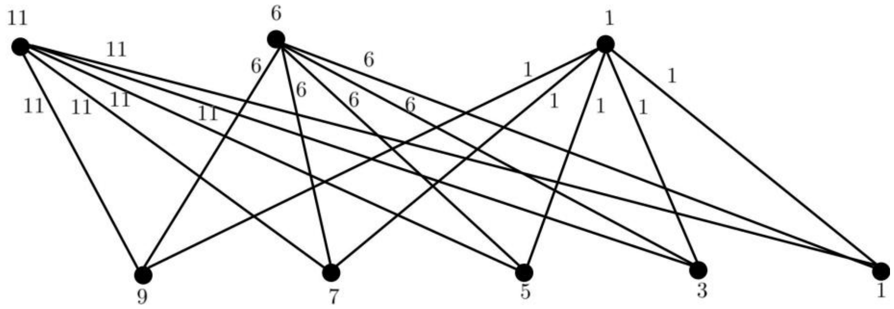

Figure 10. $(3,2)-\ tes(K_{3,5}) = 11$.

Conclusion

This article investigates $(3,2)$-total edge irregularity strength of standard graphs namely: $(3,2)-\ tes(SL_n) = 2n-1$ for any $n \geq 2$ where SL_n is a slanting ladder with $2n$ vertices, $(3,2)-\ tes(C_n) = \left\lceil\frac{2n+1}{3}\right\rceil$, $n \geq 6$ where C_n is a cycle having n vertices, $(3,2)-\ tes(TL_n) = \left\lceil\frac{8n}{3}\right\rceil - 2$ for $n \geq 2$ where TL_n is a triangular ladder. In addition, an open problem for complete bipartite graph $(3,2)-\ tes(K_{m,n}) = \begin{cases} n(m-1)+1, & 3 \leq m \leq n \\ \left\lfloor\frac{4n}{3}\right\rfloor + 1, & m = 2, \text{for any } n \geq 2 \end{cases}$ is solved affirmatively. To investigate similar results for other families of graphs is an open area of research.

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Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Government Arts and Science College, Srivilliputtur – 626 125, Tamil Nadu, India.

Authors' Contribution Statement

This work was carried out in collaboration with all authors. K. M., conceptualization. P. P., investigation. R G and I. M., methodology. K. M., super- vision. R. G. and I. M, validation. P. P., writing-original draft. K. M., writing-review and editing. All the authors read and approved the final manuscript.

References

1. Chartrand G, Jacobson MS, Lehel J, Oellermann OR, Ruiz S, Saba F. Irregular Networks. Congr Numer. 1988; 64: 187-192. https://www.scopus.com/record/display.uri?eid=2-s2.0-0039081400&origin=inward&txGid=bd47f5f74f59d6aa8d8532bd5eb8904b
2. Baca M, Jendrol S, Miller M, Ryan J. On Irregular Total Labelings. Discrete Math. 2007; 307: 1378-1388. DOI: 10.1016/j.disc.2005.11.075
3. Salama F. On Total Edge Irregularity Strength of Polar Grid Graph. J Taibah Univ Sci. 2019; 13(1): 912-916. https://doi.org/10.1080/16583655.2019.1660086
4. Yeni Susanti, Yulia Indah Puspitasari, Husnul Khotimah. On Total Edge Irregularity Strength of Staircase Graphs and Related Graphs. Iran J Math Sci Inform. 2020; 15 (1): 1-13. 10.21859/IJMSI.15.1.1
5. Ratnasari L, Susanti Y. Total Edge Irregularity Strength of Ladder-Related Graphs. Asian-Eur J Math. 2020; 13(04): 2020072. https://doi.org/10.1142/S1793557120500722
6. Muthu Guru Packiam K, Padmapriya R. (a, d) –Total Edge Irregularity Strength of Graphs. J Math Comput Sci. 2021; 11(4): 4436-4453. https://doi.org/10.28919/jmcs/5886
7. Adirasari RP, Suprajitno H, Susilowati L. The Dominant Metric Dimension of Corona Product Graphs. Baghdad Sci J. 2021 Jun. 1; 18(2): 0349. https://doi.org/10.21123/bsj.2021.18.2.0349
8. Omran AA, Oda HH. Hn-Domination in Graphs. Baghdad Sci J. 2019 Mar.17 ;16(1(Suppl.): 0242. https://doi.org/10.21123/bsj.2019.16.1(Suppl.).0242
9. Amreen J, Naduvath S. Order Sum Graph of a Group. Baghdad Sci J. 2023 Feb. 1; 20(1): 0181. https://doi.org/10.21123/bsj.2022.6480
10. Tarawneh I, Hasni R, Ahmad A. On the Edge Irregularity Strength of Grid Graphs. AKCE Int J Graphs Comb. 2020; 17(1): 414-418. https://doi.org/10.1016/j.akcej.2018.06.011
11. Gallian JA. A Dynamic Survey of Graph Labeling. Electron J Comb. 2018; 1(DynamicSurveys). https://experts.umn.edu/en/publications/a-dynamic-survey-of-graph-labeling-3

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المزيد من النتائج حول (أ ، د) - إجمالي قوة عدم انتظام الحواف للرسوم البيانية

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الخلاصة

ليكن $G = (V, E)$ رسمًا بيانيًا بسيطًا على رؤوس l وحواف m مع إجمالي - h وضع العلامات $\rho: V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,h\}$. فان ρ تسمى (ا,د)- وسم غير منتظم للحافة الإجمالية إذا وجد تطابق متقابل وليكن $\psi: E(G) \rightarrow \{a, a+d, a+2d, \ldots, a+(m-1)d\}$ معرفة بواسطة $\psi(uv) = \rho(u) + \rho(v) + \rho(uv)$ لكل $uv \in E(G)$, حيث $a \geq 3, d \geq 2$. كذلك قيمة $\psi(uv)$ يقال لها وزن الحافة uv. يشار الى (ا,د)-اجمالي قوة عدم انتظام الحواف للرسم البياني G ب $(a,d) - test(G)$ وهي اقل h التي يقبلها G للحافة -(ا,د) الغير منتظمة للعلامة-h . في هذه المقالة تم فحص, $(a,d) - test(G)$ لبعض عائلات الرسم البياني الشائعة. بالاضافة الى ذلك تم حل المسالة المفتوحة $(3,2) - test(K\_(m,n))$,$m, n > 2$ بشكل ايجابي. م تسمى (ρأ ، د) - وسم غير منتظم للحافة الإجمالية إذا كان هناك تطابق واحد لواحد ، قل ψ: E (G) → {a ،a + d ،a + 2d ،(m- 1) + a + ... د} محدد بواسطة ψ (uv) = ρ (u) + ρ (v) + ρ (uv) لجميع uv∈E (G) ، حيث a≥3 ، d≥2. أيضًا ، يُقال إن القيمة ψ (uv) هي وزن حافة الأشعة فوق البنفسجية. يشار إلى قوة عدم انتظام الحافة الإجمالية (أ ، د) للرسم البياني G بواسطة a)، (G) tes- (d وهي أقل h التي يقبلها) Gأ ، د) - علامة h غير منتظمة للحافة. في هذه المقالة ، يتم فحص (أ ، د) -(G) tes لبعض عائلات الرسم البياني الشائعة. بالإضافة إلى ذلك ، يتم حل المشكلة المفتوحة (3،2) - tes (K_ (m ،n)) ،m، n> 2 بشكل إيجابي.

الكلمات المفتاحية: (أ ، د) - وضع العلامات غير المنتظمة، وضع العلامات غير المنتظمة للحافة ، وضع العلامات غير المنتظم ، قوة المخالفة ، وضع العلامات غير المنتظمة للحافة الكلية.