Open Access
2020, 17(2):545-549

Baghdad Science Journal

P-ISSN: 2078-8665
E-ISSN: 2411-7986

DOI: http://dx.doi.org/10.21123/bsj.2020.17.2.0545

Some Properties of Finite Dimensional Fuzzy Anti-Normed Linear Spaces

Esraa Alaa Hussein [*] *Radhi I. M. Ali*

Received 16/9/2019, Accepted 8/12/2019, Published 1/6/2020

Abstract:

In this paper the definition of fuzzy anti-normed linear spaces and its basic properties are used to prove some properties of a finite dimensional fuzzy anti-normed linear space.

Key words: Fuzzy anti-norm, Finite dimensional fuzzy anti-norm, t-co-norm.

Introduction:

Fuzzy set theory was introduced in 1965(1), later on, many scientists have applied the concept of the theory in many fields of mathematics, in which data is inaccurate or quasi-occult such as, computer programming, medicine,… etc.

Defined a fuzzy norm on the linear space in a way that induced fuzzy metric space(2). In 2015(3) introduced some results of fundamental theorems for fuzzy normed spaces.

Fuzzy anti-norm defined on a linear space has been introduced (4) which reverses somehow to the concept (5) of fuzzy norm on a linear space. Mursaleen M and *et al* in 2016 (6) introduced generalized statistically convergent sequences of fuzzy numbers .In 2017(7) introduced some aspects on 2- fuzzy 2- anti Normed Linear space. Moreover, In (8) introduced some properties of Fuzzy Norm of fuzzy Bounded Operators. Recently, in 2018(9) introduced Generalized weighted statistical convergence in intuitionistic fuzzy normed linear spaces.

In this paper, we introduce the definition of fuzzy anti-norm on a linear space and study the finite dimensional fuzzy anti-normed linear space and the some properties of them.

Preliminaries

To make this paper as self-contained as possible, we give some definitions and preliminary in this section.

Definition 1(10)

A binary operation $\circ : I \times I \to I$, where $I = [0,1]$ is said to be continuous t-co-norm if for a, b, c, d in I then $\circ$ satisfies the following conditions :

(i) $\circ$ is associative and commutative ,
(ii) $a \circ 0 = a$, i.e. $\underline{0}$ is an identity element of the binary operation $\circ$,
(iii) if $a \le c$ and $b \le d$ then $a \circ b \le c \circ d$,
(iv) $\circ$ is continuous on I x I

Some examples of continuous t-co-norm are: for all $a, b, \in I$
(i) $a \circ b = a + b - ab$,
(ii) $a \circ b = \max\{a, b\}$,
(iii) $a \circ b = \min\{a + b, 1\}$.

Remark 2(10)
(a) For any $a, b \in (0,1)$ with $a > b$, there exist $c \in (0,1)$ such that $a > c \circ b$.
(b) For any $a \in (0,1)$, there exist $b \in (0,1)$ such that $b \circ b \le a$.

Definition 3(4)
Let V be a linear space over a field $F(R\ or\ C)$. A fuzzy subset N° of X x R such that
$\forall\ x, v$ in V, and c in F :
(N°1) $\forall\, t \in R$ with $t \le 0$, $N°(x, t) = 1$;
(N°2) $\forall\, t \in R$ with $t > 0$, $N°(x, t) = 0$ if and only if $x = \underline{0}$;
(N°3) $\forall\ t \in R$ with $t > 0$, $N°(cx, t) = N°(x, \frac{t}{|c|})$ if $0 \ne c \in F$;
(N°4) $\forall\, s, t \in R$,$N°(x + v, s + t) \le \max\{N°(x, s), N°(v, t)\}$;
(N°5) $N°(x, t)$ is a decreasing function of $t \in R$ and $\mathrm{Lim}_{t\to\infty} N°(x, t) = 0$.
Then we called N° be a fuzzy anti-norm on a linear space V and the pair $(V, N°)$ is called a fuzzy anti-normed linear space (in short Fa-NLS).
The following condition of fuzzy norm N° will be required later on.

Mathematics Department, College of Science for Women, University of Baghdad, Baghdad, Iraq.
[*]Corresponding author: israa.alaa1103a@csw.uobaghdad.edu.iq
[*]ORCID ID: https://orcid.org/0000-0002-1479-6578

(N°6) $\forall$ t $\in R$ with $t > 0$, $N°(x, t) < 1$ implies $x = \underline{0}$.

Example 4(4)
Suppose $(V, \|.\|)$ be a normed linear space. Define

$$N°(x,t) = \begin{cases} \frac{\|x\|}{t+\|x\|} & \text{if } t > 0,\ t \in R, x \in V \\ 1 & \text{if } t \le 0 \end{cases}$$

Then $(V, N°)$ is a Fa-NLS.

Lemma 5(4)
If $(V, N°)$ be a Fa-NLS. Then $N° (x - y, t) = N° (y - x, t)$ $\forall x, y \in V$ and $t \in (0, \infty)$.

Definition 6(4)
Let N° be a fuzzy anti-norm V satisfying (N°6). Define
$\|x\|_\alpha^* = \inf \{ t > 0 : N°(x, t) < \alpha, \alpha \in (0,1]\}$

Lemma 7(4)
If $(V, N°)$ be a Fa-NLS. $\forall \alpha \in (0,1]$ and $x \in V$. Then we have
(i) $\|x\|_{\alpha_1}^* \ge \|x\|_{\alpha_2}^*$ for $0 < \alpha_1 < \alpha_2 \le 1$,
(ii) $\|cx\|_\alpha^* = |c|\|x\|_\alpha^*$, $\forall c$.
(iii) $\|x + y\|_\alpha^* \le \|x\|_\alpha^* + \|y\|_\alpha^*$.

Theorem 8(4)
If $(V, N°)$ be a Fa-NLS. Then $\{\|x\|_\alpha^* : \alpha \in (0,1]\}$ is a decreasing family of norms on V.

Theorem 9(4)
Let V be a linear space and let $\{ \|x\|_\alpha^* : \alpha \in (0,1]\}$ be a decreasing family of norms on V. Now define a function $N_1^\circ : V \text{ x } R \to I$ as

$$N_1^\circ(x,t) = \begin{cases} \inf\{\alpha \in (0,1] : \|x\|_\alpha^* \le t\} & \text{if } (x,t) \ne 0 \\ 1 & \text{if } (x,t) = 0 \end{cases}$$

(i) N_1° is a fuzzy anti-norm on V .
(ii) $\forall x \in V$, $\exists r = r(x) > 0$ such that $N_1^\circ(x,t) = 1$.

Definition 10 (11)
Let $(V, N°)$ be a Fa-NLS. A sequence $\{x_n\}$ in V is said to be convergent to $x \in V$ if given $t > 0$, $r \in (0,1)$ there exists an integer $n_0 \in N$ such that $N° (x_n - x, t) < r$, for all $n \ge n_0$.

Theorem 11(4)
Let $(V, N°)$ be a Fa-NLS. A sequence $\{x_n\}$ converges to $x \in V$ iff $\lim_{n\to\infty} N°(x_n - x, t) = 0, \forall t > 0$.

Definition 12(11)
Let $(V, N°)$ be a Fa-NLS. A sequence $\{x_n\}$ in V is said to be a Cauchy sequence if given $t > 0$ $r \in (0,1)$, $\exists$ an integer $n_0 \in N$ such that $N°(x_{n+p} - x_n, t) < r, \forall n \ge n_0, p = 1,2, \ldots$.

Theorem 13(4)
Let $(V, N°)$ be a Fa-NLS, $\{x_n\}$ is a Cauchy sequence in V iff
$\lim_{n\to\infty} N°(x_{n+p} - x_n, t) = 0$, p = 1,2,3, ... and $t > 0$.

Theorem 14(4)
If a sequence $\{x_n\}$ in a Fa-NLS $(V, N°)$ is convergent, its limit is unique.

Theorem 15(4)
Let $(V, N°)$ be a Fa-NLS every subsequence of a convergent sequence converges to the limit of sequence.

Theorem 16(4)
If $(V, N°)$ be a Fa-NLS then every convergent sequence is a Cauchy sequence.

Definition 17(4)
Let $(V, N°)$ be a Fa-NLS. A subset U of V is said to be closed set if for any sequence $\{x_n\}$ in U converges to x in U, i.e. $\lim_{n\to\infty} N°(x_n - x, t) = 0$, $\forall t > 0$ implies that $x \in U$.

Definition 18(4)
Let $(V, N°)$ be a Fa-NLS. A subset A of V is said to be the closure of U if $\forall x \in A$, $\exists$ a sequence $\{x_n\}$ in U such that $lim_{n\to\infty} N° (x_n - x, t) = 0$ $\forall t \in R^+$,we denote the set A by $\overline{U}$.

Definition 19(4)
A subset A of a Fa-NLS is said to be bounded iff there exist $t > 0$ and $0 < r < 1$ such that
$N° (x, t) < r, \forall x \in A$.

Definition 20(4)
If $(V, N°)$ be a Fa-NLS. Then a subset F of a Fa-NLS is compact if any sequence $\{x_n\}$ in F has a subsequence converging to an element of F.

Theorem 21 (4)
If $(V, N°)$ be a Fa-NLS. Then Every Cauchy sequence F in $(V, N°)$ is bounded.

Finite dimensional fuzzy anti-normed linear space

In this section we prove the following lemma which plays the key role in studying the properties of finite dimensional fuzzy anti-normed linear spaces. Then, some properties of finite dimensional fuzzy anti- normed linear spaces will be proved.

Lemma 22
Let $(V, N°)$ be a fuzzy anti-normed linear space with the underlying t-co-norm $\circ$ continuous at (0 , 0) and $\{x_1, x_2, \ldots\ldots., x_n\}$ be a linearly independent set of vectors in V. Then $\exists$ c > 0 and $\delta \in (0,1)$ such that for any set of scalars $\{\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_n\}$;
$N° (\alpha_1 x_1 + \alpha_2 x_2 + \ldots\ldots. + \alpha_n x_n, c\sum_{j=1}^{n} |\alpha_j|) > \delta$ … (1)

Proof.

Let $s = |\alpha_1| + |\alpha_2| + ... + |\alpha_n|$. If $s = 0$ then $\alpha_j = 0\ \forall j = 1,2,\dots,n$ and the relation (1) holds for any $c > 0$ and $\delta \in (0,1)$.

Next we suppose that $s > 0$. Then (1) is equivalent to

N° $(\beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n,\ ) > \delta$ … (2)

for some $c > 0$ and $\delta \in (0,1)$, and for all scalars β's with $\sum_{j=1}^{n}|\beta_j| = 1$.

If possibly suppose that (2) does not hold, thus for each $c > 0$ and $\delta \in (0,1)$, ∃ a set of scalars $\{\beta_1, \beta_2, ..., \beta_n\}$ with $\sum_{j=1}^{n}|\beta_j| = 1$ for which

N° $(\beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n, c) \leq \delta$.

Then for $c = \delta = \frac{1}{m}$, $m = 1,2,\dots$, ∃ a set of scalars $\{\beta_1^{(m)}, \beta_2^{(m)}, ..., \beta_n^{(m)}\}$ with $\sum_{j=1}^{n}\left|\beta_j^{(m)}\right| = 1$ such that N° $(y_m\ ,\ \frac{1}{m}) \leq \frac{1}{m}$ where

$y_m = \beta_1^{(m)} x_1 + \beta_2^{(m)} x_2 + ... + \beta_n^{(m)} x_n$.

Since $\sum_{j=1}^{n}\left|\beta_j^{(m)}\right| = 1$, we have $0 \leq |\beta_j^{(m)}| \leq 1$ for $j = 1,2,\dots,n$. So for each fixed j the sequence $\{\beta_j^{(m)}\}$ is bounded and hence $\{\beta_1^{(m)}\}$ has a convergent subsequence .Say that β_1 denote the limit of that subsequence and $\{y_{1,m}\}$ will represent the subsequence corresponding to $\{y_m\}$. Using a similar argument $\{y_{1,m}\}$ has a subsequence $\{y_{2,m}\}$ for which the corresponding subsequence of scalars $\{\beta_2^{(m)}\}$ converges to β_2 (say). Continuing in this way, after n steps we obtain a subsequence $\{y_{n,m}\}$ where

$y_{n,m} = \sum_{j=1}^{n}\gamma_j^{(m)} x_j$ with $\sum_{j=1}^{n}\left|\gamma_j^{(m)}\right| = 1$ and $\gamma_j^{(m)} \to \beta_j$ as $m \to \infty$.

Let $y = \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n$. Thus we have

$\lim_{m\to\infty}$ N° $(y_{n,m} - y, t) = 0 \quad \forall t > 0$ … (3)

Now for $k > 0$, choose m such that $\frac{1}{m} < k$. We have

N° $(y_{n,m}, k)$ = N° $(y_{n,m} + \underline{0}, \frac{1}{m} + k - \frac{1}{m})$

$\leq$ N° $(y_{n,m}, \frac{1}{m})$ ∘ N° $(\underline{0}, k - \frac{1}{m})$

$\leq \frac{1}{m}$ ∘ N° $(\underline{0}, k - \frac{1}{m})$.

i.e. N° $(y_{n,m}, k) \leq \frac{1}{m}$ ∘ N° $(\underline{0}, k - \frac{1}{m})$.

i.e. $\lim\limits_{m\to\infty}$ N° $(y_{n,m}, k) \leq \frac{1}{m}$.

i.e.

$\lim\limits_{m\to\infty}$ N° $(y_{n,m}, k) = 0$. … (4)

Now

N°$(y, 2k)$ = N° $(y - y_{n,m} + y_{n,m}\ , k + k) \leq$ N° $(y - y_{n,m}, k)$ ∘ N° $(y_{n,m}, k)$

⇒ N° $(y, 2k) \leq \lim\limits_{m\to\infty}$ N° $(y - y_{n,m}, k)$ ∘ $\lim\limits_{m\to\infty}$ N° $(y_{n,m}, k)$

(by the continuity of t-co-norm ∘ at (0, 0)).

⇒ N° $(y, 2k) \leq 0 \circ 0$ by (3) & (4)

⇒ N° $(y, 2k) = 0 \circ 0 = 0$.

Since $k > 0$ is arbitrary, by (N°2) it follows that $y = \underline{0}$. Again since $\sum_{j=1}^{n}\left|\beta_j^{(m)}\right| = 1$ and $\{x_1, x_2, ..., x_n\}$ are linearly independent set of vectors, so

$y = \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n \neq \underline{0}$.

Thus we arrive at a contradiction and the lemma is proved.

Theorem 23

Every finite dimensional fuzzy anti- normed linear space $(V, \text{N°})$ with the continuity of the underlying t-co-norm ∘ at (0,0) is complete.

Proof.

Let $(V, \text{N°})$ be fuzzy anti- normed linear space and let dim $V = k$. Let $\{e_1, e_2, ..., e_k\}$ be a basis for V and $\{x_n\}$ be a Cauchy sequence in V. Let

$$x_n = \beta_1^{(n)} e_1 + \beta_2^{(n)} e_2 + ... + \beta_k^{(n)} e_k$$

where $\beta_1^{(n)}, \beta_2^{(n)}, ..., \beta_k^{(n)}$ are suitable scalars. So

$\lim\limits_{m,n\to\infty}$ N° $(x_m - x_n, t) = 0 \quad \forall t > 0$ ... (5)

Now from lemma 22, it follows that $\exists c > 0$ and $\delta \in (0,1)$ such that

N° $(\sum_{i=1}^{k}(\beta_i^{(m)} - \beta_i^{(n)})e_i\ ,\ c\sum_{i=1}^{k}|\beta_i^{(m)} - \beta_i^{(n)}|) > \delta$. … (6)

Again for $0 < \delta < 1$, from (5), it follows that ∃ a positive integer n_0,

$0 < n_0(\delta, t)$ such that

N° $\left(\sum_{i=1}^{k}(\beta_i^{(m)} - \beta_i^{(n)}\right) e_i, t) < \delta \quad \forall m, n \geq n_0(\delta, t)$ ...(7)

Now from (6) and (7), we have

N° $\left(\sum_{i=1}^{k}(\beta_i^{(m)} - \beta_i^{(n)}\right) e_i, t)$

$< \delta <$ N° $(\sum_{i=1}^{k}(\beta_i^{(m)} - \beta_i^{(n)})e_i\ ,\ c\sum_{i=1}^{k}|\beta_i^{(m)} - \beta_i^{(n)}|)$

$\forall m, n \geq n_0(\delta, t)$

$\Rightarrow c\sum_{i=1}^{k}|\beta_i^{(m)} - \beta_i^{(n)}| > t \quad \forall m, n \geq n_0(\delta, t)$

(since N° $(x, .)$ is non-increasing in t).

$\Rightarrow \sum_{i=1}^{k}|\beta_i^{(m)} - \beta_i^{(n)}| > \frac{t}{c} \quad \forall m, n \geq n_0(\delta, t)$

$\Rightarrow |\beta_i^{(m)} - \beta_i^{(n)}| > \frac{t}{c} \quad \forall m, n \geq n_0(\delta, t)$ and $i = 1,2,\dots,k$.

Since $t > 0$ is arbitrary, from above we have,
$\lim_{n,m\to\infty} |\beta_i^{(m)} - \beta_i^{(n)}| = 0$ for $i = 1,2,\ldots,k$.
$\Rightarrow \{\beta_i^{(n)}\}$ is a Cauchy sequence of scalars for each $i = 1,2,\ldots,k$.
So each sequence $\{\beta_i^{(n)}\}$ converges.
Let $\lim_{n\to\infty} \beta_i^{(n)} = \beta_i$ for $i = 1,2,\ldots,k$ and $= \sum_{i=1}^{k} \beta_i e_i$. Clearly $x \in V$.
Now $\forall\, t > 0$,
$N^\circ(x_n - x, t) = N^\circ(\sum_{i=1}^{k} \beta_i^{(n)} e_i - \sum_{i=1}^{k} \beta_i e_i, t)$
$= N^\circ(\sum_{i=1}^{k} (\beta_i^{(n)} - \beta_i) e_i, t)$
i.e.

$$N^\circ(x_n - x, t) \leq N^\circ\left(e_1, \frac{t}{k|\beta_1^{(n)} - \beta_1|}\right) \circ N^\circ\left(e_2, \frac{t}{k|\beta_2^{(n)} - \beta_2|}\right) \circ \ldots \circ N^\circ\left(e_k, \frac{t}{k|\beta_k^{(n)} - \beta_k|}\right) \quad \ldots(8)$$

When $n \to \infty$ then $\frac{t}{k|\beta_k^{(n)} - \beta_k|} \to \infty$ (since $\beta_i^{(n)} \to \beta_i$)
for $i = 1,2,\ldots,k$ and $\forall t > 0$.

From (8) we get, using the continuity of t-co-norm $\circ$ at (0 , 0),
$\lim_{n\to\infty} N^\circ(x_n - x, t) \leq 0 \circ 0 \circ \ldots \circ 0 \quad \forall\, t > 0$
$\Rightarrow \lim_{n\to\infty} N^\circ(x_n - x, t) = 0 \quad \forall\, t > 0$
$\Rightarrow x_n \to x$
$\Rightarrow V$ is complete.

Theorem 24

Let (V, N°) be a finite dimensional fuzzy anti- normed linear space in which the underlying t-co-norm $\circ$ is continuous at (0,0). Then a subset A which is said to be compact if and only if A is closed and bounded.

Proof.

First we suppose that A is compact. We have to show that A is closed and bounded. Let $x \in \overline{A}$. Then $\exists$ a sequence $\{x_n\}$ in A such that $\lim_{n\to\infty} N^\circ(x_n - x, t) = 0 \quad \forall\, t > 0$ implies $x \in A$. Since A is compact, $\exists$ a subsequence $\{x_{nk}\}$ of $\{x_n\}$ converges to a point in A. Again $\{x_n\} \to x$ so $\{x_{nk}\} \to x$ and hence $x \in A$. So A is closed.

If A is not bounded. Then $\exists\, r = r_0$, $0 < r_0 < 1$, such that for each positive integer n, $\exists\, x_n \in A$ such that $N^\circ(x_n, n) \geq r_0$. Since A is compact, $\exists$ a subsequence $\{x_{nk}\}$ of $\{x_n\}$ converging to some element $x \in A$. . Thus $\lim_{k\to\infty} N^\circ(x_{nk} - x, t) = 0 \quad \forall\, t > 0$. Also $N^\circ(x_{nk}, n_k) \geq r_0$.
Now
$r_0 \leq N^\circ(x_{nk}, n_k) = N^\circ(x_{nk} - x + x, n_k - t + t)$ where $t > 0$
$\Rightarrow r_0 \leq N^\circ(x_{nk} - x, t) \circ N^\circ(x, n_k - t)$
$\Rightarrow r_0 \leq \lim_{k\to\infty} N^\circ(x_{nk} - x, t) \circ \lim_{k\to\infty} N^\circ(x, n_k - t)$
$\Rightarrow r_0 \leq 0 \circ 0 = 0$ (using the continuity of t-co-norm at (0 , 0))
$\Rightarrow r_0 \leq 0$ which is a contradiction. Hence A is bounded.

Conversely suppose that A is closed and bounded and we have to show that A is compact. Let $\dim V = n$ and $\{e_1, e_2, \ldots, e_n\}$ be a basis for V. Choose a sequence $\{x_k\}$ in A and suppose $x_k = \beta_1^{(k)} e_1 + \beta_2^{(k)} e_2 + \ldots + \beta_n^{(k)} e_n$ where $\beta_1^{(k)}, \beta_2^{(k)}, \ldots, \beta_n^{(k)}$ are scalars. Now from Lemma 22, $\exists c > 0$ and $\delta \in (0,1)$ such that
$N^\circ(\sum_{i=1}^{n} \beta_i^{(k)} e_i, c \sum_{i=1}^{n} |\beta_i^{(k)}|) > \delta. \quad \ldots(9)$
Again since A is bounded, for $\delta \in (0 , 1)$ such that $N^\circ(x, t) < \delta \;\; \forall\, x \in A$. So
$N^\circ\left(\sum_{i=1}^{n} \beta_i^{(k)} e_i, t\right) < \delta. \quad \ldots(10)$

From (9) and (10) we get,
$N^\circ(\sum_{i=1}^{n} \beta_i^{(k)} e_i, c \sum_{i=1}^{n} |\beta_i^{(k)}|) > \delta > N^\circ\left(\sum_{i=1}^{n} \beta_i^{(k)} e_i, t\right)$
$N^\circ(\sum_{i=1}^{n} \beta_i^{(k)} e_i, c \sum_{i=1}^{n} |\beta_i^{(k)}|) > N^\circ\left(\sum_{i=1}^{n} \beta_i^{(k)} e_i, t\right)$
$\Rightarrow c \sum_{i=1}^{n} |\beta_i^{(k)}| > t$ (since $N^\circ(x, .)$ is non-increasing)
$\Rightarrow \sum_{i=1}^{n} |\beta_i^{(k)}| > \frac{t}{c}$
$\Rightarrow |\beta_i^{(k)}| \geq \frac{t}{c}$ for $k = 1,2,\ldots$ and $i = 1,2,\ldots,n$.
So each sequence $\{\beta_i^{(k)}\}$ and $(i = 1,2,\ldots,n)$ is bounded. By repeated applications of Bolzano-Weierstrass theorem, it follows that each of the sequences $\{\beta_i^{(k)}\}$ has a convergent subsequence say $\{\beta_i^{(kl)}\}$, $\forall i = 1,2,\ldots,n$. Let
$x_{kl} = \beta_1^{(kl)} e_1 + \beta_2^{(kl)} e_2 + \ldots + \beta_i^{(kl)} e_n$
and $\{\beta_1^{(kl)}\}$, $\{\beta_2^{(kl)}\}$, ..., $\{\beta_i^{(kl)}\}$ are all convergent. Let
$\beta_i = \lim_{l\to\infty} \beta_i^{(kl)}$, $i = 1,2,\ldots,n$ and $x = \beta_1 e_1 + \beta_2 e_2 + \ldots + \beta_n e_n$.
Now for $t > 0$ we have
$N^\circ(x_{kl} - x, t) = N^\circ(\sum_{i=1}^{n} (\beta_i^{(kl)} - \beta_i) e_i, t)$
$\leq N^\circ\left(e_1, \frac{t}{n|\beta_1^{(kl)} - \beta_1|}\right) \circ \cdots \circ N^\circ\left(e_n, \frac{t}{n|\beta_n^{(kl)} - \beta_n|}\right)$

$\Rightarrow \lim_{l\to\infty} \mathrm{N}^\circ(x_{kl} - x, t) \leq 0 \circ 0 \circ 0 ... \circ 0$
$(\beta_i^{(kl)} \to \beta_i$ as $l \to \infty)$

(using the continuity of t-co-norm $\circ$ at (0 , 0))

$\Rightarrow \lim_{l\to\infty} \mathrm{N}^\circ(x_{kl} - x, t) = 0.$

Since $t > 0$ is arbitrary, it follows that

$\lim_{l\to\infty} \mathrm{N}^\circ(x_{kl} - x, t) = 0 \Rightarrow \lim_{l\to\infty} x_{kl} = 0.$

i.e. $\{x_{kl}\}$ is a convergent subsequence of $\{x_k\}$ and converges to x. Since A is closed and $\{x_k\}$ is a sequence in A, it follows that $x \in A$, thus every sequence in A has a convergent subsequence and converges to an element of A. Hence A is compact.

Conclusion

In this paper, properties of fuzzy anti-normed linear space are given and they are more effective to prove some properties of a finite dimensional fuzzy anti-normed linear space for instance, a finite dimensional fuzzy anti-normed linear space is complete, etc. For future work we may study and find some characterization of infinite dimensional fuzzy anti-normed linear space.

Authors' declaration:

- Conflicts of Interest: None.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

References:

1. Zadeh L A, fuzzy S. Information and Control. 1965 ;8: 338-353.
2. Felbin C. The completion of fuzzy normed linear space. Journal of mathematical analysis and application .1993; 147(2): 428-440.
3. Twair HW. Some results of fundamental theorems for fuzzy normed spaces .Master Thesis,University of AL-Qadissiya. 2015.
4. Iqbal H, Jebril S T K. Fuzzy Anti-Normed space. Journal of Mathematics Technology, February. 2010: 66-77.
5. Bag T, Samanta SK. Finite Dimensional Fuzzy Normed Linear Spaces.
The Journal of Fuzzy Mathematics.2003; 11: 687-705.
6. Mursaleen M, Srivastava HM , Sunil K S. Generalized statistically convergent sequences of fuzzy numbers. I JFS.2016; 30 :1511–1518.
7. Thangaraj B , Beulah M. Some aspects on 2- fuzzy 2-anti Normed Linear space, GJPAM.2017;13(2):324-332.
8. Kider J R, Kadhum N A. Properties of Fuzzy Norm of fuzzy Bounded Operators. IJS. 2017; 58(3A): 1237-1281.
9. Altundaˇg S,Kamber E. Generalized weighted statistical convergence in intuitionistic fuzzy normed linear spaces. CMI. 2018; 27: 101-110.
10. Samanta T K, Iqbal H J. Finite dimensional intuitionistic fuzzy normed linear space.Int. IJOPCM. 2009; 2(4): 574-591.
11. Dinda B, Samanta TK, Jebril IH. Fuzzy Anti-norm and Fuzzy ∝-anti-convergence. DOAJM. 2012;4.

بعض خواص الفضاءات الخطية ضد المعيارية الضبابية المنتهية الابعاد

إسراء علاء حسين **راضي إبراهيم محمد علي**

قسم الرياضيات، كلية العلوم للبنات، جامعة بغداد، بغداد، العراق.

الخلاصة:

في هذا البحث تعريف الفضاءات الخطية ضد المعيارية وافضل خواصها تم استخدامها لبرهان بعض الخواص للفضاءات الخطية ضد المعيارية الضبابية المنتهية الابعاد.

الكلمات المفتاحية: المعيارية ضد الضبابية ، الفضاءات ضد المعيارية الضبابية المنتهية الابعاد ، مع معايير مشتركة