Effect of Defensive Switching of Prey and the Handling Time of The Food Web Models

Intisar Haitham Qasem*

Received 11, June, 2013
Accepted 30, September, 2013

Abstract:
In this paper we treat theoretical aspects of the dynamics of three species food web models. We consider two independent predators feeding on a single prey species with defensive switching property and introducing handling time effects. Also we get the system has stable three species coexisting equilibrium state under the condition .We study special case when handling time
equal zero .

Key word: food web-variational matrix-Lyapunov function.

Introduction:

In predator-prey environment, there is variety of ways in which potential prey attempts to avoid predators. These anti-predator behaviors include habitat selection, vigilance and other types of behaviors [1, 2, 3, 4]. The anti-predator behavior, when a prey is shared by two or more predator species, may be classified as specific or non-specific. It is known as predator-specific defense when the prey defense is effective against only one predator species. However, if each of the different behaviors is equally effective against all predator species then it is perfectly non-specific defense [5].

Many of the studies in literature have been focus on the observed patterns in anti-predator behavior in terms of costs and benefits of different levels of prey investment. However, little attention had been given to the influence of these anti-predator behaviors on the population dynamics of predator prey system [6, 7, 8, 9, 10].

The phenomena of change of habitats from one to other due to prey guards itself against the abundant predator, is called defensive switching. Later on M.Saleem et. al. (2003) analyzed simple mathematical model consisting of two predators and one prey which has the defensive switching property of predation avoidance .They assumed that the prey is growing exponentially in the absence of predators. It is observed that, the system is asymptotically settles to a volterra`s oscillation in the three dimensional space when the intensity of defensive switching equals one and the two predators have the same mortality rates.

A three species food web model consisting of two independent predators feeding on a single prey species is considered. It is assumed that the prey species growth logistically in the absence of predators', while the predators decay exponentially in the absence of prey species. The effect of prey's defensive switching on the dynamical behavior of food web and handling time effect is also discussed. Special case when handling time equal zero also discussed.

*Department of Mathematics, College of Science for Women, University of Baghdad.

In the following, we present some basic definitions of population dynamics that are needed in our study.

Definition [11]

The equilibrium point X^* of autonomous system is said to be stable if for all $\varepsilon > 0$ there exists $\delta > 0$ such that

$\|X(0) - X^*\| \le \delta \Rightarrow \|X(T) - X^*\| < \varepsilon \;\; \forall \; T \ge 0,$

with $X(0)$ is an initial value, and $X(T)$ is the solution of autonomous system at time T. The equilibrium point X^* is said to be asymptotically stable if it is stable and $\delta > 0$ can be chosen such that

$\|X(0) - X^*\| < \delta \Rightarrow \lim_{T\to\infty} \|X(T) - X^*\| = 0.$

Further more if an equilibrium point X^* is not stable, then it is called unstable.

Remark: If the equilibrium point X^* is asymptotically stable for all $X(0) \in R^n$, it is called globally asymptotically stable.

Also the equilibrium point X^* will be locally asymptotically stable if and only if the real parts of all the eigenvalues of Vaiational matrix(or Jacobin matrix) are negative, however, it is known globally unstable point if and only if all the eigenvalues of V have a positive real part, otherwise, the equilibrium point X^* is called unstable saddle point .

Definition: [12]

Let U be a neighborhood of an equilibrium point X^*. Then the local stable manifold $W^s(X^*,U)$, and the local unstable manifold $W^u(X^*,U)$ of X^* are defined, respectively, to be the following subsets of U :

$$W^s(X^*,U) \equiv \{X(0) \in U : X(T, X(0)) \in U \;\; for T \ge 0, \;\; and \; X(T, X(0)) \to X^* \; as \, T \to +\infty\},$$

$$W^u(X^*,U) \equiv \{X(0) \in U : X(T, X(0)) \in U \;\; for T \le 0, \;\; and \, X(T, X(0)) \to X^* as T \to -\infty\}.$$

Lyapunov functions: [13]

In general, Lyapunov function is an energy function associated with a dynamical system that decreases over time. The theory of Lyapunov function gives us a global approach toward determining asymptotic behavior of solutions. Further, Lyapunov functions can tell us that initial values from a long region converge to equilibrium. In addition, they can sometimes be used to determine stability of a non-hyperbolic equilibrium point.

Theorem: (Lyapunov Stability Theorem)

Let X^* be an equilibrium point of system of non linear equation and $\Omega \subseteq R^n$ a region containing X^*. If $L : \Omega \to R$ is a C^1 function such that

$L(X^*) = 0$

$L(X) > 0; \forall X \in \Omega \setminus \{X^*\}$ (ie. L is a

$L'(X) \le 0; \forall X \in \Omega$.

positive definite function)

Then X^* is stable. Furthermore, if X^* is stable and $L'(X) < 0, \forall X \in \Omega \setminus \{X^*\}$ then X^* is asymptotically stable. Not that a function L that satisfies the above three assumptions for stability is called a Lyapunov function.

2-The mathematical model:-

A mathematical model consisting of a prey species interacting with two kind's predator species living in two different habitats is considered. It is assumed that, the prey species growth logistically in the absence of predators, while the predators decay exponentially in the absence of prey species. Also the prey species exhibits group defense and handling time effect.

The food web model with this type of interaction can be represented in the following set of differential equations:

$$\left.\begin{aligned}\frac{dP_1}{dT} &= P_1[-\alpha_1 + C_1 a\frac{RP_2^n}{(P_1^n+P_2^n)(1+C_1ahP_2^n+C_2bhP_1^n)}],\\ \frac{dP_2}{dT} &= P_2[-\alpha_2 + C_2 b\frac{RP_1^n}{(P_1^n+P_2^n)(1+C_1ahP_2^n+C_2bhP_1^n)}],\\ \frac{dR}{dT} &= R[\alpha_3(1-\frac{R}{K}) - \frac{aP_1P_2^n+bP_2P_1^n}{(P_1^n+P_2^n)(1+C_1ahP_2^n+C_2bhP_1^n)}].\end{aligned}\right\}\ldots(1)$$

Where $P_1(0), P_2(0), R(0)$ is positive real number, $n>0$ is the intensity of prey defensive switching. $P_1(0), P(0)_2, R(0)$ initial point denote ,respectively, Predator's species and a prey species. of the population densities of two kinds

where

$\alpha_i\,(i=1,2,3)$, K , $C_j\,(j=1,2)$, a, b are positive constants

while $h \geq 0$.

The parameters C_1, C_2 are the conversion rates of a prey R to predators P_1 and P_2.

The parameters C_1, C_2 While a, b are the positive constants that stand for the predation coefficients of the first and second predator respectively, finally h is the handling time parameter.

Clearly, system of first order of equations (1) has a characteristic property of a prey defensive switching and handling time effects. Clearly when the population of predator becomes large and handling time become small the prey defends itself against it and switches to another predator species habitat with a relatively smaller population in order to avoid too much predation of individuate.

<u>In the case of n=1</u>

For this case the system of handling time take the simple form :-

$$\frac{dP_1}{dT} = P_1[-\alpha_1 + C_1 a\frac{RP_2}{(P_1+P_2)(1+C_1ahP_2+C_2bhP_1)}],$$

$$\frac{dP_2}{dT} = P_2[-\alpha_2 + C_2 b\frac{RP_1}{(P_1+P_2)(1+C_1ahP_2+C_2bhP_1)}], \ldots(2)$$

$$\frac{dR}{dT} = R[\alpha_3(1-\frac{R}{K}) - \frac{aP_1PP_2+bP_2P_1}{(P_1+P_2)(1+C_1ahP_2+C_2bhP_1)}].$$

The following dimensionless variable and parameters are used

$$X=\frac{C_1a}{\alpha_1}P_1,\ Y=\frac{C_2a}{\alpha_2}p_2\ ,\ Z=\frac{C_1a}{\alpha_1}R,\ t=\alpha_1T\ ,\ H=\alpha_1h,$$

$$W_1=\frac{C_2b}{C_1a}, W_2=\frac{\alpha_2}{\alpha_1}, W_3=\frac{\alpha_1}{C_1aK}, W_4=\frac{b}{a}, W_5=\frac{\alpha_3}{\alpha_1}.\quad \ldots(3)$$

Accordingly, the dimensionless system is

$$\left.\begin{aligned}\frac{dX}{dT} &= X[-1+\frac{ZY}{(X+Y)(1+HY+W_1HX)}],\\ \frac{dY}{dT} &= Y[-W_2+\frac{W_1ZX}{(X+Y)(1+HY+W_1HX)}],\\ \frac{dZ}{dT} &= Z[W_5(1-W_3Z)-\frac{(1+W_4)XY}{C_1(X+Y)(1+HY+W_1HX)}]\end{aligned}\right\}\ldots(4)$$

If H=0 the system becomes

$$\begin{aligned}\frac{dX}{dT} &= X[-1+\frac{ZY}{(X+Y)}],\\ \frac{dY}{dT} &= Y[-W_2+\frac{W_1ZX}{(X+Y)}],\ \ldots(5)\\ \frac{dZ}{dT} &= Z[W_5(1-W_3Z)-\frac{(1+W_4)XY}{C_1(X+Y)}].\end{aligned}$$

It is easy to show that the system (5) has at most three nonnegative equilibrium points, namely $(0,0,0)$, $(0,0,\frac{1}{W_3})$, $(\bar{X},\bar{Y},\bar{Z})$with $\bar{X}>0$, $\bar{Y}>0$ and $\bar{Z}>0$,since only this point represents the solution to the equation $\frac{dX}{dT}=\frac{dY}{dT}=\frac{dZ}{dT}=0$.

The boundary points $(0,0,0)$ and $(0,0,\frac{1}{W_3})$ are always exist, however the positive equilibrium points $(\bar{X},\bar{Y},\bar{Z})$ is exists if and only if there is a positive solution to the following set of algebraic equations.

$X+Y=ZY$ (6a)

$W_2(X+Y)=W_1Z\,X$ (6b)

$C_1W_5(1-W_3Z)(X+Y)=XY+W_4XY$ (6c)

Clearly, if we do that $-W_2(6a)+(6b)$ we get

$$\bar{Y}=(\frac{W_1}{W_2})\bar{X} \quad (7)$$

…

Substituting equation (7) in (6a) yields

$$\bar{Z}=1+\frac{W_2}{W_1} \quad ...(8)$$

Using equation (7) and (8) in equation (6c) we get

$$\left.\begin{aligned}\bar{X}&=\frac{c_1W_5}{(1+W_4)}(1+\frac{W_2}{W_1})(1-W_3(1+\frac{W_2}{W_1}))\\ \bar{Y}&=\frac{c_1W_5}{(1+W_4)}(\frac{W_1}{W_2}+1)(1-W_3(1+\frac{W_2}{W_1}))\end{aligned}\right\} ...(9)$$

Obviously, $\bar{Z}>0$ while $\bar{X}>0$ *and* $\bar{Y}$ if and only if the following condition hold

$$w_3<\frac{w_1}{w_1+w_2} ...(10)$$

Now, in order to study the stability at the above equilibrium points, the variation matrix G of system (5) at point (X,Y,Z) is computed.

$$G=\begin{bmatrix} b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{bmatrix}$$

Where

$$b_{11}=F_1-\frac{XYZ}{A^2},\ b_{12}=\frac{X^2Z}{A^2},\ b_{13}=\frac{XY}{A},$$

$$b_{21}=\frac{W_1Y^2Z}{A^2},\ b_{22}=F_2-\frac{W_1XYZ}{A^2},\ b_{23}=\frac{W_1XY}{A},$$

$$b_{31}=\frac{YZ(X-W_4Y-A)}{c_1A^2},$$

$$b_{32}=\frac{XZ(-X+W_4(Y-1))}{c_1A^2},\ b_{33}=F_3-W_3W_5Z.$$

Where $A=X+Y$.

Let $G_i\ (i=0,1,2)$ denotes the variational matrix G at the points $(0,0,0),(0,0,\frac{1}{W_3})$, and $(\bar{X},\bar{Y},\bar{Z})$ respectively. Then

$$G_0=\begin{bmatrix}-1 & 0 & 0\\ 0 & -w_2 & 0\\ 0 & 0 & w_5\end{bmatrix}$$

$$,G_1=\begin{bmatrix}-1 & 0 & 0\\ 0 & -w_2 & 0\\ 0 & 0 & -w_5\end{bmatrix},$$

$$G_2=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

where

$$a_{11}=\frac{-W_2}{W_1+W_2},\qquad a_{12}=\frac{W_2^{\,2}}{W_1(W_1+W_2)},$$

$$a_{13}=\frac{W_1\bar{X}}{W_1+W_2},$$

$$a_{21}=\frac{W_1^{\,2}}{(W_1+W_2)},\ a_{22}=\frac{-W_1W_2}{(W_1+W_2)},$$

$$a_{23}=\frac{W_1^{\,2}\bar{X}}{(W_1+W_2)},$$

$$a_{31}=\frac{-W_1(1+W_4)}{C_1(W_1+W_2)},$$

$$a_{32}=\frac{-(W_2^{\,2}W_4+W_1^{\,2})}{C_1W_1(W_1+W_2)},$$

$$a_{33}=-W_3W_5(1+\frac{W_2}{W_1}).$$

Accordingly, the following observations are made.

The eigenvalues of G_0 are

$$\mu_{01}=-1<0\ ,\ \mu_{02}=-w_2<0$$

and $\mu_{03}=w_5>0$

Thus the point $(0,0,0)$ is unstable saddle point with locally stable manifold in the $X-Y$ plane and with locally unstable manifold in the z direction.

The eigenvalues of G_1 are

$\mu_{11}=-1<0$, $\mu_{12}=-w_2<0$ and $\mu_{13}=-w_5<0$.

Therefore $(0,0,\frac{1}{w_3})$ is locally asymptotically stable point.
In the following, the local stability analysis of the positive equilibrium point $(\overline{X},\overline{Y},\overline{Z})$ is investigated.

Theorem: [14]
The positive equilibrium point $(\overline{X},\overline{Y},\overline{Z})$ is a locally asymptotically stable with respect to all solutions initiate in the interior of R_{+}^{3}.

3-The Existence and Stability Aanalysis of Equilibrium points If, $H \neq 0$:-

System (4) has at most three nonnegative equilibrium points, resulting from solving the following algebraic equations

$$(X+Y)(1+HY+W_1HX)=YZ \quad \ldots \quad (11a)$$
$$W_2(X+Y)(1+HY+W_1HX)=W_1ZX \quad \ldots. \quad (11b)$$
$$C_1W_5(1-W_3Z)(X+Y)(1+HY+W_1HX)=(1+W_4)XY \ldots (11c)$$

It is easy to verify that $(0,0,0)$, $(0,0,\frac{1}{W_3})$ *and* (X^*,Y^*,Z^*) *with* $X^* \succ 0, Y^* \succ 0$ *and* $Z^* \succ 0$ resent the equilibrium points of the system (4).

The boundary points $(0,0,0)$ *and* $(0,0,\frac{1}{W_3})$ are always exist, however the positive equilibrium points (X^*,Y^*,Z^*) is exists if and only if there is a positive solution to the set of algebraic equations(11 a),(11 b)and(11 c).
Clearly, equation (11 a) and equation (11 b) gives
$Z = A + BHX$ *where*

$$A=(\frac{W_2}{W_1}+1),\ B=A(\frac{W_1}{W_2}+W_1).$$

And $Y=\frac{W_1}{W_2}X.$

From equation (11 c)

$$X=\frac{C_1W_5Z(1-W_3Z)}{(1+W_4)},$$

$$0 \prec Z \prec \frac{1}{W_3}$$

By substitute Z in X we obtain

$A_2X^2+A_2X+A_3=0$ …(12)

Where
$$A_1 = C_1W_3W_5B^2H^2$$
$$A_2 = C_1W_5A(W_3A-1)$$
$$A_3 = (1+W_4)+C_1W_5BH(2W_3A-1)$$

Clearly that
$A_1 > 0, A_2 > 0$ *since* $A > \frac{1}{W_3}$ *according the condaction* $0 < Z < \frac{1}{W_3}$.

The solution of the second order algebraic equation (12) is

$$X=\frac{-A_2 \pm \sqrt{A_2^2-4A_1A_3}}{2A_1}$$ provided that $A_2 \geq 2\sqrt{A_1A_3}$.

The dynamical behavior of system (4) can be investigated locally through computing the variational matrices corresponding to each equilibrium point and then using Lyapunov function. The variational matrix Vat the point (X, Y, Z) is

$$V=\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Where

Where

$$b_{11}=F+\frac{-YXZ(\sigma+\delta W_1H)}{(\delta\sigma)^2}\quad,\quad b_{12}=\frac{XZ\delta\sigma-YZX(\sigma+\delta H)}{(\delta\sigma)^2}\quad,b_{13}=\frac{XY}{(\delta\sigma)}$$

$$b_{21}=W_1ZY\frac{\delta\sigma-X(\sigma+\delta HW_1)}{(\delta\sigma)^2}\quad,\quad b_{22}=G+\frac{-(\sigma+\delta H)ZXYW_1}{(\delta\sigma)^2},\quad b_{23}=\frac{XYW_1}{\delta\sigma}$$

$$b_{31}=\frac{(1+W_4)YZ[\delta\sigma-X(\sigma+\delta HW_1)}{(\delta\sigma)^2}\quad,\quad b_{32}=\frac{Z(1+W_4)X[\delta\sigma-Y(\sigma+H\delta)]}{C_1(\delta\sigma)^2},$$

$$b_{33}=R-ZW_3W_5$$

Where

$$\delta=X+Y\quad,\quad\sigma=1+HY+W_1$$

$$F=-1+\frac{YZ}{(X+Y)(1+HY+W_1HX)},$$

$$G=-W_2+\frac{W_1XZ}{(X+Y)(1+HY+W_1HX)}$$

$$R=W_5(1-W_3Z)-\frac{(1+W_4)XY}{C_1(X+Y)(1+HY+W_1HX)}$$

Let V_i $(i=0,1,2)$ denotes the variational matrix V at the points $(0,0,0),(0,0,\frac{1}{W_3})$, and (x^*,y^*,z^*) respectively. Then

$$V_0=\begin{bmatrix}-1&0&0\\0&-w_2&0\\0&0&w_5\end{bmatrix}$$

$$,V_1=\begin{bmatrix}-1&0&0\\0&-w_2&0\\0&0&-w_5\end{bmatrix},$$

$$V_2=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}.$$

Where

$$a_{11}=\frac{-X^*Y^*Z^*(\sigma+\delta W_1H)}{(\delta\sigma)^2}\quad,\quad a_{12}=\frac{X^*Z^*[\delta\sigma-Y^*(\sigma+\delta H)]}{(\delta\sigma)^2},a_{13}=\frac{X^*Y^*}{(\delta\sigma)}$$

$$a_{21}=W_1Y^*Z^*\frac{\delta\sigma-X^*(\sigma+\delta HW_1)}{(\delta\sigma)^2}\quad,\quad a_{22}=\frac{-(\sigma+\delta H)X^*Y^*Z^*W_1}{(\delta\sigma)^2}$$

$$,\quad a_{23}=\frac{X^*Y^*W_1}{\delta\sigma}$$

$$a_{31}=\frac{(1+W_4)Y^*Z^*[\delta\sigma-X^*(\sigma+\delta HW_1)}{(\delta\sigma)^2},a_{32}=\frac{X^*Z^*(1+W_4)[\delta\sigma-Y^*(\sigma+H\delta)]}{C_1(\delta\sigma)^2},$$

$$a_{33}=-Z^*W_3W_5.$$

Accordingly, the following observations are made.

The eigenvalues of V_0 are

$\lambda_{11}=-1<0,\lambda_{12}=-W_2<0$ and $\lambda_{13}=W_5>0.$

Thus the point $(0,0,0)$ is unstable saddle point with locally stable manifold in the $X-Y$ plane and with locally unstable manifold in the Z direction.

The eigenvalues of V_1 are

$\lambda_{21}=-1<0,\lambda_{22}=-W_2<0$ and $\lambda_{23}=-W_5<0.$

Therefore $(0,0,\frac{1}{W_3})$ is locally asymptotically stable point.

In the following, the locally stability analysis of the positive equilibrium point (X^*,Y^*,Z^*) is investigated.

Theorem(3.1):-

Suppose that the positive equilibrium (X^*,Y^*,Z^*) of system (4) exists. Then it is locally asymptotically stable if $X>\frac{\delta\sigma}{(\sigma+H\delta W_1)}$ and $Y>\frac{\delta\sigma}{\sigma+\delta H}$.

Proof:-

Consider the following positive definite function:-

$$L(X,Y,Z)=M_1U^2+M_2V^2+M_3W^2$$

Where M_1,M_2 and M_3 are positive constants to be determined.

Differentiating L with respect to time T along the solution of system (4), we get

$$\frac{dL}{dT} = 2M_1U(a_{11}U + a_{12}V + a_{13}W) + 2M_2V(a_{21}U + a_{22}V + a_{23}W) + 2M_3W(a_{31}U + a_{32}V + a_{33}W)$$
$$= 2M_1a_{11}U^2 + 2M_2a_{22}V^2 + 2M_3a_{33}W^2 + 2(M_1a_{12} + M_2a_{21})UV + 2(M_1a_{13} + M_3a_{31})UW + 2(M_2a_{23} + M_3a_{32})VW.$$

Choosing the arbitrary positive non-zero constants such that

$$M_1a_{13} + M_3a_{31} = 0 \xrightarrow{yields} \frac{M_1}{M_3} = \frac{-a_{31}}{a_{13}} \xrightarrow{yields} M_1 = \frac{-a_{31}}{a_{13}}M_3$$

$$M_2a_{23} + M_3a_{32} = 0 \xrightarrow{yields} \frac{M_2}{M_3} = \frac{-a_{32}}{a_{23}} \xrightarrow{yields} M_2 = \frac{-a_{32}}{a_{23}}M_3$$

Obtain that By choosing $M_3 = 1$ $M_1 = \frac{-a_{31}}{a_{13}}$, $M_2 = \frac{a_{32}}{a_{23}}$

Clearly that $M_1 > 0 \text{ and } M_2 > 0$ $\quad$ since $a_{31} < 0 \text{ and } a_{32} < 0$

according the condition $X > \frac{\delta\sigma}{(\sigma + H\delta W_1)}$ and $Y > \frac{\delta\sigma}{\sigma + \delta H}$.

Then:-

$$\frac{dL}{dT} = 2M_1a_{11}U^2 + 2M_2a_{22}V^2 + 2a_{33}W^2 + 2\left(\frac{-a_{31}}{a_{13}}a_{12} - \frac{a_{32}}{a_{23}}a_{21}\right)UV.$$

Since $a_{31} = \frac{1+W_4}{M_1W_1}a_{21}$, $a_{32} = \frac{1+W_4}{M_1}a_{12}$, $a_{23} = a_{13}W_1$

$$\therefore \frac{dL}{dT} = 2M_1a_{11}U^2 + 2M_2a_{22}V^2 + 2a_{33}W^2 - 2UV\frac{a_{12}a_{21}}{a_{13}}\left(\frac{1+W_4}{M_1W_1}\right).$$

Clearly $\frac{dL}{dt} \prec 0$. So, L is a Lyapunov function with respect to (x^*, y^*, z^*) and hence (x^*, y^*, z^*) is a locally asymptotically stable.

Accordingly, it is concluded that adding the defensive switching behavior and handling time effect to the food web system under consideration may have a stabilizing effect on the dynamical behavior.

Reference:

1. Eraser ,D.F. and Huntingford ,F.A. .1986. Feeding and avoidingpredation hazard :the behavioral response of the prey ,Ethology73:56-70.
2. LendremD.W. 1983.Predation risk and vigilance in the blue tit (Paruscaeruleus) , Behav . Ecol . Sociobiol. 14:9-20.
3. Milinski ,M. and Heller,R. 1978. Influenceof a predator on the optimal foraging behavior of sticklebacks. Gasterosteusaculeatus , Nature 275:650.
4. Sih,A. 1979. Stability and prey behavioral responses to predator density , J. Anim . Ecol . 48 :79-85.
5. Saleem ,M. Tripathi,A.K. and Sadiyal,A.H. 2003. Coexistence of species in a defensive switching model , Mathematical Biosciences 181:145-164.
6. Hassell,M.P. and May R.M., From individual behavior to population dynamics,in: R.M.Sibly and R.H. Smith (Eds), Behavioral Ecology :Ecological Consequences of Adaptive Behavior , Blackwell , Oford 1985.p.3.
7. Matsuda ,H.P. Abrams A., and M. Hori, M. 1993. The effect of adaptive anti-predator behavior on exploitative competition and mutualism between predators, Oikos 68:549-595.
8. Mc Nair, .N. 1986.The effects of refuges on predator – prey interactions: a reconsideration, Theor. Popul. 29:38-63.
9. Parker, G.A., 1985. population consequences of evolutionarily stable strategies , in :R.M. Sibly , R.H .Smith (Eds.), Behavioral Ecology :Ecological Consequences of Adaptive Behavior , Blackwell , Oxford , P. 33.
10. Takahashi ,S. and Hori ,M. 1994. Unstable evolutionarily stable strategy and oscillation :a model of

lateral asymmetry in scale –eating cichlids ,Am . Nat. 144:1001-1020.

11. Freedman, H.I. 1980. Deterministic mathematical models in population ecology, Marcel Dekker, Inc, New york, USA.

12. Hale ,J.and Kocak,H. 1991. Dynamics and Bifurcation, by spring- verlag, New York, Inc..

13. Beltrami, E. 1987. Mathematics for Dynamic Modeling, Academic press, Inc.

14. Kasim,I.H. 2006. Thesis"the effect of switching and group defense on the stability of Interacting Species" Department of mathematic, the college of science university of Baghdad.

تأثير التحول الدفاعي للفريسة ووقت المسك لنظم الشبكات الغذائية

انتصار هيثم قاسم*

*قسم الرياضيات-كلية العلوم للبنات- جامعة بغداد.

الخلاصة:

في هذاالبحث عالجنا نظريان حالة خاصة من الديناميك لنظام الشبكات الغذائيةالمتكونة من مفترسين مستقلين يتغذون على فريسة واحدة مع دراسة تأثير كل من التحول و وقت المسك للمفترس على السلوك الديناميكي للنظام، كما تم ايجاد شروط الوجود لهذا النظام .