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A Tile with Nested Chain Abacus

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Abstract:

This study had succeeded in producing a new graphical representation of James abacus called nested chain abacus. Nested chain abacus provides a unique mathematical expression to encode each tile (image) using a partition theory where each form or shape of tile will be associated with exactly one partition.Furthermore, an algorithm of nested chain abacus movement will be constructed, which can be applied in tiling theory.

Keywords: Abacus, Chain, Movement, Partition, Tile.

Introduction:

About 15 years after James abacus was introducedJames abacus diagram is a graphical representation of a special type of non-increasing sequence$\mu = (\mu_1, \mu_2,\ldots,\mu_b)$called the partition of $t$where b is the number of partition parts and t is any positive integer. The James abacus diagram configuration for beta numbers $\beta_1, \beta_2, \beta_3,\ldots,\beta_b$ can be created by rearranging these numbers on the runners, where $\beta_i = \mu_i + b - i$ for$1 \leq b \leq i$)[1].A new abacus was established by adding one empty column to the James abacus called $(e + 1)$-abacus. It was proven in a theorem related to the determination of the decomposition numbers for any $e$where $e$is a prime positive integer greater than or equal to 2[2]. James and Mathas investigated several relationships between the original and the new partition. Fayers (2007) contracted a similar theorem but with different weight of partition by adding 'full' column to the James abacus[3].

The parallelism idea of the previous theorem in[3] was used by considering a finite composition of weight of the partition[4,5]. Fayers, expands the use of the James abacus to compute the e-regularisation of a partition by implementing another movement. This movement is considered to be more complicated than the previous movement. A bead moving from b_k to b_{k-e} and a bead from S_{k-e} to b_k, $k = 1, \ldots, c$, where positions x_{k-e} and S_k are empty beads while positions x_{k-e} and S_k are beads as long as the volume $1, \ldots, c$where $c$is the number of themovement of the bead positions of James abacus, such as a work by Wildon (2008)who discovered a new way to find the conjugate of any partition by reflecting James abacus in its leading diagonal[6].

The literature on James abacus shows the constraction of a variety of diagrama beads on movement of the bead positions of James abacus, such as a work by[7]who discovered a new way to find the conjugate of any partition by reflecting James abacus in its leading diagonal.

Scan movement was the development in the case the bead jumps x positions to the right, which is used to prove the combinatorial definition of Schur polynomials equivalent the algebraic definition of Schur polynomials[8]. Another movement of the bead was constracted to give a combinatorial proof of a plethstic generalization of the Murnaghanâ Nakayama rule called single-step bead move. In this movement, all bead positions will be changing locations from position x to position $x - e$ above it in the same column such that $x - e \leqslant 0$ and $x - e$ is empty bead position[9].

Tingley, places all beads on the horizontal axis and then moves one bead exactly one step to the right in the corresponding row of beads, possibly jumping over other beads. Then, he put the beads into groups and rotating each group 90 degrees anticlockwise[10].

Meanwhile, a new diagram was found by applying Brauer algebra on James abacus along with a consideration of the Temperley-Lieb algebra[11]. In addition a new abacus called neated chain abacus was consract. The construction of the new diagram is developed by first converting the original James abacus diagram to matrix form. James abacus diagram positions are divided into several nested chains[12,13,14].

Many movement were construct to converts one tile (image) into another or converts one tiling of a lattice region into another[15,16].

In this work, a new abacus depending on James' abacus idea to represent any tile as a discrete object was constructed. In addition, three types of chain movement are developed. These movements have been used in tiling.

Definitions And Terminologies

This section briefly discusses some basic steps, definitions and theorems of tile using nested chain.The graphical form of tile with respect to a minimal frame, enables us to define object positions and empty object positions in terms of rows and columns of the minimal frame. Then the object positions and empty object positions as bead positions and empty bead positions respectively were redefined, which enables us to apply the concept of beads on an abacus to represent any tile on a nested chain abacus.

Definition 1(15): A tile is a plane geometric figure formed by one or more ominoes.

Definition 2(**17**): A minimal frame is a minimal rectangle containing the tile itself, s.t. each column and row contains at least one omino where$n < re$ for n ominoes, r rows and e columns.

Definition3(17): Let tile in a minimal frame have e columns and r rows. The function $f(m,j): Z \times Z \to Z$ such that, if location (m, j) in the minimal frame contains an ominoes, then

$$w_k = f(m,j) = (m-1)e + (j-1)$$

for$k = 1,2,\dots,n$ where e and r refer to the number of the rows and columns of the minimal frame respectively, for $1 \leqslant j \leqslant e$ and $1 \leqslant m \leqslant r$.

Definition 4[12]: A nested chain abacus is an abacus with$e$ columns, r rows and $n$of bead positions which satisfy the following conditions:

1. The columns are labelled from left to right as $0,1,\dots,e-1$.
2. The rows are labelled from up to down as $0,1,\dots,r-1$.
3. The connected bead locations are labelled with numbers $0,1,\dots,er-1$ across the rows from left to right beginning from the number 0 in the top-leftmost location until the number $er-1$ is in the bottom-rightmost location.
4. Each column and row has at least one bead position.

Table 1 corresponding placement of position numbers on the nested chain abacus with e columns number from 1 to $e-1$ and r rows number from 1 to $r-1$.

Table 1. Placement of position numbers on the nested chain abacus with x positions where $x = r \times e$

col.0	col.1	col.2	...	col. $e-1$
0	1	2	...	e-1
e	e +1	e +2	...	2e-1
.	.	.	...	.
.	.	.	...	.
.	.	.	...	.
$e(r-2)$	$e(r-2)+1$	$e(r-2)+2$	...	$e(r-1)-1$
$e(r-1)$	$e(r-1)+1$	$e(r-1)+2$	...	$re-1$

Nested Chain Abacus[17]

Following anew representation of n-connected ominoes called nested chain abacus construction

Step-I: Establishing a graphical form of tile w.t.r. minimal frame.

1- The abacus with e columns and r rows is considered a tile (picture) with the minimal frame (rectangular forms), and the bead as well as empty bead positions which are considered squares in different colors where the bead positions constitute the image of the picture and the empty bead positions constitute the background.

2- Identify the first column (leftmost row), last column (rightmost row), first row (topmost column) and last row (bottommost column) with at least one ominoe as a minimal frame. The columnsnumbered from the leftmost, worked from left to right 1,2,…,$e$and numbered the rows from topmost to bottommost $1,2,\dots,r$in the minimal frame where r and e number of rows and columns respectively.

Step-II: Mathematical expression for tile.

In this step, a direction to obtain a nested chain abacus with respect to the minimal frame is created.Identify the first ominoe located in the top-leftmost, from left to right, working down from the top-leftmost ominoe to the bottom in the minimal frame.

Step-III: Connected partition of nested chain abacus.

following Definition 3 bead positions on the nested chain abacus has been disputed. According to Step II, beginning at the top-leftmost ominoe of the minimal frame and the rest of the ominoes.

Step IV: Constructing a connected partition of the nested chain abacus

Using thew_k's obtained from Step 3, aconnected partitionwhich represents the nested chain abacus with n beads,e columns and r rows for$k = 1,2, \dots, n$called connected partition is produced.

$\mu_n = w_1$, $\mu_{n-g} = w_{g+1} - w_g + \mu_{n-g+1} - 1$ where $1 \leqslant g \leqslant n-1$.

Then, $\mu^{(e,r)} = (\mu_1^{t_1}, \mu_2^{t_2}, \dots, \mu_b^{t_b})$ is a connected partition with e columns and r rows where$\mu_1 \geqslant \mu_2 \geqslant \dots \geqslant \mu_b$ and $\sum_{b'=1}^{b} t_{b'} = n$. Consider Figure 1, a. $w_1 = 0, w_2 = 1, w_1 = 2, w_1 = 3$(Based on definition) then , $\mu_4 = w_1 = 0, \mu_3 = 0, \mu_2 = 0,\ \mu_1 = 0$

The result of the previous algorithm is shown in Fig. 1where each shape is associated by a unique nested chain abacus and represented by a unique connected partition.

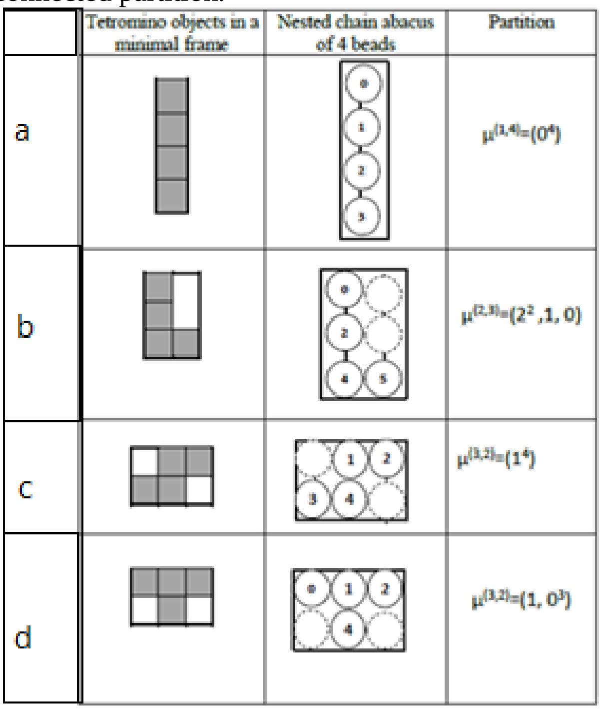

Figure1.Modeling the 4 shapes of family of Tetrominos using nested chain abacus

The nested chain is considered as tile (picture) with rectangular forms and the empty bead as well as bead positions are considered squares in two colors where the bead position constitutes the tile (image) in the two colors where the bead position constitutes the image of the tile (image) and the empty bead position constitutes the background.

Nested chain movement

This proposed method is called nested chain movement. That is, any bead position as well as empty position can be used as an initial bead to be moved from one location to another location and the rest of the beads will follow to form a chain. The number of chains in a diagram depends on column e and row r. In new movement any bead position can be bypass one location or more. The proposer movement is used to convert any tiling into any other by means of these movement.

Definition 5: Chain movement (Ch) is a moving movement when

$$Ch(m-1)e + (j-1) \to \begin{cases} (m-1)e + (j+d-1), & d \in \mathbb{Z} \\ (m+c-1)e + (j-1), & c \in \mathbb{Z}, \end{cases}$$

which is in anticlockwise direction for all bead positions in chain i in the nested chain abacus with e columns and r rows where$0 \leqslant m \leqslant r-1$and $0 \leqslant j \leqslant e-1$.

Definition 6: Let the matrix$A_{r \times e}$represents bead positions and empty bead positions in the nested chain abacus with $e$columns and$r$rows. Then

- Vertical-path chain is an arrangement of bead and empty bead positions in column $\frac{e+1}{2}$ in the nested chain abacus such that $[(r-i+1)e+i] - [ie+i] = 0$ and the elements in

$$chain\left(\frac{e+1}{2}\right) = \left\{a_{m\left(\frac{e+1}{2}\right)} : \frac{e-1}{2} \leqslant m \leqslant \frac{2r-e+3}{2}\right\}$$

where $e < r$, e is odd and c is a positive integer.

- Horizontal-path chain is an arrangement of bead and empty bead positions in row $\frac{r+1}{2}$ in the nested chain abacus such that $[ie + (e-i+1)] - [ie+i] = 0$ and the elements in

$$chain\left(\frac{r+1}{2}\right) = \left\{a_{\left(\frac{r+1}{2}\right)j} : \frac{r-1}{2} \leqslant m \leqslant \frac{2e-r+3}{2}\right\}$$

where $r < e$, r is odd, and c is a positive integer.

Remark 7: In the chain movement the beads will move by a specific distance. A bead located in the chain i and in the column i will move to the downwards if c is a positive integer. While, a bead located in the row *r-i+1* will move to the rightwards if d is a positive integer. Meanwhile, the bead located in the column *e-i+1* will move to the upwards if c is a negative integer. Finally, the bead located in row i will move to the leftwards if d is a positive integer.

Remark 8: Leta_{mj}be an element of a matrix$A_{r \times e}$which represents the bead positions and empty bead positions in the nested chain abacus with$e$columns, $r$rows and $c$chains. Based on Definition 5, then, the chain movement$(Ch)$is

$$Ch: a_{mj} \rightarrow \begin{cases} a_{m(j+d)}, & d \in \mathbb{Z} \\ a_{(m+c)j}, & c \in \mathbb{Z}, \end{cases}$$

which is in anticlockwise direction for all bead positions in chain in the nested chain abacus where $1 \leqslant m \leqslant r$ and $1 \leqslant j \leqslant e$.

Lemma 9: Let a_{mj} be an element of a matrix $A_{r\times e}$ which represents the bead positions and empty bead positions in the nested chain abacus with e columns, r rows and c chains. Then,

1. the bead positions in the rectangle chain i are located in columns $\{i, e-i+1\}$ and rows $\{i, r-i+1\}$.
2. the direction of the positions are such that
   - a_{ii} through $a_{(r-i+1)i}$ is downwards.
   - $a_{(r-i+1)i}$ through $a_{(r-i+1)(e-i+1)}$ is rightwards.
   - $a_{(r-i+1)(e-i+1)}$ through $a_{i(e-i+1)}$ is upwards.
   - $a_{i(e-i+1)}$ through a_{ii} is leftwards.

Proof.

1- The elements of the rectangle chain are $\{a_{mi}, a_{m(e-i+1)}, a_{ij}, a_{(r-i+1)j}: i \leqslant m \leqslant (r-i+1), i \leqslant j \leqslant (e-i+1)\}$. Thus, the bead positions of the rectangle chain i are located in columns $\{i, e-i+1\}$ and rows $\{i, r-i+1\}$.

2- Since the chain movement is in anticlockwise direction for all bead positions in chain i in the nested chain abacus then,

- $Ch(a_{ii}) \rightarrow a_{(i+c)i}, \ldots, Ch(a_{(r-i)i}) \rightarrow a_{(r-i+1)i}$ where c is a positive integer. Then, the position will move downward.
- $Ch(a_{(r-i+1)i}) \rightarrow a_{(r-i+1)(i+d)}, \ldots, Ch(a_{(r-i+1)(e-i)}) \rightarrow a_{(r-i+1)(e-i+1)}$ where d is a positive integer. Then, the position will move rightwards.
- $Ch(a_{(r-i+1)(e-i+1)}) \rightarrow a_{(r-i+1+c)(e-i+1)}, \ldots, Ch(a_{(i+1)(e-i+1)}) \rightarrow a_{(i)(e-i+1)}$ where c is a negative integer. Then the position will move upwards.
- $Ch(a_{i(e-i+1)}) \rightarrow a_{i(e-i+1+d)}, \ldots, Ch(a_{i(i-1)}) \rightarrow a_{ii}$ where d is a negative integer. Then the position will move leftwards.

In our movement a position in the chain as the initial point was selected. When this point is moved rotationally anticlockwise to a new location, all other positions in the rectangle chain will move rotationally to a new location accordingly.

Definition 10: Nested chain abacus movement is a chain movement in one or more chains in the nested chain abacus.

Next the movement of the bead positions inside the chains is construct. Lemma 9 provides the basic concept for bead position movements.

movement in Chains

There are three cases of movements based on three types of chains: movement in rectangle chain, movement in path chain and movement in singleton chain. Next, a new movements in nested chain abacus is constructed.

The movement in rectangle chain is a chain movement Ch employed in rectangle chain. First, the movement is (Ch) if the beads skip one position anticlockwise in rectangle chain $\{a, b\} = \pm 1$ was constructed(see Definition 5)

Lemma 11: Let a_{mj} be an element in matrix $A_{r\times e}$ which represents bead positions and empty bead positions in nested chain abacus with e columns, r rows and c chains such that the bead positions a_{mj} located in rectangle chain. Then, movement Ch:

$$Ch(a_{mj}) \rightarrow \begin{cases} a_{(m-1)j} & if \quad li+1 \leqslant m \leqslant (r-i+1), j = e-i+1 \\ a_{(m+1)j} & if \quad li \leqslant m \leqslant (r-i), j = i \\ a_{m(j-1)} & if \quad lm = i, i+1 < j \leqslant e-i+1 \\ a_{m(j+1)} & if \quad lm = (r-i+1), i \leqslant j < e-i \end{cases}$$

where $1 \leqslant i \leqslant c$.

Proof: All positions located on rectangle chain i belong to two columns $\{i, e-i+1\}$ and two rows $\{r, r-i+1\}$. Based on Definition 5 and $\{a, b\} = \pm 1$ then the bead position in column i will skip one position downward so $Ch(a_{mj}) \rightarrow a_{(m+1)j}$ where $1 \leqslant m < r$. *S*ince the direction of the positions' movement anticlockwise then the positions $a_{(r-i+1)j}$ *w*ill skip to $a_{(r-i+1)(j+1)}$ one position where $i \leqslant j < e-i+1$. Since the direction of the positions move anticlockwise then the bead position location a_{mj} on the column $e-i+1$ will skip to $a_{(m-1)j}$ one position where $1 < m \leqslant r$. If $a_{mj} \in \{a_{mj} | m = i, i \leqslant j < e-i+1\}$ then $a_{mj} \rightarrow a_{m(j-1)}$.

In the following theorem, the maximal x number of movement (Ch^x) in rectangle chain was established.

Theorem 12: Let $\mathfrak{N}$ be a nested chain abacus with e columns, r rows and c chains. Then, the positions' movement number of movement in rectangle chain is $2e + 2r - 8i + 3$ where $1 \leqslant i \leqslant c$.

Proof. Let a_{mj} be an initial position in rectangle chain i for the nested chain abacus with e columns, r rows and c chain. a_{mj} can be moved downwards, upwards, rightwards or leftwards depending on m and j where $1 \leqslant m \leqslant r$ and $1 \leqslant j \leqslant e$. If $j = i$ then the initial position will be moved downwards along the column i until location $a_{(r-i+1)i}$ after skipping $r-i+1-m$ positions. Then, the position $a_{(r-i+1)i}$ will move from left to the right where the last location of the initial position is $a_{(r-i+1)(e-i+1)}$

after skipping $e-2i+1$ positions. Since $a_{(r-i+1)(e-i+1)}$ in column $e-i+1$ then the initial in position $a_{(r-i+1)(e-i+1)}$ will move up and skip $r-2i+1$ positions to get to the position $a_{i(e-i+1)}$. Furthermore, the initial position will move from the right to the left and they are skip $e-2i+1$ positions until it reaches location a_{ii}. Finally, the initial positions a_{ii} skip $m-i$ positions to return back. Thus the initial position a_{mj} will skip $2e+2r-4$ positions to move and return to its original position. The same applies if the initial position is $a_{(r-i+1)j}$ or $a_{m(e-i+1)}$ or a_{ij}. Hence, the maximum of chain movement for each position in the rectangle chain is $2e+2r-8i+3$.

The following corollary describes the number of possible movements for a positions in outer chain.

Corollary 13: Let $\mathfrak{N}$ be the nested chain abacus with e columns, r rows and c chains. Then, the maximum number of movement in the outer chain is $2e+2r-5$.

Proof: From Theorem 11, the maximal number of movement in a rectangle chain is $2e+2r-8i+3$. Since the outer chain is chain 1 in the nested chain abacus then the maximal number of movement in the outer chain is $2e+2r-5$

Not: Notation $a_{mj} \Rightarrow b$ means a_{mj} will skip b positions

Theorem 14 the movement in rectangle chain (Ch^x) was formulated if the beads skips x positions in four cases depending on the location of the beads in the rectangle chain where x is the maximal number of movement such that $1 \leqslant x \leqslant 2e+2r-8i+3$. T_1, T_2, T_3, T_4 describe the location of the bead positions where

$T_1 = \{a_{mi} | i \leqslant m < r-i+1\}$,
$T_2 = \{a_{(r-i+1)j} | i \leqslant j < e-i+1\}$,
$T_3 = \{a_{m(e-i+1)} | i \leqslant m < r-i+1\}$ and
$T_4 = \{a_{ij} | i \leqslant j \leqslant e-i+1\}$.

Theorem 14: *Let* a_{mj} be an element in a rectangle chain in the nested chain abacus with e columns, r rows and c chains represented by matrix $A_{r\times e}$. Then, movement number x is Ch^x:

Case one

$$a_{mi} \rightarrow \begin{cases} a_{(m+x)i} & if \quad li \leq m+x < r-i+1 \\ a_{(r-i+1)(x-r+2i+m-1)} & if \quad lr-i+1 \leq m+x < r+e-3i+2 \\ a_{(2r-x-m+e-4i+3)(e-i+1))} & if \quad le+r-3i+2 \leq m+x < 2r+e-5i+3 \\ a_{i(2e+2r-m-x-6i+4)} & if \quad l2r+e-5i+3 \leqslant m+x < 2r+2e-7i+4 \\ a_{(x-2r-2e+m+8i-4)i} & if \quad l2r+2e-7i+4 \leq m+x < 3r+2e-9i+5 \end{cases}$$

if $a_{mj} \in T_1$ where $T_1 = \{a_{mj} | j = i, l \leq m < -i+1\}$.

Case two

$$a_{mn} \rightarrow \begin{cases} a_{m(j+x)} & if \quad (j+x) \leq e-i+1 \\ a_{(r+e-2i-x-j+2)(e-i+1)} & if \quad le-i+1 < j+x \leq e+r-3i+2 \\ a_{i(2e+r-4i-x-j+3)} & if \quad le+r-3i+2 < j+x \leq 2e+r-5i+3 \\ a_{(x-2e-r+6i+j-3)i} & if \quad l2e+r-5i+3 < j+x \leq 2e+2r-7i+4 \\ a_{f(x-2e-2r+8i+j-4)} & if \quad l2e+2r-7i+4 < n+x \leq 3e+2r-9i+5 \end{cases}$$

if $a_{mj} \in T_2$ where $T_2 = \{a_{mj} | m = r-i+1, i \leqslant j < e-i+1\}$,

Case three

$$a_{mj} \rightarrow \begin{cases} a_{(m-x)(e-i+1)} & if \quad lm-x \geqslant i \\ a_{i(m+e-2i-x+1)} & if \quad l3i-e-1 \geqslant m-x \geqslant i \\ a_{(x+4i-m-e-1)i} & if \quad l5i-e-r-2 \geqslant m-x \geqslant 3i-e-1 \\ a_{(r-i+1)(x-m-e-r+6i-2)} & if \quad l7i-2e-r-3 \geqslant m-x \geqslant 5i-e-r-2 \\ a_{(2r+2e-8i-x+4+m)(e-i+1)} & if \quad l9i-2e-2r-4 \geqslant m-x \geqslant 7i-2e-r-3 \end{cases}$$

if $a_{mj} \in T_3$ where $T_3 = \{a_{mj} | j = e-i+1, i < m \leqslant r-i+1\}$,

Case four:

$$a_{mj} \rightarrow \begin{cases} a_{i(j-x)} & if \quad lj - x \geqslant i \\ a_{(x-j+2i)i} & if \quad li > j - x \geq 3i - r - 1 \\ a_{(r-i+1)(x-j-r+4i-1)} & if \quad l3i - r - 1 > j - x \geqslant 5i - e - r - 2 \\ a_{(j-x+2r+e-6i+3)(e-i+1)} & if \quad l5i - e - r - 2 > n - x \geq 7i - e - 2r - 3 \\ a_{i(j-x+2r+2e-8i+4)} & if \quad l7i - e - 2r - 3 > n - x \geq 9i - 2e - 2r - 4 \end{cases}$$

if $a_{mj} \in T_4$ where $T_4 = \{a_{mj} | i \leqslant j \leqslant e - i + 1, m = i\}$, for $1 \leqslant i < c$ and $1 \leqslant x \leqslant 2e + 2r - 8i + 3$.

Proof.

Case one:

1. If $m + x \leqslant r - i + 1$.

a_{mi} will move downwards and skip x positions. Since $x \leqslant r - i + 1 - m$ then $Ch(a_{mi}) \in T_1$ and $Ch: a_{mi} \rightarrow a_{(m+x)i}$.

2. If $r - i + 1 < m + x \leqslant r + e - 3i + 2$.

Based on Definition 5 and Lemma 9a_{mi} will move downwards from a_{mi} through $a_{(r-i+1)i}$. Since $x > r - i + 1 - m$ and $a_{mi} \in T_1$ then, a_{mi} will skip $r - i - m + 1$ positions and then move rightwards to a new location after skipping $x - r + i + m - 1$ positions (See Lemma 9). Thus

Ch: $a_{mi} \rightarrow a_{(r-i+1)(x-r+2i+m-1)}$.

3. If $e + r - 3i + 2 < m + x \leqslant 2r + e - 5i + 3$.

Based on Definition 5 and Lemma 9a_{mi} will move downwards to $a_{(r-i+1)i}$. Since $x > (r - i + 1 - m) + (e - 2i + 1)$ then the new location of a_{mi} belong toT_3. Thus, a_{mi} will move rightwards until $a_{(r-i+1)(e-i+1)}$ and then upward. Based on Lemma 10,

$$Ch: a_{mi} \xrightarrow{r-i-m+1} a_{(r-i+1)i}$$
$$Ch: a_{(r-i+1)i} \xrightarrow{e-2i+1} a_{(r-i+1)(e-i+1)}$$
$$Ch: a_{(r-i+1)(e-i+1)} \xrightarrow{x-e-r+m+3i-2} a_{i(e+2r-m-x-4i+3)}$$

4. If $2r + e - 5i + 3 < m + x \leqslant 2r + 2e - 7i + 4$.

Since $x > (r - i + 1 - m) + (e - 2i + 1) + (r - 2i + 1)$ and based on Definition 5 and Lemma 9, the new location $\in T_4$. Thus, a_{mi} will move downwards to $a_{(r-i+1)i}$ and from $a_{(r-i+1)i}$ through $a_{(r-i+1)(e-i+1)}$ rightwards and then upwards until $a_{i(e-i+1)}$. From $a_{i(e-i+1)}$ the direction of bead positions is left. Based on Lemma 10,

$$Ch: a_{mi}\underline{r - i - m + 1}a_{(r-i+1)i}\underline{e - 2i + 1}a_{(r-i+1)(e-i+1))}\underline{(r - 2i + 1)}a_{i(e+2r-m-5i+3)}\underline{x - 2r - e + 5i + m - 3}a_{(2r+2e-x-6i-m+4)i}$$
.

5. If $2r + 2e - 7i + 4 \leq m + x < 3r + 2e - 9i + 5$.

Since $x > (r - i + 1 - m) + (e - 2i + 1) + (r - 2i + 1) + (e - 2i + 1)$ and based on Definition 5 and Lemma 9, the new location is $\in T_5$. Thus a_{mi}will be moved downwards to $a_{(r-i+1)i}$ and from $a_{(r-i+1)i}$ through $a_{(r-i+1)(e-i+1)}$ rightwards and then upwards until $a_{i(e-i+1)}$. From $a_{i(e-i+1)}$ through a_{ii} the bead positions will move leftwards, and at a_{ii} the bead position will move downward. Based on Lemma 10,

$$Ch: a_{mi}\underline{r - i - m + 1}a_{(r-i+1)i}\underline{e - 2i + 1}a_{(r-i+1)(e-i+1))}\underline{(r - 2i + 1)}\left(a_{i(e+2r-m-5i+3)}\right)$$
$$\underline{(e - 2i + 1)}a_{(x-2r-2e+m+7i-3)i}\underline{(x - 2r - 2e + 7i + m - 4x)}a_{(x-2r-2e+m+8i-4)i}$$

The proofs for case two, case three and case four are similar as case one.

Corollary 15:Let a_{mj} be an element in a rectangle chain in the nested chain abacus with 2 columns, r rows and c chains represented by matrix $A_{r\times 2}$. Then, movement number x is Ch^x

$$a_{mj} \rightarrow \begin{cases} a_{(m+x)1} & if \quad lj = 1, m + x \leqslant r \\ a_{(m-x)2} & if \quad lj = 2, m - x \geqslant 1 \\ a_{(x-m+1)1} & if \quad lj = 2, 1 - r \leqslant m - x < 1 \\ a_{(2r-x+m)2} & if \quad lj = 2, 1 - 2r \leqslant m - x \leqslant -r \\ a_{(2r-x-m+1)2} & if \quad lj = 1, r < m + x \leqslant 2r \\ a_{(x-2r+m)1} & if \quad lj = 1, 2r < m + x \leqslant x - 2r - 1 \end{cases}$$

where $1 \leqslant x \leqslant 2e + 2r - 8i + 3$ for $1 \leqslant m \leqslant r$ and $j = 1, 2e$.

Proof. It follows immediately from the proof in Theorem 13.

Movement in Path Chain

Movement in Path Chain is a chain movement where $d = 0$. Based on Definition 6 there are two designs of path chains. Thus there are two types of movements in path chain; in Lemma 16 the movement in vertical path chain (Ch) was construct if the beads skip one position anticlockwise.

Lemma 16: Let a_{mj} be an element in a vertical-path chain in the nested chain abacus with e columns, r rows and c chains represented by matrix $A_{r\times 2}$. Then, movement number x is Ch^x

$$a_{mj}\rightarrow\begin{cases} a_{(m+1)j} & \text{if } \frac{e+1}{2}\leqslant m<\frac{2r-e+1}{2}, j=\frac{e+1}{2} \\ a_{\left(\frac{e+1}{2}\right)\left(\frac{e+1}{2}\right)} & \text{if } m=\frac{2r-e+1}{2}, j=\frac{e+1}{2} \end{cases}$$

Proof:

Let T_c be the position of the vertical-path chain c where $T_c = \{a_{cc}, a_{(c+1)c}, \dots, a_{(r-c+1)c}\}$ and $a_{cc} = a_{\left(\frac{e+1}{2}\right)\left(\frac{e+1}{2}\right)}$ such that $a_{(r-c+1)c} = a_{\left(\frac{2r-e+12}{2}\right)\left(\frac{e+12}{2}\right)}$ since a_{mj} is a bead position in the path chain and in the bead movement is anticlockwise then, bead position $\{a_{cc}, \dots, a_{(r-c)c}\}$ will skip one position downward and $a_{mj} \longrightarrow a_{(m+`1)j}$ thus

$$a_{\left(\frac{e+1}{2}\right)\left(\frac{e+1}{2}\right)} \longrightarrow a_{\left(\frac{2r-e+1}{2}\right)\left(\frac{e+1}{2}\right)}.$$

Where $\frac{e+1}{2} \leqslant m < \frac{2r-e+1}{2}$ and $j = \frac{e+1}{2}$

Theorem 17: Let a_{mj} be an element in the vertical rectangle-path chain in the nested chain abacus with e columns, r rows and c chains represented by the matrix $A_{r\times e}$. Then, ch^x

$$a_{mj}\rightarrow\begin{cases} a_{(m+x)j} & \text{if } m+x\leq\frac{2r-e+1}{2}, j=\frac{e+1}{2} \\ a_{\left(\frac{2x-2r+2e+2m-2}{2}\right)\left(\frac{e+1}{2}\right)} & \text{if } m+x>\frac{2r-e+1}{2}, j=\frac{e+1}{2} \end{cases}$$

Where $1 \leq x \leq r - e + 1$.

Proof:

LetT_c be a set of the positions of vertical-path chain e for $T_c = \{a_{cc}, a_{(c+1)c}, \dots, a_{(r-c+1)c}\}$and $c = \frac{e+1}{2}$ where x refers to the number of positions that the bead positions will skip to get to the new location. Based on Lemma 5.2.1, the direction of bead position movement is downwards, upwards and downwards. If $m + x \in T_c$ then the bead positions will move downwards and skip x positions. Then

$$a_{(mj)} \xrightarrow{\frac{2r-e-m+1}{2}} a_{\left(\frac{2r-e+1}{2}\right)} \xrightarrow{1} a_{\left(\frac{e+1}{2}\right)\left(\frac{e+1}{2}\right)}$$

and

$$a_{\left(\frac{e+1}{2}\right)} \xrightarrow{\left(\frac{2x-2r+e+2m-3}{2}\right)} a_{\left(\frac{2x-2r+2e+2m-2}{2}\right)\left(\frac{e+1}{2}\right)}$$

Where $m + x \notin T_c$.

Next, the movement in the horizontally-path chain was constructed. First, movement Ch, in other words the beads, will skip one position.

Corollary 18 Let $a_{(mj)}$ be an element in the horizontal rectangle-path chain in the nested chain abacus with e columns, r rows and c chains represented by matrix $A_{r\times e}$. Then, Ch:

$$a_{(mj)}\longrightarrow\begin{cases} a_{m(j+x)} & \text{if } \frac{r+1}{2}\leq j<\frac{2e-r+12}{2}, m=\frac{r+1}{2} \\ a_{\left(\frac{2x-2e+2r+2m-2}{2}\right)\left(\frac{r+1}{2}\right)} & \text{if } m=\frac{r+1}{2}, j=\frac{2e-r+1}{2} \end{cases}$$

where $1 \leq x \leq e - r + 1$.

Proof : it follows immediately from Lemma 16 and theorem 17

In the following theorem the number of possible movements in the horizontal-path chain were determined.

Theorem 19: Let $a_{m\ j}$ be an element in the horizontal-path chain in the nested chain abacus with e columns, r rows and c chains represented by matrix $A_{r\times e}$. Then, the total number of movements of each position in the path-chain is$e - r + 1$

Proof. Let $a_{m\ j}$ be a position in the horizontal-path for nested chain abacus with e columns, r rows and c chains. Based on Lemma 44, $a_{m\ j}$ can be moved rightwards and leftwards depending on m. Therefore

$$a_{\left(\frac{r+1}{2}\right)\left(\frac{r+1}{2}\right)} \rightarrow a_{\left(\frac{r+1}{2}\right)\left(\frac{r+3}{2}\right)} \rightarrow \cdots \rightarrow a_{\left(\frac{r+1}{2}\right)\left(\frac{2e-r+1}{2}\right)} \rightarrow a_{\left(\frac{r+1}{2}\right)\left(\frac{r+1}{2}\right)}$$

Then $a_{\left(\frac{r+1}{2}\right)\left(\frac{2e-r+1}{2}\right)} \rightarrow a_{\left(\frac{r+1}{2}\right)\left(\frac{r+1}{2}\right)}$ where $\frac{e+1}{2} \leq m \leq \frac{2e-r+1}{2\rightarrow}$and $= c$, thus $a_{m\ j}$ will be move $\frac{2e-r+1}{2} - \frac{r+1}{2} + 1 = e - r + 1$Next, movement Ch^x where the beads will skip x positions was developed.

Theorem 20: Let $a_{m\ j}$ be an element in matrix $A_{r\times e}$ which represents the beadand empty bead positions in the horizontal-rectangle path chain with e columns and $r$rows. Then, the chain movement Ch^x is:

$$a_{mj}\rightarrow\begin{cases} a_{m(j+x)} & \text{if } \frac{r+1}{2}<j<\frac{2e-r+1}{2}, m=\frac{r+1}{2} \\ a_{\left(\frac{2x-2e+2r+2m-2}{2}\right)\left(\frac{r+1}{2}\right)} & \text{if } m=\frac{r+1}{2}, j=\frac{2e-r+1}{2} \end{cases}$$

Where $1 \leq x \leq r - e + 1$.

Theorem 2:1 Let $a_{m\ j}$ be an element in the horizontal-path chain in the nested chain abacus with e columns, r rows and c chains represented by matrix $A_{r\ e}$. Then, the total number of movement of each position in the path-chain is$r - e + 1$.

Proof. Let $a_{m\ j}$ be a position in the horizontal-path for nested chain abacus with e columns, r rows and c chains. Based on Lemma 44, $a_{m\ j}$ can be moved rightwards and leftwards depending on m. Therefore

$$a_{\left(\frac{e+1}{2}\right)\left(\frac{e+1}{2}\right)} \to a_{\left(\frac{e+1}{2}\right)\left(\frac{e+3}{2}\right)} \to \cdots \to a_{\left(\frac{e+1}{2}\right)\left(\frac{2r-e+1}{2}\right)} \to a_{\left(\frac{e+1}{2}\right)\left(\frac{e+1}{2}\right)}$$

Thus $a_{m\,j}$ will skip

$$\frac{2r-e+1}{2} - \frac{e+1}{2} + 1 = e - r + 1$$

Nested Chain Abacus Movement Algorithm

This section begins by formulating the conceptual framework used to structure a new abacus. This is done by converting nested chain abacus to matrix form where any bead position as well as empty bead position in the nested chain abacus is represented as the element of a matrix and then applying the new movement.

This is followed by the development of three different types of nested chain abacus movement which are single nested chain abacus movement with $e = 2$ (SNC2-Movement), stratum nested chain abacus movement with $e > 2$ (SNC-Movement) and multiple chain movement (MNC-Movement).

SNC2-Movement

A movement of a nested chain abacus with a chain is called SNC2-Movement construction.

Step 1: Convert the nested chain abacus with n beads and one chain into $A_{r\times 2}$.

Step 2: Select a_{mj} as an initial point where a_{mj} is an element in the $r \times e$ matrix.

Step 3: Generate different nested chain abacus $\mathfrak{N}$ with one chain by employing Ch^x with 2 columns (see Corollary 14), based on Theorem 11 with $e = 2, 1 \leqslant x \leqslant 2r$.

Make SNC2-Movement $\boldsymbol{Ch^3}$ inchain 2 as illustrated in Fig. 2.

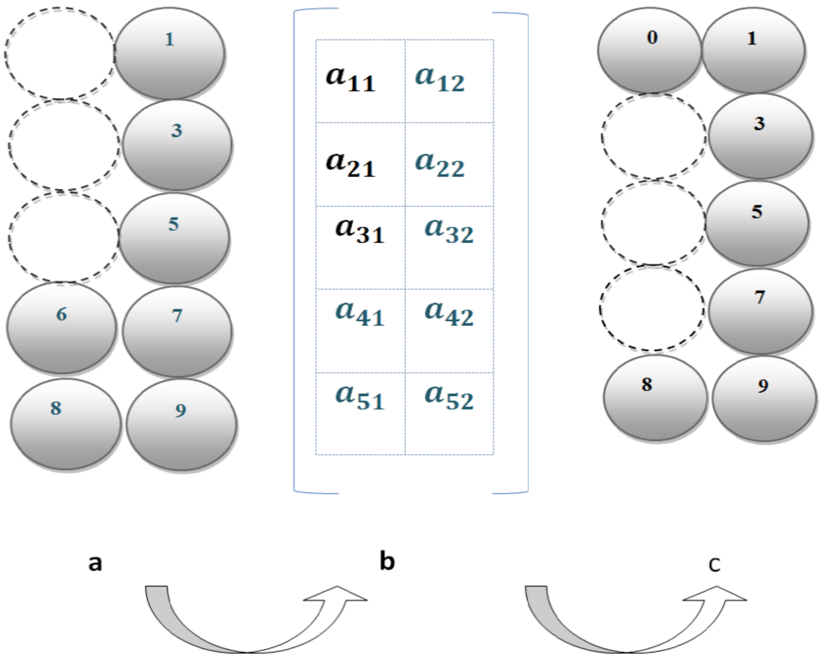

Figure2.Nested chain abacus with one chain in (a) and the result of applying $\mathbf{Ch^1}$ in (c)

Consider Fig.2 of 10 beads the movement in rectangle chain Ch^x, where $x = 9$was employed. Based on Corollary15, then, the new location of beads is as shown in Table 2. For example a_{41}move by Ch to a_{51}, by Ch^2 to a_{52}, by Ch^3 to a_{42}, by Ch^4 to a_{32}, by Ch^5 to a_{22}, by Ch^6 to a_{12}, by Ch^7 to a_{11}, by Ch^8 to a_{21} and by Ch^9 to a_{31}.

Table 2. New location of the positions in the nested chain abacus by SNC2-Movement

a_{mj}	$\rightarrow$	Ch	Ch^2	Ch^3	Ch^4	Ch^5	Ch^6	Ch^7	Ch^8	Ch^9
a_{41}	$\rightarrow$	a_{51}	a_{52}	a_{42}	a_{32}	a_{22}	a_{12}	a_{11}	a_{21}	a_{31}
a_{51}	$\rightarrow$	a_{52}	a_{42}	a_{32}	a_{22}	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}
a_{12}	$\rightarrow$	a_{11}	a_{21}	a_{31}	a_{41}	a_{51}	a_{52}	a_{42}	a_{32}	a_{22}
a_{22}	$\rightarrow$	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}	a_{51}	a_{52}	a_{42}	a_{32}
a_{32}	$\rightarrow$	a_{22}	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}	a_{51}	a_{52}	a_{42}
a_{42}	$\rightarrow$	a_{32}	a_{22}	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}	a_{51}	a_{52}
a_{52}	$\rightarrow$	a_{42}	a_{32}	a_{22}	a_{12}	a_{11}	a_{21}	a_{31}	a_{31}	a_{51}

SNC-Movement

A movement of a nested chain abacus with c chains is called SNC-Movement construction. In SNC-Movement, the chain movementhas been employed in only one chain.

Step 1: Convert the nested chain abacus into $A_{r\times e}$.

Step 2: Select the chain i and then a_{mj} as an initial point where $i \leqslant j \leqslant e - i + 1$ and $i \leqslant m \leqslant r - i + 1$.

step 3: Generate different nested chain abacus with c chain by employing

- Theorem 13 to find the movement Ch^x in chain i if chain i is rectangle chain, based on Theorem 11, $1 \leqslant x \leqslant 2e + 2r - 8i + 3$ or,
- Theorem 16 to find the movement Ch^x in chain c if chain c is vertical-path chain, based on Theorem 19, $1 \leqslant x \leqslant r - e + 1$ or,
- Corollary 5.2.3 to find the movement Ch^x in chain c if chain c is horizontal-path chain, based on Theorem 5.2.4, $1 \leqslant x \leqslant e - r + 1$ or,
- If chain c is singleton chain then$a_{mj} \rightarrow a_{mj}$.

Make SNC-Movement $\mathbf{Ch^3}$ in chain 2 as illustrated in Fig. 3.

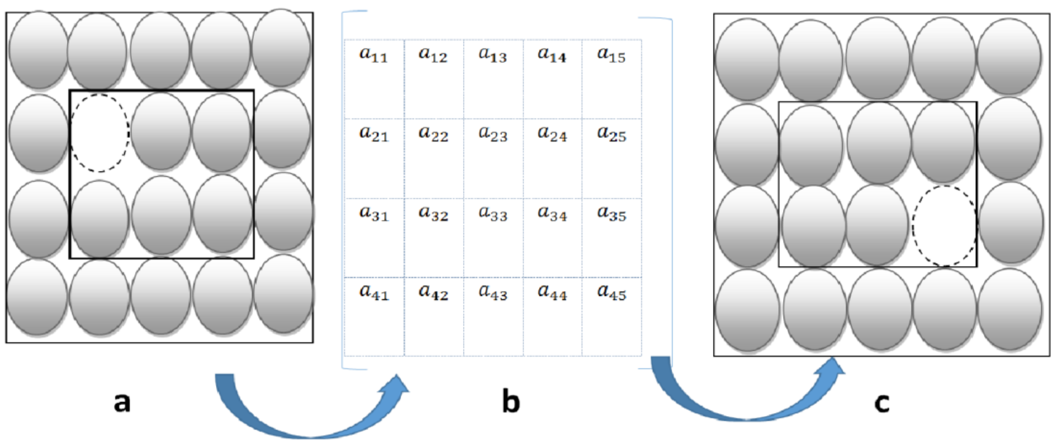

Figure 3. a- Nested chain abacus of $\mu^{(5,4)}$ b- convert nested chain abacus to matrix c- Nested chain abacus of $\mu^{(5,4)}$.

Consider Fig. 3 of 19 beads the new location of beads is as shown in Table 3.

Table 3. New location of the positions in the nested chain abacus by SNC-Movement

a_{mj}	$\rightarrow$	Ch	Ch^2	Ch^3
a_{22}	$\rightarrow$	a_{32}	a_{33}	a_{34}
a_{32}	$\rightarrow$	a_{33}	a_{34}	a_{24}
a_{33}	$\rightarrow$	a_{34}	a_{24}	a_{23}
a_{34}	$\rightarrow$	a_{24}	a_{23}	a_{22}
a_{24}	$\rightarrow$	a_{23}	a_{22}	a_{32}
a_{23}	$\rightarrow$	a_{22}	a_{32}	a_{33}

MNC-Movement

A movement of a nested chain abacus with c chains called MNC-Movement construction.

Step 1: Converted the nested chain abacus into $A_{r\times e}$.

Step 2: Select c element in c chains as an initial point where:

Step 3: Generate different nested chain abacus with c chain by employing

- Theorem 13 to find the movement Ch^x in all chains if $\mathfrak{N}$ is rectangular nested chain abacus, based on Theorem 5.1.2, $1 \leqslant x \leqslant 2e + 2r - 8i + 3$ or,
- Theorem 13 to find the movement Ch^x in chain i where $1 \leqslant i < c$, based on Theorem

11, $1 \leqslant x \leqslant 2e + 2r - 8i + 3$. In addition, employ Theorem 16to find the movement Ch^x in chain c, based on Theorem 18, $1 \leqslant x \leqslant r - e + 1$, if $\mathfrak{N}$ is vertical rectangle-path nested chain abacus or,

- Theorem 13 to find the movement Ch^x in chain i where $1 \leqslant i < c$, based on Theorem 11, $1 \leqslant x \leqslant 2e + 2r - 8i + 3$. In addition, employ Theorem 19 to find the movement Ch^x in chain c, based on Theorem 18,$1 \leqslant x \leqslant e - r + 1$, if $\mathfrak{N}$ is horizontal-path rectangle nested chain abacus or,
- Theorem 13 to find the movement Ch^x in chain i where $1 \leqslant i < c$, based on Theorem 11, $1 \leqslant x \leqslant 2e + 2r - 8i + 3$.

Make MNC-Movement Ch^9 in nested chain abacus with 10-beads as illustrated in Fig. 4.

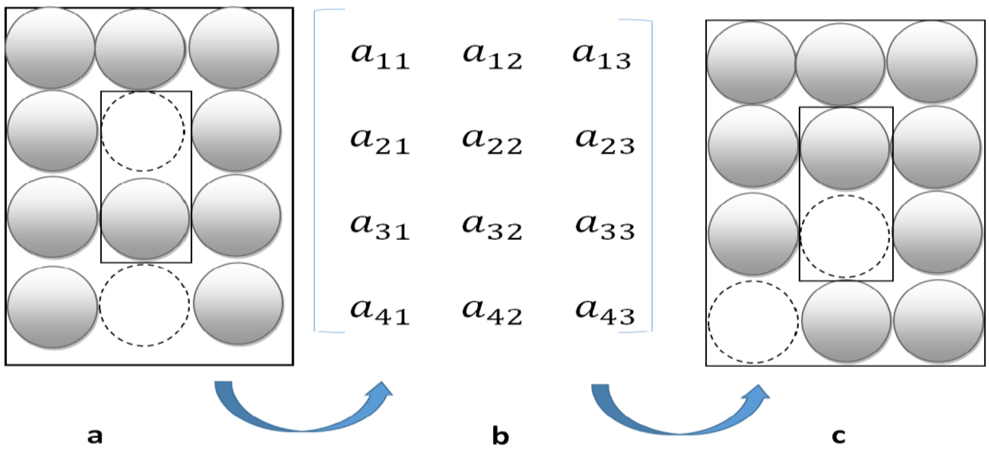

Figure 4. a- Nested chain abacus and b- convert nseted chain abacus to matrix c- Nested chain abacus after employ Ch^9

Consider Fig. 4 of 14 beads movement Ch^9in chain 1 and movement Ch^1 in chain 2 were employed. The new location of beads is as shown in Tabel4.

Table 4. The new location of beads 10 beads where Ch^9 movement employ in chain 1 and Ch^9 movement employed in chain 2

a_{mj}	$\longrightarrow$	Ch^1	Ch^2	Ch^3	Ch^4	Ch^5	Ch^6	Ch^7	Ch^8	Ch^9
a_{11}	$\longrightarrow$	a_{21}	a_{31}	a_{41}	a_{42}	a_{43}	a_{33}	a_{23}	a_{13}	a_{12}
a_{21}	$\longrightarrow$	a_{31}	a_{41}	a_{42}	a_{43}	a_{33}	a_{23}	a_{13}	a_{12}	a_{11}
a_{31}	$\longrightarrow$	a_{41}	a_{42}	a_{43}	a_{33}	a_{23}	a_{13}	a_{12}	a_{11}	a_{21}
a_{41}	$\longrightarrow$	a_{42}	a_{43}	a_{33}	a_{23}	a_{13}	a_{12}	a_{11}	a_{21}	a_{31}
a_{42}	$\longrightarrow$	a_{43}	a_{33}	a_{23}	a_{13}	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}
a_{43}	$\longrightarrow$	a_{33}	a_{23}	a_{13}	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}	a_{42}
a_{33}	$\longrightarrow$	a_{23}	a_{13}	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}	a_{42}	a_{43}
a_{23}	$\longrightarrow$	a_{13}	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}	a_{42}	a_{43}	a_{33}
a_{13}	$\longrightarrow$	a_{12}	a_{11}	a_{21}	a_{31}	a_{41}	a_{42}	a_{43}	a_{33}	a_{32}
a_{12}	$\longrightarrow$	a_{11}	a_{21}	a_{31}	a_{41}	a_{42}	a_{43}	a_{33}	a_{23}	a_{13}
a_{22}	$\longrightarrow$	a_{32}	a_{22}	a_{32}	a_{22}	a_{32}	a_{22}	a_{32}	a_{22}	a_{32}

algorithm of nested chain abacus movement

File Number one

\clc kkk=0;clear; nt=0; v=1;

global Tmat; r = input(’ numbers of rows ’) c = input(’ numbers of columns ’) mat = ones(r,c); Tmat=mat; r,c = size(mat) ; ch=21; path=0; tn=0; fl=[r,c] ; tmpv=min(fl)/2; tpn1=ceil(tmpv) while path 0) break end n=n+1; if (tn>=totn) break end mat(i,j)=v; tn=tn+1; pt(tn,4)=j; pt(tn,3)=i; pt(tn,1)=tn; pt(tn,2)=path; if ((i0)) i=i+1; else if((j0)); j=j+1; sti=-1; else if (i>si) i=i-1; stj=-1; else if (j>sj) j=j-1; if ((i==si)(j==sj)) sti=1;stj=1; end end end end end i; j; A1 = path; A2 = n; end end s=size(pt); c=s(1); cp=1 i=1; clc k=1; pathStart(1)=1; for i=1:s if (i>1) if pt(i,2) =pt(i-1,2) k=k+1; pathStart(k)=i; end endend k=k+1; pathStart(k)=s(1) k(1:tpn1)=0; clc p01=1 No=0; pt(:,5)=1; totalNss =size(pt) tpn=max(pt(:,2)); for i=1:tpn i pn(i)=input(’ÚÏÏ ÇáÚäÇÕÑáááãÓÇÑ ’) end for p=1:length(pn) for i=1:totalN if(pt(i,2)==p)(k(p) pt(i,5)=11; k(p)=k(p)+1; end ii=pt(i,3); jj=pt(i,4); mat(ii,jj)= pt(i,5); end end Ax=0 ii=0; jj=0; **ccc=length(x1) Ax(1:pathStart(2)-

1,1:5,1:3) =-1 k=0 for i=1:tpn i1=pathStart(i) if (i==tpn) i2=pathStart(i+1) else i2=pathStart(i+1)-1 end * x1=pt(pathStart(1):pathStart(2)-1,:) x=pt(i1:i2,:) k=k+1; rrr=size(x) pL(k)=rrr(1) Ax(1:pL(k),:,i)=x; * Ax(1:length(x),:,i)=x;
end Ax(1:pL(i),:,i) clc ** x1=Ax(1:pL(1),:,1); ** x2=Ax(1:pL(2),:,2); ** x3=Ax(1:pL(3),:,3); s=0 N=0 mTem=0 i=1 global Nx1 Nx1=0; LoopF(i,Ax,pL,mat)
F3
File Number Two
global Nx1 global Tmat for t=1:sxv ii=xt(t,3); jj=xt(t,4); tt=xt(t,5) mat(ii,jj)= tt; end
mat r,c = size(mat) ; if (isequal(mTem,mat)) dlmwrite('Rtxt',mat,'-append','delimiter',' ','roffset',1);
****
imagesc((1:c)+0.5,(1:r)+0.5,mat); colormap(winter); axis equal ; N=N+1 ; set(gca,'XTick',1:(c),'YTick',1:(r),... 'XLim',[1 c+1],'YLim',[1 r+1],... 'GridLineStyle','-','XGrid','on','YGrid','on'); rndd1 = 1 rndd2 = 1 Nx1=Nx1+1 Tmat(:,:,Nx1)=mat; s=sprintf('000 saveas(gcf,s);
** end
mTem=mat; File Number Three a b w =size(Tmat) Tmat2=Tmat(:,:,:)
Tmat3=Tmat(:,:,1) k=0; m1=Tmat(:,:,1); m2=Tmat2(:,:,1); kk=1 for i=1:w t=1 t=0 for j=i+1:w kk=kk+1 m1=Tmat(:,:,i); m2=Tmat2(:,:,j); if Tmat(:,:,i)== Tmat2(:,:,j) t=1; end if k==150 nnn=2 end end if (t==0) k=k+1 Tmat3(:,:,k)= Tmat(:,:,i)

Conclusions:

In this research a new combinatorial interpretation called nested chain abacus was used to encode any tile. Furthermore, the research presented and formated three type of movement. Based on theses movementsan Algorithems of SNC2-Movement, SNC2-Movement and MNC-Movementwere developed to constract new abacus (tile).

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Author's declaration:

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Besides, the Figures and images, which are not mine, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Mustansiriyah.

Authors' contributions statement:

E.F. contributeda new combinatorial interpretation called nested chain abacus which used to formated three type of movement **.**

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البلاط باستخدام سلسلة متداخلة للمعداد

ايمان فاضل محمد

قسم الرياضيات، كلية التربية، الجامعة المستنصرية، بغداد، العراق.

الخلاصة:

هذه الدراسة نجحت في ايجاد تمثيل جديد لمعداد James يدعى معداد السلاسل المتداخلة. المعداد الجديد وفر لنا تمثيل رياضي وحيد ل لتشفير اي بلاطه (صوره) باستخدام نظرية التجزئة بحيث ان كل شكل او صوره للبلاط سوف يقترن بتجزئة واحدة فقط بالإضافة الى هذا انشاء خوارزمية لحركة معداد السلاسل المتداخلة وهذه الحركة ممكن الاستفادة منها في نظرية التبليط.

الكلمات المفتاحية: المعداد، السلاسل، الحركات، التجزئة، البلاط