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Some Properties of Fuzzy Neutrosophic Generalized Semi Continuous Mapping and Alpha Generalized Continuous Mapping

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Abstract:

In the current study, the definition of mapping of fuzzy neutrosophic generalized semi-continuous and fuzzy neutrosophic alpha has generalized mapping as continuous. The study confirmed some theorems regarding such a concept. In the following, it has been found relationships among fuzzy neutrosophic alpha generalized mapping as continuous, fuzzy neutrosophic mapping as continuous, fuzzy neutrosophic alpha mapping as continuous, fuzzy neutrosophic generalized semi mapping as continuous, fuzzy neutrosophic pre mapping as continuous and fuzzy neutrosophic γ mapping as continuous.

Keywords: Fuzzy neutrosophic topological space, Fuzzy neutrosophic generalized semi closed set, Fuzzy neutrosophic alpha generalized closed set, Fuzzy neutrosophic generalized semi mapping as continuous, Fuzzy neutrosophic alpha generalized mapping as continuous.

Introduction:

Zadeh (1965) introduced fuzzy concept set [1]. Chang [2] later made use of this concept to present the concept of fuzzy topological space in 1968. Atanassov in 1986 [3] developed the fuzzy concept set into the concept of fuzzy intuitionistic set. At this new study, the concept gives membership degree and non-membership degree functions. In 1997 [4], the intuitionistic fuzzy topological space concept was presented via Cokor relying on the concept of intuitionistic fuzzy set. In 2013 the concept of fuzzy neutrosophic set was presented via Arockiarani Sumathi and Martina Jency, this set provides membership degree, non-membership degree and indeterminacy degree. Additionally, the fuzzy neutrosophic topological space has isen defining via them in 2014 [5]. The concept of intuitionistic fuzzy alpha generalized mapping as continuous and intuitionistic fuzzy alpha generalized irresolute mapping has is studied in 2014via Sakthivel [6].

At the current study, the concept of fuzzy neutrosophic generalized semi closed set, fuzzy neutrosophic alpha generalized closed set, fuzzy neutrosophic generalized semi- mapping as continuous and fuzzy neutrosophic alpha generalized mapping as continuous has been introduced.

Preliminaries:

Definition 1. [7]**:** Suppose X is a non-empty constant set. The fuzzy set as neutrosophic (Briefly, S_{FN}), β_N is an object with form $\beta_N = \{< \omega, \mu_{\beta N}(\omega), \sigma_{\beta N}(\omega), \nu_{\beta N}(\omega) >: \omega \in X\}$ while the functions $\mu_{\beta N}$, $\sigma_{\beta N}$, $\nu_{\beta N} : X \to [0, 1]$ denoted membership function degree (named $\mu_{\beta N}(\omega)$), indeterminacy function degree (named $\sigma_{\beta N}(\omega)$) and non-membership degree (named $\nu_{\beta N}(\omega)$) respectively of every element $\omega \in X$ to the set β_N and $0 \leq \mu_{\beta N}(\omega) + \sigma_{\beta N}(\omega) + \nu_{\beta N}(\omega) \leq 3$, for every $\omega \in X$.

Remark 1. [7]**:** S_{FN} $\beta_N = \{< \omega, \mu_{\beta N}(\omega), \sigma_{\beta N}(\omega), \nu_{\beta N}(\omega) >: \omega \in X\}$ might be identwheny to an ordered triple $< \omega, \mu_{\beta N}(\omega), \sigma_{\beta N}(\omega), \nu_{\beta N}(\omega) >$ in [0, 1] on X.

Definition 2. [7]**:** Suppose X is a set of non-empty and the Ss_{FN} β_N and γ_N is in the form:

$\beta_N = \{< x, \mu_{\beta N}(\omega), \sigma_{\beta N}(\omega), \nu_{\beta N}(\omega) >: \omega \in X\}$ and, $\gamma_N = \{< \omega, \mu_{\gamma N}(\omega), \sigma_{\gamma N}(\omega), \nu_{\gamma N}(\omega) >: \omega \in X\}$ on X. So that:

i. $\beta_N \subseteq \gamma_N$, whenf $\mu_{\beta N}(\omega) \leq \mu_{\gamma N}(\omega)$, $\sigma_{\beta N}(\omega) \leq \sigma_{\gamma N}(\omega)$ and $\nu_{\beta N}(\omega) \geq \nu_{\gamma N}(\omega)$ for all $\omega \in X$.

ii. $\beta_N = \gamma_N$, whenf $\beta_N \subseteq \gamma_N$ and $\gamma_N \subseteq \beta_N$.
iii. $(\beta_N)^c = \{< \omega, \nu_{\beta N}(\omega), 1 - \sigma_{\beta N}(\omega), \mu_{\beta N}\ \omega) >: \omega \in X\}$.
iv. $\beta_N \cup \gamma_N = \{< \omega, Max(\mu_{\beta N}(\omega), \mu_{\gamma N}(\omega)), Max(\sigma_{\beta N}(\omega), \sigma_{\gamma N}(\omega)), Min(\nu_{\beta N}(\omega), \nu_{\gamma N}(x))>: \omega \in X\}$.
v. $\beta_N \cap \gamma_N = \{< \omega, Min(\mu_{\beta N}(\omega), \mu_{\gamma N}(\omega)), Min(\sigma_{\beta N}(\omega), \sigma_{\gamma N}(\omega)), Max(\nu_{\beta N}(\omega), \nu_{\gamma N}(\omega))>: \omega \in X\}$.
vi. $0^N = < \omega, 0, 0, 1>$ and $1^N = < \omega, 1, 1, 0 >$.

Definition 3. [7]**:** Fuzzy neutrosophic topology (Briefly, T_{FN}) on a set of non-empty X is a family τ^N of fuzzy neutrosophic subsets in X satisfying axioms as follows:
i. $0^N, 1^N \in \tau^N$.
ii. $\beta_{N1} \cap \beta_{N2} \in \tau^N$ for every $\beta_{N1}, \beta_{N2} \in \tau^N$.
iii. $\cup\ \beta_{Ni} \in \tau^N, \forall\{\beta_{Ni}: i \in J\} \subseteq \tau^N$.

In such case, the pair (X, τ^N) is said to be fuzzy neutrosophic space topological (Briefly, TS_{FN}). The elements of τ^N are fuzzy neutrosophic open set (Briefly, OS_{FN}). The complement of OS_{FN} in the TS_{FN} (X, τ^N) is fuzzy neutrosophic closed set (Briefly, CS_{FN}).

Definition 4. [7]**:** Suppose (X, τ^N) is TS_{FN} and $\beta_N = < \omega, \mu_{\beta N}(\omega), \sigma_{\beta N}(\omega), \nu_{\beta N}(\omega) >$ is S_{FN} in X. So that, the fuzzy β_N neutrosophic closure (Briefly, Cl_{FN}) and fuzzy β_N neutrosophic interior (Briefly, In_{FN}) are defining via:
$Cl_{FN}(\beta_N) = \cap\ \{C_N: C_N$ is CS_{FN} in X and $\beta_N \subseteq C_N\}$,
$In_{FN}(\beta_N) = \cup\ \{O_N: O_N$ is OS_{FN} in X and $O_N \subseteq \beta_N\}$.
Known that, $Cl_{FN}(\beta_N)$ be CS_{FN} and $In_{FN}(\beta_N)$ be OS_{FN} in X. Moreover.
i. β_N is CS_{FN} in X, whenf $Cl_{FN}(\beta_N) = \beta_N$.
ii. β_N is OS_{FN} in X, whenf $In_{FN}(\beta_N) = \beta_N$.

Proposition 1. [7]**:** Suppose (X, τ^N) be TS_{FN} and β_N, γ_N are Ss_{FN} in X. Then, those properties follow hold:
i. $In_{FN}(\beta_N) \subseteq \beta_N$ and $\beta_N \subseteq Cl_{FN}(\beta_N)$.
ii. $\beta_N \subseteq \gamma_N \Longrightarrow In_{FN}(\beta_N) \subseteq In_{FN}(\gamma_N)$ and $\beta_N \subseteq \gamma_N \Longrightarrow Cl_{FN}(\beta_N) \subseteq Cl_{FN}(\gamma_N)$.
iii. $In_{FN}(In_{FN}(\beta_N)) = In_{FN}(\beta_N)$ and $Cl_{FN}(Cl_{FN}(\beta_N)) = Cl_{FN}(\beta_N)$.
iv. $In_{FN}(\beta_N \cap \gamma_N) = In_{FN}(\beta_N) \cap In_{FN}(\gamma_N)$ and $Cl_{FN}(\beta_N \cup \gamma_N) = Cl_{FN}(\beta_N) \cup Cl_{FN}(\gamma_N)$.
v. $In_{FN}(1^N) = 1^N$ and $Cl_{FN}(1^N) = 1^N$.
vi. $In_{FN}(0^N) = 0^N$ and $Cl_{FN}(0^N) = 0^N$.

Definition 5. [8]**:** The S_{FN} β_N in TS_{FN} (X, τ^N) is called:
i. Fuzzy neutrosophic closed regular set (Briefly, RCS_{FN}), when $\beta_N = Cl_{FN}(In_{FN}(\beta_N))$.
ii. Fuzzy neutrosophic pre closed set (Briefly, PCS_{FN}), when $Cl_{FN}(In_{FN}(\beta_N)) \subseteq \beta_N$.
iii. Fuzzy neutrosophic semi open set (Briefly, SOS_{FN}), when $\beta_N \subseteq Cl_{FN}(Int_{FN}(\beta_N))$.
iv. Fuzzy neutrosophic semi closed set (Briefly, SCS_{FN}), when $In_{FN}(Cl_{FN}(\beta_N)) \subseteq \beta_N$.
v. Fuzzy neutrosophic α open set (Briefly, αOS_{FN}), when $\beta_N \subseteq In_{FN}(Cl_{FN}(In_{FN}(\beta_N)))$.
vi. Fuzzy neutrosophic α closed set (Briefly, αCS_{FN}), when $Cl_{FN}(In_{FN}(Cl_{FN}(\beta_N))) \subseteq \beta_N$.

Definition 6. [9]**:** Fuzzy neutrosophic sub set β_N of TS_{FN} (X, τ^N) is named fuzzy neutrosophic generalized closed set (Briefly, GCS_{FN}), when $Cl_{FN}(\beta_N) \subseteq U_N$ wherever, $\beta_N \subseteq U_N$ and U_N is OS_{FN} in X. And β_N be said as fuzzy neutrosophic generalized open set (Briefly, GOS_{FN}), when the complement $(\beta_N)^c$ is GCS_{FN} set in (X, τ^N).

Definition 7. [10]**:** Let $\beta_N = \{< \xi, \mu_{\beta N}(\xi), \sigma_{\beta N}(\xi), \nu_{\beta N}(\xi) >: \xi \in Y\}$ is S_{FN} in Y. So that, the β_N inverses image under f, $(f^{-1}(\beta_N))$ is S_{FN} in X defined:
$f^{-1}(\beta_N) = \{< \omega, f^{-1}(\mu_{\beta N})(\omega), f^{-1}(\sigma_{\beta N})(\omega), f^{-1}(\nu_{\beta N})(\omega) >: \omega \in X\}$ where,
$f^{-1}(\mu_{\beta N})(\omega) = \mu_{\beta N} f(\omega)$, $f^{-1}(\sigma_{\beta N})(\omega) = \sigma_{\beta N} f(\omega)$ and $f^{-1}(\nu_{\beta N})(\omega) = \nu_{\beta N} f(\omega)$.

Definition 8. [10 - 12]**:** Suppose $(X, \tau^{N\omega})$ and **(**$Y, \tau^{N\xi}$) are TSs_{FN}. So that, a mapping $f : (X, \tau^{N\omega}) \rightarrow (Y, \tau^{N\xi})$ is called :
i. Fuzzy neutrosophic continuous ($Con._{FN}$), such that $f^{-1}(\beta_N)$ is CS_{FN} in $(X, \tau^{N\omega})$ for every CS_{FN} in $(Y, \tau^{N\xi})$.
ii. Fuzzy neutrosophic pre continuous ($Pcon._{FN}$), such that $f^{-1}(\beta_N)$ is PCS_{FN} in (X, τ^{Nx}) for every CS_{FN} in (Y, τ^{NY}).
iii. Fuzzy neutrosophic α continuous ($\alpha\ con._{FN}$), such that $f^{-1}(\beta_N)$ is αCS_{FN} in (X, τ^{Nx}) for every CS_{FN} in (Y, τ^{NY}).

Fuzzy Neutrosophic Generalized Semi Closed Set and Fuzzy Neutrosophic Alpha Generalized Closed Set.

In this section, the concept of fuzzy neutrosophic generalized semi closed set and the fuzzy neutrosophic alpha generalized closed set was introduced and studied some of its properties.

Definition 9. :
i. Suppose (X, τ^N) is TS_{FN} and $\beta_N = <\omega, \mu_{\beta N}(\omega), \sigma_{\beta N}(\omega), \nu_{\beta N}(\omega) >$ is S_{FN} in X. So that the fuzzy neutrosophic semi closure of β_N (Briefly, SCl_{FN}) and fuzzy neutrosophic semi interior of β_N (Briefly, SIn_{FN}) are defining via:
$SCl_{FN}(\beta_N) = \cap\{C_N: C_N$ is SCS_{FN} in X and $\beta_N \subseteq C_N\}$,
$SIn_{FN}(\beta_N) = \cup\{O_N: O_N$ is SOS_{FN} in X and $O_N \subseteq \beta_N\}$.
ii. Suppose (X, τ^N) is TS_{FN} and $\beta_N = <\omega, \mu_{\beta N}(\omega), \sigma_{\beta N}(\omega), \nu_{\beta N}(\omega) >$ is S_{FN} in X. So that the fuzzy neutrosophic β_N alpha closure (Briefly, αCl_{FN}) and fuzzy neutrosophic β_N alpha interior (Briefly, αIn_{FN}) are defining via:
$\alpha Cl_{FN}(\beta_N) = \cap\{C_N: C_N$ is αCS_{FN} in X and $\beta_N \subseteq C_N\}$,
$\alpha In_{FN}(\beta_N) = \cup\{O_N: O_N$ is αOS_{FN} in X and $O_N \subseteq \beta_N\}$.

Know that αCl_{FN} (β_N) is CS_{FN} and αIn_{FN} (β_N) is OS_{FN} in X. Moreover.

a. β_N is αCS_{FN} in X whenf αCl_{FN} (β_N) = β_N.

b. β_N is αOS_{FN} in X whenf αIn_{FN} (β_N) = β_N.

Definition 10. : Fuzzy neutrosophic subset β_N of TS_{FN} (X, τ^N) is called:

i. Fuzzy neutrosophic γ closed set (Briefly, γCS_{FN}), such that $Cl_{FN}(In_{FN}$ (β_N)) ∩ $In_{FN}(Cl_{FN}$ (β_N))⊆ β_N.

ii. Fuzzy neutrosophic generalized semi closed set (Briefly, $GSCS_{FN}$), when SCl_{FN} (β_N) ⊆ U_N such that, β_N ⊆ U_N and U_N is OS_{FN} in X.

iii. Fuzzy neutrosophic alpha generalized closed set (Briefly, αGCS_{FN}), when αCl_{FN} (β_N) ⊆ U_N such that, β_N ⊆ U_N and U_N is OS_{FN} in X.

Theorem 1. : For every Ss_{FN}, the statements as following are true in general:

i. Every CS_{FN} is αGCS_{FN}.

ii. Every RCS_{FN} is αGCS_{FN}.

iii. Every αCS_{FN} is αGCS_{FN}.

iv. Every αGCS_{FN} is $GSCS_{FN}$.

Proof:

i. Suppose β_N = < ω, $\mu_{\beta N}$ (ω), $\sigma_{\beta N}$ (ω), $\nu_{\beta N}$ (ω) > be CS_{FN} in the TS_{FN} (X, τ^N).
So that via definition 4. (i). Hence Cl_{FN} (β_N) = β_N … (1).
Now, suppose the U_N is OS_{FN} such that, β_N ⊆ U_N. Since αCl_{FN} (β_N) ⊆Cl_{FN} (β_N) via definition 4. and definition 9. ii. So, αCl_{FN} (β_N) ⊆ Cl_{FN} (β_N) = β_N ⊆ U_N. Hence, β_N is αGCS_{FN} in (X,τ^N).

ii. Suppose β_N = < ω, $\mu_{\beta N}$ (ω), $\sigma_{\beta N}$ (ω), $\nu_{\beta N}$ (ω) > , be RCS_{FN} in the TS_{FN} (X, τ^N). So that,
Cl_{FN} (In_{FN}(β_N)) = β_N … (1)
This implies, Cl_{FN} (In_{FN} (β_N)) = Cl_{FN} (β_N) … (2).
Suppose the U_N be OS_{FN} such that, β_N ⊆ U_N. From (1) and (2), Cl_{FN} (β_N) = β_N.
That β_N is CS_{FN} in X. So by (i), αCl_{FN} (β_N) ⊆ Cl_{FN} (β_N) = β_N ⊆ U_N. Hence, β_N is αGCS_{FN} in (X,τ^N).

iii. Suppose β_N = < ω, $\mu_{\beta N}$ (ω), $\sigma_{\beta N}$ (ω), $\nu_{\beta N}$ (ω) > is αCS_{FN} in the TS_{FN} (X, τ^N).
So that, αCl_{FN} (β_N) = β_N. Now, assume the U_N is OS_{FN} i.e., β_N ⊆ U_N. So, αCl_{FN} (β_N) = β_N ⊆ U_N.
Hence, β_N is αGCS_{FN} in (X, τ^N).

iv. Suppose β_N = < ω, $\mu_{\beta N}$ (ω), $\sigma_{\beta N}$ (ω), $\nu_{\beta N}$ (ω) > be αGCS_{FN} in the TS_{FN} (X, τ^N). So that, αCl_{FN} (β_N) ⊆ U_N. Where, U_N is OS_{FN} such that, β_N ⊆ U_N.
So, Cl_{FN} (In_{FN} (Cl_{FN} (β_N))) ⊆ U_N. That is In_{FN} (Cl_{FN} (β_N)) ⊆ U_N. Therefore, SCl_{FN} (β_N)⊆U_N. Where, U_N is OS_{FN} such that, β_N ⊆ U_N. Hence, β_N is $GSCS_{FN}$ in (X, τ^N).

Remark 2. : The foregoing theorem converse is not true as illustrated in the following examples:

Example 1.:

i. Suppose X= {c, d} defined S_{FN} β_N in X as follows:
β_N = <ω, (0.2$_{(c)}$, 0.2$_{(d)}$), (0.5$_{(c)}$, 0.5$_{(d)}$), (0.6$_{(c)}$,0.6$_{(d)}$) >. The family τ^N = {0^N, 1^N, β_N} is T_{FN}. When take, Φ_N= <ω, (0.3$_{(c)}$, 0.2$_{(d)}$), (0.5$_{(c)}$, 0.5$_{(d)}$), (0.6$_{(c)}$, 0.6$_{(d)}$) >. So that, ω_N is αGCS_{FN} , but not CS_{FN}.

ii. Take **i**. So that, Φ_N is αGCS_{FN}, but not RCS_{FN}.

iii. Also take (i**)**. So that, ω_N is αGCS_{FN} , but not αCS_{FN}.

iv. Assume X= {c, d} defined by S_{FN} β_N in X as following:
β_N = < ω, (0.2$_{(c)}$, 0.3$_{(d)}$), (0.5$_{(c)}$, 0.5$_{(d)}$), (0.4$_{(c)}$,0.5$_{(d)}$) >. The family τ^N = {0^N, 1^N, β_N} is T_{FN}. When take, Φ_N =< ω, (0.1$_{(c)}$, 0 $_{(d)}$), (0.5$_{(c)}$, 0.5$_{(d)}$), (0.5$_{(c)}$, 0.6$_{(d)}$) >. So that, Φ_N is $GSCS_{FN}$, but not αGCS_{FN}.

Fuzzy Neutrosophic Generalized Semi Mapping as continuous and Fuzzy Neutrosophic Alpha Generalized Mapping as continuous

In the current section, the concept of fuzzy neutrosophic generalized semi mapping as continuous and fuzzy neutrosophic alpha generalized mapping as continuous has been introduced.

Definition 11. : Suppose (X, $\tau^{N\omega}$) and **(**Y, $\tau^{N\xi}$) are 2 TSs_{FN}. So, a mapping
f:(X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) is called:

i. Fuzzy neutrosophic γ continuous (γ con. $_{FN}$), when f^{-1}(β_N) is γ CS_{FN} in (X, $\tau^{N\omega}$) for each CS_{FN} in (Y, $\tau^{N\xi}$).

ii. Fuzzy neutrosophic generalized semi continuous (GScon. $_{FN}$), when f^{-1}(β_N) is $GSCS_{FN}$ in (X, $\tau^{N\omega}$) for each CS_{FN} in (Y, $\tau^{N\xi}$).

iii. Fuzzy neutrosophic α generalized continuous (αGcon. $_{FN}$), when f^{-1}(β_N) is αGCS_{FN} in (X, $\tau^{N\omega}$) for each CS_{FN} in (Y, $\tau^{N\xi}$).

Theorem 2.:

i. Every Con.$_{FN}$ is αGcon.$_{FN}$.

ii. Every αCon.$_{FN}$ is αGcon.$_{FN}$.

iii. Every αGcon.$_{FN}$ is GScon.$_{FN}$.

Proof:

i. Suppose f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) is con.$_{FN}$ mapping. Take β_N is CS_{FN} in Y.
Therefore f is con.$_{FN}$ mapping f^{-1}(β_N) is CS_{FN} in X (definition 8. i). Since every CS_{FN} is αGCS_{FN} (Theorem 2. i). So, f^{-1}(β_N) is αGCS_{FN} in X. Thus, f is αGcon.$_{FN}$.

ii. Suppose f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) is αcon.$_{FN}$ mapping. Take β_N is CS_{FN} in Y. So that, via hypothesis f^{-1}(β_N) is αCS_{FN} in X (definition 8. iii). Since every αCS_{FN} is αGCS_{FN} (Theorem 2. iii). So, f^{-1}(β_N) is αGCS_{FN} in X. Hence, f is αGcon.$_{FN}$.

iii. Suppose f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) is αGcon.$_{FN}$ mapping. Take β_N is CS_{FN} in Y. So that, via hypothesis $f^{-1}(\beta_N)$ is αGCS_{FN} in X (definition 10. iii). Since every αGCS_{FN} is $GSCS_{FN}$ (Theorem 2. Iv). So, $f^{-1}(\beta_N)$ is $GSCS_{FN}$ in X. Hence, f is $GScon._{FN}$.

Remark 3. : The foregoing theorem converse is not true as illustrated in the following example:

Example 2. :

i. Suppose X= {c, d}, Y ={e, g}.
Defined, β_{N1} = <ω, (0.2$_{(c)}$, 0.2$_{(d)}$), (0.5$_{(c)}$, 0.5$_{(d)}$), (0.6$_{(c)}$, 0.6$_{(d)}$) > and
β_{N2} = < ξ, (0.6$_{(e)}$, 0.6$_{(g)}$), (0.5$_{(e)}$, 0.5$_{(g)}$), (0.3$_{(e)}$, 0.2$_{(g)}$) >. So that, the family $\tau^{N\omega}$ = {0^N, 1^N, β_{N1}} and $\tau^{N\xi}$ = {0^N, 1^N, β_{N2} } are T_{FN} on X and Y respectively.
Define a mapping f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) via f(c) = e and f(d) = g.
Assume that Φ_N =< ξ, (0.3$_{(e)}$, 0.2$_{(g)}$), (0.5$_{(e)}$, 0.5$_{(g)}$), (0.6$_{(e)}$, 0.6$_{(g)}$) > is CS_{FN} in Y.
So that, $f^{-1}(\Phi_N)$ =<ω, (0.3$_{(c)}$, 0.2$_{(d)}$), (0.5$_{(c)}$, 0.5$_{(d)}$), (0.6$_{(c)}$, 0.6$_{(d)}$) > is αGCS_{FN} in X. Hence, f is $\alpha Gcon._{FN}$, but not $con._{FN}$.

ii. Suppose X= {c, d} , Y ={e, g}.
Define, β_{N1} = <ω, (0.3$_{(c)}$, 0.3$_{(d)}$), (0.3$_{(c)}$, 0.4$_{(d)}$), (0.7$_{(c)}$, 0.7$_{(d)}$) >,
β_{N2} = <ω, (0.8$_{(c)}$, 0.8$_{(d)}$), (0.7$_{(c)}$, 0.8$_{(d)}$), (0.3$_{(c)}$, 0.3$_{(d)}$) > and
β_{N3} = <ξ, (0.3$_{(e)}$, 0.3$_{(g)}$), (0.7$_{(e)}$, 0.6$_{(g)}$), (0.6$_{(e)}$, 0.6$_{(g)}$) >. So that, the $\tau^{N\omega}$ = {0^N, 1^N, β_{N1}, β_{N2}} and
$\tau^{N\xi}$ = {0^N, 1^N, β_{N3}} are T_{FN} on X, Y respectively.
Define a mapping f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) via f(c) = e and f(d) = g.
Assume Φ_N = <ξ, (0.6$_{(e)}$, 0.6$_{(g)}$), (0.3$_{(e)}$, 0.4$_{(g)}$), (0.3$_{(e)}$, 0.3$_{(g)}$) > is CS_{FN} in Y.
So that, $f^{-1}(\Phi_N)$ = <ω, (0.6$_{(c)}$, 0.6$_{(d)}$), (0.3$_{(c)}$, 0.4$_{(d)}$), (0.3$_{(c)}$, 0.3$_{(d)}$) > is αGCS_{FN} in X. Hence, f is $\alpha Gcon._{FN}$, but not $\alpha con._{FN}$.

iii. Suppose X= {c, d}, Y ={e, g}.
Defining, β_{N1} = <ω, (0.2$_{(c)}$, 0.3$_{(d)}$), (0.5$_{(c)}$, 0.5$_{(d)}$), (0.4$_{(c)}$, 0.5$_{(d)}$) > and
β_{N2} = <ξ, (0.5$_{(e)}$, 0.6$_{(g)}$), (0.5$_{(e)}$, 0.5$_{(g)}$), (0.1$_{(e)}$, 0$_{(g)}$) >.
So that, the family $\tau^{N\omega}$ = {0^N, 1^N, β_{N1}} and $\tau^{N\xi}$ = {0^N, 1^N, β_{N2}} are T_{FN} on X and Y respectively.
Define a mapping f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) via f(c) = e and f(d) = g.
Assume Φ_N = <ξ, (0.1$_{(e)}$, 0$_{(g)}$), (0.5$_{(e)}$, 0.5$_{(g)}$), (0.5$_{(e)}$, 0.6$_{(g)}$) > is CS_{FN} in Y. So that, $f^{-1}(\Phi_N)$ = <ω, (0.1$_{(c)}$, 0$_{(d)}$), (0.5$_{(c)}$, 0.5$_{(d)}$), (0.5$_{(c)}$, 0.6$_{(d)}$) > is $GSCS_{FN}$ in X. Hence, f is $GScon._{FN}$, but not $\alpha Gcon._{FN}$.

Theorem 3. : A mapping f: X→ Y is $\alpha Gcon._{FN}$ if and only if the inverse image of each OS_{FN} in Y is αGOS_{FN} in X.

Proof: Suppose β_N is OS_{FN} in Y. Such implicates $(\beta_N)^c$ is CS_{FN} in Y. Therefore, f is $\alpha Gcon._{FN}$. So that $f^{-1}(\beta_N)^c$ is αGCS_{FN} in X. Therefore, $f^{-1}(\beta_N)^c$ = $(f^{-1}(\beta_N))^c$. Hence, $f^{-1}(\beta_N)$ is αGOS_{FN} in X.

Theorem 4. : Suppose f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) is mapping and $f^{-1}(\beta_N)$ is RCS_{FN} in X for every CS_{FN} in Y. Then f is $\alpha Gcon._{FN}$ mapping.

Proof: Suppose β_N is CS_{FN} in Y. So that $f^{-1}(\beta_N)$ is RCS_{FN} in X. Since, Every RCS_{FN} is αGCS_{FN} (Theorem 2. ii). So, $f^{-1}(\beta_N)$ is αGCS_{FN} in X. Hence, f is $\alpha Gcon._{FN}$ mapping.

Remark 4. :

i. The relationship between $Pcon._{FN}$ and $\alpha Gcon._{FN}$ is independent.

ii. The relationship between γ $con._{FN}$ and $\alpha Gcon._{FN}$ is independent.

And this can be illustrated in the next example:

Example 3. :

i. 1- Suppose X= {c, d}, Y ={e, g}.
Defining, β_{N1} = <ω, (0.1$_{(c)}$, 0.8$_{(d)}$), (0.4$_{(c)}$, 0.6$_{(d)}$), (0.5$_{(c)}$, 0.2$_{(d)}$) > and
β_{N2} = <ξ, (0.8$_{(e)}$, 0.8$_{(g)}$), (0.6$_{(e)}$, 0.4$_{(g)}$), (0.1$_{(e)}$, 0.4$_{(g)}$) >.
So that, the family $\tau^{N\omega}$ = {0^N, 1^N, β_{N1}} and $\tau^{N\xi}$ = {0^N, 1^N, β_{N2}} are T_{FN} on X and Y, respectively.
Defining a mapping f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) via f(c) = e and f(d) = g.
Suppose Φ_N = <ξ, (0.1$_{(e)}$, 0.4$_{(g)}$), (0.4$_{(e)}$, 0.6$_{(g)}$), (0.8$_{(e)}$, 0.8$_{(g)}$) > is CS_{FN} in Y.
So, $f^{-1}(\Phi_N)$ = <ω, (0.1$_{(c)}$, 0.4$_{(d)}$), (0.4$_{(c)}$, 0.6$_{(d)}$), (0.8$_{(c)}$, 0.8$_{(d)}$) > is PCS_{FN} in X.
Hence, f is $Pcon._{FN}$, but not $\alpha Gcon._{FN}$.
2- Suppose X= {c, d}, Y ={e, g}. Define, β_{N1} = <ω, (0.2$_{(c)}$, 0.2$_{(d)}$), (0.3$_{(c)}$, 0.4$_{(d)}$), (0.5$_{(c)}$, 0.6$_{(d)}$) > and
β_{N2} = <ξ, (0.4$_{(e)}$, 0.5$_{(g)}$), (0.4$_{(e)}$, 0.3$_{(g)}$), (0.3$_{(e)}$, 0.2$_{(g)}$) >.
So that, the family $\tau^{N\omega}$ = {0^N, 1^N, β_{N1}} and $\tau^{N\xi}$ = {0^N, 1^N, β_{N2}} are T_{FN} on X and Y, respectively.
Defining a mapping f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) via f(c) = e and f(d) = g.
Assume Φ_N = <ξ, (0.3$_{(e)}$, 0.2$_{(g)}$), (0.6$_{(e)}$, 0.7$_{(g)}$), (0.4$_{(e)}$, 0.5$_{(g)}$) > is CS_{FN} in Y. So that, $f^{-1}(\Phi_N)$ = <ω, (0.3$_{(c)}$, 0.2$_{(d)}$), (0.6$_{(c)}$, 0.7$_{(d)}$), (0.4$_{(c)}$, 0.5$_{(d)}$) > is αGCS_{FN} in X. Hence, f is $\alpha Gcon._{FN}$, but not $Pcon._{FN}$.

ii. 1- Suppose X= {c, d}, Y ={e, g}.
Defining, β_{N1} = <ω, (0.4$_{(c)}$, 0.6$_{(d)}$), (0.3$_{(c)}$, 0.4$_{(d)}$), (0.2$_{(c)}$, 0.2$_{(d)}$) > and
β_{N2} = <ξ, (0.7$_{(e)}$, 0.2$_{(g)}$), (0.7$_{(e)}$, 0.6$_{(g)}$), (0.3$_{(e)}$, 0.3$_{(g)}$) >.
So that, the family $\tau^{N\omega}$ = {0^N, 1^N, β_{N1}} and
$\tau^{N\xi}$ = {0^N, 1^N, β_{N2}} are T_{FN} on X and Y, respectively.
Let defined a mapping f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) via f(c) = e and f(d) = g.

Assume $\Phi_N = <\xi, (0.3_{(e)}, 0.3_{(g)}), (0.3_{(e)}, 0.4_{(g)}), (0.7_{(e)}, 0.2_{(g)}) >$ is CS_{FN} in Y. So, $f^{-1}(\Phi_N) = <\omega, (0.3_{(c)}, 0.3_{(d)}), (0.3_{(c)}, 0.4_{(d)}), (0.7_{(c)}, 0.2_{(d)}) >$ is γ CS_{FN} in X. Hence, f is γ $con._{FN}$, but not $\alpha Gcon._{FN}$.

2- Suppose X= {c, d}, Y ={e, g}. Defining, $\beta_{N1} = <\omega, (0.6_{(c)}, 0.2_{(d)}), (0.2_{(c)}, 0.4_{(d)}), (0.4_{(c)},0.8_{(d)}) >$ and $\beta_{N2} = <\xi, (0.3_{(e)}, 0.2_{(g)}), (0.8_{(e)}, 0.6_{(g)}), (0.6_{(e)}, 0.7_{(g)}) >$. So that, the family $\tau^{N\omega} = \{0^N, 1^N, \beta_{N1}\}$ and $\tau^{N\xi} = \{0^N, 1^N, \beta_{N2}\}$ are T_{FN} on X and Y, respectively. Defining a mapping f : (X, $\tau^{N\omega}$) → (Y, $\tau^{N\xi}$) via f(c) = e and, f(d) = g. Suppose $B_N = <\xi, (0.6_{(e)}, 0.7_{(g)}), (0.2_{(e)}, 0.4_{(g)}), (0.3_{(e)},0.2_{(g)}) >$ is CS_{FN} in Y. So that, $f^{-1}(B_N) = <\omega, (0.6_{(c)}, 0.7_{(d)}), (0.2_{(c)}, 0.4_{(d)}), (0.3_{(c)}, 0.2_{(d)}) >$ is αGCS_{FN} in X. Hence, f is $\alpha Gcon._{FN}$, but not γ $con._{FN}$.

Remark 5. : Fig. 1 shows the relationships when deferent fuzzy neutrosophic continuous in the fuzzy neutrosophic topology spaces and in general the converse is not true.

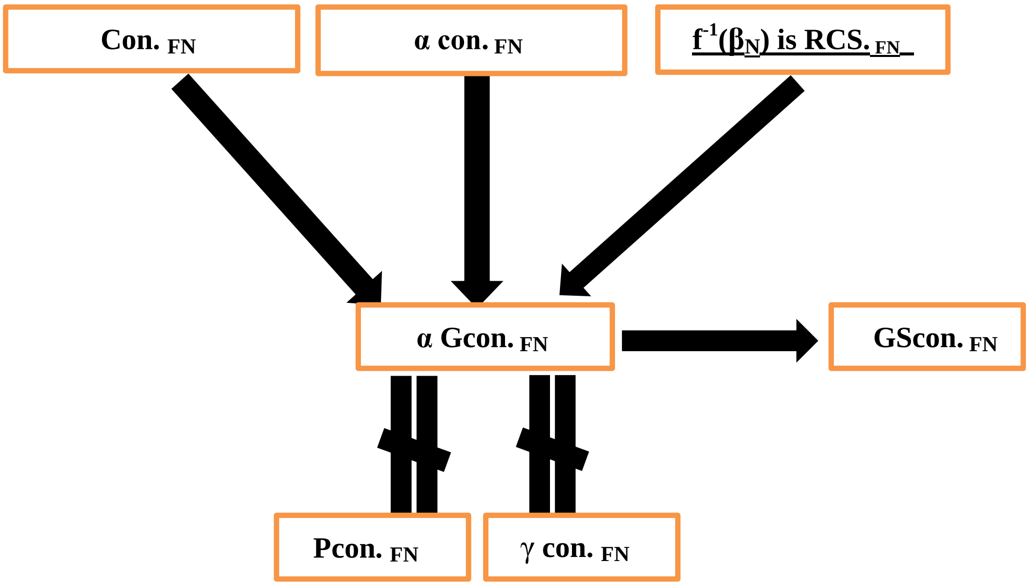

Figure 1. Relationship with α Gcon. FN.

Conclusion:

The aim of this research is to identify some new generalized mapping of fuzzy neutrosophic as continuous and tried to solve the trouble of branching or splitting with definitions of the generalized mapping of fuzzy neutrosophic and showing that generalized semi-continuous and fuzzy neutrosophic alpha generalized mapping as continuous is independent according to the examples and Figure 1. Founded some relationships that connect it and the reverse is not true always.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Tikrit.

Authors' contributions statement:

S.F. Matar and A.A. Hijab contributed to the study and development most proofs, to the relations structure; revision, and approved the original manuscript.

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بعض خواص الدوال شبه المستمر المعممة- النايتروسوفيك الضبابية والدوال آلفا المستمر المعممة- النايتروسوفيك الضبابية

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الخلاصة:
في هذا البحث عرفنا الدوال شبه المستمرة المعممة النيتروسوفك الضبابية والدوال آلفا المستمرة المعممة نيتروسوفك الضبابية، وإثبات بعض المبرهنات المتعلقة بهذا المفهوم ومن ثم إيجاد العلاقة بين الدوال آلفا المستمرة المعممة نيتروسوفك الضبابية. والتي تضمنت الدوال المستمرة نيتروسوفك الضبابية والدوال آلفا المستمرة نيتروسوفك الضبابية والدوال شبه المستمرة نيتروسوفك الضبابية والدوال Pre المستمرة نيتروسوفك الضبابية ودوال γ المستمرة نيتروسوفك الضبابية.

الكلمات المفتاحية: مجموعة نيتروسوفك الضبابية، فضاء التبولوجيا نيتروسوفك الضبابية، المجموعات شبه المغلقة المعممة نيتروسوفك الضبابية، المجموعات ألفا المغلقة المعممة نيتروسوفك الضبابية، الدوال شبه المستمرة المعممة نيتروسوفك الضبابية، الدوال آلفا المستمرة المعممة نيتروسوفك الضبابية.