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A Comparison Between Two Shape Parameters Estimators for (Burr-XII) Distribution

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Abstract

This paper deals with defining Burr-XII, and how to obtain its p.d.f., and CDF, since this distribution is one of failure distribution which is compound distribution from two failure models which are Gamma model and weibull model. Some equipment may have many important parts and the probability distributions representing which may be of different types, so found that Burr by its different compound formulas is the best model to be studied, and estimated its parameter to compute the mean time to failure rate. Here Burr-XII rather than other models is consider because it is used to model a wide variety of phenomena including crop prices, household income, option market price distributions, risk and travel time. It has two shape-parameters (α, r) and one scale parameter (λ) which is considered known. So, this paper defines the p.d.f. and CDF and derives its Moments formula about origin, and also derive the Moments estimators of two shapes parameters (α, r) in addition to maximum likelihood estimators as well as percentile estimators, the scale parameter (λ) is not estimated (as it is considered known). The comparison between three methods is done through simulation procedure taking different sample size (n=30, 60, 90) and different sets of initial values for (α, r, λ).It is observed that the moment estimators $\hat{r}_{mom}$ and $\hat{\alpha}_{mom}$ are the best estimator with percentage (46%) ,(42%) respectively compared with other estimators.

Key word: Burr-XII failure model, Maximum likelihood estimator, Moments estimator, Percentile estimator.

Introduction

Twelve different methods of cumulative distribution functions are presented by Burr on the data of the lifetime modeling or the data of the survival (1). It is worthy to mention that there are two types from these twelve methods mentioned above which are considered as the most important methods due to their application in the study of biological, industrial, reliability and life testing, and several industrial and economic experiments, these types are Burr Type XII and Burr Type X (2).
The Burr type XII distribution became a vital research area for many authors and many studies. Evans and Ragab. 1983 (3) present a Bayes that estimates the shape parameter (α) and the reliability function based on type-II censored samples. Saracoglu et al. 2013 (4) progressive type-II right censored samples are used to obtain the maximum likelihood, weighted least squares ,ordinary least squares, and best linear unbiased estimators for the shape parameter α . According to Abuzaid 2015 (5),middle-censoring is considered as a modern general scheme of censoring and studying the analysis of middle-censored data with Burr-XII distribution which is considered one of the most popular and flexible distributions for modeling stochastic events and lifetime for many products. Nasser and et al.2016(6) introduced an adaptive type-II progressive hybrid censoring scheme that is used to obtain the maximum likelihood and Bayesian estimation for the unknown parameters of the Burr type XII distribution and Bayes estimates of the unknown parameters

The objective of this paper is to estimate two parameters (α, r) ,where scale parameter (λ) is considered known of the Burr XII distribution by the three different types of estimators Moments, Maximum likelihood as well as percentile estimators . The paper is presented as follows: Section 2, gives an introduction about (Burr-XII), finding this p.d.f. and its cumulative CDF, and discuss the Moments, Maximum likelihood as well as percentile estimators for the two parameters (α, r) ,the scale parameter (λ) is not estimated (considered known). Section 3 focuses on the results and compares between three methods through simulation procedure. Section 4 covers some conclusions from the results.

Theoretical Aspect

The compound p.d.f. of Burr-XII distribution can be obtained by compounding (p.d.f. of Gamma)distribution with (p.d.f. of weibull) distribution.
The formula for the probability density function of the Weibull distribution is

$f_Y(y) = \beta\, \alpha\, y^{\alpha-1} e^{-\beta y^{\alpha}}$, y>0, α, β >0 … (1) ,
i.e. y be r.v ~weibull(α, β)

where, α is the shape parameter and β is the scale parameter
and, the formula for the probability density function of the gamma distribution is

$f(\beta) = \frac{\lambda^r}{\Gamma(r)} \beta^{r-1} e^{-\lambda\beta}$, $\beta > 0$, λ, r > 0 …(2)
, i.e. β be r.v ~Gamma(r,λ)

where, r is the shape parameter and λ is the scale parameter
since y be r.v ~ weibull(α, β) and one of its parameter β be r.v ~ Gamma(r,λ) then , y has a compound density function is (7)

$$f(y) = \int_{\beta} f(y|\beta).f(\beta)\, d\beta$$

Where, $f(y|\beta)$ is a conditional density function depending on the parameter β

∴f(y | β, r, λ) =

$$\int_0^{\infty} [\beta\, \alpha\, y^{\alpha-1} e^{-\beta y^{\alpha}} \frac{\lambda^r}{\Gamma(r)} \beta^{r-1} e^{-\lambda\beta}]d\beta$$

$$= \frac{\alpha\lambda^r}{\Gamma(r)} \int_0^{\infty} \beta^r y^{\alpha-1} e^{-\beta(\lambda + y^{\alpha})} d\beta$$

$$= \frac{\alpha\lambda^r}{\Gamma(r)} y^{\alpha-1} \int_0^{\infty} \beta^r e^{-\beta(\lambda + y^{\alpha})} d\beta \qquad (3)$$

After some steps,

$$f(y \mid \alpha, \lambda, r) = \frac{r\alpha}{\lambda} y^{\alpha-1} [1 + \left(\frac{y^{\alpha}}{\lambda}\right)]^{-r-1} \qquad y>0 \qquad (4)$$

the p.d.f. in equation (4) is (Burr-XII) distribution with (λ) is scale parameter and (r, α) are shape parameters.
Also, the C.D.F of (Burr-XII) corresponding to p.d.f. in equation (4) is given in equation (5):

$$F(y \mid \alpha, r, \lambda) = 1- [1 + \frac{y^{\alpha}}{\lambda}]^{-r} \qquad y>0 \qquad (5)$$

Moments derivation

The (mth) moments formula about origin is

$$\mu'_m = E(y^m) = \int_0^{\infty} y^m f(y) dy$$

$$= \int_0^{\infty} y^m \frac{r\,\alpha}{\lambda} y^{\alpha-1} \left[1 + \frac{y^{\alpha}}{\lambda}\right]^{-r-1} dy$$

Applying formula

$$\beta(a,b) = \int_0^{\infty} \frac{z^{a-1}}{(1+z)^{a+b}} dz \qquad (6)$$

Assume $z = \frac{y^{\alpha}}{\lambda}$

$$y = (\lambda z)^{\frac{1}{\alpha}}$$

$$dy = \frac{1}{\alpha}(\lambda z)^{\frac{1}{\alpha}-1} \lambda\, dz$$

$$\mu'_m = \int_0^{\infty} [((\lambda z)^{\frac{1}{\alpha}})]^{m+\alpha-1} \frac{r\alpha}{\lambda} [1 + z]^{-r-1} \frac{\lambda}{\alpha} (\lambda z)^{\frac{1}{\alpha}-1} dz$$

Then:

$$E(y^m) = r\lambda^{\frac{m}{\alpha}} \beta\left(\frac{m}{\alpha} + 1, r - \frac{m}{\alpha}\right)$$

Then

$$\text{Mean}= E(Y)= r\, \lambda^{\frac{1}{\alpha}} \beta\left(\left(\frac{1}{\alpha} + 1, r - \frac{1}{\alpha}\right) \qquad (7)$$

and $E(Y^2) = r\lambda^{\frac{2}{\alpha}} \beta\left(\frac{2}{\alpha} + 1, r - \frac{2}{\alpha}\right)$
this gives

$$variance = S^2 = r\lambda^{\frac{2}{\alpha}} \beta\left(\frac{2}{\alpha} + 1, r - \frac{2}{\alpha}\right) - \left[r\lambda^{\frac{1}{\alpha}} \beta\left(\frac{1}{\alpha} + 1, r - \frac{1}{\alpha}\right)\right]^2 \qquad (8)$$

And from equations:

$$\left.\begin{aligned} E(Y^2) &= \frac{\sum y_i^2}{n} \\ E(Y) &= \frac{\sum y_i}{n} \end{aligned}\right\} \qquad (9)$$

According to given values of λ equation (9) can be solved to obtain $\hat{\alpha}_{MOM}$ and $\hat{r}_{MOM}$

Maximum Likelihood Estimator

Maximum likelihood estimation (MLE) is a procedure of finding the value of one or more

parameters of a statistical model given observations, by finding the parameter values that maximize the likelihood of making the observations give the parameters. The maximum likelihood estimator is widely used in practice largely because of its conceptual simplicity.
Now, let $y_1, y_2, \ldots. y_n$ be a r.s. from p.d.f in equation (4), then:

$$L=\prod_{i=1}^{n} f(y_i\ ,\alpha,\lambda,r)$$

$$L=\alpha^n\lambda^{-n}r^n\prod_{i=1}^{n}(y_i{}^{\alpha-1})\prod_{i=1}^{n}[(1+\frac{y_i{}^{\alpha}}{\lambda})]^{-r-1}$$

$$\log L=n\log r+n\log\alpha-n\log\lambda+(\alpha-1)\sum_{i=1}^{n}\log y_i-(r+1)\sum_{i=1}^{n}\log\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]$$

Then

$$\frac{\partial\log L}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\log y_i-(r+1)\sum_{i=1}^{n}\frac{\left(\frac{y^{\alpha}}{\lambda}\right)\log[y_i]}{1+\frac{y_i{}^{\alpha}}{\lambda}}=0$$

$$\frac{n}{\hat{\alpha}}=(r+1)\sum_{i=1}^{n}\frac{\frac{y_i^{\alpha}}{\lambda}\log y_i}{(1+\frac{y_i^{\alpha}}{\lambda})}-\sum_{i=1}^{n}\log y_i$$

$$\hat{\alpha}_{MLE}=\frac{n}{(r+1)\sum_{i=1}^{n}\frac{\frac{y_i^{\alpha}}{\lambda}\log y_i}{(1+\frac{y_i^{\alpha}}{\lambda})}-\sum_{i=1}^{n}\log y_i} \quad (10)$$

$$\frac{\partial logl}{\partial r}=\frac{n}{r}-\sum_{i=1}^{n}\log\left[1+\frac{\mathrm{yi}^{\alpha}}{\lambda}\right]=0$$

$$\hat{r}_{MLE}=\frac{n}{\sum_{i=1}^{n}\log\left[1+\frac{\mathrm{yi}^{\alpha}}{\lambda}\right]} \quad (11)$$

Since the scale parameter (λ) known is considered so , $\frac{\partial logl}{\partial\lambda}$ do not find and $(\hat{\lambda})$ may be found from mean time to failure according to given values α , r.

Percentile estimators

The estimation by this method is obtained from minimizing the total sum squares of difference between cumulative distribution function and its **non-Parametric** estimator:

$\hat{F}(y_i|\alpha,r,\lambda)=\frac{i}{n+1}$, then

$$T=\sum_{i=1}^{n}\left[F(y_i|\alpha,r,\lambda)-\hat{F}(y_i|\alpha,r,\lambda)\right]^2$$

$$T=\sum_{i=1}^{n}\left[1-\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right]^2 \quad (12)$$

Here the scale parameter (λ) is constant and estimating (α,r) by moments and maximum likelihood method and percentile method
Now the percentile estimator obtained is:

$$\frac{\partial T}{\partial\alpha}=2\sum_{i=1}^{n}\left[1-\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right]*[(-r)(-\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]^{-r-1}(y_i{}^{\alpha}(1)\log(y_i)]$$

$$\frac{\partial T}{\partial\alpha}=2\sum_{i=1}^{n}\left[1-\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right]*[(r)\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]^{-r-1}(\frac{y_i{}^{\alpha}}{\lambda}(1)\log(y_i)]$$

$$\frac{\partial T}{\partial\alpha}=2\sum_{i=1}^{n}\left[1-\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right][1-\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]^{-r}(-1)\log\left[1+\frac{y_i{}^{\alpha}}{\lambda}\right]$$

$$\frac{\partial T}{\partial\alpha}=2\sum_{i=1}^{n}\left[1-\left[1+\frac{y_i^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right]r[1-\left(1+\frac{y_i^{\alpha}}{\lambda}\right)^{-r-1}\left(\frac{y_i^{\alpha}}{\lambda}(1)\log y_i\right)$$

$$\frac{\partial T}{\partial\alpha}=0\ \rightarrow\ \sum_{i=1}^{n}\left[1-\left[1+\frac{y_i^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right]^{-r-1}\left(\frac{y_i^{\alpha}}{\lambda}\log y_i\right)\ \rightarrow\ \sum_{i=1}^{n}y_i^{\alpha}\log y_i\left[1-\left[1+\frac{y_i^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right]^{-r-1}=0\ \ldots \quad (13)$$

Solved numerically to obtain $\hat{\alpha}_{pec}$.
While the percentile estimator of r is obtained from:

$$\frac{\partial T}{\partial r}=2\sum_{i=1}^{n}\left[1-\left[1+\frac{y_i^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right](-r)[1-\left(1+\frac{y_i^{\alpha}}{\lambda}\right)^{-r-1}\left(\frac{y_i^{\alpha}}{\lambda}\log(1+\frac{y_i^{\alpha}}{\lambda}\right)$$

$$\frac{\partial T}{\partial r}=0\ \rightarrow\sum_{i=1}^{n}\left[1-\left[1+\frac{y_i^{\alpha}}{\lambda}\right]^{-r}-\frac{i}{n+1}\right]\left(1+\frac{y_i^{\alpha}}{\lambda}\right)^{-r-1}\left(\log(1+\frac{y_i^{\alpha}}{\lambda}\right) \quad (14)$$

Solved numerically to $\hat{r}_{pec}$

From solving $\frac{\partial T}{\partial \alpha} = 0, \frac{\partial T}{\partial r} = 0$ by Newton – Raphson techniques, $\hat{\alpha}_{pec}$ and $\hat{r}_{pec}$ are obtained while (λ is constant).

Simulation Procedure

In this section, Monte Carlo simulation results have been conducted to examine and compare the performance of three Methods (moment ,maximum likelihood and Percentile) for the unknown shape parameters(α, r) considering scale parameter λ constant respecting to their MSE values with different cases and different sample sizes n=30,60,90.For a given values of(r, α, λ) ,generated a random sample ,say y as Burr-XII distribution through the adoption inverse transformation method

$$y_i = \lambda^{\frac{1}{\alpha}} \left[(1 - U_i)^{\frac{1}{r}} - 1 \right]^{\frac{1}{\alpha}}$$

The results of the simulation study are summarized and tabulated in Table 1, Table 2 and 3 of the three estimators for all sample sizes and (r, α, λ)values

Table 1: Comparing estimators of (α) by three Methods

n	λ	α	r	$\hat{\alpha}_{mom}$	$\hat{\alpha}_{mle}$	$\hat{\alpha}_{pec}$
30	1.5	2	2.5	2.09633	2.29803	2.06934
		2	4	2.66050	2.59070	2.77532
		4	2.5	2.53821	2.57762	2.63645
		4	4	2.48650	2.53870	2.61693
30	0.8	2	2.5	2. 53786	2.54610	2.61684
		2	4	2.41586	2.00670	2.73040
		4	2.5	2.57764	2.57860	2.66050
		4	4	2.82110	2.57782	2.77630
60	1.5	2	2.5	2.02647	2.04130	2.06060
		2	4	2.56560	2.58970	2.56070
		4	2.5	2.52050	2.56760	2.48750
		4	4	2.41329	2.44560	2.53802
60	0.8	2	2.5	2.52906	2.51862	2.45910
		2	4	2. 37015	2.00577	2.63621
		4	2.5	2.56604	2.56064	2.63620
		4	4	2.7631	2.5741	2.57736
90	1.5	2	2.5	2.01950	2.01408	2.01298
		2	4	2.54633	2.47230	2.55616
		4	2.5	2.51776	2.56607	2.46361
		4	4	2.40770	2.40380	2.40120
90	0.8	2	2.5	2.36678	2.34351	2.39020
		2	4	2.35776	2.004701	2.62878
		4	2.5	2.51362	2.40059	2.62861
		4	4	2.75085	2.29360	2.31820
30	1.5	2	2.5	2.36450	2.28670	2.57030
		2	4	2.28790	2.62201	2.55310
		4	2.5	2.57630	2.65630	2.63020
		4	4	2.63190	2.66450	2.77430
30	0.8	2	2.5	2.63020	2.28780	2.48670
		2	4	2.59130	2.59135	2.59630
		4	2.5	2.47060	2.47050	2.66390
		4	4	2.57310	2.47056	2.33060
60	1.5	2	2.5	2.30280	1.98767	1.99560
		2	4	1.99760	1.98720	1.88500
		4	2.5	1.63560	1.64670	1.89320
		4	4	1.77320	1.63020	1.94900
60	0.8	2	2.5	2.29850	2.25699	2.25389
		2	4	2.39670	2.57749	2.56637
		4	2.5	2.35776	2.46887	2.65778
		4	4	2.46356	2.46671	2.18940
90	1.5	2	2.5	1.77030	1.86210	1.74610
		2	4	1.77310	1.55820	1.87994
		4	2.5	1.62641	1.637521	1.88100
		4	4	1.76791	1.62631	1.55420
90	0.8	2	2.5	2.28886	2.25577	2.25238
		2	4	2.6605	2.5590	2.4853
		4	2.5	2.34786	2.26161	2.26163
		4	4	2.45773	2.15582	2. 15542

Table (2): MSE for $\hat{\alpha}$

n	λ	α	r	$\hat{\alpha}_{mom}$	$\hat{\alpha}_{mle}$	$\hat{\alpha}_{pec}$	Best
30	1.5	2	2.5	0.38282	0.20844	0.18389	PEC
		2	4	0.38047	0.21518	0.39860	MLE
		4	2.5	0.32414	0.21664	0.33450	MLE
		4	4	0.30340	0.09630	0.20537	MLE
30	0.8	2	2.5	0.33802	0.82013	0.64490	MOM
		2	4	0.33425	0.44630	0.59317	MOM
		4	2.5	0.27560	0.45020	0.55680	MOM
		4	4	0.15340	0.13040	0.48790	MLE
60	1.5	2	2.5	0.15047	0.08091	0.06052	PEC
		2	4	0.35007	0.09560	0.05530	PEC
		4	2.5	0.20844	0.08332	0.05082	PEC
		4	4	0.211580	0.06654	0.05091	PEC
60	0.8	2	2.5	0.08984	0.07981	0.05002	PEC
		2	4	0.07076	0.04457	0.05031	MLE
		4	2.5	0.04117	0.03950	0.05006	MLE
		4	4	0.03534	0.03267	0.05001	MLE
90	1.5	2	2.5	0.01039	0.03626	0.06043	MOM
		2	4	0.00625	0.03024	0.04898	MOM
		4	2.5	0.00234	0.01937	0.04670	MOM
		4	4	0.00122	0.00971	0.04117	MOM
90	0.8	2	2.5	0.00139	0.00380	0.00366	MOM
		2	4	0.00128	0.00170	0.00347	MOM
		4	2.5	0.00962	0.00781	0.00583	PEC
		4	4	0.00127	0.00976	0.00970	MOM

Table (3): MSE of $\hat{r}$

n	λ	α	r	$\hat{r}_{mom}$	$\hat{r}_{mle}$	$\hat{r}_{pec}$	Best
30	1.5	2	2.5	0.00641	0.00218	0.00360	MLE
		2	4	0.00128	0.01276	0.03290	MOM
		4	2.5	0.00636	0.00624	0.00223	PEC
		4	4	0.00312	0.00209	0.00241	MLE
30	0.8	2	2.5	0.00322	0.00309	0.00110	PEC
		2	4	0.00139	0.00151	0.00130	PEC
		4	2.5	0.00102	0.00135	0.00145	MOM
		4	4	0.00125	0.00277	0.00672	MOM
60	1.5	2	2.5	0.00141	0.00203	0.00263	MOM
		2	4	0.00127	0.00330	0.00329	MOM
		4	2.5	0.00136	0.00224	0.00225	MOM
		4	4	0.00129	0.00126	0.00189	MLE
60	0.8	2	2.5	0.00305	0.00102	0.00104	MLE
		2	4	0.00138	0.00125	0.00127	MLE
		4	2.5	0.00101	0.00100	0.00142	MLE
		4	4	0.00123	0.00241	0.00589	MOM
90	1.5	2	2.5	0.00102	0.00202	0.00243	MOM
		2	4	0.00126	0.00122	0.00125	MLE
		4	2.5	0.00125	0.00223	0.00224	MOM
				0.00121	0.00124	0.00123	MOM
90	0.8	2	2.5	0.00135	0.00101	0.00103	MLE
		2	4	0.00112	0.00115	0.00107	PEC
		4	2.5	0.00031	0.00092	0.00022	PEC
		4	4	0.00122	0.00235	0.00360	MOM

Conclusion

1. $\hat{r}_{mom}$ is the best estimator with percentage (46%) and $\hat{r}_{mle}$ also with percentage (33%) while $\hat{r}_{pec}$ is dominated with (21%).
2. For α (shape parameter), $\widehat{\alpha}_{mom}$ is dominated with percentage 42% and $\widehat{\alpha}_{mle}$ (7/24) = 29%, while $\hat{\alpha}_{pec}$ is best percentage 29%.
3. The compound (Burr-XII) model is important for estimating time to failure of distribution that represents the time of failure for compound model of many parts especially for big system and equipment.

Estimate the expected mean time to failure of this compound distribution from applying E(x) = r λ Beta $(\frac{1}{\alpha}+1, r-\frac{1}{\alpha})$ when Beta (a,b)= $\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ after applying the best estimators of (r, α) (λ is known) and also estimating $E(x^2)$, where these are necessary to obtain estimated (variance), which is important for finding confidence internal of estimators.

Author's declaration:

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Besides, the Figures and images, which are not mine, have been given the permission for re-publication attached with the manuscript.
- The author has signed an animal welfare statement.
- Ethical Clearance: The project was approved by the local ethical committee in University of Mustansiriyah.

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مقارنة بين المقدرات لمعلمتي الشكل من توزيع(Burr-XII)

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الملخص

يتناول هذا البحث تعريف Burr-XII , و كيفية الحصول على دالة كثافة الاحتمال و دالة التوزيع له, نظراً لانه احد توزيعات الفشل والذي يركب من نموذجيين للفشل هما نموذج كاما ونموذج ويبل ,المعدات تحتوي علئ العديد من الاجزاء الهامة وقد يكون هناك انواع مختلفة من توزيع بور الاحتمالية الذي يمثلها , بواسطة صيغه المركبة المختلفة ,لذلك وجدنا انه هو افضل نموذج للدراسة, وتقدير المعلمة لحساب متوسط الزمن لمعدل الفشل.والتي نعتبرها معلومة.

ولاجل ما سبق تم استخدام Burr-XII بدلا من النماذج الاخرى ,وهذا التوزيع له معلمتين للشكل ومعلمة قياس واحدة لذلك في هذا البحث عرفنا دالة الكثافة ودالة التوزيع وانشانا العزم حول نقطة الاصل , وكذلك انشانا مقدرات العزوم والامكان الاعظم كذلك التقدير المئوي لمعلمتي الشكل بافتراض معلمة القياس معلومة.

تم المقارنة بين الطرق الثلاثة من خلال المحاكاة باخذ حجوم عينة مختلفة ومجموعات مختلفة من القيم الابتدائية. لوحظ ان مقدرات العزوم $\hat{r}_{mom}$ and $\widehat{\alpha}_{mom}$ قدما افضل تقدير بنسب 42%و46% على التوالي مقارنة مع المقدرات الاخرى.

الكلمات المفتاحية: نموذج فشل Burr-XII , مقدر العزوم, مقدر الامكان الاعظم, المقدر المئوي.