Open Access | Baghdad Science Journal | P-ISSN: 2078-8665
2020, 17(1) Supplement (March):371-377 | E-ISSN: 2411-7986

DOI: https://dx.doi.org/10.21123/bsj.2020.17.1(Suppl.).0371

On Light Mapping and Certain Concepts by Using $m_X N$-Open Sets

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Received 4/3/2019, Accepted 15/1/2020, Published 18/3/2020

Abstract:

The aim of this paper is to present a weak form of m-light functions by using $m_X N$-open set which is mN-light function, and to offer new concepts of disconnected spaces and totally disconnected spaces. The relation between them have been studied. Also, a new form of m-totally disconnected and inversely m-totally disconnected function have been defined, some examples and facts was submitted.

Key words: $m_X N$-disconnected space, $m_X N$-Hausdorff space, mN-light function, $m_X N$-open set, mN-totally disconnected function

Introduction:

In (2016) Abass and Ali (1) introduced the definition of m-light function, Humadi and Ali (2) presented the $m\widehat{\omega}$-light function. Al Ghour and Samarah (3) defined N-open set. In this research we defined the set $m_X N$-open set, we submitted a new type of functions by using $m_X N$-open sets, it is weaker than m-light function and we named it mN-light function. In (4) Carlos Carpintero, Jackeline Pacheco, Nimitha Rajesh, Ennis Rafael Rosas and S. Saranyasri defined N-connected space, by the same manner m-disconnected, $m_X N$-disconnection, mN-disconnected, mN-connected and $m_X N$-totally disconnected spaces have been defined, additionally, many types of functions in m-structure spaces such as mN-totally disconnected, mN^*-totally disconnected, mN^{**}-totally disconnected, inversely mN-totally disconnected function have been introduced. In (5) Enas Ridha Ali, Raad Aziz Hussain introduced the definition of N-hausdorff, and in the same way, mN-hausdorff has been defined. Also mNT_1-spaces and zero dimension m-spaces have been provided. The relation between these concepts has been discussed. Moreover the relation between m-homeomorphism functions (6) and the mN-light functions has been illustrated. Examples, theorems and some facts supported our study.

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Main Results:

In this section, mN-totally disconnected, mN-light functions and some spaces by using $m_X N$-open sets have been presented.

Definition 1 (7), (8)

A subcollection m_X of the power set $P(X)$ of a non-empty set X is called a minimal structure on X if $\emptyset, X \in m_X$, the pair (X, m_X) is called m-structure space (in short m-space). Each element in m_X is said to be m_X-open set and its complement is said to be m_X-closed set.

Remark 1 (9)

Every topological space (X, Υ) is m-space, but not conversely, because $\emptyset, X$ belong to Υ.

Example 1

If X= {n, m, f} and m_X= {$\emptyset$, X, {n}, {m}}, we observe that m_X is not a topology, since {n}∪{m} ={n, m}$\notin m_X$.

Definition 2 (10), (11)

The m_X-closure to a subset B of m-space (X, m_x) is the intersection of all closed sets $\mathcal{F}$ in X which containing B and we denote it by m_X-$cl(B)$, by symbols m_X-$cl(B)$=$\cap\{\mathcal{F}: B \subseteq \mathcal{F}\}$, where $\mathcal{F}$ is m_x-closed subset of X. While the m_X-interior to a subset B of m-space (X, m_x) is the union of all open sets K in X which contained in B and we denote it by m_X-$Int(B)$, by symbols m_X-$Int(B)$=$\cup\{K: K \subseteq B\}$ where K is m_X-open set in X.

Definition 3

A subset B of m-space X is called $m_X N$-open set if for each element $a \in B$ there exists an m_X–open set K in X containing a such that K-B is finite, the complement of $m_X N$-open set is called $m_X N$-closed. The family of all $m_X N$-open sets in X is symbolized as m_N.

Example 2

Any subset of a finite m-structure space (X, m_X) is m_X N-open and m_X N-closed set.

Lemma1

If $\{K_i \setminus i \in I\}$ is a collection of $m_X N$-open subsets of m-space X, then $\bigcup_{i\in I} K_i$ is $m_X N$-open too.

Proof

Consider $x \in \bigcup_{i\in I} K_i$, so there is an $m_X N$-open set K_j containing x for some $j \in I$, so W_j-K_j is finite, where W_j is m_X-open subset of X containing x, then W_j-$\bigcup_{i\in I} K_i$ is also finite since W_j-$\bigcup_{i\in I} K_i$ $\subseteq W_j$-K_j, (a subset of finite set is finite), therefore $\bigcup_{i\in I} K_i$ is $m_X N$-open set.

Definition 4 (1)

An m-space X is said to be m-disconnected, if there are non-empty m_X-open sets H and L in X such that $H \cup L = X$ and $H \cap L = \emptyset$, if X is not m-disconnected space then it is called m-connected space.

Example 3

The discrete m-space (Z, m_D), is m-disconnected space.

Definition 5

Let (X, m_X) be an m-space and H, L are two non-empty $m_X N$-open subsets of X, we call $H \cup L$ to be $m_X N$-disconnection to X, if $H \cup L = X$ and $H \cap L = \emptyset$. In example 3 Z-$\{x\}$ and $\{x\}$ where $x \in Z$, are $m_X N$-disconnection to Z.

Definition 6

An m-space X is mN-disconnected if we can find an $m_X N$-disconnection to it, if there is no such mN-disconnected so X is mN-connected space.

Example 4

The finite indiscrete m-space (X, m_{ind}) is mN-disconnected, but not mN-connected.

Proposition 1

An m-space X is mN-disconnected if and only if there is a non-empty $m_X N$-clopen subset G in X such that $G \neq X$.

Proof

Suppose G is a non-empty $m_X N$-clopen subset of X such that $G \neq X$. Let $U = G^c$, so U is a subset of X and $U \neq \emptyset$ (because $G \neq X$, and $G \cup U = X$, $G \cap U = \emptyset$). Also U is $m_X N$-clopen because G is $m_X N$-clopen, therefore X is mN-disconnected space. Conversely, if X is mN-disconnected space, so there is an $m_X N$-disconnection $G \cup U$ to X, hence $G = U^C$ which implies G is $m_X N$-closed subset of X, therefore G is $m_X N$-clopen subset of X and $G \neq X$ since U is non-empty subset of X, and then G is a non-empty $m_X N$-clopen subset of X such that $G \neq X$.

Proposition 2

An m-space X is mN-connected space if and only if $\emptyset$ and X are the only $m_X N$-clopen set in X.

Proof

If X is an mN-connected space, and U is a non-empty proper $m_X N$-clopen subset of X, then U^c is also $m_X N$-clopen subset of X, and since $U \cup U^c = X$, where $U^c \neq \emptyset$, therefore X is mN-disconnected space and that is a contradiction, so $\emptyset$ and X are the only $m_X N$-clopen set in X. Conversely, suppose X is mN-disconnected space, so there is $m_X N$-disconnection $L \cup H$ to X, but L is $m_X N$-closed (since $L = H^c$) which is a contradiction, therefore X is mN-connected.

Definition 7

The m-space (X, m_X) is called an mN-totall disconnected space. If for every pair of distinct points a and b in X, there are two $m_X N$-open sets N, M such that $N \neq \emptyset$, $M \neq \emptyset$, $a \in N$, $b \in M$, $N \cup M = X$ and $N \cap M = \emptyset$.

Example 5

For any distinct points x, y in the discrete m-space $(Z,\ m_D)$, the sets $\{x\}$ and $(Z$-$\{x\})$ are $m_Z N$-open sets containing x, y respectively such that $\{x\} \cap (Z$-$\{x\}) = \emptyset$ and $\{x\} \cup (Z$-$\{x\}) = Z$, so (Z, m_D) is mN-totally disconnected space.

Remark 2

Let X be an m-space, then:-

1- Every m_X–open subset of X is $m_X N$-open, but the converse is not true, since if K is m_X-open subset of X, then for each $x \in K$ there is an m_X-open subset M of X, pick $M = K$ then M containing x and M-$K = \emptyset$ (finite), so K is $m_X N$-open set.

2- Every m_X-closed subset of X is m_X N-closed.

Example 6

Let $K = \mathcal{R}$-$\{0\}$ be a subset of the indiscrete m-space $(\mathcal{R}, m_{ind})$, then K is $m_{\mathcal{R}} N$-open set, but not $m_{\mathcal{R}}$-open set.

Remark 3

I- Every mN-connected space is m-connected but the converse is not true, since if (X, m_X) is an mN-connected space, and suppose it is m-disconnected space then there is m_X-disconnection $N \cup M$ to X, and then it is mN-disconnected (by Remark 2) which is a contradiction, hence X is m-connected.

II- Every m-disconnected space is mN–disconnected, but the converse is not true, since if $(X,\ m_X)$ is m-disconnected space, then there is m_X-disconnection $M \cup N$ for X, and by Remark 2 it is $m_X N$-disconnection, therefore X is mN-disconnected space.

Example 7

The finite indiscrete m-space $(X,\ m_{ind})$ is m-connected and mN-disconnected space, but neither mN-connected nor m-disconnected space.

Proposition 3

A subset G of m-space X is $m_X N$-disconnected if and only if there is $m_X N$-open subsets N and M of

X with $G \subseteq N \cup M, N \cap G \neq \emptyset$, $M \cap G \neq \emptyset$, and $N \cap M \cap G = \emptyset$.

Proof

If G is an m_X N-disconnected subset of X so there is $m_X N$-disconnection $S \cup T$ to G, and then there are $m_X N$-open sets N and M in X such that $S = N \cap G$ and $T = M \cap G$, therefore $G \subseteq N \cup M$, $N \cap G \neq \emptyset$, $M \cap G \neq \emptyset$ and $N \cap M \cap G = \emptyset$. Conversely, since $N \cap G$ and $M \cap G$ are separate G, so G is $m_X N$-disconnected subset of X.

Definition 8

The $m_X N$–closure for a subset K of m-space X is the intersection of all $m_X N$-closed sets of X which containing K and it is denoted by $m_X N\text{-}cl(K)$. And the $m_X N$-interior for a subset K of m-space X is the union of all $m_X N$-open sets of X which containing in K and it is denoted by $m_X N\text{-}Int(K)$.

Example 8

In the indiscrete m-space (Q, m_{ind}). If K=Q-{1}, then $m_X N\text{-}cl(K)$) =Q, and $m_X N\text{-}Int(K)$= Q-{1}.

Proposition 4

Let K be a non-empty subset of an m-space (X, m_X), then $m_X \omega\text{-}cl(K)$= $K \cup (m_X \omega\text{-}d(K))$

Proof

Assume that $x \in K \cup (m_X N\text{-}d(K))$ and $x \notin m_X N\text{-}cl(K)$, so there is an $m_X N$-closed set D such that $K \subseteq D$ with $x \notin D$, put W=X-D, hence W is $m_X N$-open set containing x, then $W \cap K = \emptyset$, thus $x \notin m_X N\text{-}d(K)$ and since $x \notin K$ (because $x \notin D$ and $K \subseteq D$), so that $x \notin K \cup (m_X N\text{-}d(K))$ and that is a contradiction, therefore $x \in m_X N\text{-}cl(K)$, which implies $K \cup (m_X N\text{-}d(K)) \subseteq m_X N\text{-}cl(K)$. Conversely, if $x \in m_X N\text{-}cl(K)$ and assume that $x \notin K \cup (m_X N\text{-}d(K))$, then there exists an $m_X N$-open set S in X containing x and $S \cap K = \emptyset$, also C=X-S is $m_X N$-closed set in which $K \subseteq C$ and $x \notin C$, so $x \notin m_X N\text{-}cl(K)$ C!, thus $x \in K \cup (m_X N\text{-}d(K))$, and then $m_X N\text{-}cl(K) \subseteq K \cup (m_X N\text{-}d(K))$, which implies $m_X N\text{-}cl(K)$ =$K \cup$ $m_X N\text{-}d(K)$.

Proposition 5

If K is a subset of an m-space X, then K is $m_X N$-open set if and only if any point in K is an $m_X N$–interior point of it.

Proof

Consider K is an $m_X N$-open set and $x \in K$, since K is a subset of itself, so x is an $m_X N$-interior point. Conversely, since K is a union of all its points which are $m_X N$-interior point, so for each x in K there is an $m_X N$-open set W in X with $x \in W \subseteq K$, then $K = \bigcup_{x_\alpha \in K} W_{x_\alpha}$, for each $\alpha \in \wedge$, , and by lemma 1 we get K is $m_X N$-open set.

Proposition 6

Let K be a subset of an m-space X, then K is $m_X N$-closed if and only if $m_X N\text{-}d(K) \subseteq K$.

Proof

Suppose K is $m_X N$-closed set in X, and assume that $x \in m_X N\text{-}d(K)$ with $x \notin K$, hence K^c is $m_X N$-open subset of X containing x, and since $K^c \cap K = \emptyset$, we get that x is not $m_X N$-limit point to K, that implies $x \notin m_X N\text{-}d(K)$ which is a contradiction, therefore $x \in K$, and hence $m_X N\text{-}d(K) \subseteq K$. Conversely, if $m_X N\text{-}d(K) \subseteq K$ take $x \in X$ and $x \notin K$, then $x \in K^c$, hence x is not $m_X N$-limit point for K, so there is an $m_X N$-open set G containing x, with $G \cap K = \emptyset$, then $G \subseteq K^c$, therefore x is $m_X N$-interior point for K^c, thus K^c is $m_X N$-open subset in X, which implies K is $m_X N$-closed.

Definition 9 (6)

An m-function f from m-space X into m-space Y is called an m-continuous function if and only if $f^{-1}(M)$ is m_X-open set in X, for every m_Y-open set M in Y.

Proposition 7

The m-continuous image of m_X-connected set in X is m_Y-connected set in Y.

Proof

Let $f: X \longrightarrow Y$ be an m-continuous function and T is m_X-connected set in X, and suppose that $f(T)$ is not m_Y-connected, so it is m_Y-disconnected, so there is an m_Y-disconnection $N \cup M$ to $f(T)$, since f is m-continuous function, then $f^{-1}(N)$ and $f^{-1}(M)$ are m_X-open sets in X, with $f^{-1}(N) \cup f^{-1}(M)$ =T, so $T \subseteq f^{-1}(N) \cup f^{-1}(M)$, and $N \cap M \cap f(T) = \emptyset$, then $f^{-1}(N)$ and $f^{-1}(M)$ are disjoint and separation of T, that is a contradict the hypothesis that T is m-connected set in X, so $f(T)$ is m_Y-connected set.

Note 1

An m-space X is called is mNT_1-space if for each two distinct points a, b in X there are two non-empty
$m_X N$-open sets N and M such that N containing a but not b and M containing b but not a.

Example 9

The co-finite m-space $(\mathcal{R}, m_{cof})$ is mNT_1-space.

Remark 4

Every mT_1-space is mNT_1-space.

Definition 10

An m-space X is called is mN-Hausdorff space if for each distinct points a, b in X there are two non-empty $m_X N$-open sets N and M in X such that $a \in N$, $b \in M$ and $N \cap M = \emptyset$.

Example 10

The discrete m-space $(\mathcal{R}, m_D)$ is mN-Hausdorff space.

Remark 5

1- Every mN-Hausdorff space is mNT_1-space, but not conversely, since if X is mN-Hausdorff space, then there are two disjoint $m_X N$-open sets N and M

in X, such that $N \cap M = \emptyset$, and $a \in N$, $b \in M$, since $N \cap M = \emptyset$, so $b \notin N$ and $a \notin M$, hence X is mNT_1-space.

Example 11

Let $(Z,\ m_{ind})$ be the indiscrete m-space, let $x, y \in Z$ with $x \neq y$, then we can find two $m_Z N$-open sets U and V in Z such that $U = Z\text{-}\{x\}$ which containing y but not x, and $V = Z\text{-}\{y\}$ which containing x but not y, so (Z, m_{ind}) is $mN\ T_1$-space, but not $mN\ T_2$-space since $(Z\text{-}\{x\}) \cap (Z\text{-}\{y\}) \neq \emptyset$. Also $(\mathcal{R}, m_{cof})$ is mNT_1-space but not mNT_2-space.

Remark 6

Every mN-totally disconnected space is mN-Hausdorff space, but the converse is not true, since if X is mN-totally disconnected space then for each distinct points a, b in X, we can find two $m_X N$-open sets M, N containing a, b respectively with $N \cap M = \emptyset$ and $N \cup M = X$, so X is mN-Hausdorff space.

Example 12

Let $(\mathcal{R},\ m_u)$ be the usual m-space, it is mN-Hausdorff space, but not mN-totally disconnected.

Remark 7

Every m-Hausdorff space is mN-Hausdorff, but the converse is not true, since if X is m-Hausdorff space, so there are m_X-open sets N and M in X, such that $N \neq \emptyset$, $M \neq \emptyset$, and $a \in N$, $b \in M$, by Remark 2 X is mN-Hausdorff.

Example 13

Let $X = \{1, 2, 3\}$ and $m_X = \{\emptyset, X, \{1,2\}, \{2\}, \{3\}\}$, 1 and 2 are distinct points in X, and there exist $m_X N$-open sets $U = \{1\}$ and $V = \{2\}$ in X containing 1, 2 respectively, and $U \cap V = \emptyset$, also 1 and 3 are distinct points in X, there exist $m_X N$-open sets $U = \{1\}$ and $V = \{3\}$ in X containing 1, 3 respectively, and $U \cap V = \emptyset$, by the same way 2 and 3 are distinct points in X, there exist $m_X N$-open sets $U = \{2\}$ and $V = \{3\}$ in X containing 2, 3 respectively, and $U \cap V = \emptyset$ so (X, m_X) is mNT_2-space which is not mT_2-space since there is no two disjoint m_X-open sets containing 1, 2 respectively.

Remark 8

Every m-totally disconnected space is mN-disconnected but the converse is not true, since if X is m-totally disconnected space, then for any two points $a, b \in X$ where $a \neq b$ we can find m_X-open sets N and M in X, with $N \neq \emptyset$, $M \neq \emptyset$, $N \cap M = \emptyset$, and they containing a, b respectively such that $N \cup M = X$, so X is m-disconnected and then mN-disconnected (by remark 4)).

Example 14

Let X = {a, b, c} and m_X = {$\emptyset$, X, {a}, {b, c}}, then X is mN-disconnected space and not m-totally disconnected.

Remark 9

Let $f:(X, m_X) \longrightarrow (Y, m_Y)$ be an m-continuous function and K be $m_X N$-totally disconnected subset of X, then $f(K)$ is not $m_Y N$-totally disconnected subset of Y.

Example 15

Let $I_Z: (Z, m_D) \longrightarrow (Z, m_{cof})$ where I_Z is the identity function, (Z, m_D) is mN–totally disconnected space, while (Z, m_{cof}) is not mN-totallly disconnected.

Definition 11

The m-function $f: (X, m_X) \longrightarrow (Y, m_Y)$ is called mN-totally disconnected if the image of each m_X-totally disconnected set in X is $m_Y N$-totally disconnected in Y.

Definition 12

The m-function $f: (X, m_X) \longrightarrow (Y, m_X)$ is called mN^*-totally disconnected if the image of each $m_X N$-totally disconnected set in X is m_Y-totally disconnected in Y.

Definition 13

The m-function $f:(X,\ m_X) \longrightarrow (Y,\ m_Y)$ is called mN^{**}-totally disconnected if the image of each $m_X N$-totally disconnected set in X is $m_Y N$-totally disconnected in Y.

The following Example satisfying Definitions 11, 12 and 13.

Example 16

The identity m-function $I_X:(X,\ m_X) \longrightarrow (X,\ m_D)$ is mN-totally disconnected function.

Definition 14

The surjective m-function $f: (X,\ m_X) \longrightarrow (X, m_X)$ is called mN-light function if the inverse image of any $b \in Y$ is $m_X N$-totally disconnected set in X.

Example 17

The identity m-function $I_{\mathcal{R}}:(\mathcal{R}, m_D) \longrightarrow (\mathcal{R}, m_{cof})$ is mN-light function.

Remark 10

Every m-light function is mN-light function, but the converse is not true, since if f:(X, m_X) $\longrightarrow$(Y, m_Y) is m-light function, then $f^{-1}(b)$ is m_X-totally disconnected for any b in Y, then it is $m_X N$-totally disconnected set in X (by Remark 2), so f is mN-light function.

Example 18

The m-function $f:(X, m_{ind}) \longrightarrow (X, m_{cof})$, which defined by $f(x)$ = c, for each $x \in X$, where X= {1, 2, 3}, is mN-light function but not m-light function.

Remark 11

Every m-homeomorphism function is mN-light function, but the converse is not true, since if f :(X, m_X) $\longrightarrow$(Y, m_Y) is m-homeomorphism function, then for any b in Y there is a unique a in X where $f(a)$= b (since f is bijective), so $f^{-1}(b)$={a} which is m_X-totally disconnected, so {a} is $m_X N$-

totally disconnected (by Remark 2), and then f is mN-light.

Example 19

The function f:(X, m_D) ⟶(Y, m_Y), where X= {a, b, c, d, e, f} and Y= {g, h, i} such that f(a) = f(b)= g, f(c)=f(d)=h, f(e)=f(f)=i, is mN-light function but not m-homeomorphism.

Theorem 1

If f:(X, m_X) ⟶(Y, m_Y) is mN-light function and $G \subseteq X$, so $f|_G:G \longrightarrow f(G)$ is mN-light function too.

Proof

If $g \in f(G)$, so $g \in Y$ (because $f(G) \subseteq Y$), and since f is mN-light function so $f^{-1}(g)$ is $m_X N$-totally disconnected set in X. To prove that $f^{-1}(g) \cap G$ is $m_G N$-totally disconnected set in G for any $g \in f(G)$. Let $a, b \in f^{-1}(g) \cap G$, then $a, b \in f^{-1}(g)$, since $f^{-1}(g)$ is $m_X N$-totally disconnected set in X, then there is an $m_X N$-disconnection $N \cup M$ to $f^{-1}(g)$ with $(N \cap f^{-1}(g)) \cup (M \cap f^{-1}(g)) = f^{-1}(g)$ and $(N \cap f^{-1}(g)) \cap (M \cap f^{-1}(g)) = \emptyset$, such that N and M are $m_X N$-open sets in X, and $a \in N$, $b \in M$. To show that $f^{-1}(g) \cap G$ is $m_G N$-totally disconnected set in G. Since $((G \cap f^{-1}(g)) \cap N) \cup ((G \cap f^{-1}(g)) \cap M) = (G \cap (f^{-1}(g) \cap N)) \cup (G \cap (f^{-1}(g) \cap M)) = G \cap ((f^{-1}(g) \cap N) \cup (f^{-1}(g) \cap M)) = G \cap f^{-1}(g)$, and $((G \cap f^{-1}(g)) \cap N) \cap ((G \cap f^{-1}(g)) \cap M) = (G \cap (f^{-1}(g) \cap N)) \cap (G \cap (f^{-1}(g) \cap M)) = G \cap ((f^{-1}(g) \cap N) \cap (f^{-1}(g) \cap M)) = G \cap \emptyset = \emptyset$, such that $a \in (G \cap f^{-1}(g)) \cap N$ and $b \in (G \cap f^{-1}(g)) \cap M$, hence $(G \cap f^{-1}(g)) \cap M$ and $(G \cap f^{-1}(g)) \cap N$ are disjoint $m_G N$-open sets and the union of them is equal to $f^{-1}(g) \cap G$, so $f^{-1}(g) \cap G$ is $m_G N$-totally disconnected set in G, therefore $f|_G$ is mN-light function.

Definition 15

A surjective m-function f:(X, m_X) ⟶(Y, m_Y) is called inversely mN-totally disconnected function if the inverse image of any $m_Y N$-totally disconnected set in Y is $m_X N$-totally disconnected set in X.

Example 20

The identity m-function I_x:(X, m_{ind})⟶(X, m_X), where X is a finite set is inversely mN-totally disconnected function.

Proposition 8

Every inversely mN-totally disconnected function is mN-light function.

Proof

Let f:(X, m_X) ⟶(Y, m_Y) be inversely mN-totally disconnected function and $b \in Y$, since f is surjective m-function (since it is inversely mN-totally disconnected) and $f^{-1}(\{b\})$ is $m_X N$-totally disconnected set in X, where {b} is $m_Y N$-totally disconnected set in Y which implies f is mN-light function.

Proposition 9

The m-function h:(X, m_X)⟶ (Y, m_Y), where h=$g \circ f$ is mN-light function if $f: (X, m_X) \longrightarrow (Z, m_Z)$ is inversely mN-totally disconnected function and $g: (Z, m_Z) \longrightarrow (Y, m_Y)$ is mN- light function.

Proof

Let $b \in Y$, so $h^{-1}(b)$ =$(g \circ f)^{-1}$ (b) = $f^{-1}(g^{-1}(b))$, but $g^{-1}(b)$ is $m_Z N$-totally disconnected set (because g is mN-light function), and then $f^{-1}(g^{-1}(b))$ is m_X N-totally disconnected set in X (since f is inversely mN-totally disconnected function), so that $h^{-1}(b)$ is $m_X N$-totally disconnected set in X, which means h is mN-light function.

Proposition 10

If $g: (Z, m_Z) \longrightarrow (Y, m_Y)$ is bijective m-function and f:(X, m_X)⟶(Z, m_Z) is mN-light function, then the surjective m-function h:(X, m_X) ⟶(Y, m_Y) where h=$g \circ f$ is mN- light function.

Proof

Let $b \in Y$, then there is an element $z \in Z$ such that $g(z)$ =b (since g is bijective m-function), now $h^{-1}(b)$ = $(g \circ f)^{-1}(b)$ = $f^{-1}(g^{-1}(b))$ = $f^{-1}(g^{-1}(g(z))$ = $f^{-1}(z)$, but $f^{-1}(z)$ is $m_X N$-totall disconnected set in X (because f is mN-light function), which implies $h^{-1}(b)$ is $m_X N$-totally disconnected set in X, so that h is mN-light function.

Proposition 11

If g:(Z, m_Z) ⟶ (Y, m_Y) is one-to-one m-function, f:(X, m_X) ⟶(Z, m_Z) is m-function and h:(X, m_X) ⟶(Y, m_Y) is mN-light function such that h=$g \circ f$, then f is mN-light function.

Proof

Since $g(z) \in Y$, for each $z \in Z$ and $h^{-1}(g(z))$ is $m_X N$-totally disconnected set in X (because h is mN-light function), and since $h^{-1}(g(z))$ = $(g \circ f)^{-1}$ $(g(z))$ = $f^{-1}(g^{-1}(g(z)))$ = $f^{-1}(z)$, so $f^{-1}(z)$ is $m_X N$-totally disconnected set in X, hence f is mN-light function.

Proposition 12

If f:(X, m_X) ⟶(Z, m_Z) is mN-totally disconnected function and h:(X, m_X)⟶(Y, m_Y) is a surjective mN-light function such that h=$g \circ f$, then g:(Z, m_Z)⟶(Y, m_Y) is mN-light function.

Proof

Since $h^{-1}(y)$ is $m_X N$-totally disconnected set in X for each $y \in Y$ (because h is mN-light function), and $f(h^{-1}(y))$ is $m_Z N$-totally disconnected set in Z (since f is mN-totally disconnected function), but $f(h^{-1}(y))$ = $f((g \circ f)^{-1}(y))$ = $f(f^{-1}(g^{-1}(y))$ =

$g^{-1}(y)$, hence $g^{-1}(y)$ is $m_Z\omega$-totally disconnected set in Z,so that g is mN-light function.

Definition 16

The m-space (X, m_X) is called a zero dimension m-space if it has a base of $m_X\omega$-clopen sets.

Lemma 2

Every zero dimension metric m-space is mN-totally disconnected space.

Proof

Let X be a zero dimension metric m-space and a, b are points in X with $a{\neq}b$, then X is m-Hausdorff space and since it is metric m-space, then a has a neighbourhood K with $b \notin K$, then there exists a basic m_X-open set W which is also m_X-closed set in X (since X is zero dimensional m-space) and then W is m_XN-clopen set (by Remark 2 and since the complement of m_XN-open set is m_XN-closed set), where $a \in W \subseteq K$, and W^C is m_XN-clopen set in X such that $b \in W^C$, X=$W{\cup}W^C$ and $W{\cap}W^C$=$\emptyset$, so X is mN-totally disconnected space.

Proposition 13

Let X, Y be metric m-spaces and f:$(X, m_X) \longrightarrow (Y, m_Y)$ be a surjective m-function where X is mN-compact space, then f is mN-light function if the inverse image for each $b \in Y$ is a zero dimension a subspace of X.

Proof

Let $b \in Y$, so $f^{-1}(b)$ is zero dimension metric m-subspace of X (since metric is hereditary property), so it is m_XN-totally disconnected subspace of X (by lemma (3-63)) and so that f is mN-light function.

New subjects and future work.

Definition 17 (12)

A subset F of m-space X is said to be m_X-g-closed if for each m_X-open set U with $F \subseteq U$, then m_X-cl $(F) \subseteq U$.

Definition 18

A subset G of m-space X is said to be m_X-g-open if $F \subseteq m_X$-Int (G) for each m_X-closed set F with $F \subseteq G$.

Definition 19

A subset A of m-space X is said to be m_X-Ng-open set if for each $x \in A$, there exists m_X-g-open set U containing x such that U-A is finite. The complement of m_X-Ng-open set is m_X-Ng-closed set.

There is a relation between Definition 19 and m_X-N-open set as follows.

Remark 12

Every m_X-N-open set is m_X-Ng-open, but the converse is not true in general.

Example 21

The subset $\{x\}_{x\in\mathcal{R}}$ in $(\mathcal{R}, m_{ind})$ is m_X-Ng-open but it is neither m_X-open nor m_X-N-open set.

Question 1

Is there a relation between Definition 19 and m_X-open set? if there is a relation, is there an example to the converse?

Question 2

If we use m_X-Ng-open set instead of m_XN-open in this research, will we get approach results?

Now we will use the previously presented set to define another type of m-disconnected space, which is:-

Definition 20

An m-space X is said to be m-Ng-disconnected if it is union of two disjoint m_X-Ng-open sets.

Question 3

What is the relation between m-N-disconnected and m-Ng-disconnected space?

In a same way and by using m_X-Ng-open set, new type of m-light function have been defined, which is:-

Definition 21

A function f from m-space X into m-space Y is said to be m-Ng-light if for every $y \in Y$, $f^{-1}(y)$ is m-Ng-totally disconnected.

Question 4

What is the relation between mN-light and m-Ng-light function?

Remark 13

There is a definition in the topological space to Nadia Kadum Humadi (13), we can exploit it by using the definition of m_X-ωg-open set.

Conclusions:

In this research, new spaces namely mN-disconnected, mN-totally disconnected, mN-Hausdorff, mNT_1-spaces, have been defined and mN-light and inversely mN-totally disconnected functions have been introduced.

Acknowledgements:

The authors would like to thank Mustansiriyah University (www.uomustansiriyah.edu.iq) Baghdad – Iraq for its support in the present work. We also express our gratitude to the referees for carefully reading the paper and for their valuable comments.

Conflicts of Interest: None.

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حول التطبيقات الواهنة وأنماط من الفضاءات بأستخدام المجموعات المفتوحة- $m_X N$

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الخلاصة:
قدمنا صيغه ضعيفة من الدوال m–واهنه بأستخدام المجموعة $m_X N$- المفتوحة والتي هي الدالةmN - الواهنة، وقدمنا مفاهيم جديدة الفضاءات غير المترابطة والغير مترابطة كلياً، العلاقة بينهما قد درست. كذلك عرفنا صيغة جديدة من الدوال m-غير المترابطة كلياً و الدوال العكسياً m-غير المترابطة كلياً قد عرفت، أعطينا بعض الامثلة والحقائق.

الكلمات المفتاحية: الفضاء غير المتصل – $m_x N$ ، الفضاء هاوسدورف– $m_x N$ ، الدالة الواهنة – mN ، المجموعة المفتوحة– $m_x N$ ، الدالة غير المتصل كليا– mN.