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New Concepts of Fuzzy Local Function

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Abstract:

The main idea of this paper is to define other types of a fuzzy local function and study the advantages and differences between them in addition to discussing some definitions of finding new fuzzy topologies. Also in this research, a new type of fuzzy closure has been defined, where the relation between the new type and different types of fuzzy local function has been studied.

Key words: Fuzzy Closure, Fuzzy Local Function, Locally-Fuzzy Closure, Weakly- Local Function.

Introduction:

Since Zadeh introduced his paper in 1965 related to fuzzy set, his study has helped to remove ambiguity from a large number of mathematical exercises (1). Zadeh's approach offered an accurate description of thing where we have the ability to deal with any set traditionally or fuzzily. Since then research has been launched to investigate fuzzy set involved in almost all different branches of mathematics and adopt the traditional definition based on fuzzy set. Consequently, scientists and researchers impressive and surprising findings. In addition, it has been involved in other disciplines such as computer, electrical, mechanical, and encryption science and other applied sciences.

Chang is considered as the first researcher who established the notion of fuzzy topology in 1968 (2). Lowen in 1976 introduced the definition of fuzzy topology (3). As well as Pupo-Ming and Ying-Ming 1980 presented the first definition for neighborhood system based on fuzzy set and fuzzy point (4). M.K Chakraborty 1992 introduced the concept of quasi-coincidence between fuzzy set (5).

In1997, Sarkar introduced the concept of fuzzy ideal and idea of fuzzy local function based on quasi- fuzzy neighborhood in fuzzy topology (6). Additionally, the fuzzy ideal has received increasing attention by several researchers such as Yuksel, El Naschie, Nasef, Salama, and other researchers who introduced an extensive study in fuzzy topological spaces. However, their studies focused on different fuzzy family (7, 8, 9, 10, 11).

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In 2018, the fuzzy family has been classified (12). Therefore, our research will focus on the broad family in the classification of the fuzzy family. In this paper, we introduced the broader fuzzy local function definition than the definition of Sarkar, also introduced other definitions that are narrower than the definition of Sakar as well as studying the relationships between them. We also introduce the new definition of fuzzy closure by utilizing the quasi-coincident and we discuss its types of fuzzy local function.

In (13, 14) different types of local function were studied in ideal topological space and soft topological space. We can develop the compactness, continuity with respect to fuzzy set theory.

Preliminaries

In this paper, we believed that the space that we work is known by a broad family of families of fuzzy sets which depends on several definitions such continuous functions, or non-continuous of every subset of X (12). We will symbolize for universal fuzzy set 1^{z}, for the family of all fuzzy sets is denoted by Γ^{X}, and any subset of it as A^{z} as an appreciation of Zadeh.

Definition 1: (12) Let the membership $\mathrm{M}(\mathrm{X},\mathrm{I}) = \{f; f: \mathrm{X} \to \mathrm{I}\}$ where X any set, I= [0, 1]. A fuzzy set A^{z} of the space $\mathrm{X} \times \mathrm{I}$ is form,

$\mathrm{A}^{z} = \{ (x, f_A(x)), \forall\, x \in \mathrm{X} \}$ where

$$f_{\mathrm{A}}(x) = \begin{cases} f(x) & for\ \ x \in \mathrm{A} \\ 0 & for\ \ x \notin \mathrm{A} \end{cases}$$

Example 1: Let $\mathrm{X} = \{1, 2, 3\}$, $\mathrm{A} = \{2\}$ and $\mathrm{B} = \{1, 3\}$, when the memberships of $\mathrm{A}^{z}, \mathrm{B}^{z}$ are: $f_{\mathrm{A}}(x) = \frac{1}{x}$, $g_{\mathrm{B}}(x) = \frac{1}{x^2}$. Hence

$A^z=\{(1,0),(2,0.5),(3,0)\}$,
$B^z=\{(1,1),(2,0),(3,0.11)\}$.

Definition 2: A fuzzy point in Γ^X with support $x \in X$ is fuzzy set and denoted by P_x^λ for ($0<\lambda \le 1$) and the membership is $P_x^\lambda(z)=\begin{cases}\lambda & \text{if } z=x\\ 0 & \text{if } z\ne x\end{cases}$, P_x^λ contained in A^z iff $\lambda \le f_A(x)$. A fuzzy set A^z is called fuzzy subset of fuzzy set B^z if and only if $f_A(x)\le g_B(x)\ \forall\, x\in X$ (4).

Definition 3: (4) A^z is called quasi-coincident with a fuzzy set B^z denoted by $A^z q B^z$ iff there exists $y \in X$ such that $f_A(y)+g_B(y)>1$. Otherwise, we called not quasi-coincident if $f_A(x)+g_B(x)\le 1\ \forall x\in X$ and denoted by $A^z \wp B^z$.

Definition 4: Let A^z, B^z any two fuzzy sets. The membership of fuzzy standard intersection, fuzzy standard union and fuzzy complement sequentially define by $A^z \wedge B^z=\{(x, min\{f_A(x), g_B(x)\}), \forall x\in X\}$, $A^z \vee B^z=\{(x, max\{f_A(x), g_B(x)\}), \forall x \in X\}$, $1^z - A^z$ or $(A^z)^c=\{(x, 1-f_A(x)), \forall x\in X\}$ where f_A and g_B are memberships of A^z and B^z

In fuzzy set, there are two means of obtaining the difference,

I. Simple difference: the membership function is defined as $f_{A-B}(x)=min\{f_A(x), 1-f_B(x)\}$.

II. Bounded difference: the membership function is defined as $f_{A-B}(x)=max\{f_A(x)-f_B(x), 0\}$ (15).

Definition 5: (2) Let $(1^z, \tau)$ be a fuzzy topology space as mentioned in Chang (in short, $F\mathcal{T}S$) then,

I. A fuzzy set A^z in $F\mathcal{T}S$ is called a fuzzy neighborhood (in short, f.nbd) of a fuzzy point P_x^λ iff $\exists$ fuzzy open set $\mathcal{U}^z$ s.t $P_x^\lambda \in \mathcal{U}^z \le A^z$. We will denote the set of all f.nbd of P_x^λ in $F\mathcal{T}S$ by $\mathcal{N}(P_x^\lambda)$.

II. A fuzzy set A^z in $F\mathcal{T}S$ is called a quasi- fuzzy neighborhood (in short, q-f.nbd) of a fuzzy point P_x^λ iff there exists a fuzzy open set $\mathcal{U}^z$ s.t $P_x^\lambda q \mathcal{U}^z$ and $\mathcal{U}^z \le A^z$. We will denote the set of all q-f.nbd of P_x^λ in $F\mathcal{T}S$ by $q-\mathcal{N}(P_x^\lambda)$. (4, 5)

Any fuzzy set $A^z \in \tau$ is called a fuzzy open set.
A^z is fuzzy closed iff the complement is fuzzy open. The closure of a fuzzy set $A^z \in \Gamma^X$ denoted by $cl(A^z)$ is the collection fuzzy point P_x^λ such that $\forall\, \mathcal{U}^z \in q-\mathcal{N}(P_x^\lambda)$ then $\mathcal{U}^z q A^z$.

Definition 6: (6) A sub collection on $\mathcal{J}^z$ of Γ^X is called fuzzy ideal if the following properties are achieved.

I. If $A^z \in \mathcal{J}^z$ and $B^z \le A^z$ then $B^z \in \mathcal{J}^z$ (Heredity).

II. If $A^z \in \mathcal{J}^z$ and $B^z \in \mathcal{J}^z$ then $(A^z \vee B^z) \in \mathcal{J}^z$ (Finite additivity).

Definition 7: (6) Let $(1^z, \tau)$ be $F\mathcal{T}S$ and $\mathcal{J}^z$ be a fuzzy ideal, a fuzzy local function of a fuzzy set A^z denoted by $A^{z*1}(\mathcal{J}^z, \tau)$ is defined by:
$A^{z*1}(\mathcal{J}^z,\tau)=\vee\{P_x^\lambda;\ \forall\, \mathcal{U}^z \in q-\mathcal{N}(P_x^\lambda), \exists\, y \in X$ s.t $f_{\mathcal{U}}(y)+g_A(y)-1>h_j(y)$ for every $j^z \in \mathcal{J}^z\}$. Also, we denoted the fuzzy local function of A^z by A^{z*1} or $A^{z*1}(\mathcal{J}^z)$.

Note that if $P_x^\lambda \notin A^{z*1}(\mathcal{J}^z,\tau)$ there is at least one $\mathcal{V}^z \in q-\mathcal{N}(P_x^\lambda)$ s.t $\forall\, x\in X$, $f_{\mathcal{V}}(x)+g_A(x)-1\le h_J(x)$ for some $J^z \in \mathcal{J}^z$. In order to unify the formulas, we have written the definition of Sarkar as above. Also, to unify terms, we have called Sarkar's definition the first type of fuzzy local function and we symbolized it as A^{z*1}.

Types of fuzzy local function

Sarkar defined the fuzzy local function in fuzzy ideal topology and obtained new fuzzy topology. In this section, we introduce different types of fuzzy local function in fuzzy ideal topology and discuss their characteristics.

Definition 8: Let $(1^z, \tau)$ be $F\mathcal{T}S$ and $\mathcal{J}^z$ be a fuzzy ideal. A fuzzy local function of A^z of the second type $A^{z*2}(\mathcal{J}^z,\tau)$ is defined by:
$A^{z*2}(\mathcal{J}^z,\tau)=\vee\{P_x^\lambda;\ \forall\, \mathcal{U}^z \in q-\mathcal{N}(P_x^\lambda), \exists\, y \in X$ s.t $min\{f_{\mathcal{U}}(y), g_A(y)\}>h_j(y)$ for every $j^z \in \mathcal{J}^z\}$. Denoted the fuzzy local function of A^z by A^{z*2} or $A^{z*2}(\mathcal{J}^z)$ for $A^{z*2}(\mathcal{J}^z,\tau)$.

Therefore any $P_x^\lambda \notin A^{z*2}(\mathcal{J}^z,\tau)$ there is at least one $\mathcal{V}^z \in q-\mathcal{N}(P_x^\lambda)$ s.t $\forall\, x\in X$ $min\{f_{\mathcal{V}}(x), g_A(x)\}\le h_J(x)$ for some $J^z \in \mathcal{J}^z$.

Throughout the Definition 7and 8 we notice that the first type of fuzzy local function is sub set of the second type. The following Proposition demonstrates.

Proposition 1: Every fuzzy local function of the first type for any fuzzy set is subset of the local function of the second type for it's that fuzzy set.

Proof

Let $P_x^\lambda \in A^{z*1}$ this implies for each $\mathcal{U}^z \in$ q_$\mathcal{N}(P_x^\lambda)\ \exists\, y \in X\ s.t$ $f_{\mathcal{U}}(y)+g_A(y)-1>h_j(y)$ for every $j^z \in \mathcal{J}^z$.
If $min\{f_{\mathcal{U}}(y), g_A(y)\}=f_{\mathcal{U}}(y)$ this mean $f_{\mathcal{U}}(y)\le g_A(y)$. Since $g_A(y)-1\le 0$, $f_{\mathcal{U}}(y)+(g_A(y)-1)\le f_{\mathcal{U}}(y)$, thus $min\{f_{\mathcal{U}}(y), g_A(y)\}=f_{\mathcal{U}}(y)\ge f_{\mathcal{U}}(y)+g_A(y)-1>h_j(y)$, hence $min\{f_{\mathcal{U}}(y), g_A(y)\}>h_j(y)$.
Again, if $min\{f_{\mathcal{U}}(y), g_A(y)\}=g_A(y)$,

This implies $g_A(y) \le f_{\mathcal{U}}(y)$, since $f_{\mathcal{U}}(y) - 1 \le 0$, $g_A(y) + f_{\mathcal{U}}(y) - 1 \le g_A(y)$,
thus $min\{f_{\mathcal{U}}(y), g_A(y)\} = g_A(y) \ge g_A(y) + f_{\mathcal{U}}(y) - 1 > h_j(y)$ So, $min\{f_{\mathcal{U}}(y), g_A(y)\} > h_j(y)$. Therefore $P_x^\lambda \in A^{z*2}$.

The converse of the above Theorem is not true. The following Example explain that.

Example 2: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and X= {1, 2, 3} the memberships of A^z, B^z, C^z and j^z are:

$$f_A(x) = \begin{cases} \frac{1}{2} & \text{if } x \text{ is odd} \\ \frac{3}{10} & \text{if } x \text{ is even} \end{cases} \quad \forall\, x \in A \quad , \quad g_B(x) = \begin{cases} \frac{1}{2} & \text{if } x \text{ is odd} \\ \frac{7}{10} & \text{if } x \text{ is even} \end{cases} \quad \forall\, x \in B\ , \ K_C(x) = \frac{x^2}{10}\ \forall\, x \in C,$$

$h_j(x) = \frac{2x+1}{10}$ $\forall\, x \in j$. Where A= {1, 2}, B= {2, 3}, C= {1, 3} and j = {2, 3}.
A^z={(1,0.5), (2, 0.3), (3,0)},
B^z={(1, 0), (2, 0.7), (3,0.5)},
C^z={(1, 0.1), (2, 0), (3,0.9)}.
put $\tau = \{ 0^z, 1^z, B^z, C^z, B^z \wedge C^z, B^z \vee C^z\}$ and $\mathcal{J}^z = \{0^z, j^z\} \cup \{\zeta^z; \zeta^z \le j^z\}$.Where j^z= {(1, 0), (2, 0.5), (3, 0.7)}.
Then, A^{z*2}= {(1, 1), (2, 0.3), (3,0.5)}. But A^{z*1}= {(1, 0.9), (2, 0.3), (3,0.1)}.

The following Theorem shows that the attributes of a fuzzy local function of the second type achieve all attributions of fuzzy local function in the first type.

Theorem 1: Let $(1^z, \tau)$ be $F\mathcal{TS}$. Let $\mathcal{J}^z, \mathcal{G}^z$ two fuzzy ideal and A^z, B^z any two fuzzy sets. Then,
I. $A^z \le B^z \implies A^{z*2} \le B^{z*2}$.
II. $\mathcal{J}^z \le \mathcal{G}^z \implies A^{z*2}(\mathcal{G}^z) \le A^{z*2}(\mathcal{J}^z)$.
III. $0^{z*2} = 0^z$.
IV. $j^z \in \mathcal{J}^z \implies j^{z*2} = 0^z$.
V. $(A^z \vee B^z)^{*2} = A^{z*2} \vee B^{z*2}$.
VI. $j^z \in \mathcal{J}^z \implies (A^z - j^z)^{*2}_{(max)\text{or}(min)} \le (A^z \vee j^z)^{*2} = A^{*2}$.
VII. $(A^z \wedge B^z)^{*2} \le A^{z*2} \wedge B^{z*2}$.
VIII. $A^{z*2}(\mathcal{J}^z \vee \zeta^z) \le A^{z*2}(\mathcal{J}^z) \vee A^{z*2}(\zeta^z)$
IX. $A^{z*2}(\mathcal{J}^z \wedge \zeta^z) = A^{z*2}(\mathcal{J}^z) \vee A^{z*2}(\zeta^z)$ for each $\mathcal{J}^z$and $\zeta^z \in \mathcal{J}^z$.
X. $(A^{z*2})^{*2} \le A^{z*2}$.
XI. $A^{z*2} = cl(A^{z*2})$.

Proof: We prove only VI, IX, X and XI.
VI) Let $P_x^\lambda \in (A^z - j^z)^{*2}_{(max)\text{or}(min)}$, for every $\mathcal{U}^z \in q\_\mathcal{N}(P_x^\lambda)\ \exists\, y \in X$ s.t $min\{f_{\mathcal{U}}(y), g_{(A^z - j^z)}(y)\} > h_J(y)$ for every $J^z \in \mathcal{J}^z$, since $A^z - j^z \le A^z$ this means $min\{f_{\mathcal{U}}(y), g_A(y)\} > h_J(y)$ for every $J^z \in \mathcal{J}^z$ is also true, thus $P_x^\lambda \in A^{z*2}$. Evident $A^{*2} = (A^z \vee j^z)^{*2}$.

IX) $\mathcal{J}^z \wedge \mathcal{G}^z \le \mathcal{J}^z$ this leads $A^{z*2}(\mathcal{J}^z) \le A^{z*2}(\mathcal{J}^z \wedge \mathcal{G}^z)$,
$\mathcal{J}^z \wedge \mathcal{G}^z \le \mathcal{G}^z$ this leads $A^{z*2}(\mathcal{G}^z) \le A^{z*2}(\mathcal{J}^z \wedge \mathcal{G}^z)$,
thus $A^{z*2}(\mathcal{J}^z) \vee A^{z*2}(\mathcal{G}^z) \le A^{z*2}(\mathcal{J}^z \wedge \mathcal{G}^z)$.
Again, let $P_x^\lambda \in A^{z*2}(\mathcal{J}^z \wedge \mathcal{G}^z)$ this implies for each $\mathcal{U}^z \in q\_\mathcal{N}(P_x^\lambda)\ \exists\, y \in X$ s.t $min\{f_{\mathcal{U}}(y), g_A(y)\} > min\{h_j(y), q_J(y)\}$ for every $j^z \in \mathcal{J}^z$ and $J^z \in \mathcal{G}^z$,
but $min\{h_j(y), q_J(y)\} = \begin{cases} h_j(y) \text{ if } h_j(y) \le q_J(y) \\ q_J(y) \text{ if } h_j(y) \ge q_J(y) \end{cases}$,
hence $min\{f_{\mathcal{U}}(y), g_A(y)\} > h_j(y)$
or $min\{f_{\mathcal{U}}(y), g_A(y)\} > q_J(y)$,
this implies $P_x^\lambda \in A^{z*2}(\mathcal{J}^z) \vee A^{z*2}(\mathcal{G}^z)$.

X) Let $P_x^\lambda \in (A^{z*2})^{*2}$ this leads $\forall\, \mathcal{U}^z \in q\_\mathcal{N}(P_x^\lambda)\ \exists\, y \in X$ such that $min\{(f_{\mathcal{U}}(y), g_{A^{z*2}}(y)\} > h_j(y)$ for every $j^z \in \mathcal{J}^z$, this $mean$ $f_{\mathcal{U}}(y) > h_j(y)$ and $g_{A^{z*2}}(y) > h_j(y)$ for every $j^z \in \mathcal{J}^z$,
if possible $P_x^\lambda \notin A^{z*2}$ this implies $\exists\, \mathcal{V}^z \in q\_\mathcal{N}(P_x^\lambda), \forall\, x \in X$ then, $min\{k_{\mathcal{V}}(x), g_A(x)\} \le h_J(x)$ for some $J^z \in \mathcal{J}^z$,
this mean either $k_{\mathcal{V}}(x) \le h_J(x)$ this contradiction, or $g_A(x) \le h_J(x)$ this implies $A^{z*2} = 0^z$ that is also contradiction.
XI) Obviously $A^z \le cl(A^z)$, this implies $A^{z*2} \le cl(A^{z*2})$
Let $P_x^\lambda \in cl(A^{z*2})$ for each $\mathcal{U}^z \in q - \mathcal{N}(P_x^\lambda)\ \exists\, y \in X$ s.t $f_{\mathcal{U}}(y) + g_{A^{z*2}}(y) > 1$,
this mean $g_{(A^{z*2})}(y) \ne 0$, then there exists $\sigma \in (0,1]$ s.t $g_{(A^{z*2})}(y) = \sigma$, let $P_y^\sigma \in A^{z*2}$ which mean that for each $\mathcal{V}^z \in q - \mathcal{N}(P_y^\sigma)\ \exists\, \acute{y} \in X$ s.t $min\{k_{\mathcal{V}}(\acute{y}), g_A(\acute{y})\} > h_J(\acute{y})$ for every $\mathcal{J}^z \in \mathcal{J}^z$, but $f_{\mathcal{U}}(y) + \sigma > 1$, this mean $\mathcal{U}^z \in q - \mathcal{N}(P_y^\sigma)$ so, we get that $min\{f_{\mathcal{U}}(\acute{y}), g_A(\acute{y})\} > h_J(\acute{y})$ but $\mathcal{U}^z$ is also $q - \mathcal{N}(P_x^\lambda)$ therefore, $P_x^\lambda \in A^{z*2}$.

According to this property, we conclude that whenever we expand the scope of the fuzzy local function, the relationship will be unstable there will be three possibilities of the relationship between the closure fuzzy set and fuzzy local function. The following Example illustrates this,

Example 3: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and X= {1, 2, 3} the memberships of j^z, A^z, B^z and C^z are:

$h_j(x)=\frac{x+2}{10}$, $f_A(x)=\frac{x^2}{10}$, $g_B(x)=\frac{x}{10}$, $K_C(x)=\frac{1}{10}$, $\forall\, x\in X$
$A^z=\{(1,0.1), (2, 0.4), (3,0.9)\}$
$B^z=\{(1, 0.1), (2, 0.2), (3,0.3)\}$
$C^z=\{(1,0.1), (2, 0.1), (3,0.1)\}$
Put $\tau=\{0^z, 1^z, B^z, C^z\}$. Let $\mathcal{J}^z$ be a fuzzy ideal of all fuzzy subset of j^z where $j^z=\{(1, 0.3), (2, 0.4), (3,0.5)\}$
$A^{z*2}=\{(1,0.9), (2,0.8), (3,0.7)\}=cl(A^{z*2})$ but $cl(A)=\{(1,0.9), (2,0.9), (3,0.9)\}$ thus $A^{z*2}=cl(A^{z*2})\le cl(A^z)$, but by Example 2 $A^{z*2}=cl(A^{z*2})=\{(1,1), (2,0.3), (3,0.5)\}$, $cl(A^z)=\{(1,0.9), (2,0.3), (3,0.1)\}$ this mean $cl(A^z)\le A^{z*2}=cl(A^{z*2})$.

We know that not all the results in the general topology can be achievable in fuzzy topology. There is an important Lemma achieved in the general topological space "if $\mathcal{U}\in\tau$ then $\mathcal{U}\cap A^*\subseteq(\mathcal{U}\cap A)^*$" where * is the local function with respect to ideal and general topological space $(X,\tau,\mathcal{J})$, but was not achieved in the fuzzy topological space, despite the fact that the researcher in 1997 proved that it can be achieved and many of the researchers have agreed with that (16). However the following Example contrasts this.

Example 4: Let $(1^z,\tau)$ be $F\mathcal{TS}$ and $X=\{1, 2, 3\}$ the memberships of A^z, $\mathcal{U}^z$, j^z are:

$$g_{\mathcal{U}}(x)=\begin{cases}\frac{2x}{3x+1} & \text{if x is odd}\\ \frac{x^2}{10} & \text{if x is even}\end{cases},\quad f_A(x)=\begin{cases}\frac{7}{10} & \text{if } x=1\\ \frac{1}{10} & \text{otherwise}\end{cases},\quad h_j(x)=\begin{cases}\frac{1}{2} & \text{if } x \text{ is odd}\\ 1 & \text{if } x \text{ is even}\end{cases}$$

$,\forall x\in X$
$A^z=\{(1,0.7), (2, 0.1), (3,0.1)\}$,
$\mathcal{U}^z=\{(1, 0.5), (2, 0.4), (3,0.6)\}$
put $\tau=\{0^z, 1^z, \mathcal{U}^z\}$, let $\mathcal{J}^z$ be a fuzzy ideal is all fuzzy subset of j^z where $j^z=\{(1, 0.5), (2, 1), (3,0.5)\}$
Obviously, $A^{z*1}=\{(1, 0.5), (2, 0.6), (3, 0.4)\}$, $\mathcal{U}^z\in\tau$ then $\mathcal{U}^z\wedge A^{z*1}=\{(1,0.5), (2, 0.4), (3,0.4)\}$, but $(\mathcal{U}^z\wedge A^z)^{*1}=0^z$ this implies $(\mathcal{U}^z\wedge A^z)^{*1}\le\mathcal{U}^z\wedge A^{z*1}$.

Also, the Lemma is achieved by the general topological space "if $\mathcal{U}\in\tau$, then $\mathcal{U}\cap clA^*\subseteq cl^*(\mathcal{U}\cap A)$". Where $cl^*(A)=A\cup A^*$ which has been approved by adopting the above Lemma, although achieved some researchers (17), but also has not been achieved in fuzzy topology space by adopting the following Example,
$\mathcal{U}^z\wedge cl(A^{z*1})=\{(1,0.5), (2, 0.4)(3,0.4)\}$, but $cl^{*1}(\mathcal{U}^z\wedge A^z)=(\mathcal{U}^z\wedge A^z)^{*1}\vee(\mathcal{U}^z\wedge A^z)$, this implies $cl^{*1}(\mathcal{U}^z\wedge A^z)=\{(1,0.5), (2, 0.1), (3,0.1)\}$.

Definition 9: Let $(1^z,\tau)$ be $F\mathcal{TS}$ and $\mathcal{J}^z$ be a fuzzy ideal. A^z any fuzzy set is called:
I. $\mathcal{J}^z$- Fuzzy dense iff $A^{z*2}=1^z$.
II. $\mathcal{J}^z$-Fuzzy open iff $A^z\le int(A^{z*2})$.
III. Fuzzy Locally in $\mathcal{J}^z$ iff $A^z\wedge A^{z*2}=0^z$.

Example 5: Let $(1^z,\tau)$ be $F\mathcal{TS}$ and $\mathcal{J}^z$ be a fuzzy ideal. Let $X=\{1, 2, 3\}$ the memberships of j^z, A^z, D^z , C^z and B^z are:
$h_j(x)=\frac{6-x}{10}$ $\forall\, x\in j$, $f_A(x)=\frac{3}{10}\forall\, x\in A$, $g_D(x)=\frac{3x}{10}\forall\, x\in D$, $K_C(x)=\frac{7-x}{10}$ $\forall\, x\in C$, $g_B(x)=\frac{3}{10}$ $\forall\, x\in B$.
Where $A=\{1, 2\}=D$, $C=\{1, 3\}=j$, $B=\{1\}$.
$A^z=\{(1,0.3), (2, 0.3), (3,0)\}$,
$D^z=\{(1, 0.3), (2, 0.6), (3,0)\}$,
$C^z=\{(1, 0.6), (2, 0), (3,0.4)\}$,
$B^z=\{(1, 0.3), (2, 0), (3,0)\}$. put $\tau=\{0^z, 1^z, D^z, C^z, D^z\wedge C^z, D^z\vee C^z\}$ and $\mathcal{J}^z=\{0^z, j^z\}\cup\{\zeta^z; \zeta^z\le j^z\}$.
Where $j^z=\{(1, 0.5), (2, 0), (3, 0.3)\}$ this implies $A^{z*2}(\mathcal{J}^z)=\{(1, 0.4), (2, 1), (3,0.6)\}$, $int(A^{z*2})=\{(1, 0.3), (2, 0.6), (3,0)\}$, thus A^z is $\mathcal{J}^z$- fuzzy open. $B^{z*2}=0^z$ this implies $B^z\wedge B^{z*2}=0^z$, thus B^z fuzzy Locally in $\mathcal{J}^z$.

The following Theorem is achieved in the general topology. We will prove that it is also true for the second type of fuzzy local function.

Theorem 2: Let $(1^z,\tau)$ be $F\mathcal{TS}$ and $\mathcal{J}^z$ be a fuzzy ideal, A^z any fuzzy set. The following statements are equivalent,
I. $\tau\cap\mathcal{J}^z=0^z$.
II. If $J^z\in\mathcal{J}^z$ then, $int(J^z)=0^z$.
III. For every $\mathcal{U}^z\epsilon\,\tau$ then, $\mathcal{U}^z\le\mathcal{U}^{z*2}$.
IV. $1^z=1^{z*2}$.

Proof

I) ⟹ II) Let $\tau\cap\mathcal{J}^z=0^z$, and $J^z\in\mathcal{J}^z$. If possible $int(J^z)\neq 0^z$, there exists $P_x^\lambda\in int(J^z)$, such that $\exists\,\mathcal{U}^z\in\tau(P_x^\lambda)$ and $\mathcal{U}^z\le J^z$, this contradiction, since $\mathcal{U}^z\in\tau$, thus $int(J^z)=0^z$.

II) ⟹ III) Let $\mathcal{U}^z\epsilon\,\tau$ and $P_x^\lambda\in\mathcal{U}^z$.If possible $P_x^\lambda\notin\mathcal{U}^{z*2}$, $\exists\,\mathcal{V}^z\epsilon$ q_$\mathcal{N}(P_x^\lambda)$, such that $\forall\, x\in X$, $min\{f_{\mathcal{V}}(x), g_{\mathcal{U}}(x)\}\le h_J(x)$ for some $J^z\in\mathcal{J}^z$, this means $\mathcal{V}^z\wedge\mathcal{U}^z\in\mathcal{J}^z$, but $\mathcal{V}^z\wedge\mathcal{U}^z\in\tau$ this lead to $P_x^\lambda\in int(\mathcal{V}^z\wedge\mathcal{U}^z)=int(J^z)\neq 0^z$ this contradiction, thus $\mathcal{U}^z\le\mathcal{U}^{z*2}$.

IIII) ⟹ IV) $1^{z*2}\le 1^z$ always is true. So, $1^z\in\tau$ by part (3) we get $1^z\le 1^{z*2}$, thus $1^z=1^{z*2}$.

IV) ⟹ I) If possible $\tau\cap\mathcal{J}^z\neq 0^z$, there exists $0^z\neq\mathcal{U}^z\in\tau$ and $\mathcal{U}^z\in\mathcal{J}^z$ there exists $P_x^\lambda\in$

$\mathcal{U}^{z}$ this implies $min\{f_{\mathcal{U}}(x),1\} \leq h_J(x)$ for some $J^z \in \mathcal{J}^z$, this means $P_x^\lambda \notin 1^{z*2}$ this contradiction, since $1^z = 1^{z*2}$, thus $\tau \cap \mathcal{J}^z = 0^z$.

Definition 10: Let $(1^z, \tau)$ be $F\mathcal{TS}$ with $\mathcal{J}^z$ a fuzzy ideal, A^z be any fuzzy set. The fuzzy closure of A^z with respect to τ and $\mathcal{J}^z$ denoted by $cl^{*2}(A^z)$ and defined by $cl^{*2}(A^z) = A^z \vee A^{z*2}$.

By Example 4 $cl^{*2}(A^z) =\{(1,0.7),(2,0.6)(3,0.4)\}$.

Theorem 3: Let $(1^z, \tau)$ be $F\mathcal{TS}$ with $\mathcal{J}^z$ a fuzzy ideal, for any two fuzzy set A^z and B^z. The following statements are hold,
I. $cl^{*2}(1^z)=1^z$ and $cl^{*2}(0^z)=0^z$.
II. $A^z \leq B^z \Rightarrow cl^{*2}(A^z) \leq cl^{*2}(B^z)$.
III. $cl^{*2}(A^z \vee B^z) = cl^{*2}(A^z) \vee cl^{*2}(B^z)$.
IV. $cl^{*2}(A^z \wedge B^z) \leq cl^{*2}(A^z) \wedge cl^{*2}(B^z)$.
V. $cl^{*2}(cl^{*2}(A^z)) = cl^{*2}(A^z)$.
VI. $A^z \leq cl^{*2}(A^z)$.

Proof
Clear from the Definition 10 and Theorem 1.

Definition 11: (3) Let $(1^z, \tau)$ be $F\mathcal{TS}$ with $\mathcal{J}^z$ a fuzzy ideal, A^z be any fuzzy set. A fuzzy closure operator with respect to $\mathcal{J}^z$ define by
$cl^{*2}: \Gamma^X \longrightarrow \Gamma^X$, satisfying the following four conditions:
I. $cl^{*2}(0^z)=0^z$.
II. $A^z \leq cl^{*2}(A^z)$.
III. $cl^{*2}(A^z \vee B^z) = cl^{*2}(A^z) \vee cl^{*2}(B^z)$.
IV. $cl^{*2}(cl^{*2}(A^z)) = cl^{*2}(A^z)$. Where A^z, B^z any fuzzy sets in Γ^X.

Theorem 4: Let $(1^z, \tau)$ be $F\mathcal{TS}$ with $\mathcal{J}^z$ a fuzzy ideal, and let cl^{*2} be the fuzzy closure operator, $cl^{*2}: \Gamma^X \longrightarrow \Gamma^X$ satisfying the four conditions in Definition 11, then τ^{*2} is a fuzzy topology such that $\tau^{*2} = \{A^z ; cl^{*2}(A^z)^c = (A^z)^c\}$.

Proof
Direct from τ^{*2} and the Definition of a fuzzy topology.

Definition 12: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and $\mathcal{J}^z$ be a fuzzy ideal. A^z be any fuzzy set is called,
I. Fuzzy τ^{*2}-dense in itself iff $A^z \leq A^{z*2}$.
II. Fuzzy τ^{*2}-dense iff $cl^{*2}(A^z) = 1^z$.

Remark 1:
I. If $\mathcal{J}^z = \{0^z\}$, then $cl(A^z) \leq A^{z*2}$ for any fuzzy set.
II. If $\mathcal{J}^z = \Gamma^X$, then $A^{z*2} = 0^z$ for any fuzzy set.

Theorem 5: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and $\mathcal{J}^z = \{0^z\}$ be a fuzzy ideal. For any fuzzy set A^z the following statements hold,
I. $A^z \leq A^{z*2}$.
II. Any fuzzy set is fuzzy τ^{*2}-dense in itself.
III. A^z is fuzzy τ^{*2}-dense iff A^z is fuzzy $\mathcal{J}^z$- dense.
IV. Any fuzzy set is not fuzzy locally in $\mathcal{J}^z$.

Proof
Directly from the Definition 12 and Remark 1.

Note 1: Every fuzzy $\mathcal{J}^z$- dense is a fuzzy τ^{*2}-dense.

Theorem 6: Let τ_1 and τ_2 be two fuzzy topologies s.t $\tau_1 \subseteq \tau_2$ and A^z be any fuzzy set. For any fuzzy ideal $\mathcal{J}^z$,
I. $A^{z*2}(\tau_2, \mathcal{J}^z) \leq A^{z*2}(\tau_1, \mathcal{J}^z)$.
II. ${\tau_1}^{*2}(\mathcal{J}^z) \subseteq {\tau_2}^{*2}(\mathcal{J}^z)$.

Proof
I) Let $P_x^\lambda \in A^{z*2}(\tau_2, \mathcal{J}^z)$, then every $\mathcal{U}^z \in \tau_2$ s.t $\mathcal{U}^z \in q\_\mathcal{N}(P_x^\lambda)$, $\exists$ y $\in$ X s.t $min\{f_{\mathcal{U}}(y), g_A(y)\} > h_J(y)$ for every $J^z \in \mathcal{J}^z$, this also true for all $\mathcal{V}^z \in q\_\mathcal{N}(P_x^\lambda)$ in τ_1 because $\tau_1 \subseteq \tau_2$, we have that $P_x^\lambda \in A^{z*2}(\tau_1, \mathcal{J}^z)$.

II) Let $\mathcal{U}^z \in {\tau_1}^{*2}(\mathcal{J}^z)$, this implies $\tau_1\text{-} cl^{*2}(\mathcal{U}^z)^c = (\mathcal{U}^z)^c$, since $\tau_1 \subseteq \tau_2$, then by part (1) we have $\tau_2\text{-} cl^{*2}(\mathcal{U}^z)^c \leq \tau_1\text{-} cl^{*2}(\mathcal{U}^z)^c = (\mathcal{U}^z)^c$, but $(\mathcal{U}^z)^c \leq \tau_2\text{-} cl^{*2}(\mathcal{U}^z)^c$ that is $\tau_2\text{-} cl^{*2}(\mathcal{U}^z)^c = (\mathcal{U}^z)^c$, thus $\mathcal{U}^z \in {\tau_2}^{*2}(\mathcal{J}^z)$.

Theorem 7: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and $\mathcal{J}^z$ be a fuzzy ideal, A^z be any fuzzy set. The following statements hold,
I. $(cl^{*2}(A^z))^{*2}= A^{z*2}$.
II. If $A^z \leq A^{z*2}$ then $cl(A^z) \leq cl(A^{z*2}) = A^{z*2} = cl^{*2}(A^z)$.

Proof
Directly from the Definition 10 and Theorem 1.

Through the following definition, we will minimize the scope of fuzzy local function and we will investigate its attributes and features during this period.

Definition 13: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and $\mathcal{J}^z$ be a fuzzy ideal, A^z fuzzy set in Γ^X the fuzzy local function of the third type by $A^{z*3}(\mathcal{J}^z, \tau)$ is defined by:
$A^{z*3}(\mathcal{J}^z, \tau) = \vee \{P_{\dot{x}}^\lambda;\ \forall\, \mathcal{U}^z \in q\_\mathcal{N}(P_{\dot{x}}^\lambda)$ and $\dot{x} \in$ X s.t $f_{\mathcal{U}}(\dot{x}) + g_A(\dot{x}) - 1 > h_J(\dot{x})$ for every $J^z \in \mathcal{J}^z\}$.

Noted, if $P_{\dot{x}}^\lambda \notin A^{z*3}(\mathcal{J}^z, \tau)$ there is at a least one $\mathcal{V}^z \in q\_\mathcal{N}(P_{\dot{x}}^\lambda)$ and $\dot{x} \in$ X s.t $f_{\mathcal{V}}(\dot{x}) + g_A(\dot{x}) - 1 \leq h_j(\dot{x})$ for some $j^z \in \mathcal{J}^z$. We will occasionally write A^{z*3}or $A^{z*3}(\mathcal{J}^z)$ for $A^{z*3}(\mathcal{J}^z, \tau)$.

By Example 2 the fuzzy local function $A^{z*3} = \{(1, 0.9), (2, 0), (3,0)\}$.

Theorem 8: Let $(1^{z},\tau)$ be F$\mathcal{TS}$ and let $\mathcal{J}^{z}$ fuzzy ideal. For any two fuzzy sets A^{z} and B^{z} we have the following:

I. $(A^{z} \wedge B^{z})^{*3} = A^{z*3} \wedge B^{z*3}$.

II. $A^{z*3} \leq cl(A^{z*3}) \leq cl(A^{z})$.

Proof

I) Let $P_{\dot{x}}^{\lambda} \in (A^{z*3} \wedge B^{z*3})$ this leads $P_{\dot{x}}^{\lambda} \in A^{z*3}$ and $P_{\dot{x}}^{\lambda} \in B^{z*3}$ this mean $\forall\, \mathcal{U}^{z} \in$ q_$\mathcal{N}(P_{\dot{x}}^{\lambda})$, $\dot{x} \in X$ s.t $f_{1\mathcal{U}}(\dot{x}) + g_{1A}(\dot{x}) - 1 > h_{j}(\dot{x})$ for every $j^{z} \in \mathcal{J}^{z}$. So, $\forall\, \mathcal{V}^{z} \in$ q_$\mathcal{N}(P_{\dot{x}}^{\lambda})$ and $\dot{x} \in X$ s.t $f_{2\mathcal{V}}(\dot{x}) + g_{2B}(\dot{x}) - 1 > h_{I}(\dot{x})$ for every $I^{z} \in \mathcal{J}^{z}$. If possible $P_{\dot{x}}^{\lambda} \notin (A^{z} \wedge B^{z})^{*3}$, this mean $\exists\, \mathcal{W} \in$ q_$\mathcal{N}(P_{\dot{x}}^{\lambda})$, $\dot{x} \in X$ s.t $f_{3\mathcal{W}}(\dot{x}) + min\{g_{1A}(\dot{x}), g_{2B}(\dot{x})\} - 1 \leq h_{\mathcal{J}_1}(\dot{x})$ for some $\mathcal{J}_1{}^{z} \in \mathcal{J}^{z}$. If $min\,\{g_{1A}(\dot{x}), g_{2B}(\dot{x})\} = g_{1A}(\dot{x})$ this imply that $f_{3\mathcal{W}}(\dot{x}) + g_{1A}(\dot{x}) - 1 \leq h_{\mathcal{J}_1}(\dot{x})$ for some $\mathcal{J}_1{}^{z} \in \mathcal{J}^{z}$ this contradiction. Also in the case that $min\,\{g_{1A}(\dot{x}), g_{2B}(\dot{x})\} = g_{2B}(\dot{x})$, this imply that $f_{3\mathcal{W}}(\dot{x}) + g_{2B}(\dot{x}) - 1 \leq h_{\mathcal{J}_1}(\dot{x})$ for some $\mathcal{J}_1{}^{z} \in \mathcal{J}^{z}$ this also contradiction. Hence $P_{\dot{x}}^{\lambda} \in (A^{z} \wedge B^{z})^{*3}$.

II) Directly from the definition of fuzzy closure.

Example 6: Let $(1^{z}, \tau)$ be a F$\mathcal{TS}$ and $X = \{1, 2, 3\}$, the memberships of A^{z}, B^{z}, j^{z} are:

$f_A(x) = \frac{x}{5}$, $g_B(x) = \frac{x^2-1}{10}$, $h_j(x) = \frac{x^2}{10}$, $\forall\, x \in X$

$A^{z}=\{(1, 0.2), (2, 0.4), (3, 0.6)\}$

$B^{z}=\{(1, 0), (2, 0.3), (3, 0.8)\}$

$\tau = \{0^{z}, 1^{z}, B^{z}\}$,

$\mathcal{J}^{z} = \{0^{z}, j^{z}\} \cup \{I^{z}, I^{z} \leq j^{z}\}$. Where $j^{z}=\{(1, 0.1), (2, 0.4), (3, 0.9)\}$.

Then $A^{z*3}= \{(1, 1), (2, 0), (3, 0)\}$ and $cl(A^{z*3})=\{(1, 1), (2, 0.7), (3,0.2)\}$ and $cl(A^{z})=\{(1, 1), (2, 1), (3, 1)\}$. This implies $A^{z*3} \leq cl(A^{z*3}) \leq cl(A^{z})$.

We notice that the other characteristics of the Theorem 1 is achieved for the third type of fuzzy local function.

Lemma 1: Let $(1^{z},\tau)$ be F$\mathcal{TS}$ with fuzzy ideal $\mathcal{J}^{z}$. Then, $A^{z*3} \leq A^{z*1}$ for any fuzzy set A^{z}.

Proof

Let $P_{\dot{x}}^{\lambda} \in A^{z*3}$, this implies $\forall\, \mathcal{U}^{z} \in$ q_$\mathcal{N}(P_{\dot{x}}^{\lambda})$ and $\dot{x} \in X$ then $f_{\mathcal{U}}(\dot{x}) + g_A(\dot{x}) - 1 > h_j(\dot{x})$ for every $j^{z} \in \mathcal{J}^{z}$, but $\dot{x}$ is a point in X, this mean $\exists\, x = \dot{x} \in X$ s.t $f_{\mathcal{U}}(x) + g_A(x) - 1 > h_j(x)$ for every $j^{z} \in \mathcal{J}^{z}$, thus $P_{\dot{x}}^{\lambda} \in A^{*1}$.

Theorem 9: Let $(1^{z},\tau)$ be F$\mathcal{TS}$ and let $\mathcal{J}^{z}$ be fuzzy ideal then, $A^{z*1} = cl(A^{z*3})$.

Proof

By Lemma 1 $A^{z*3} \leq A^{z*1}$, this implies $cl(A^{z*3}) \leq cl(A^{z*1}) = A^{z*1}$.

Again, let $P_{x}^{\lambda} \in A^{z*1}$ this mean $\forall\, \mathcal{U}^{z} \in$ q_$\mathcal{N}(P_{x}^{\lambda})\ \exists\, y \in X$ s.t $f_{1\mathcal{U}}(y) + g_A(y) - 1 > h_J(y)$ for every $J^{z} \in \mathcal{J}^{z}$. If $P_{x}^{\lambda} \notin cl(A^{z*3})$,

$A^{z*3} \leq cl(A^{z*3})$, this implies $P_{x}^{\lambda} \notin A^{z*3}$ thus $\forall\, x \in X\ \exists\, \mathcal{V}^{z} \in$ q_$\mathcal{N}(P_{x}^{\lambda})$ s.t $f_{2\mathcal{V}}(y) + g_A(y) - 1 \leq h_J(y)$ this contradiction.

Definition 14: Let $(1^{z},\tau)$ be F$\mathcal{TS}$ with a fuzzy ideal $\mathcal{J}^{z}$, $A^{z} \in \Gamma^{X}$. A fuzzy local of the fourth type symbolizes them $A^{z*4}(\mathcal{J}^{z},\tau)$ is defined by:

$A^{z*4}(\mathcal{J}^{z},\tau) = \vee\,\{ P_{\dot{x}}^{\lambda};\ \forall\, \mathcal{U}^{z} \in$ q_$\mathcal{N}(P_{\dot{x}}^{\lambda})$, $\dot{x} \in X$ s.t $min\{f_{\mathcal{U}}(\dot{x}), g_A(\dot{x})\} > h_J(\dot{x})$ for every $J^{z} \in \mathcal{J}^{z}\}$. We denoted the fuzzy local function of A^{z} by A^{z*4} or $A^{z*4}(\mathcal{J}^{z})$.

Therefore, any $P_{\dot{x}}^{\lambda} \notin A^{z*4}(\mathcal{J}^{z},\tau)$ there is at least one $\mathcal{V}^{z} \in$ q_$\mathcal{N}(P_{\dot{x}}^{\lambda})$ s.t $\dot{x} \in X$ then $min\{f_{\mathcal{V}}(\dot{x}), g_A(\dot{x})\} \leq h_j(\dot{x})$ for some $j^{z} \in \mathcal{J}^{z}$.

By Example 2 the fuzzy local function $A^{z*4} = \{(1,1), (2, 0), (3,0)\}$.

Definition 15: Let $(1^{z},\tau)$ be F$\mathcal{TS}$ with a fuzzy ideal $\mathcal{J}^{z}$, $A^{z} \in \Gamma^{X}$. A fuzzy local function of the fifth type symbolizes them $A^{z*5}(\mathcal{J}^{z},\tau)$ is defined by:

$A^{z*5}(\mathcal{J}^{z},\tau) = \vee\,\{P_{x}^{\lambda};\ \forall\, \mathcal{U}^{z} \in q\_\mathcal{N}(P_{x}^{\lambda}), \exists\, y \neq x \in X$ s.t $f_{\mathcal{U}}(y) + g_A(y) - 1 > h_j(y)$ for every $j^{z} \in \mathcal{J}^{z}\}$.

By Example 2 the fuzzy local function $A^{z*5}(\mathcal{J}^{z},\tau) = \{(1,0), (2, 0.3), (3,0.1)\}$.

Definition 16: Let $(1^{z},\tau)$ be F$\mathcal{TS}$ with a fuzzy ideal $\mathcal{J}^{z}$. The fuzzy local function of the sixth type symbolizes them $A^{z*6}(\mathcal{J}^{z},\tau)$ is defined by:

$A^{z*6}(\mathcal{J}^{z},\tau) = \vee\,\{P_{x}^{\lambda};\ \forall\, \mathcal{U}^{z} \in$ q-$\mathcal{N}(P_{x}^{\lambda})$, there exist $y \neq x \in X$ s.t $min\{f_{\mathcal{U}}(y), g_A(y)\} > h_j(y)$ for every $j^{z} \in \mathcal{J}^{z}\}$.

By Example 2 the fuzzy local function $A^{z*6}(\mathcal{J}^{z},\tau) = \{(1,0), (2, 0.3), (3,0.5)\}$.

The last definitions, as well as Sarkar's definition, we adopted the concept of quasi-fuzzy neighborhood, in the following definition will depend only on the concept of the fuzzy neighborhood.

Definition 17: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and $\mathcal{J}^z$ be a fuzzy ideal, A^z be any fuzzy set. A weakly- fuzzy local function denoted by $A^{z*w}(\mathcal{J}^z, \tau)$ is defined by:
$A^{z*w}(\mathcal{J}^z, \tau) = \vee \{P_x^\lambda;\ \forall\, \mathcal{U}^z \in \mathcal{N}(P_x^\lambda)\ , \exists\, y \in X \text{ s.t } f_U(y) + g_A(y) - 1 > h_J(y)$ for every $J^z \in \mathcal{J}^z\}$.

Therefore, any $P_x^\lambda \notin A^{z*w}(\mathcal{J}^z, \tau)$ there is at a least one $\mathcal{V}^z \in \mathcal{N}(P_x^\lambda), \forall\ y \in X \text{ s.t } f_V(y) + g_A(y) - 1 \le h_j(y)$ for some $j^z \in \mathcal{J}^z$.

Remark 2: The membership of every weakly- fuzzy local function A^z is equal one or zero.

Example 7: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and the membership of A^z, B^z and J^z are:

$$f_A(x) = \begin{cases} 1 & \text{if } x = 3 \\ \frac{1}{10} & \text{other wis} \end{cases},$$

$$h_J(x) = \begin{cases} \frac{1}{x+1} & \text{if } x \text{ is odd} \\ 1 & \text{other owis} \end{cases},$$

$$g_B(x) = \frac{1}{x^2}, E_C(x) = 1 - \frac{x}{10}\ ,\ \forall\, x \in X.$$

$A^z = \{(1,0.1), (2, 0.1), (3,1)\}$
$B^z = \{(1, 1), (2, 0.25), (3,0.11)\}$
$C^z = \{(1,0.9), (2, 0.8), (3,0.7)\}$
Put $\tau = \{0^z, 1^z, B^z, C^z, B^z \wedge C^z, B^z \vee C^z\}$. Let $\mathcal{J}^z$ be a fuzzy ideal of all fuzzy subset of J^z where $J^z = \{(1, 0.5), (2, 1)(3,0.25)\}$
Then, $A^{z*w} = \{(1, 0), (2, 1), (3,1)\}$.

Locally-Fuzzy closure

In this section, we introduce a special type of fuzzy closure set based on the concept of quasi-coincident, and study its various property.

Definition 18: Let A^z be a nonzero fuzzy set. The locally fuzzy closure of A^z denoted by L-$cl(A^z)$ is the intersection of all closed fuzzy set quasi-coincident with A^z and containing A^z.
i.e., L-$cl(A^z) = \wedge\{F^z;\ \ A^z q F^z$ and $A^z \le F^z, (F^z)^c \in \tau\}$. Therefore if $A^z = 0^z$ then, L-$cl(A^z) = 0^z$.

Example 8: Let $(1^z, \tau)$ be $F\mathcal{TS}$ and $X = \{1, 2\}$.
The memberships of A^z, B^z and C^z are:

$$f_A(x) = \frac{x^2}{10},\ g_B(x) = 1 - \frac{x}{5}, E_C(x) = \frac{x}{10}\ ,$$
$\forall\, x \in X$,
$A^z = \{(1, 0.1), (2, 0.4)\}$, $B^z = \{(1, 0.8), (2, 0.6)\}$
$C^z = \{(1,0.1), (2, 0.2)\}$, Put $\tau = \{0^z, 1^z, B^z, C^z\}$,
Then L-$clA^z = \{(1,0.9), (2, 0.8)\}$.

Note, the fuzzy closure is not necessarily a locally fuzzy closure since {(1, 0.2), (2, 0.4)} is a fuzzy closure but it's not locally fuzzy closure, since the locally fuzzy closure is the intersection of only the quasi-coincident closed set, while the fuzzy closure is the intersection of a quasi-coincident closed set or not quasi-coincident.

Note also, since $f_A(x) + g_F(x) > 1$ for some $x \in X$, and $f_A(x) \le g_F(x)\ \forall x \in X$, this mean must the value of $g_F(x) > 0.5$, if not $g_F(x) \le 0.5$ this implies that $f_A(x) + g_F(x) \le 1$ which contradicts with Definition 18.

Definition 19: A fuzzy set A^z is said to be locally fuzzy closed if and only if L-$cl(A^z) = A^z$.

Proposition 2: Every locally fuzzy closure set is fuzzy closed set.

The proof of the Proposition is obvious, since the locally fuzzy closure is smallest fuzzy closed set content A^z.

Theorem 10: Let A^z and B^z are fuzzy set then,
I. Every locally fuzzy closure set is locally fuzzy closed set
II. $A^z \le$ L-$cl(A^z)$.
III. $cl(A^z) \le$ L-$cl(A^z)$.
IV. If $A^z \le B^z$ then, L-$cl(A^z) \le$ L-$cl(B^z)$.
V. L-$cl(A^z \vee B^z) =$ L-$cl(A^z) \vee$ L-$cl(B^z)$.
VI. L-$cl(A^z \wedge B^z) \le$ L-$cl(A^z) \wedge$ L-$cl(B^z)$.
VII. Every locally fuzzy closed is fuzzy closed set.
VIII. L-$cl($L-$cl(A^z)) =$ L-$cl(A^z)$.
IX. L-$cl(1^z) = 1^z$.

Lemma 2: Let A^z any fuzzy sets, then L-$cl(A^z)$ = L-$cl(cl(A^z))$.

Proof
Since $A^z \le cl(A^z)$ and by Theorem 10 part (IV) we have that L-$cl(A^z) \le$ L-$cl(cl(A^z))$.
Also by Theorem 10 part (III) we get, $cl(A^z) \le$ L-$cl(A^z)$ this implies
L-$cl(cl(A^z)) \le L$-$cl(L$-$cl(A^z)) = L$-$cl(A^z)$,
thus L-$cl(A^z)$ = L-$cl(cl(A^z))$.

The following Theorem shows the relationship of locally -fuzzy closure and the fuzzy local function.

Theorem 11:
Let A^z any fuzzy sets. Then,
I. $A^{z*1} \le$ L-$cl(A^{z*1}) \le$ L-$cl(A^z)$.
II. $A^{z*2} \le$ L-$cl(A^{z*2})$.
III. $A^{z*3} \le$ L-$cl(A^{z*3}) \le$ L-$cl(A^Z)$.
IV. $A^{z*4} \le$ L-$cl(A^{z*4})$.

Proof
I) By Theorem 10 part (II) we get $A^{z*1} \le$ L-$cl(A^{z*1})$.
$cl(A^{z*1}) \le cl(A^z)$, and by Theorem 10 part (IV) we have L-$cl(cl(A^{z*1})) \le$ L-$cl(cl(A^z))$, by Lemma 2, that is L-$cl(A^{z*1}) \le$ L-$cl(A^z)$.

II) By Theorem 10 part (I) we get $A^{z*2} \leq$ L- $cl(A^{z*2})$.

III) Similarity by 1.

IV) Similarity by 2.

Conclusion:

The results presented in this paper indicate that expansion can readily define fuzzy local function in fuzzy ideal topological space so that we get a different values for them, therefore construct a new type of fuzzy topology.

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مفاهيم جديدة للدالة الموضعية الضبابية

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الخلاصة:

الفكرة الأساسية من البحث هو إيجاد تعاريف أخرى للدالة الموضعية الضبابية ودراسة مميزاتها والاختلاف فيما بينها ، مع مناقشة بعض التعاريف في إيجاد تبلوجي ضبابي جديد. أيضا في هذا البحث تم تعريف نوع جديد من الانغلاق الضبابي ، حيث تم دراسة العلاقة بين النوع الجديد من الانغلاق والأنواع المختلفة من الدوال الموضعية الضبابية.

الكلمات المفتاحية: الانغلاق الضبابي، الدالة الموضعية الضبابية، الانغلاق الضبابي المحلي، دالة موضعية ضعيفة.