Nuclear Structure Study of ^{164}Er Isotope in IBM-1and IBM-1$_{CQF}$

دراسة التشوه النووي للنظير ^{164}Er بأستخدام IBM_1 و IBM-1$_{CQF}$

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Abstract

The interacting boson model has been used to calculate the positive parity states of stable and neutron rich isotope ^{164}Er .A simple parameterization has been used which corresponds to a description close to the SU(3) limit of the model. The energy values, B(E2) values and potential energy surfacewere calculated. The resultshave reasonable agreement with the experimental energies and B(E2) values.The ^{164}Er isotope has shown its membership to the rotational SU(3)limit. The IBM-1 predicted the energy levels of (1.935 and2.056 MeV) with spin and parity 3^+ and 4^+,respectively in βγ-band, , also the energy level of (2.255 MeV) was limited with spin and parity 6^+in ββ-band under IBM-1.

Key words: Consistent Q formalism of the IBM. Energy and B(E2) predictions, contour plots, and potential energy surface.

الخلاصة

تم استخدام نموذج البوزونات المتفاعلة لحساب المستويات ذات التماثل الموجب للنظير ^{164}Er المستقر والغني بالنيترونات. تم استخدام ثوابت بسيطة تقابل وصف مقارب للحد SU(3). وقد حسب كل من مستويات الطاقة، قيم B(E2) وسطح طاقة الجهد. وكانت نتائج كل من الطاقات وقيم B(E2) متوافقة مع العملي. اظهر النظير ^{164}Er انتمائه للحد SU(3) الدوراني. تنبأ IBM-1 بمستويات الطاقة (1.935 و MeV2.056) بزخم وتماثل 3^+ و4^+ على التوالي ضمن الحزمة γβ، وكذلك تم تحديدمستوي الطاقة (2.255 MeV) لزخم وتماثل 6^+ في الحزمة ββ من خلال IBM-1.

1. Introduction

Low-lying states in Er isotopes have been studied by H. Yazar and I.Uluer (2005) [1] which established a correspondence between IBA-1 and IBA-2 model space by using the microscopic background of the IBA-2 model, they explored the energy levels, the electric quadrupole transition probabilities $B(E2; I_i \rightarrow I_f)$ and γ -ray E2/M1 mixing ratios for selected transitions of $^{162\text{-}170}$Er isotopes, but they failed to described ^{162,164}Er isotopes.

ZANG Jin-Fu and LU Li-Jun (2009) [2] studied the energy levels and E2 transition rates for the $^{160-170}$Er isotopes in the framework of the interacting boson model, and found these nuclei belong the transitional region U(5) - SU(3). As a result of this study, the gamma band was above the beta band, while experimental values of the β bandshould be above the γ band for all these nuclei except ^{170}Er.

S.N. Abood and M.A. Al-Jubbori (2013) [3] used IBM-2 to determine the Hamiltonian for $^{158\text{-}168}$Er isotopes with new idea for calculating bosons number at N = 64.They calculated energy levels, electromagnetic transition probabilities (B(E2), B(M1)) and mixing ratios (δ(E2/M1)).

The aim of this work is calculate the energy levels and B(E2) values for deformed ^{164}Er isotope using normal IBM-1 and IBM-1$_{CQF}$, and to compare the results with the experimental data, also to calculate its potential energy surface.

2. Theoretical Framework

2.1. Interacting Boson Model-1

One of the main feature of the interacting boson model-1 (IBM-1) is the ability to describe the changing collective properties of nuclei across an entire major shell within the framework of the IBM-1 Hamiltonian, in terms of the symmetries U(5), SU(3), and O(6) associated with its group theoretical foundations. However, the calculations in deformed nuclei require the use of a simple form [4]:

$$\boldsymbol{H} = a_1 \boldsymbol{L}.\boldsymbol{L} + a_2 \boldsymbol{Q}.\boldsymbol{Q} + a_0 \boldsymbol{p}^{\dagger}.\boldsymbol{p}\ , \qquad (1)$$

the first and second terms define the SU(3) limit and the third is the dominant term of the O(6) limit. The corresponding *E2* operator is given by [5]:

$$\boldsymbol{T}(E2) = \alpha_2 \left[\left[\boldsymbol{d}^{\dagger} \times \tilde{\boldsymbol{s}} + \boldsymbol{s}^{\dagger} \times \tilde{\boldsymbol{d}} \right]^{(2)} + \chi \left[\boldsymbol{d}^{\dagger} \times \tilde{\boldsymbol{d}} \right]^{(2)} \right], \qquad (2)$$

where χ is a free parameter with no prior restriction [5] and α_2 is found out from [6]:

$$B(E2;\ 2_1^+ \rightarrow 0_1^+) = \alpha_2^2 \frac{1}{5} N(2N+3). \qquad (3)$$

which is limited for SU(3) limit, and β_2 is defined [6]:

$$\beta_2 = \chi \alpha_2, \qquad (4)$$

The quadrupole moments of the 2_1^+ state is[6]:

$$Q_{2_1^+} = -\alpha_2 \sqrt{\frac{16\pi}{40}} \frac{2}{7} (4N+3)\ . \qquad (5)$$

2.2 Interacting Boson Model-1 In A Consistent Q Formalism

Same parametrization of the boson quadrupole operator is used in the consistent-Q formalism. This approach indeed produces the perturbation to the SU(3) symmetry required to reproduce the properties of deformed nuclei without the need for an additional symmetry breaking term ($\mathbf{p}^{\dagger}.\mathbf{p}$). This framework then involves one less free parameter than the earlier one and provides equivalent or improved agreement with the data. Thus the Hamiltonian take the form [7]:

$$\boldsymbol{H} = a_1 \boldsymbol{L}.\boldsymbol{L} + a_2 \boldsymbol{Q}.\boldsymbol{Q}. \qquad (6)$$

and the corresponding *E2* operator is the same Eq. (2), α_2 is found out from [8]:

$$B(E2;\ 2_1^+ \rightarrow 0_1^+) \approx \alpha_2^2 \frac{(N+1)^2(1-0.1\chi)}{2} \qquad (7)$$

2.3. Potential Energy Surface

The geometric properties of the interacting boson model are particularly important since they allow one to connect this model to the description of states in nuclei by shape variables introduced by Bohr and Mottelson. For discuss these geometric properties it is convenient to use set of coherent (or intrinsic) states [8].The energy functional, E(N, β, γ), associated with the Casimir invariant of the group chain II for deformed nuclei is [9]:

$$E(N;\beta,\gamma) = a_2 \frac{N(N-1)}{(1+\beta^2)^2} \left(4\beta^2 + 2\sqrt{2}\beta^3 \cos 3\gamma + \frac{1}{2}\beta^4 \right), \qquad (8)$$

3. Results and Discussion

3.1. Energy Levels

The isotope ^{164}Er, with N=96 and Z=68, has the ratio $R_{4/2}$ equals 3.277 and the beta band of this isotope above the gamma band which is contrary to SU(3) limit, thus it was convenient to applied SU(3) Hamiltonian and breaking it with pairing term of Eq.(1), where the $\mathbf{P}^{\dagger}.\mathbf{P}$term push β-band above γ-band. Also it can apply IBM-$1_{CQF}$of Eq. (6) (the easiest way)to get equivalent or better results than IBM-1without need for an additional term, χ is found out from Fig. (1). The present theoretical values of the energy levels are shown in Fig. (2) which is in good agreement with experiment value for the low-lying positive parity states.The parameters of ^{164}Er are shown in Table (1).

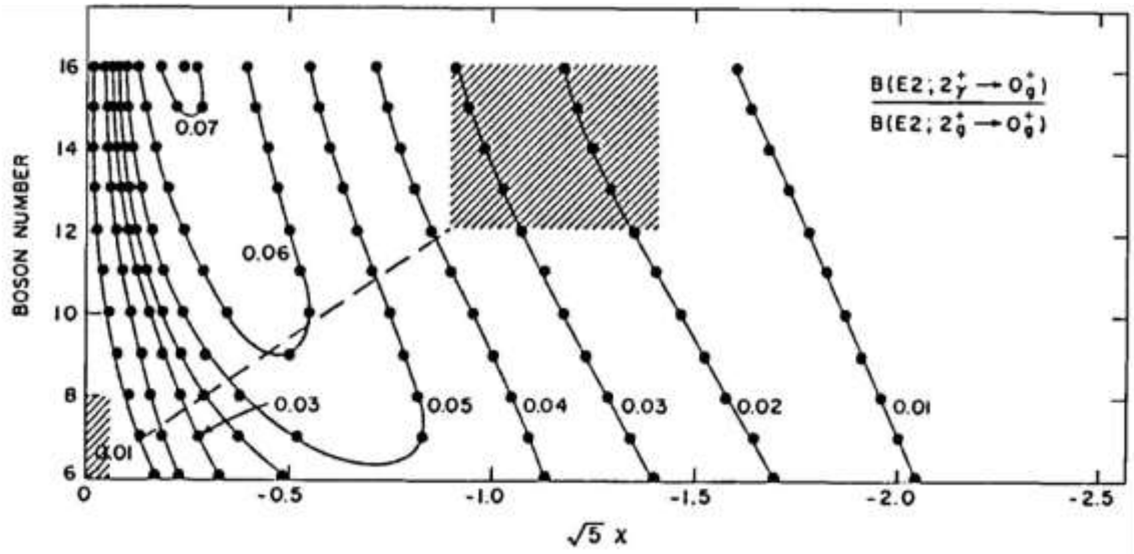

Fig. (1): Contour plot of the indicated B(E2) ratio in the CQF as a function of N and χ. Taken from [7].

Table. (1): The parameters are used for calculation energies in ^{164}Er with normal IBM-1 and IBM-1_{CQF}.

parameters	a_0	a_1	a_2	χ
IBM-1	0.05	0.0117	-0.0095	-1.310
IBM-1_{CQF}	0	0.0072	- 0.0214	- 0.485

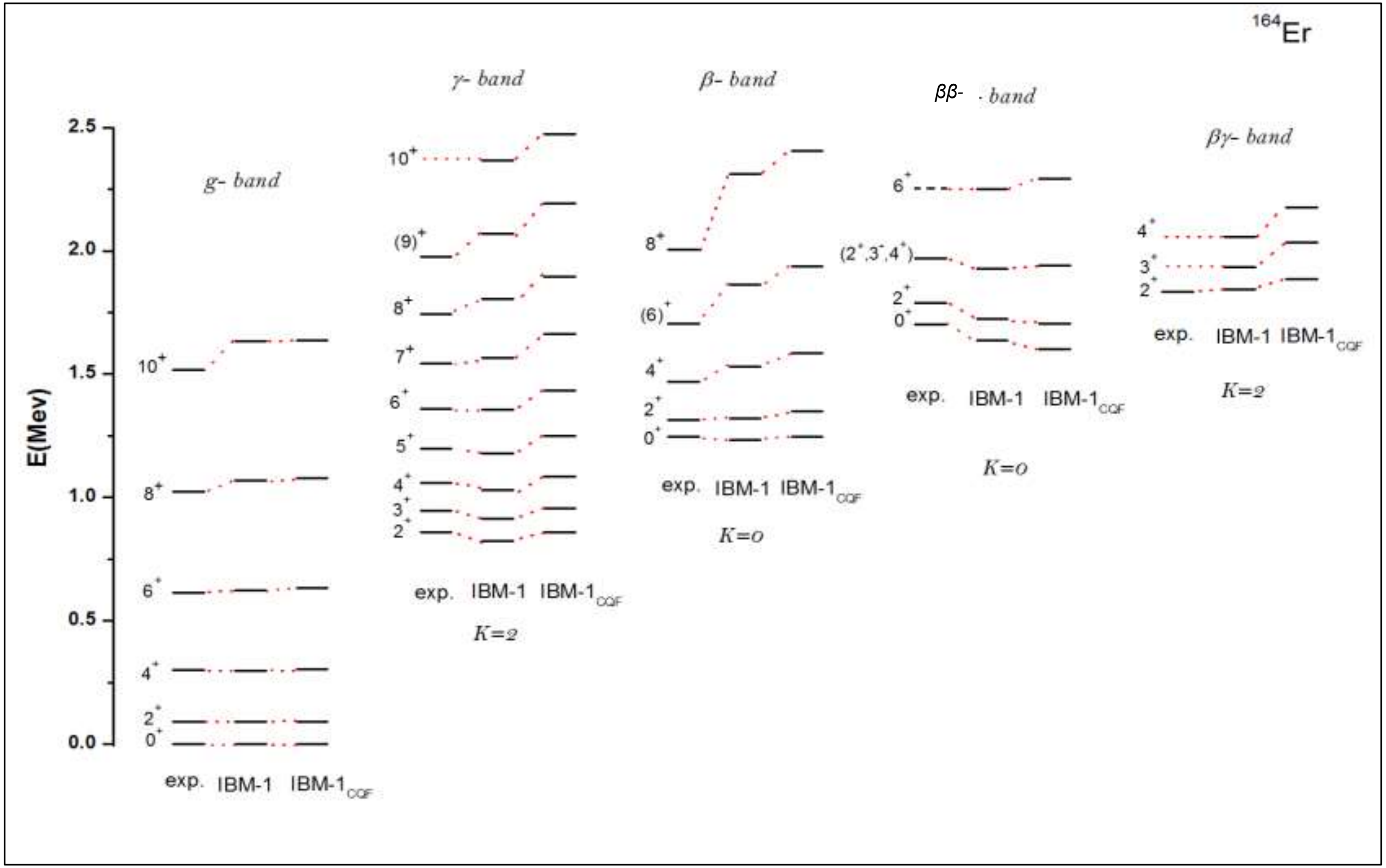

Fig. (2): Comparison of the experimental low-lying positive parity states of ^{164}Er with the predictions of both normal IBM-1 and IBM-1 in a consistent Q framework. Experimental data are taken from Ref. [10].

3.2. Reduced Transition Probabilities B(E2) And Electric Quadrupole Moment

For the calculations of the absolute B(E2) values in ^{164}Er, the α_2=0.1157 and 0.0993 eb in IBM-1 and IBM-1_{CQF}, respectively.The χ parameter in IBM-1_{CQF} is the same for energy levels, but in IBM-1, it was necessary to change the χ parameter from $-\sqrt{7}/2$to relax the rigorous selection rule for SU(3) limit to reproduce empirical B(E2) strengths in deformed nuclei [4] as shown in Fig.(3). Therefore, the better value of χ parameter for B(E2) values in ^{164}Er was -0.3.

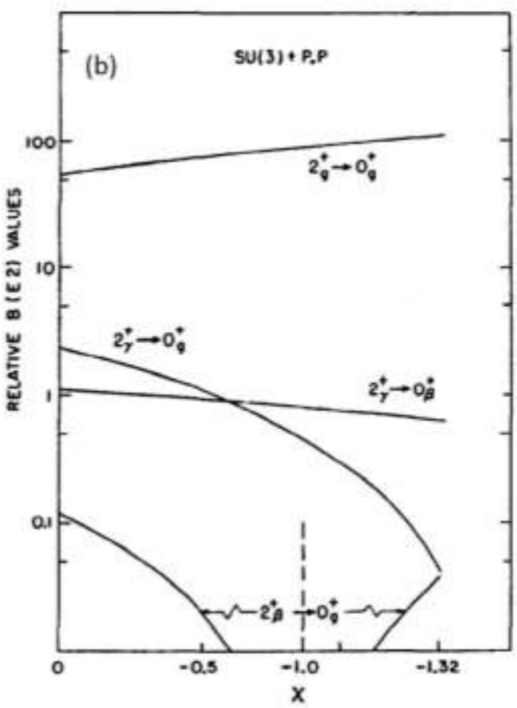

Fig (3): Relative B(E2) values involving the β, γ and ground bands plotted as a function of the constant χ for the SU(3) Hamiltonian with a perturbation by $\boldsymbol{p}^{\dagger}.\boldsymbol{p}$ term. Figure is taken from [11].

The transition probabilities of B(E2) values are calculated and normalized to the previous experimental value as well as electric quadrupole moment and presented in Table (2). The calculated values reported in Table (2) are in good agreement with the experimental data in IBM-1 and IBM-1_{CQF} with relative difference not exceed the limits. The quadrupole moment has negative sign, thus the ^{164}Er is a prolate.

Table (2): Comparison of the experimental absolute B(E2) in ^{164}Er with the predictions of both of the normal IBM-1 and IBM-1 in a consistent Q framework. Experimental data are taken from Ref. [12].

	Transition	B(E2) e^2b^2		
		exp.	IBM-1[a]	IBM-1$_{CQF}$ [a]
^{164}Er	$2_g^+ \rightarrow 0_g^+$	1.162	1.162	1.162
	$4_g^+ \rightarrow 2_g^+$	1.376	1.64	1.647
	$6_g^+ \rightarrow 4_g^+$	-------	1.766	1.785
	$2_\gamma^+ \rightarrow 4_g^+$	0.009	0.003	0.005
	$2_\gamma^+ \rightarrow 2_g^+$	0.061	0.05	0.058
	$2_\gamma^+ \rightarrow 0_g^+$	0.028	0.033	0.03
	$3_\gamma^+ \rightarrow 2_\gamma^+$	-------	1.76	1.71
	$3_\gamma^+ \rightarrow 4_g^+$	-------	0.027	0.039
	$3_\gamma^+ \rightarrow 2_g^+$	-------	0.057	0.053
	$8_g^+ \rightarrow 6_g^+$	1.829	1.788	1.822
	$4_\gamma^+ \rightarrow 4_g^+$	-------	0.059	0.069
	$4_\gamma^+ \rightarrow 2_g^+$	-------	0.018	0.011
	$0_\beta^+ \rightarrow 2_\gamma^+$	-------	0.168	0.197
	$0_\beta^+ \rightarrow 2_g^+$	-------	0.003	0.002
	$10_g^+ \rightarrow 8_g^+$	1.909	1.756	1.806
	$Q_{2_1^+}$ (eb)	< 0	-2.186	-1.875

[a] Normalized to the $2_g^+ \rightarrow 0_g^+$ transition.

3.3. Potential Energy Surface

The potential energy surfaces (PES) are appeared in Fig. (4), which are calculated depending on Eq. (7), the PES shows the ^{164}Er isotope has a prolate deformed shape.

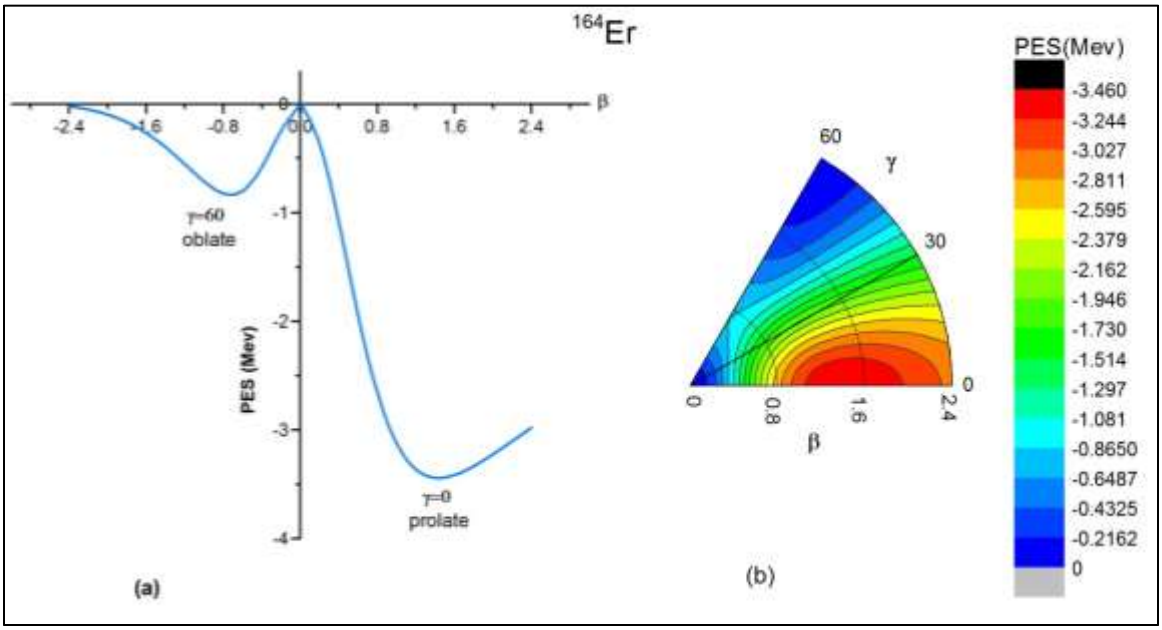

Fig. (4): a. Potential Energy Surface for ^{164}Er as a function of a β. b. The corresponding β-γ plot for γ=0.

4. Conclusions

The nuclear structure has been studied in ^{164}Er isotope via the IBM-1 and IBM-1$_{CQF}$.As a result of study electric quadrupole moment ($Q_{2_1^+}$) and potential energy surface for ^{164}Er, indicated that isotope is prolate. To get well agreement with an experiment in B(E2) values must be reduce the χ value.

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