The Soft Ideal of Soft BH-algebra

المثالية الناعمة في جبر–BH الناعم

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Abstract:

In this paper, the concept of soft ideal of a soft BH-algebra is introduced , several examples are provided, some of their properties and structural characteristics are discussed and studied. The intersection, union, ⋁-union, ⋀-intersection of soft BH-algebra and concept the idealistic are established. Also, the theorems of homomorphic image and homomorphic pre-image of soft sets are given.

Keywords: BH-algebra, Soft set, Soft BH-algebra, Soft BH-subalgebra, ideal, soft ideal, idealistic Soft BH-algebra.

الخلاصة

في هذا البحث قدمنا مفهوم المثالية الناعمة في جبر-BH الناعم وذكرنا بعض الامثلة, وبعض صفاتها والخصائص الهيكلية التي تمت دراستها. التقاطع، الاتحاد, الاتحاد-⋁ والتقاطع-⋀ في جبر BH الناعم وانشئ مفهوم المثالي. أيضا، أعطينا نظريات الصورة وعكس الصورة في دالة حفظ العملية للمجموعات الناعمة.

1. INTRODUCTION

To solve complicated problem in economics, engineering, and environment, we can't successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which were pointed out in Molodtsov suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties In 1999, Molodtsov introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly[2]. In 2002, Maji et al. described the application of soft set theory to a decision making problem [6]. In 2003, Maji et al. studied several operations on the theory of soft sets[7]. In 2005, Chen et al. presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors[1]. In 1998 ,Y. B. Jun, E. H. Roh and H .S. Kim introduced the notion of BH-algebra [10]. In 2015 ,L. S. Mahdi introduced the notion of soft BH-algebra and Soft BH-subalgebra[4]. In this paper, we deal with the ideal structure of BH-algebra by applying soft set theory. We introduce the notion of soft ideals in BH-algebra and idealistic soft BH-algebra, and give several examples. We give relations between an ideal and an idealistic soft BH-algebra. We establish the characterization of idealistic soft BH-algebra. We also discuss the intersection, union, "AND" operation, and "OR" operation of ideals and idealistic soft BH-algebra.

2. Preliminaries

In this section, we give some basic concepts about BH-algebra, Soft sets, Soft BH-subalgebra, intersection and union of soft sets, and some other concepts that we need in our work.

Definition (2.1):[10]

A ***BH-algebra*** is a nonempty set X with a constant 0 and a binary operation"*" satisfying the following conditions:

i. $x*x=0$, for all $x\in X$.

ii. $x*y=0$ and $y*x=0$ imply $x = y$, for all $x, y\in X$.

iii. $x*0=x$, for all $x\in X$.

Definition (2.2):[10]

Let X a BH-algebra and $S \subseteq X$. Then S is called a ***subalgebra*** of X if $x*y\in S$ for all $x,y\in S$.

Remark (2.3):[8]

Let X and Y be BH-algebras. A mapping $f: X\rightarrow Y$ is called a homomorphism if $f(x*y)=f(x)*f(y)$ for all $x,y\in X$. A homomorphism f is called a monomorphism (resp., epimorphism) if it is injective (resp., surjective). A bijective homomorphism is called an isomorphism. Two BH-algebras X and Y are said to be isomorphic, written $X \cong Y$, if there exists an isomorphism $f:X\rightarrow Y$. For all homomorphism $f:X \rightarrow Y$, the set $\{x\in X: f(x)=0\}$ is called the kernel of f, denoted by Ker(f), and the set $\{f(x):x\in X\}$ is called the image of f, denoted by Im(f). Notice that $f(0)= 0^{\prime}$, for all homomorphism f, and $f^{-1}(Y) =\{x\in X: f(x) = y, \text{ for some } y\in Y\}$

Definition (2.4):[10]

Let I be a nonempty subset of a BH-algebra X. Then I is called an **ideal** of X if it satisfies:

i. $0\in I$.

ii. $x*y\in I$ and $y\in I$ imply $x\in I$, for all $x\in X$.

Definition (2.5):[2]

The notion of a soft set defined in the following way: Let U be an initial universe set and E a set of parameters. The power set of U is denoted by $\mathcal{P}(U)$ and A is a subset of E. A pair (F,A) is called a soft set over U, where F is a mapping $F : A \rightarrow \mathcal{P}(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $x \in A$, F(x) may be considered as the set of x-approximate elements of the soft set (F,A). Clearly, a soft set is not just a subset of U.

Definition (2.6):[2]

Let (F, A), (G,B) be soft sets over a common universe U.

(i) (F,A) is said to be a soft subset of (G,B), denoted by $(F,A) \tilde{\subseteq} (G,B)$, if $A \subseteq B$ and $F(a) \subseteq G(a)$ for all $a \in A$,

(ii) (F,A) and (G, B) are said to be soft equal, denoted by (F, A) = (G, B), if $(F,A) \tilde{\subseteq}$ ((G,B) and $(G,B) \tilde{\subseteq} (F,A)$.

Definition (2.7):[5]

i. The intersection of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set $(H,C) = (F,A) \tilde{\cap} (G,B)$, where $C = A \cap B \neq \emptyset$, and $H(c) = F(c) \cap G(c)$, for all $c \in C$. (ii) The intersection of a nonempty family soft sets $\{(F_i,A_i) \mid i\in\alpha\}$over a common universe U is defined as the soft set $(H,B) = \tilde{\cap}_{i\in\alpha} (F_i,A_i)$, where $B = \cap_{i\in\alpha} A_i \neq \emptyset$, and $H(x) = \cap_{i\in\alpha} F_i(x)$, for all $x\in B$.

Definition (2.8):[5]

The union of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set (H,C)= (F,A) $\widetilde{\cup}$ (G,B), where C =A∪B ≠ ∅, and H(c) = F(c) ∪ G(c), for all c ∈ C.

Definition (2.9):[9]

The union of a nonempty family soft sets {(F_i, A_i) | i∈α} over a common universe U is defined as the soft set (H,B) = $\widetilde{\cup}_{i\in\alpha}$ (F_i,A_i), where B =$\cup_{i\in\alpha}$ A_i ≠ ∅, and H(x) = $\cup_{i\in\alpha}$ F_i(x), for all x∈ B.

Definition (2.10):[3]

(i) The ∧-intersection of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set (H,C) = (F,A) $\widetilde{\wedge}$ (G,B), where C=A×B, and H(a,b)=F(a)∩G(b), for all(a,b)∈A×B

(ii) The ∧-intersection of a nonempty family soft sets {(F_i, A_i) | i∈α}over a common universe U is defined as the soft set (H,B) = $\widetilde{\wedge}_{i\in\alpha}$ (F_i, A_i), where B = $\prod_{i\in\alpha}$ A_i and H(x) = $\cap_{i\in\alpha}$ F_i(x_i), for all x=$(x_i)_{i\in\alpha}$ ∈ B.

Definition (2.11):[3]

(i) The ∨-union of two soft sets (F,A) and (G,B) over a common universe U is defined as the soft set (H,C) = (F,A) $\widetilde{\vee}$ (G,B), where C= A × B, and H(a,b)=F(a)∪G(b), for all (a, b) ∈ A × B.

(ii) The ∨-union of a nonempty family soft sets {(F_i, A_i) | i∈α }over a common universe U is defined as the soft set (H,B)= $\widetilde{\vee}_{i\in\alpha}$($F_i$,$A_i$), where B=$\prod_{i\in\alpha}A_i$ and H(x) = $\cup_{i\in\alpha}$ F_i(x_i), for all x = $(x_i)_{i\in\alpha}$∈ B.

Definition (2.12):[4]

If X is a BH-algebra and A a nonempty set, a set-valued function F : A→ $\mathcal{P}$(X) can be defined by : F(x) ={y ∈ X | (x, y)∈R}, x∈A, where R is an arbitrary binary relation from A to X, that is a subset of A×X. The pair (F,A) is then a soft set over X. The soft sets in the examples that follow are obtained by making an appropriate choice for the relation R. For a soft set (F,A). The set Supp (F,A) = {x∈A | F(x) ≠ ∅} is called the support of the soft set (F,A), and the soft set (F,A) is called a non-null if Supp (F,A)≠ ∅.

Definition (2.13):[4]

Let (F,A) be a non-null soft set over X a BH-algebra. Then (F,A) is called a soft BH-algebra over X if F(x) is a BH-subalgebra of X, for all x∈Supp (F,A).

Example (2.14):

Consider the BH-algebra X={0,1,2,3,4} with binary operation"*" defined as follows:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	1	0	0
4	4	3	3	3	0

For A = X, let f : A → $\mathcal{P}$(X) be a set-valued function defined by f(x)={y∈X | y*(y*x) ∈{0,1}} for all x∈A. Then f(0) = X, f(1) = {0, 1, 2, 3} and f(2) = f(3) = f(4) = {0,1} are subalgebras of X, and so (f,A) is a soft BH-algebra of X.

Definition(2.15):[4]

Let (F,A) and (G,B) are two soft BH-algebras over X. Then(G,B) called is a soft BH-subalgebra of (F,A), denoted by (G, B) $\widetilde{<}_s$ (F,A), if it satisfies the following conditions:

(i) B ⊆ A,

(ii) G(x) is a BH-subalgebra of F(x), for all x ∈ Supp (G,B).

3. The Main Results

In this section, we introduce the concepts of soft ideal of soft BH-algebra and soft idealistic BH-algebra. Also, we state and prove some theorems and examples about these concepts.

Definition (3.1):

Let S be a subalgebra of a BH-algebra X. A subset I of X is called an ideal of X related to S (briefly, S-ideal of X), denoted by I⊲ S, if it satisfies:

(i) 0 ∈ I.

(ii) (∀x∈S) (∀y∈I) ((x * y∈I) ⇒ x ∈ I).

Note that if S is a subalgebra of X and I is a subset of X that contains S, then I is a S-ideal of X. Obviously, every ideal of X is a S-ideal of X for every subalgebra S of X and hence every ideal of BH-algebra X is a S-ideal of X for some subalgebra S of X. But the converse is not true in general as seen in the following example.

Example (3.2):

Consider the BH-algebra X={0,1,2,3,4} with binary operation"*" defined as follows:

*	0	1	2	3	4
0	0	0	2	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	3	0	4
4	4	1	2	3	0

Then S={0,2} is subalgebra of X and I={0,4}⊲S, but I is not an ideal of BH-algebra since 3∈X, 4∈I such that 3*4 = 4∈I, but 3∉I.

Remark(3.3):

If S_1 and S_2 are subalgebras of X such that $S_1 \subset S_2$, then every S_2-ideal of X is a S_1-ideal of X. But the converse is not true in general as seen in the following example.

Example (3.4):

Consider the BH-algebra X={0,1,2,3,4} with binary operation"*" defined as follows:

*	0	1	2	3	4
0	0	1	2	3	0
1	1	0	2	0	0
2	2	2	0	3	1
3	3	1	2	0	2
4	4	4	4	4	0

Note that S_1 = {0,1,2} and S_2 = {0, 1, 2, 3} are subalgebras of X. Let I ={0,1} Then I is a S_1-ideal of X, but I is not a S_2-ideal of X, since $3\in S_2$ and $3*1 = 1\in I$ and$1\in I$, but $3\notin I$.

Definition(3.5):

Let (f,A) be a soft BH-algebra of X. A soft set (g,I) of X is called a soft ideal of (f,A), denoted by (g,I) $\tilde{\lhd}$ (f,A), if it satisfies:

(i) $I \subset A$.

(ii) $(\forall x\in I)$ $(g(x) \lhd f(x))$.

Let us illustrate this definition using the following example.

Example (3.6):

Consider the BH-algebra X={0,1,2,3,4} with binary operation"*" defined as follows:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	2	0	0
4	4	3	3	3	0

For A = X, let $f : A \rightarrow \mathcal{P}(X)$ be a set-valued function defined by $f(x)=\{y\in X \mid y*(y*x) \in\{0,1\}\}$, for all $x \in A$. Then f(0) = X, f(1) = {0, 1, 2, 3} and f(2) = f(3) = f(4) = {0,1} are subalgebras of X, and so (f,A) is a soft BH-algebra of X. Now consider I = {0,1} ⊂ A and define a set-valued function $g : I \rightarrow \mathcal{P}(X)$ by $g(x) =\{y\in X \mid y*x\in\{0\}\}$, for all $x\in I$. We can verify that $g(0) =0 \lhd f(0)=X$ and $g(1) =\{0,1\} \lhd f(1) =\{0,1,2,3\}$ Hence (g,I) $\tilde{\lhd}$ (f,A). And the subset J = {1,2}⊂ A and define a set-valued function $h : J \rightarrow \mathcal{P}(X)$ by $h(x) =\{y\in X \mid y*(y*x)\in\{0\}\}$, for all $x\in J$. We can verify that h(1)={0,2,3} is not a soft ideal of X. Since $1\in f(1)$ and $3\in h(1)$ such that $1*3=0\in h(1)$, but $1\notin h(1)$.

Remark(3.7):

Every soft ideal is a soft BH-subalgebra, but the converse is not true in general as seen in the following example.

Example(3.8):

Consider the BH-algebra X={0,1,2,3,4,5} with binary operation " * " defined as follows:

*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	0	0	0	0	1
2	2	2	0	0	1	1
3	3	2	1	0	1	1
4	4	4	4	4	0	1
5	5	5	5	5	5	0

Consider A = X and define a set-valued function $f : A \to \mathcal{P}(X)$ by $f(x)=\{y\in X \mid (y*x)*x \in\{0\}\}$, for all $x\in A$. Then (f,A) is a soft BH-algebra of X, since f(0) = {0}, f(1) ={0,1}, f(2) = f(3) ={0,1,2,3}, f(4) ={0,1,2,3,4}and f(5)= {0,5} are subalgebras of X. Now take I ={2,4}$\subset$ A and let $g : I \to \mathcal{P}(X)$ be a set-valued function by $g(x) =\{y\in X \mid y*x\in\{0\}\}$, for all $x\in I$. Then g(2) ={0,1,2}and g(4) ={0,1,4} are subalgebras of X and hence (g,I) is also a soft BH-algebra of X. Obviously, g(2) and g(4) are subalgebras of f(2) and f(4), respectively. Thus (g,I) $\tilde{<}$ (f,A). But g(I) is not soft ideal of X, since 2*4=1$\in$g(4) and 4$\in$g(4) but 2$\notin$g(4).

Proposition(3.9):

Let (f, A) be soft BH-algebra of X. For any soft sets (g_1, I_1) and (g_2, I_2) of X, where $I_1\cap I_2 \neq\emptyset$, we have $(g_1, I_1) \tilde{\lhd} (f,A), (g_2, I_2) \tilde{\lhd} (f,A) \Rightarrow (g_1, I_1) \tilde{\cap} (g_2, I_2) \tilde{\lhd} (f,A)$.

Proof:

Using definition (2.7) (i) , we can write $(g_1, I_1) \tilde{\cap} (g_2, I_2)=(g,I)$, where $I=I_1\cap I_2$, and $g(x)=g_1(x)$ or $g_2(x)$, for all $x\in I$. Obviously, $I \subset A$ and $g : I \to \mathcal{P}(X)$ is a mapping . Hence (g, I) is a soft set of X. Since $(g_1, I_1) \tilde{\lhd} (f,A)$ and $(g_2, I_2) \tilde{\lhd} (f,A)$, we know that $g(x)=g_1(x) \lhd f(x)$or $g(x)= g_2(x) \lhd f(x)$, for all $x\in I$ Hence $(g_1, I_1) \tilde{\cap} (g_2, I_2) \tilde{\lhd} (f,A)$.
This completes the proof.

Proposition(3.10):

Let$\{(F_i, A_i) \mid i\in\alpha\}$ be a nonempty family of soft ideal of X. Then the intersection $\tilde{\cap}_{i\in\alpha} (F_i,A_i)$ is a soft ideal of X.

Proof:

Straightforward from definition (2.7) (ii).

Proposition(3.11):

Let (f, A) be soft BH-algebras of X. For any soft sets (g, I) and (h, J) of X in which I and J are disjoint, we have $(g, I) \tilde{\lhd} (f,A), (h, J) \tilde{\lhd} (f,A)\Rightarrow(g, I) \tilde{\cup} (h, J) \tilde{\lhd}(f,A)$.

Proof:

Assume that $(g, I) \tilde{\lhd} (f,A)$ and $(h, J) \tilde{\lhd}(f,A)$. By definition (2.8) **,** we can write $(g, I) \tilde{\cup} (h, J) =(k,V)$, where $V = I\cup J$, for all $x \in V$.

$$k(x) = \begin{cases} g(x) & \text{if } x \in I\setminus J , \\ h(x) & \text{if } x \in J\setminus I , \\ g(x) \cap h(x) & \text{if } x \in I \cap J . \end{cases}$$

Since $I \cap J = \emptyset$, either $x \in I\setminus J$ or $x \in J\setminus I$, for all $x \in V$. If $x \in I\setminus J$, then $k(x)= g(x) \tilde{\lhd} f(x)$, since $(g, I) \tilde{\lhd} (f,A)$. If $x \in J\setminus I$, then $k(x)= h(x) \tilde{\lhd} f(x)$, since $(h, I) \tilde{\lhd} (f,A)$. Thus $k(x) \tilde{\lhd} f(x)$, for all $x \in V$ and so $(g, I) \tilde{\cup} (h, J) = (k, V) \tilde{\lhd} (f,A)$.

Remark(3.12):

If I and J are not disjoint in proposition above, then proposition (3.11) is not true in general as seen in the following example.

Example(3.13):

Consider the BH-algebra in example (2.14). Let A = X and define a set-valued function $f : A \to \mathcal{P}(X)$ by $f(x)=\{y\in X \mid y*(x*y) \in\{0,3\}\}$, for all $x\in A$. Then (f,A) is a soft BH-algebra of X, since f(0) = f(1) = f(2) ={0, 3}, f(3) ={0,1,3}, and f(4) ={0,1,2,3} are subalgebras of X. Now take I ={2,3}$\subset$ A and let $g : I \to \mathcal{P}(X)$ be a set-valued function by $g(x) =\{y\in X \mid y*(y*x)\in\{0\}\}$, for all $x\in I$. Then we can verify $g(2) = \{0,1\} \lhd f(2)$and $g(3) = \{0\} \lhd f(3)$ and hence $(g,I) \tilde{\lhd} (f,A)$. Now let (h,J) be a soft set of X, where J= {2} $\subset$ A and let $h : J \to \mathcal{P}(X)$ is a set-valued function by $h(x) =\{y\in X \mid (y*x)*x \in\{0\}\}$, for all $x\in J$. Then $h(2) =\{0,2\} \lhd f(2)$, which hence that $(h,J) \tilde{\lhd} (f,A)$. But $g(2) \cup h(2)=\{0,1,2\}$ is not a soft ideal of X. Since $3*2=1\in g(2) \cup h(2)$ and $2\in g(2) \cup h(2)$ but $3\notin g(2)\cup h(2)$.

Proposition(3.14):

Let$\{(F_i, A_i) \mid i\in\alpha\}$ be a nonempty family of soft ideal of X such that A_i is disjoint for all $i\in\alpha$. Then the union $\widetilde{\cup}_{i\in\alpha} (F_i,A_i)$ is a soft ideal of X.

Proof:

Straightforward from definition (2.9).

Proposition (3.15):

Let (f,A) and (g,B) are soft ideals of X. Then the (f,A) $\widetilde{\wedge}$ (g,B) is a soft ideal of X.

Proof:

By definition (2.10) (i), we can write (f,A) $\widetilde{\wedge}$ (g,B) =(h,C), where C=A×B, and h(x,y) =f(x)∩f(y), for all (x,y)∈A×B. Since f(x), f(y) are a soft ideals of X. Then f(x) ∩ f(y) is also soft ideal of X. Hence h(x,y) is also soft ideal of X, for all (x,y) ∈ A×B. Therefore, (f,A) $\widetilde{\wedge}$ (g,B) = (h,C) is a soft ideal of X.

Proposition (3.16):

Let$\{(F_i, A_i) \mid i\in\alpha\}$ be a nonempty family of soft ideal of X. Then $\widetilde{\wedge}_{i\in\alpha} (F_i,A_i)$ is a soft ideal of X.

Proof:

Straightforward from definition (2.10) (ii).

Proposition (3.17):

Let (f,A) and (g,B) are soft ideals of X such that A and B are disjoint. Then the (f,A) $\widetilde{\vee}$ (g,B) is a soft ideal of X.

Proof:

By definition (2.11) (i), we can write (f,A) $\widetilde{\vee}$ (g,B) =(h,C), where C=A×B, and h(x,y) =f(x)∪f(y), for all (x,y)∈A×B. Since f(x), f(y) are a soft ideals of X. Then f(x) ∪ f(y) is also soft ideal of X. Hence h(x,y) is also soft ideal of X, for all (x,y)∈A×B. Therefore (f,A) $\widetilde{\vee}$ (g,B) = (h,C) is a soft ideal of X.

Proposition (3.18):

Let$\{(F_i, A_i) \mid i\in\alpha\}$ be a nonempty family of soft ideal of X such that A_i is disjoint for all $i\in\alpha$. Then the $\widetilde{\vee}_{i\in\alpha} (F_i,A_i)$ is a soft ideal of X.

Proof:

Straightforward from definition (2.11) (ii).

Definition(3.19):

Let (f,A) be a soft set of a BH-algebra X. Then (f,A) is called an ***idealistic*** soft BH-algebra of X, if f(x) is an ideal of X, for all x ∈ A.

Let us illustrate this definition using the following example.

Example(3.20):

Consider the BH-algebra X={0,1,2,3,4} with binary operation"*" defined as follows:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

Let A = X and define a set-valued function f : A → $\mathcal{P}$(X) by f(x)={y∈X | y*(y*x) ∈{0}}, for all x∈A. Then f(0) = X, f(1) =f(4)= {0, 2}, f(2) = {0,1,4}and f(3) = {0}. We can verify that f(x) ⊲ X, for all x∈A and hence (f,A) is an idealistic soft BH-algebra of X .

Proposition (3.21):

Let (f,A) and (f,B) are soft sets of X, where B ⊆ A ⊆ X. If (f,A) is an idealistic soft BH-algebra of X, then (f,B) is an idealistic soft BH-algebra of X.

Proof:

Straightforward.

Remark(3.22):

The converse of proposition (3.21) is not true in general as seen in the following example.

Example(3.23):

Consider the BH-algebra X={0,1,2,3,4,5} with binary operation " * " defined as follows:

*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	0	0	0	0	2
2	2	2	0	0	4	3
3	3	2	2	0	4	0
4	4	4	4	4	0	5
5	5	5	5	5	5	0

Consider A = X and define a set-valued function f : A → Ƥ(X) by f(x)={y∈X | (y*x) *x∈{0}}, for all x∈A. Then (f,A) is a soft BH-algebra of X, since f(0) = {0}, f(1)={0, 1}, f(2)= f(3)={0,1,2,3}, f(4)={0,1,2,3,4}and f(5)={0,2,3,4,5} are subalgebras of X. Now take I ={0,1,2,3,4}⊂ A,we can verify that f(0),f(1),f(2),f(3),f(4) are ideals of X. Thus (f,I) is an idealistic soft BH-algebra of X . But (f,A) is not an idealistic soft BH-algebra of X , since 1*5=2∈f(5) and 5∈ f(5), but 1∉f(5).

Proposition (3.24):

Let (f,A) and (g,B) are two idealistic soft BH-algebras of X. If A∩B ≠∅, then (f,A) $\tilde{\cap}$ (g,B) is an idealistic soft BH-algebra of X.

Proof:

By definition (2.7) (i) , we can write (f,A) $\tilde{\cap}$ (g, B)=(h,C) , where C=A∩B, and h(x)=f(x) or g(x), for all x∈C. Note that h : C → Ƥ(X) is a mapping . Hence (h, C) is a soft set of X. Since (f, A) and (g,B) are idealistic soft BH-algebra of X, it follows that h(x)=f(x) is an ideal of X or h(x)= g(x) is an ideal of X, for all x∈C. Hence (f,A) $\tilde{\cap}$ (g, B)=(h,C) is an idealistic soft BH-algebra of X.

Proposition (3.25):

Let{(F_i, A_i) | i∈α} be a nonempty family of idealistic soft BH-algebra of X. Then $\tilde{\cap}_{i\in\alpha}$ (F_i,A_i) is an idealistic soft BH-algebra of X.

Proof:

Straightforward from definition (2.7) (ii).

Proposition (3.26):

Let (f, A) and (g,B) be two idealistic soft BH-algebras of X. If A and B are disjoint, then (f,A) $\tilde{\cup}$ (g,B) is an idealistic soft BH-algebra of X .

Proof:

Using definition (2.8), we can write (f,A) $\widetilde{\cup}$ (g,B) = (h,V),where V= A∪ B,for all x∈V.

$$h(x) = \begin{cases} f(x) & \text{if } x \in A \setminus B, \\ g(x) & \text{if } x \in B \setminus A, \\ f(x) \cap g(x) & \text{if } x \in A \cap B\,. \end{cases}$$

Since A∩B = ∅, either x ∈ A\ B or x ∈ B\A , for all x ∈ V. If x ∈ A\ B , then h(x)= f(x) is an ideal of X, since (f,A) idealistic soft BH of X . If x ∈ B\A , then h(x)= g(x) is an ideal of X, since (g,B) is an idealistic soft BH of X. Hence (f,A) $\widetilde{\cup}$ (g,B) =(h,V) is an idealistic soft BH-algebra of X.

Remark(3.27):

If A and B are not disjoint in proposition above, then proposition (3.26) is not true in general as seen in the following example.

Example(3.28):

Let X={0,1,2,3,4} be a BH-algebra define in example (3.20). Consider idealistic soft BH-algebra (f,A) of X, if we take B={2,4} then B is not disjoint with A = X, define a set-valued function g : B ⟶ Ƥ(X) by f(x)={y∈X | y*x∈{0}}, for all x∈B. We obtain that g(2) = {0,2} ⊲ X, and g(4) ={0,1,4} ⊲ X.

This means that(g,B) is an idealistic soft BH-algebra of X. Now, let (f,A) $\widetilde{\cup}$ (g,B) =(h,C). Then h(2) = f(2)∪g(2) = {0,1,4}∪{0,2} = {0,1,2,4} and h(4) = f(4)∪g(4) = {0,2}∪{0 ,1,4} = {0,1,2,4}, are but h(2) and h(4) is not idealistic soft BH-algebra, since 3*4=2 ∈ h(2)∪h(4) and 4 ∈h(2)∪ h(4), but 3 ∉ h(2)∪h(4). Hence (f,A) $\widetilde{\cup}$ (g,B) is not an idealistic soft BH-algebra of X.

Proposition(3.29):

Let{(F_i, A_i) | i∈α} be a nonempty family of idealistic soft BH-algebra of X such that A_i is disjoint for all i∈α. Then the union $\widetilde{\cup}_{i\in\alpha}$ (F_i,A_i) is a idealistic soft BH-algebra of X.

Proof:

Straightforward from definition (2.9).

Proposition (3.30):

Let(f,A) and (g,B) be idealistic soft BH-algebras of X. Then (f,A) $\widetilde{\wedge}$ (g,B) is an idealistic soft BH-algebras of X.

Proof:

By definition (2.10) (i), we can write (f,A) $\widetilde{\wedge}$ (g,B) = (h, A×B), where h(x,y) = f(x) ∩ g(y), for all (x,y)∈A× B. Hence h(x,y) is an ideal of X, for all (x,y)∈A×B.

Therefore, (f,A) $\widetilde{\wedge}$ (g,B) = (h, A × B) is an idealistic soft BH-algebra of X.

Proposition (3.31):

Let{(F_i, A_i) | i∈α} be a nonempty family of idealistic soft BH-algebras of X. Then $\widetilde{\wedge}_{i\in\alpha}$ (F_i,A_i) is a soft ideal of X.

Proof:

Straightforward from definition (2.10) (ii).

Proposition (3.32):

Let (f,A) and (g,B) are idealistic soft BH-algebras of X such that A and B are disjoint. Then the (f,A) $\widetilde{\vee}$ (g,B) is an idealistic soft BH-algebras of X.

Proof:

By definition (2.11) (i), we can write (f,A) $\widetilde{\vee}$ (g,B) =(h,C), where C=A×B, and h(x,y)=f(x)∪ f(y), for all (x,y)∈A×B. Since f(x), f(y) are a soft ideals of X. Then f(x) ∪ f(y) is also soft ideal of X. Hence h(x,y) is also soft ideal of X, for all (x,y)∈A×B.

Therefore (f,A) $\widetilde{\vee}$ (g,B) = (h,C) is a soft ideal of X.

Proposition (3.33):

Let{(F_i, A_i) | i∈α} be a nonempty family of idealistic soft BH-algebras of X such that A_i is disjoint for all i∈α. Then the $\widetilde{\vee}_{i\in\alpha}$ (F_i,A_i) is an idealistic soft BH-algebras of X.

Proof:

Straightforward from definition (2.11) (ii).

Definition(3.34):

An idealistic soft BH-algebra (f,A) of X is said to be trivial if f(x) = 0 and said to be whole if f(x) = X, for all x ∈ A.

Example (3.35):

Consider the BH-algebra X={0,1,2,3,4} with binary operation"*" defined as follows:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

Let A ={0,1,2}⊂ X and define a set-valued function f : A → Ƥ(X) by f(x)={y∈X | y*(x*y) ∈{0}}, for all x∈A. Then f(0) = f(1) = f(2) = {0}and so (f,A) is a trivial idealistic soft BH-algebra of X. Now, let g : A → Ƥ(X) by f(x)={y∈X | x*y ∈{0,x}}, for all x∈A. Then g(0) = g(1)=g(2)=X. Hence (g,A) is a whole idealistic soft BH-algebra of X.

Proposition (3.36):

Let f : X → Y be an onto homomorphism of BH-algebras and let (g,A) be an idealistic soft BH-algebra of X.

(i) If g(x) ⊆ ker(f), for all x ∈ A, then (f(g),A) is the trivial idealistic soft BH-algebra of Y.

(ii) If (g,A) is whole, then (f(g),A) is the whole idealistic soft BH-algebra of Y.

Proof:

(i) Let g(x) ⊆ ker(f) for all x∈A. Then f(g)(x) = f(g(x)) = {0_Y}, for all x ∈A. Hence (f(g),A) is the trivial idealistic soft BH-algebra of Y.

(ii) Let (g,A) is whole. Then g(x) = X, for all x∈A and so f(g)(x) = f(g(x)) = f(X) = Y, for all x ∈A. It follows (f(g),A) is the whole idealistic soft BH-algebra of Y.

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