

\*

(  $c_j, b_i, a_{ij}$  )

$a_{ij}$

## Stochastic zero – one programming

### Abstract

zero - one programming case from integer linear programming where the variable's are equal to zero or one, the decision factor uses this kind from programming when he meets him problems of the kind yes or no.

The stochastic zero-one programming construction formed is used when one or all parameters of model( $c_j, b_i, a_{ij}$ ) are random variable taken mathematical distribution.

In this research we discuss stochastic zero-one programming problem where ( $a_{ij}$ ) random variable (construction and solution) and use it in practical application on some vegetative crops in Iraq.

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**Email : ali\_alzubiadi@yahoo.com**

2008/ 12/ 24 :

2008/ 1/30 :

Integer Linear )

(Programming

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Integer Linear Programming (3,4)

:2-1

(LP)

(ILP)

	(LP)
(ILP)	( )
:	
(Pure integer programming )	-1
.	
(Mixed integer programming)	-2
.	
<b>Zero – One Programming</b>	<b><u>3-1</u></b>
-	

### Integer Linear Programming

:

$$x_j = \begin{cases} 1 & \text{إذا كان القرار } i \text{ هو نعم} \\ 0 & \text{إذا كان القرار } j \text{ هو لا} \end{cases} \dots\dots\dots(1)$$

( ILP )

:

$$\begin{aligned} x_i &\leq 1 \\ x_j &\geq 0 \end{aligned} \dots\dots\dots(2)$$

: ( ILP )

$$\sum_{j=1}^n x_j = 1 \dots\dots\dots(3)$$

:



$$P_r(\sum_{j=1}^n a_{ij} x_j \leq b_i) \geq u_i \quad i = 1, 2, \dots, m \quad \dots\dots\dots(5)$$

$u_i$

$$P_r \left( \frac{\sum a_{ij} x_j - U_i(x)}{v_i(x)} \leq \frac{b_i - U_i(x)}{v_i(x)} \right) \geq u_i$$

$$\phi \left( \frac{b_i - U_i(x)}{v_i(x)} \right) \geq u_i \Rightarrow \frac{b_i - U_i(x)}{v_i(x)} \geq \phi^{-1}(u_i) = \tau_u$$

$$b_i - U_i(x) \geq v_i(x) \tau_u \quad \dots\dots\dots(6)$$

$\tau_u$

$X : U_i(x), v_i(x)$

$U_i(\chi) = \sum_{j=1}^n M_{ij} \chi_j$  Mean

$v_i(\chi) = (\sum_{j=1}^n S_{ij}^2 \chi_j^2)^{1/2}$  Variance

$\tau_u = 0 \quad a_{ij} \sim N(M, \sigma^2) \quad u_i = 0.50$

(6)

$U_i(x) \leq b_i$

$$P_r \left( \sum_{j=1}^n a_{ij} x_j \geq b_i \right) \geq u_i \quad i = 1, 2, \dots, n$$

$$P_r \left( \frac{\sum_{j=1}^n a_{ij} x_j - U_i(x)}{v_i(x)} \geq \frac{b_i - U_i(x)}{v_i(x)} \right) \geq u_i$$

$$R \left( \frac{b_i - U_i(x)}{v_i(x)} \right) \geq u_i$$

:

$$\frac{b_i - U_i(x)}{v_i(x)} \leq \phi^{-1}(1 - u_i) = \tau_N$$

$$u_i = 0.50 \quad a_{ij} \sim N(M, \sigma^2)$$

**Application Side**

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6-4

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,0.0002 ,0.002 ,0.02)

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(0.00002

**construct of model : 6-1**

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**Variables -1**

- .0.02 :  $\chi_1$
- .0.002 :  $\chi_2$
- .0.0002 :  $\chi_3$
- .0.00002 :  $\chi_4$
- :  $\chi_5$

**Function of Target -2**

$Max \quad Z = 78\chi_1 + 80.5\chi_2 + 78.5\chi_3 + 80.2\chi_4 + 81.3\chi_5$

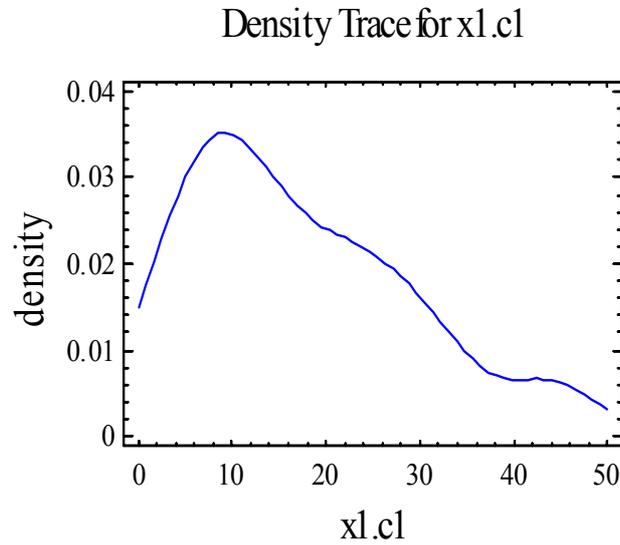
**Constraints -3**

$a_{1j}$

:

6.2	5.9	13.5	26.8	12.9	5.5	25.9	23.9	11	42.3	$a_{11}$
10.1	8.4	13.6	31.7	20.3	11.7	26.6	24.2	11.7	39.4	$a_{12}$
9.7	9	12.5	32.2	20	12.4	31.7	23.7	15.6	39.8	$a_{13}$
9.1	10.1	12	31.2	22.8	17	25.6	23.6	37.6	40.6	$a_{14}$
8	18	25.1	41.3	19.5	13.9	21.9	25.9	31.4	41	$a_{15}$

Statgraph



Analysis Summary

Data variable: x1.c1 (a<sub>11</sub>)  
 10 values ranging from 5.5 to 42.3  
 Fitted normal distribution:

    mean = 17.39  
     standard deviation = 12.012

Goodness-of-Fit Tests for a<sub>11</sub>  
 Chi-Square Test

Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chi-Square
at or below	14.3468	6	4.00	1.00
14.3468	20.4332	0	2.00	2.00
above	20.4332	4	4.00	0.00

.Insufficient data to conduct Chi-Square test  
 Estimated Kolmogorov statistic DPLUS = 0.226974  
 Estimated Kolmogorov statistic DMINUS = 0.161124  
 Estimated overall statistic DN = 0.226974  
 Approximate P-Value = 0.681514

EDF Statistic	Value	Modified Form	P-Value
Kolmogorov-Smirnov D	0.226974	0.776493	>0.10*
Anderson-Darling A <sup>2</sup>	0.492879	0.540935	0.1652*

\*Indicates that the P-Value has been compared to tables of critical values  
 .specially constructed for fitting the currently selected distribution  
 .Other P-values are based on general tables and may be very conservative

#### The StatAdvisor

This pane shows the results of tests run to determine whether a11  
 can be adequately modeled by a normal distribution. The chi-square  
 .test was not run because the number of observations was too small  
 Since the smallest P-value amongst the tests performed is greater  
 than or equal to 0.10, we can not reject the idea that a11 comes  
 .from a normal distribution with 90% or higher confidence

" " (a<sub>11</sub>)

%90

(0.226974) Kolmogorov-Smirnov

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D<sub>0.10,10</sub>=0.369

#### Density Trace for x2.c1

#### Analysis Summary

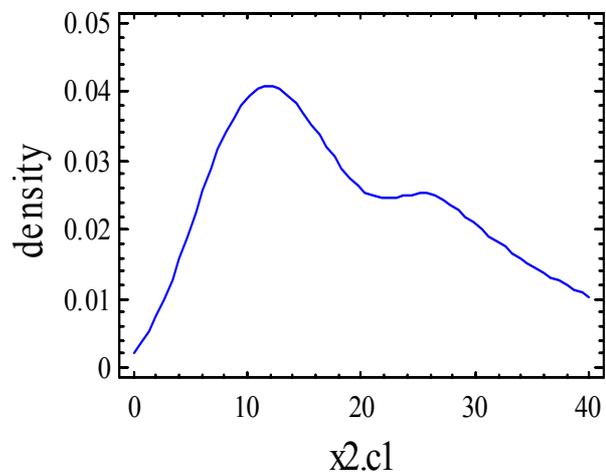
Data variable: x2.c1(a<sub>12</sub>)

10 values ranging from 8.4 to 39.4

Fitted normal distribution:

mean = 19.77

standard deviation = 10.4633



## Analysis Summary

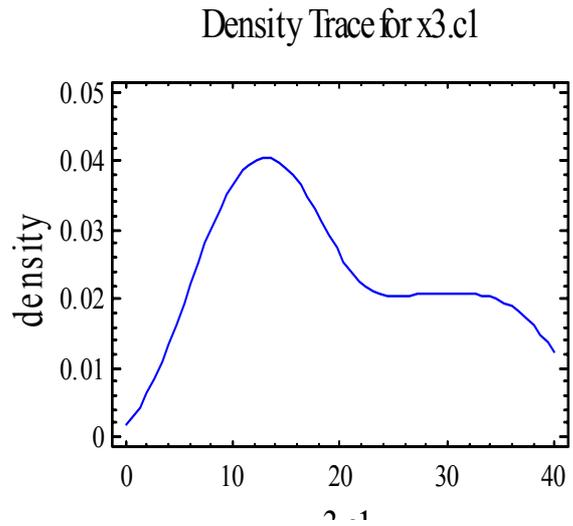
Data variable: x3.c1(a<sub>13</sub>)

10 values ranging from 9.0 to 39.8

Fitted normal distribution:

mean = 20.66

standard deviation = 10.7887



## Analysis Summary

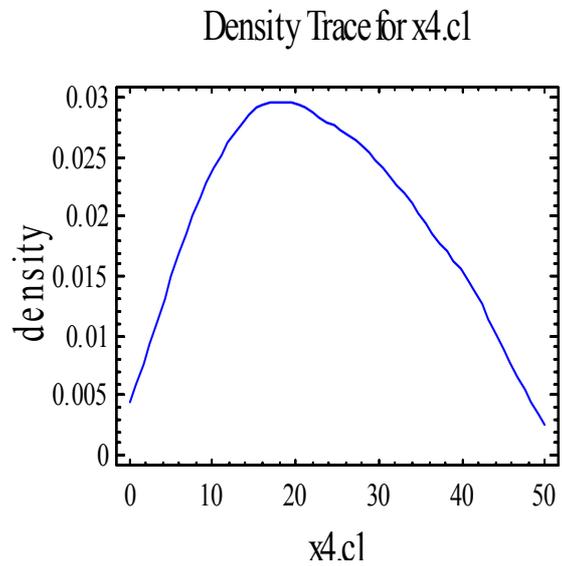
Data variable: x4.c1(a<sub>14</sub>)

10 values ranging from 9.1 to 40.6

Fitted normal distribution:

mean = 22.96

standard deviation = 11.1252



## Analysis Summary

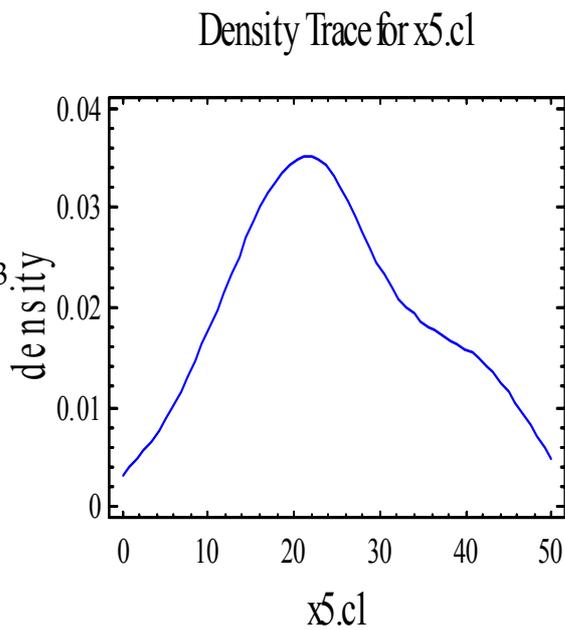
Data variable: x5.c1(a<sub>15</sub>)

10 values ranging from 8.0 to 41.3

Fitted normal distribution:

mean = 24.6

standard deviation = 10.8676



$$: u_i=0.50$$

$$17.39\chi_1 + 19.77\chi_2 + 20.66\chi_3 + 22.96\chi_4 + 24.6\chi_5 \leq 27.18$$

27.18

$a_{2j}$

:( 15 )

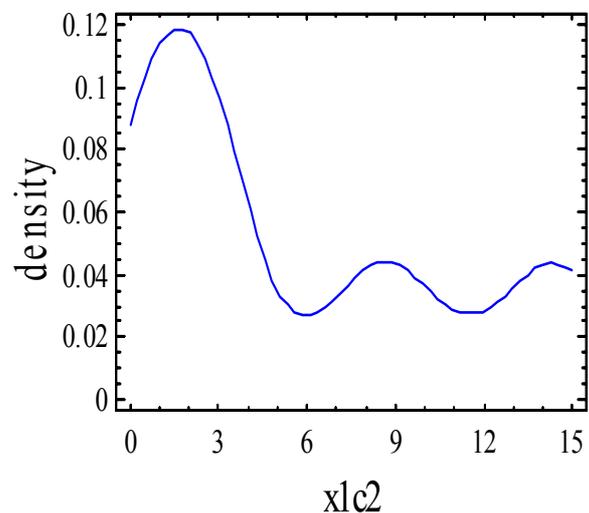
0.7	1.8	3.3	13.9	8.9	2.6	0.7	8.4	1.1	14.7	$a_{21}$
1.5	2.7	3.4	21.4	14.2	8.9	0.8	6.9	0.9	12.8	$a_{22}$
1.3	3.3	3.4	22.7	13.9	8.2	1	8.7	1.3	14	$a_{23}$
1	4.9	3.1	22.6	15	15.2	0.7	8.3	4.4	14.9	$a_{24}$
1.5	9	4.1	25.6	19.9	7.8	1	9.2	3.3	15.1	$a_{25}$

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### Statgraph

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Density Trace for x1c2



#### Analysis Summary

Data variable: x1.c2( $a_{21}$ )

10 values ranging from 0.7 to 14.7

Fitted normal distribution:

mean = 5.61

standard deviation = 5.45109

## Analysis Summary

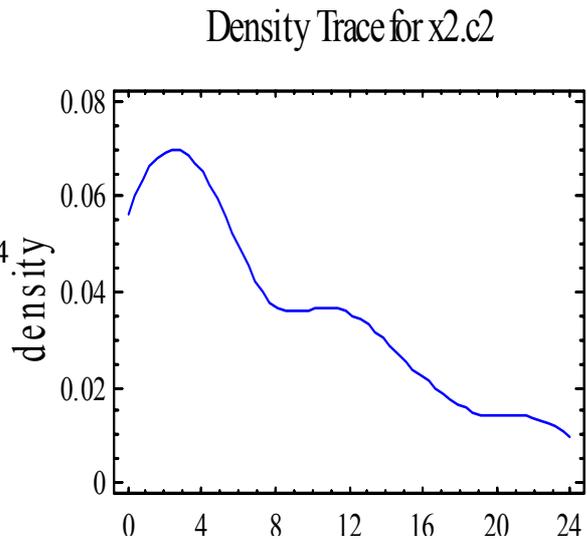
Data variable: x2.c2(a<sub>22</sub>)

10 values ranging from 0.8 to 21.4

Fitted normal distribution:

mean = 7.35

standard deviation = 6.93289



## Analysis Summary

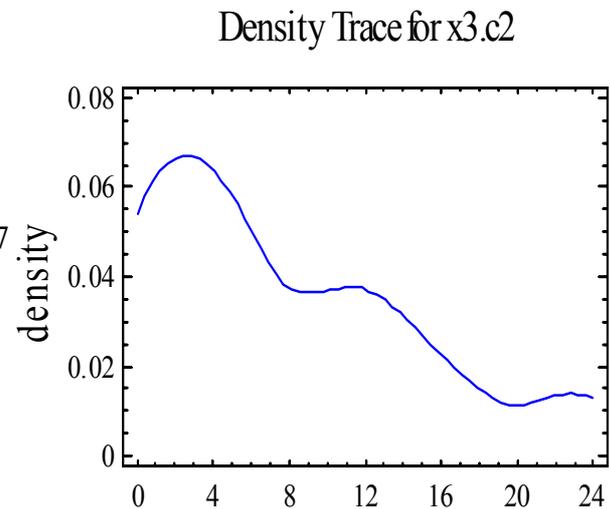
Data variable: x3.c2(a<sub>23</sub>)

10 values ranging from 1.0 to 22.7

Fitted normal distribution:

mean = 7.78

standard deviation = 7.21862



## Analysis Summary

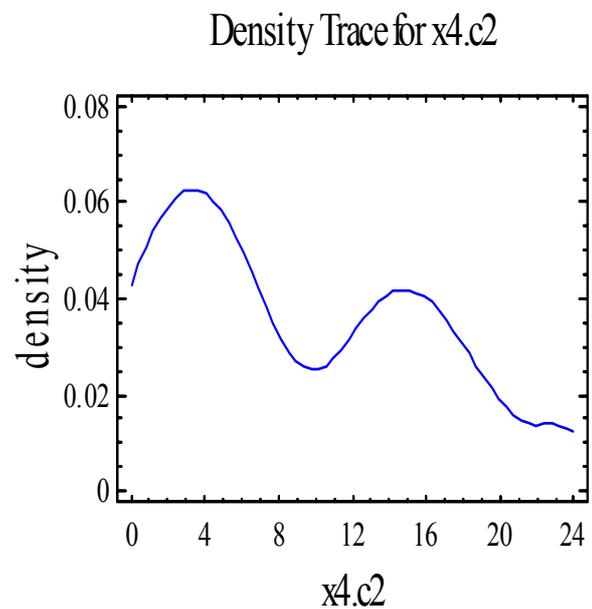
Data variable: x4.c2(a<sub>24</sub>)

10 values ranging from 0.7 to 22.6

Fitted normal distribution:

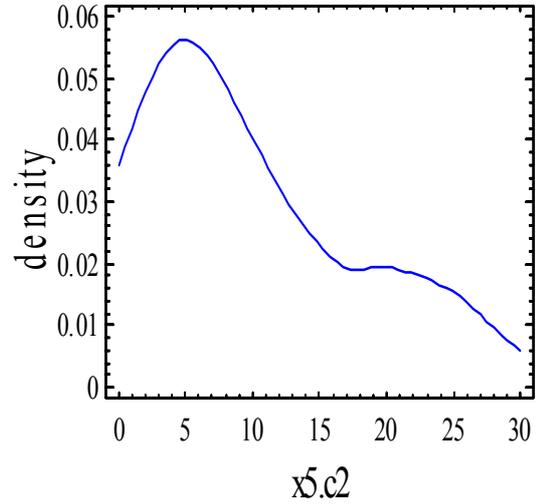
mean = 9.01

standard deviation = 7.45631



Analysis Summary  
 Data variable: x5.c2(a<sub>25</sub>)  
 10 values ranging from 1.0 to 25.6  
 Fitted normal distribution:  
 mean = 9.65  
 standard deviation = 8.20288

Density Trace for x5.c2



: u<sub>i</sub>=0.50

$$5.61\chi_1 + 7.35\chi_2 + 7.78\chi_3 + 9.01\chi_4 + 9.65\chi_5 \leq 10.49$$

a<sub>3j</sub>

:

2.4	7	9.4	10.8	3.2	1.9	6.1	3.9	4.3	17.3	a <sub>31</sub>
8.5	10	11	15.9	11.3	8.9	5.2	5.5	4.3	18.7	a <sub>32</sub>
8.9	13.1	10.6	16.7	10.5	10.6	6.6	6.1	4.7	19.7	a <sub>33</sub>
9.1	16.1	9	16	12.5	14.2	6	7.2	21.2	19.3	a <sub>34</sub>
12	8	8.6	19.5	10.5	5.5	13.6	2.7	12.1	10.5	a <sub>35</sub>

## Statgraph

## Analysis Summary

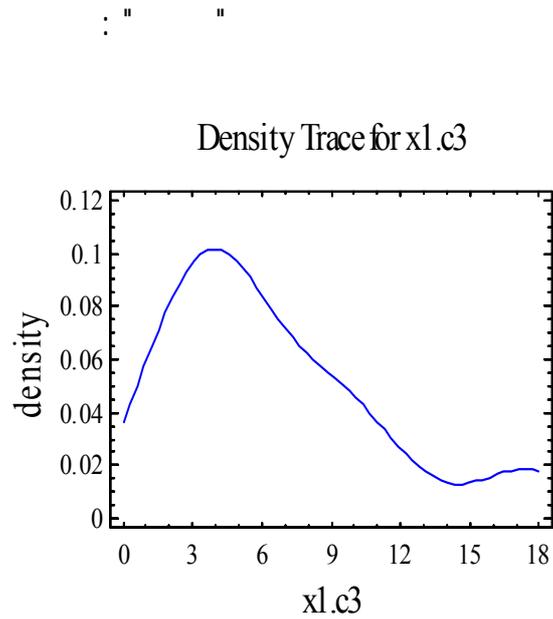
Data variable: x1.c3(a<sub>31</sub>)

10 values ranging from 1.9 to 17.3

Fitted normal distribution:

mean = 6.63

standard deviation = 4.76376



## Analysis Summary

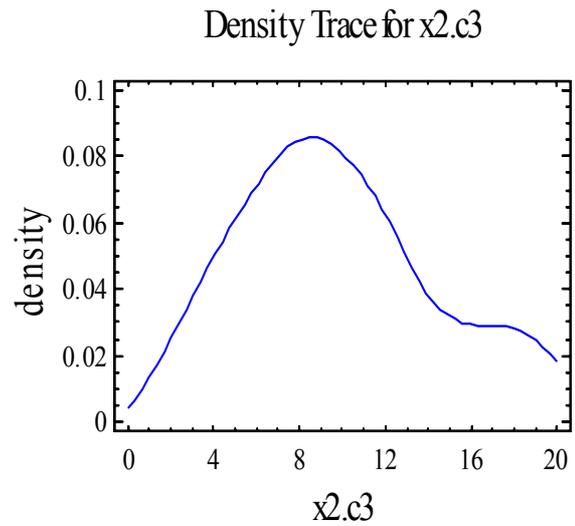
Data variable: x2.c3(a<sub>32</sub>)

10 values ranging from 4.3 to 18.7

Fitted normal distribution:

mean = 9.93

standard deviation = 4.62338



## Analysis Summary

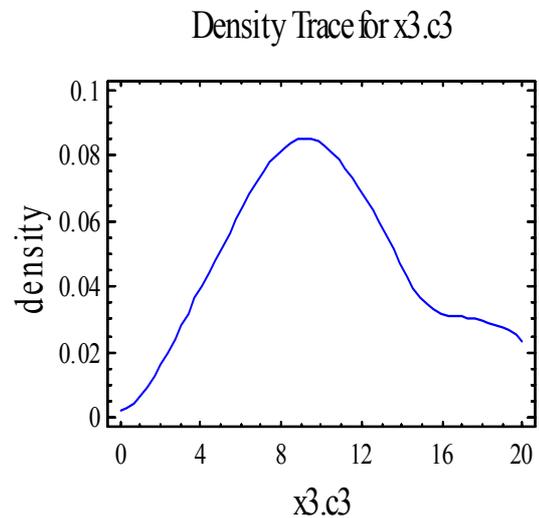
Data variable: x3.c3(a<sub>33</sub>)

10 values ranging from 4.7 to 19.7

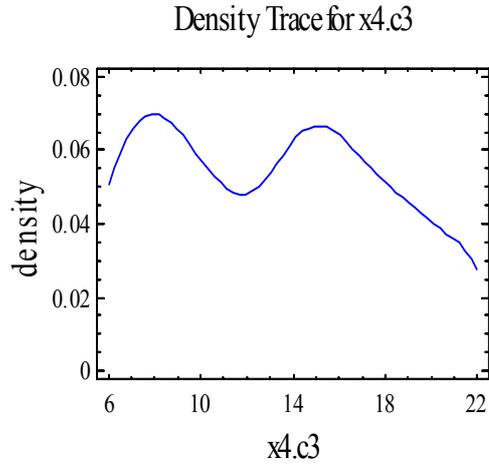
Fitted normal distribution:

mean = 10.75

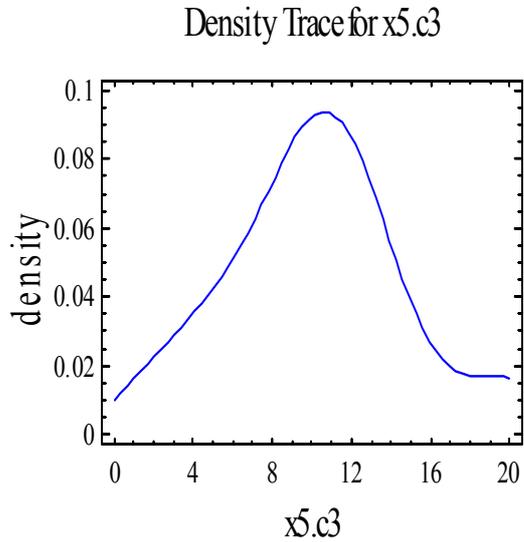
standard deviation = 4.7141



Analysis Summary  
 Data variable: x4.c3(a<sub>34</sub>)  
 10 values ranging from 6.0 to 21.2  
 Fitted normal distribution:  
 mean = 13.06  
 standard deviation = 5.1752



Analysis Summary  
 Data variable: x5.c3(a<sub>35</sub>)  
 10 values ranging from 2.7 to 19.5  
 Fitted normal distribution:  
 mean = 10.3  
 standard deviation = 4.60338



: u<sub>i</sub>=0.50

$$6.63\chi_1 + 9.93\chi_2 + 10.75\chi_3 + 13.06\chi_4 + 10.3\chi_5 \leq 14.7$$

a<sub>4j</sub>

:( 15 )

0.1	1.4	1	1.9	0.5	1.8	0.3	2	0.4	9.3	a <sub>41</sub>
0.4	3.2	1.2	4	1.8	2.2	0.2	3.2	0.5	9.6	a <sub>42</sub>
0.3	3.6	1.5	4.3	1.7	2.3	0.3	2.6	0.6	8.8	a <sub>43</sub>
0.4	4.3	1.5	5.1	2.5	4.2	0.2	3.5	2	10	a <sub>44</sub>
0.4	1.2	1.2	3.8	2.9	1.5	1.3	1.4	1.6	9.7	a <sub>45</sub>

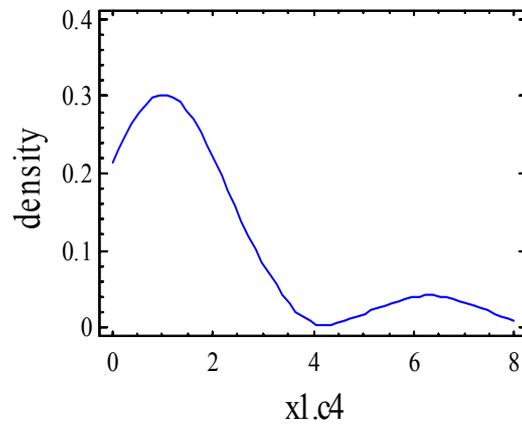
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Statgraph

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Density Trace for x1.c4

Analysis Summary  
 Data variable: x1.c4(a<sub>41</sub>)  
 10 values ranging from 0.1 to 6.3  
 Fitted normal distribution:  
 mean = 1.57  
 standard deviation = 1.80619



## Analysis Summary

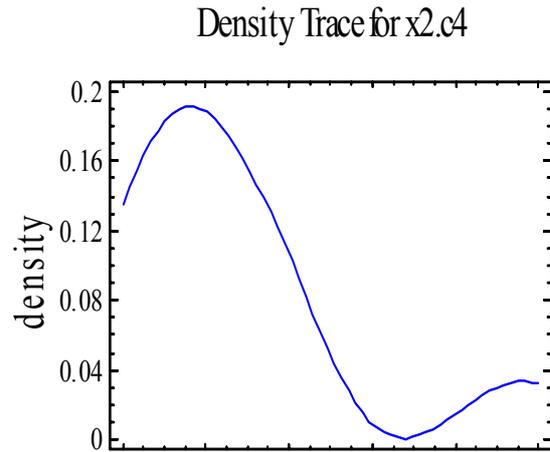
Data variable: x2.c4(a<sub>42</sub>)

10 values ranging from 0.2 to 8.8

Fitted normal distribution:

mean = 2.63

standard deviation = 2.77



## Analysis Summary

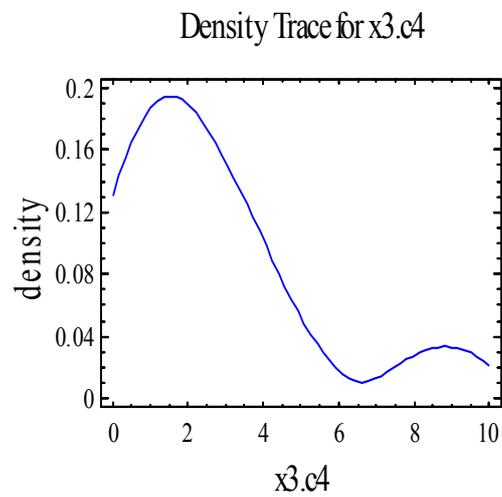
Data variable: x3.c4(a<sub>43</sub>)

10 values ranging from 0.3 to 8.8

Fitted normal distribution:

mean = 2.606

standard deviation = 2.56531



Analysis Summary

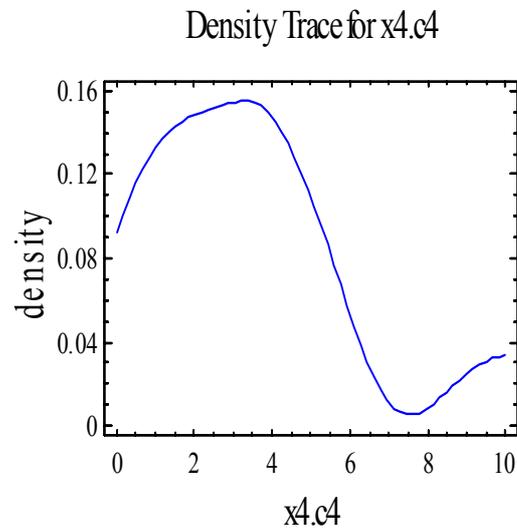
Data variable: x4.c4(a<sub>44</sub>)

10 values ranging from 0.2 to 10.0

Fitted normal distribution:

mean = 3.37

standard deviation = 2.85815



Analysis Summary

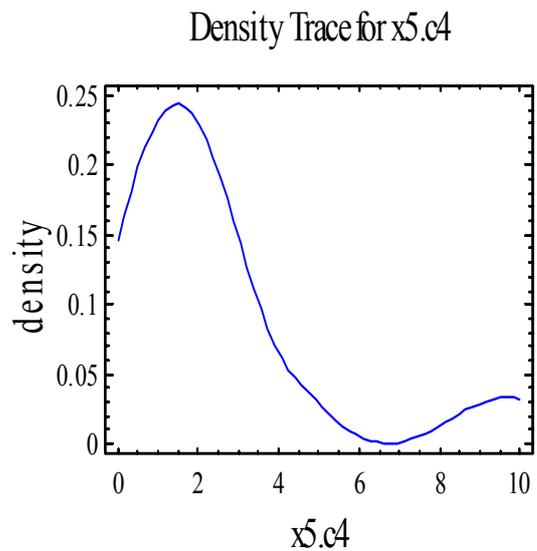
Data variable: x5.c4(a<sub>45</sub>)

10 values ranging from 0.4 to 9.7

Fitted normal distribution:

mean = 2.5

standard deviation = 2.70678



$$1.57\chi_1 + 2.63\chi_2 + 2.606\chi_3 + 3.37\chi_4 + 2.5\chi_5 \leq 3.52 \quad : u_i=0.50$$

:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0 \text{ and integer}$$

:

$$\text{Max } Z = 78x_1 + 80.5x_2 + 78.5x_3 + 80.2x_4 + 81.3x_5$$

S.t

$$17.39x_1 + 19.77x_2 + 20.66x_3 + 22.96x_4 + 24.6x_5 \leq 27.18$$

$$5.61x_1 + 7.35x_2 + 7.78x_3 + 9.01x_4 + 9.65x_5 \leq 10.49$$

$$6.63x_1 + 9.93x_2 + 10.75x_3 + 13.06x_4 + 10.3x_5 \leq 14.7$$

$$1.57x_1 + 2.63x_2 + 2.606x_3 + 3.37x_4 + 2.5x_5 \leq 3.52$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0 \text{ and integer}$$

**solution of model****: 7-1**

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Win QSB

$$x_1, x_2, x_3, x_4 = 0$$

$$x_5 = 1$$

$$Z = 81.3$$

**Conclutions****:8-1**

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3. Kwak , A.K - 1973 - .Mathematical Programming with business applications – MC Graw – Hill , Inc.
4. Liebrman & Hillier – 1990 – Introduction the operational Research – Holden – Day , Inc.