The interaction of heigh frequency electromagnetic field with the electrons of atom using relativistic quantum mechanics.

تفاعل مجال كهرمغناطيسي ذو تردد عالي مع الكترونات الذرة بأستخدام ميكانيك الكم النسبي .

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Abstract;
when electromagnetic wave interact with electron of an atom the electron can be translat between two discrete levels The initial and final state belongs to discrete eigin values.
But if the frequency of the incident wave is large enough to free the electron, the final state will be belongs to continuous spectrum.

There are two cases. If the frequency is very high in such a way that the free electron emit's from the atom have very high speed and can be compared with the velocity of light in this case we must use Dirac equation and we must find the cross-section of interaction of the electromagnetic wave with the electron and use Dirac-eq to find the final wave function of electron .i.e we must use relativistic quantum –mechanics to do that. In the case that the frequency of the electromagnetic wave is small we can use classical quantum mechanics where many scientist's did that and find result's agree with experiment's

الخلاصة:
عندما تتفاعل موجة كهرومغناطيسية مع الكترون ذرة , فان الالكترون يمكن ان ينتقل بين مستويات طاقة محددة تكون الدالة الموجبة للحالة الابتدائية والنهائية ذات مستويات طاقة محددة. ولكن اذا كان تردد الموجة الساقطة (المتفاعله مع الالكترون) كبير بما يكفي لتحرير الالكترون فان الحالة النهائية تنتمي الى منطقة الطيف المستمر للطاقة اي ان الالكترون يمكن ان يملك اي قيمة للطاقة – هناك حالتان عند ذلك إذا كان تردد الموجة الكهرومغناطيسية الساقطة عالي جدا بحيث ان الالكترون المنبعث من الذرة يملك سرعة عالية مقارنة بسرعة الضوء في هذة الحالة يجب استخدام معادلة ديراك ولايجاد المقطع العرضي التفاضلي للتفاعل بين الموجة الكهرومغناطيسية والالكترون يستلزم ان نستخدم مفاهيم ميكانيك الكم النسبي وهذا ماسنفعله في موضوع البحث المقدم وفي الحالة الثانية اي عندما يكون تردد الموجة الساقطة صغير ممكن استخدام ميكانيك الكم اللانسبي اي معادلة شرودنكر حيث قام علماء كثيرون ببحث ذلك وتوصلوا الى نتائج تتطابق مع التجارب.

Introduction;

The case that the frequency of the incident electromagnetic wave is very largei.e$\hbar\omega \gg I$where $(I)$is the ionization potential$(\omega)$is the frequency of the incident wave differs from the case of obsorbtion of electromagnetic wave of small frequency, that is in the first the final state belongs to continuous spectrum. The cross-section in this case can be calculated in an exact analytical form for Hydrogen atom and Hydrogen-like ion .In the initial state the electron is at discreet level $\varepsilon_i = -I$(where I is the ionization potential of the atom) and the photon has definite momentum $(k^{\rightarrow})$. In the final state the electron has momentum $(p^{\rightarrow})$(and energy $\varepsilon_f = \varepsilon$) since $(p^{\rightarrow})$ takes acuminous series of values the cross-section for this process [1].

$d\sigma = 2\pi|V_{fi}|\delta(-1 + \omega\hbar - \varepsilon)d^3p^{\rightarrow}/(2\pi)^3$……(1)$|V_{fi}|$ are the elements of Heisenberg matricies for the transition probability under the action of perturbation.
$d^3p^{\rightarrow}$is the volame element in $p^{\rightarrow}$ - space
δ; is Direc Delta function
The wave function of the final state of electron is normalized to (one) particle per unit volume. The wave function of photon is normalized in the same way .
There are two cases which differ as regard the magnitude of the photon energy $\omega\hbar > I, \omega\hbar \ll mc^2$and the second case is $\omega\hbar \gg I, \omega\hbar > mc^2$.

The first case many scientists studied it like Schwinger 1939 [2].
The second case livi-civita[3]. Studied relativisticaly H-atom and He- Ion only
In the nonrelativistic case i.e.$\omega\hbar \ll mc^2$

$$d\sigma = 2^7\pi\alpha a^2(\frac{I}{hw})^4\frac{e^{-4vcotv^{-1}}}{1-e^{-2\pi v}}(n^{\rightarrow}.e^{\rightarrow})^2 do[3] \quad ….. \quad .. \quad ….(2)$$

$n^{\rightarrow}$is the unit vector in the direction of electron velocity (v) and $e^{\rightarrow}$ is unit vector in the direction of polarization of electromagnetic wave.

In 2012 A.A.Ali[4]studied the interaction of magnetic field with free electron using relativistic quantum mechanics[4].

$\alpha = \hbar^2/mZe^2 = \frac{1}{137}$ fine structure constant

$a_0 = \hbar^2/mc^2 I_0 = e^4m/2\hbar^2$Is the Ionization energy of the hydrogen atom

$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$ fine structure constant.

re; is the effective electron radius.

$r_e = \frac{e^2}{m} = \frac{e^2}{mc^2} = 2.818 \times 10^{-13} cm$ $\qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad 1\leq \gamma < \infty.$

The aim of this research is to derive arelation between y and σ using relativistic Quantum mechanics

where $y = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

v ; is velocity of electron
c ; velocity of light
σ ; is the cross section for the interaction of electromegnetic field with electron of atom.

Theory;

Let us now consider the case

$\omega\hbar \gg I.$ ….. (3)

Here we have also $\varepsilon = \omega\hbar - I \gg I$, and the influence of the Coulomb field of the nucleus on the wave function ψ' of the emitted electron can be taken into account by perturbation theory. We write;[5]

$$\psi' = \frac{1}{\sqrt{(2\varepsilon)}}\left(u'e^{ip.r} + \psi^{(1)}\right)……(4)$$

The electron may be relativistic, and therefore the unperturbed function in(4) is written as relativistic plane wave .
Although the electron is non-relativistic in the initial state, it's wave function ψ must nevertheless, for reasons to be explained below, include the relativistic correction ($\sim Ze^2$). This function is written as [5].

$$\psi = \left(1 - \frac{i}{2m}y^0\boldsymbol{y}.\Delta\right)\frac{u}{\sqrt{(2mc^2)}}\psi_{non-r,}……(5)$$

where $\psi_{non-r,}$ is the non-relativistic bound-state function and (u) is the bispinor amplitude of the electron at rest, normalized by the usual condition $\bar{u}u = 2mc^2$. We substitute the functions (4),(5) in the matrix element .

$$M_{fi} = \frac{1}{2\sqrt{(m\varepsilon c^2)}}\int\{\bar{u}'(y.e)[(1-\frac{i}{2mc^2}y^0\boldsymbol{y}.\Delta)u\psi_{non-r}]e^{-i(\boldsymbol{p}-\boldsymbol{k}).\boldsymbol{r}} + $$
$$+\psi^{(1)}(\boldsymbol{y}.\boldsymbol{e})e^{ik.r}u\psi_{\boldsymbol{non-r}}\}d^3x \dots\dots.(6)$$

In order to derive the first term of the expansion of this quantity in powers of Ze^2, we can replace ψ_{non-r} in the second term in the braces by the constant $(Ze^2m)^{3/2}/\sqrt{\pi}$. The first term would vanish if treated in this way when **p-k≠ 0**, and it is for this reason that the first relativistic correction, proportional toZe^2, has to be included in ψ.When$v\sim c$ this correction gives a contribution to the cross-section that is of the same order as the contribution of the next term in the expansion of ψ_{non-r} in powers of Ze^2.

Integration the first term in (6) by parts, transferring the action of the operator (Δ) from ψ_{non-r} to the exponential factor. The result is

$$M_{fi} = \frac{(Ze^2mc^2)^{3/2}}{2(\pi m\varepsilon)^{1/2}}\left\{\bar{u}'(\boldsymbol{y.e})\left[1+\frac{1}{2mc^2}y^0\boldsymbol{y}.(\mathbf{p-k})\right]u(e^{-Ze^2mc^2})_{\boldsymbol{p-k}}+\psi_k^{(1)}(\boldsymbol{y.e})u\right\},\ldots(7)$$

Where the vector suffix denotes the spatial Fourier component. As far as the Ze^2 term we have

$$(e^{-Ze^2mc^2})_{\boldsymbol{p-k}} = \frac{8\pi Ze^2mc^2}{(\boldsymbol{p-k})^4}\ldots\ldots(8)$$

To calculate the Fourier components $\psi_k^{(1)}$, we write down the equation satisfied by the function $\psi^{(1)}$;[5]

$$(y^0\varepsilon + \boldsymbol{iy.\Delta} - \mathrm{mc}^2)\psi^{(1)} = \mathrm{e}(\mathrm{y}^\mu\mathrm{A}_\mu)\mathrm{u}'\mathrm{e}^{\mathrm{ip.r}}$$
$$= -\frac{Ze^2}{r}y^0u'e^{ip.r},$$

applying the operator $y^o\varepsilon + \boldsymbol{iy.\Delta} + \mathrm{m}c^2$ to both sides, we find where$(y^{\cdot})$ and $(y\,)$ are Dirac matrices.[6].

$$(\Delta + p^2)\psi^{(1)} = -Ze^2(y^0\varepsilon + \boldsymbol{iy.\Delta} + \mathrm{mc}^2)(\mathrm{y^o u'})\frac{1}{\mathrm{r}}e^{ip.r}$$

Multiplying this equation by $e^{-ik.r}$ and integrating with respect to $d^3x,$ with the usual integration by parts in the terms containing (Δ) operator, gives

$$(\boldsymbol{p}^2 - \boldsymbol{k}^2)\psi_{\boldsymbol{k}}^{(1)} = -Ze^2(y^o\varepsilon - \boldsymbol{y.k} + \mathrm{mc}^2)(\mathrm{y^o u'})\left(\frac{1}{\mathrm{r}}\right)_{\mathrm{k-p}}$$

$$=-Ze^2\big(2\varepsilon y^o - \boldsymbol{y}.(\boldsymbol{k-p})\big)(\mathrm{y^o u'})\frac{4\pi}{(\mathbf{k-p})^2}.$$

In the last line we have used the fact that the amplitude u'satisfies the equation

$(\varepsilon y^o - \boldsymbol{p.y}\text{-}mc^2)u' = 0$,or$(\varepsilon y^o + \boldsymbol{p} - \boldsymbol{y} - \mathrm{mc}^2)\mathrm{y^o u'} = 0$.

Hence

$$\psi_k^{(1)} = \psi_k^{(1)*}y^0 = 4\pi Ze^2\bar{u}'\frac{2\varepsilon y^0 + \boldsymbol{y}.(\boldsymbol{k-p})}{(\boldsymbol{k}^2-\boldsymbol{p}^2)(\boldsymbol{k-p})^2}y^0\ldots\ldots(9)$$

Substituting (8) and (9) in the matrix element (7), we can write it as

$$M_{fi} = \frac{4\pi^{1/2}(Ze^2mc^2)^{5/2}}{(\varepsilon mc^2)^{\frac{1}{2}}(\boldsymbol{k-p})^2}\bar{u}'Au,$$

where

$A = a(\boldsymbol{y.e}) + (\boldsymbol{y.e})y^0(\boldsymbol{y.b}) + (\boldsymbol{y.c})y^0(\boldsymbol{y.e})$, **[6]**

$a = \frac{1}{(\boldsymbol{k-p})^2} + \frac{\varepsilon}{mc^2}\frac{1}{-\boldsymbol{p}^2}\boldsymbol{b} = -\frac{\boldsymbol{k-p}}{2mc^2(\boldsymbol{k-p})^2}$, $\quad \boldsymbol{c} = \frac{\boldsymbol{k-p}}{2\mathrm{m}(\mathbf{k}^2-\mathbf{p}^2)}$.**[6]**

The cross-section is [6]

$$d\sigma = \frac{8e^2(Ze^2mc^2)^5\,|\boldsymbol{p}|}{\omega(\boldsymbol{k-p})^4\boldsymbol{mc^2}}(\bar{u}'Au)(\bar{u}\bar{A}u')do,[6]$$

Where $\bar{A} = y^0A^+y^0$. This expression has to be summed over final directions and averaged over initial directions of the electron spin, using the polarization density matrices of the initial and final states;

$$\rho = \frac{1}{2}m(y^0+1), \rho' = \frac{1}{2}(y^0\varepsilon - \boldsymbol{y}.\boldsymbol{\rho} + \mathrm{mc}^2);$$

In the initial state, **p**$= 0$ and $\varepsilon = m$.The resulting expression is

$$d\sigma = \frac{16e^2(Ze^2mc^2)^5|\boldsymbol{p}|}{m\omega(\boldsymbol{k-p})^4}\,tr\,(\rho'A\rho\,\bar{A})do.$$

$tr(\rho' A\rho\ ) = \frac{mc^2}{\varepsilon+mc^2}[\boldsymbol{ap} - (\boldsymbol{b}-\boldsymbol{c})(\varepsilon + \mathrm{mc}^2)]^2 + 4(\mathbf{b}.\mathbf{e})[(\varepsilon + \mathrm{mc}^2)(\mathbf{c}.\mathbf{e}) + \mathrm{a}(\mathbf{p}.\mathbf{e})]$;
The vector **e** is assumed real, i.e. the photon is assumed to be linearly polarized [7]. The formula for cross-section will be put in its final form by using the polar angle θ and the azimuth $\emptyset$ of the direction of **p** when the direction of **k** is in the z-axis and the plane of **k** and **e** is the xz plane (so that $\boldsymbol{p}.\boldsymbol{e} = |\boldsymbol{p}|\cos\emptyset\ \ \sin\theta$).When $\omega\hbar \gg I$, the conservation of energy may be written in the form ε-mc^2=$\omega\hbar$instead of

$\varepsilon - mc^2 = \omega\hbar - I$.

We then easily see that;

$$\boldsymbol{k}^2 - \boldsymbol{p}^2 = -2mc^2(\varepsilon - mc^2), (\boldsymbol{k}-\boldsymbol{p})^2 = 2\varepsilon(\varepsilon - mc^2)(1 - v\cos\theta),$$

Where $\boldsymbol{v} = \boldsymbol{pc^2}/\varepsilon$ is the velocity of the electron . A simple calculation gives finally

$$d\sigma = Z^5\alpha^4 r_e^2 \frac{v^3(1-v^2)^3 sin^2\theta}{[1-\sqrt{(1-v^2)}]^5(1-v\,cos\theta)^4}\times$$

$$\times\left\{\frac{[1-\sqrt{(1-v^2)}]^2}{2-(1-v^2)^{\frac{3}{2}}d\sigma}(\,1 - v\cos\theta) + \left[2 - \frac{[1-\sqrt{(1-v^2)}](1-v\,cos\theta)}{1-v^2}\right]cos^2\emptyset\right\}do, \ldots.(10) \quad [7]$$

Where $r_e = e^2/m$.
We put (c) as unit of velocity
In the ultra-relativistic case ($\varepsilon \gg m$), the cross-section has a sharp peak at small angles θ~√(1-v^2) , i.e. the electrons are emitted predominantly in the direction of incidence of the photon . Near the maximum

$$1 - v\cos\theta \approx \frac{1}{2}[(1-v^2) + \theta^2],$$

and the leading terms in (10) give

$d\sigma \approx 4Z^5\alpha^4 r_e^2 \frac{(1-v^2)^{3/2}\theta^3}{(1-v^2+\theta^2)^3}\, d\theta\, d\emptyset.$ …..(11)

The integration of (10) over angles is elementary but lengthy, and leads to the following expression for the total cross-section

$\sigma = 2\pi Z^5\alpha^4 r_e^2 \frac{(\gamma^2-1)^{3/2}}{(\gamma-1)^5}\left\{\frac{4}{3} + \frac{\gamma(\gamma-2)}{\gamma+1}(\,1 - \frac{1}{2\gamma\sqrt{\gamma^2-1}}\log\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}})\right\}, \ldots.(12)$

Where the "Lorentz factor"(γ) is used for brevity ;

$\gamma = \frac{1}{\sqrt{(1-v^2/c^2)}} = \frac{\varepsilon}{mc^2} \approx \frac{mc^2+\omega}{mc^2} \ldots..(13)$

Results and calculation;

In the ultra-relativistic case, equation (12) reduces to the simple expression
$\sigma = 2\pi Z^5\alpha^4 r_e^2/\gamma$In the case $I \ll \omega\hbar \ll mc^2$, the limit of small (-1) (12) yields the already known result

$$\sigma = \frac{2^8}{3}\,\alpha\, a_0^2 Z^5 (\frac{I_0}{\hbar\omega})^{7/2}$$

Graphs' have been plooted for the relation between (σ) and (z) for some values (γ) we also can ploot graphs between (σ) and (γ) for identified valus for (z). we can give tables for (σ) and (γ) for (z) from 1→137.
Tables(1-4) give values of (σ) and (γ)$z = (1 \rightarrow 4)$.
In the tables we gave the valus of σ from eq(12) for different values of (z) and we found that there is agood concidense of this values with international values of (σ) given by [8], and we found

agood agreement of the result given by the curves with the emperical values given by [8].
The curves represent the theoretical relation between y and σ

Table (1) give values of (σ) from eq (12) and emperically from [8] for z=1

γ	σ(from eq(12)$X10^{-33}cm^2$	σ emprically [8]$X10^{-33}cm^2$
1	1.4169	1.4357
1.1	1.28813	1.27788
1.2	1.18075	1.16767
1.3	1.0899	1.0786
1.4	0.9446	0.9887
1.5	0.8855	0.8878
1.6	0.8334	0.8354
1.7	0.787	0.778
1.8	0.7457	0.7487
1.9	0.7084	0.7098
2		

Table (2) give values of (σ) from eq (12) and emperically from [8] for z=2

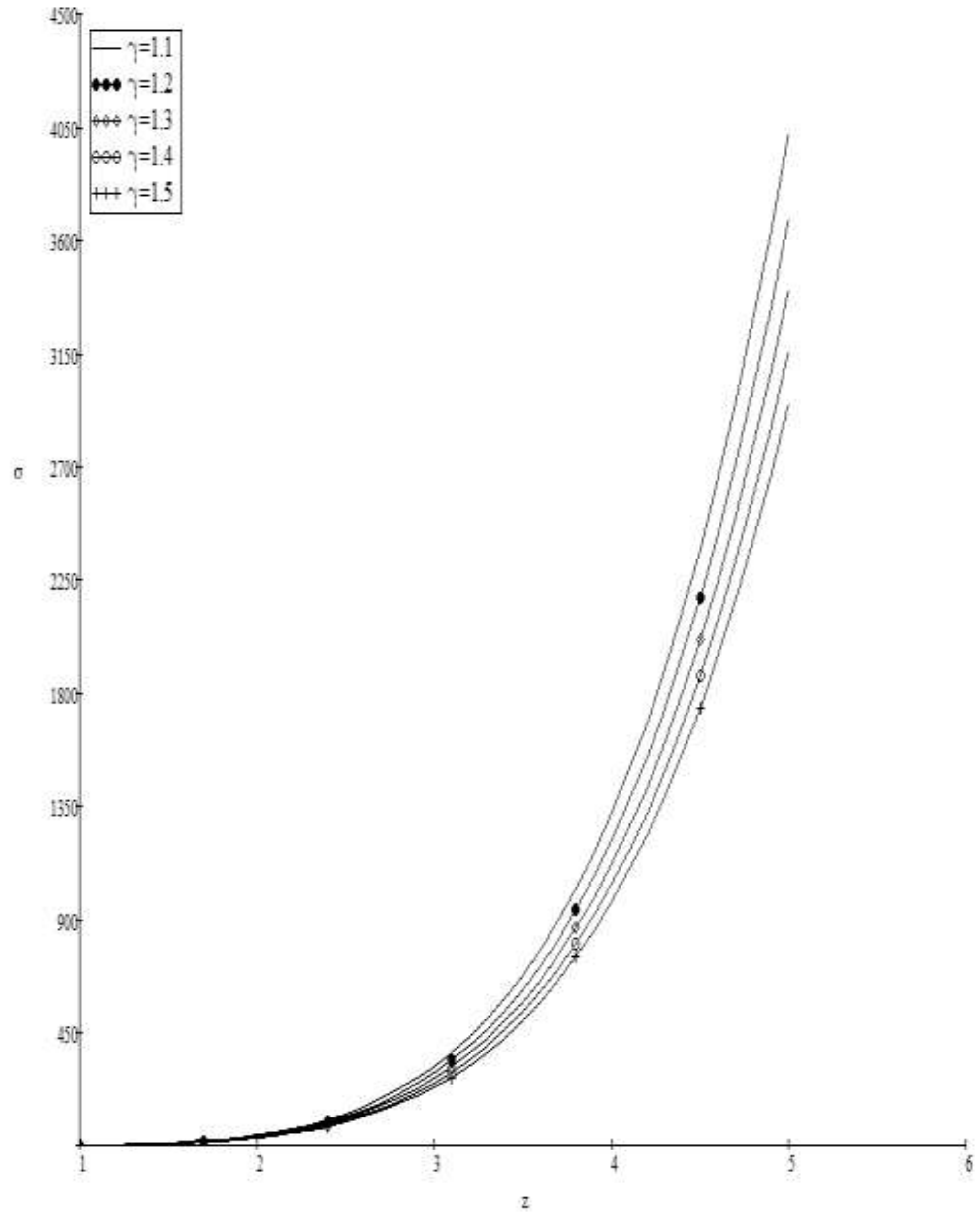
γ	σ(from eq(12)$X10^{-33}cm^2$	σ emprically [8]$X10^{-33}cm^2$
1	45.3408	45.3564
1.1	41.2192	41.2325
1.2	37.7824	37.7654
1.3	34.8777	34.8979
1.4	32.384	32.384
1.5	30.2272	30.2343
1.6	28.336	28.345
1.7	26.6688	26.6765
1.8	25.184	25.123
1.9	23.8624	23.8765
2	22.6688	22.6768

γ	σ(from eq(12)$X10^{-33}cm^2$	σ emprically [8]$X10^{-33}cm^2$
1	344.3067	344.3197
1.1	313.0083	313.0123
1.2	286.922	286.952
1.3	264.8457	264.8757
1.4	245.9330	245.9730
1.5	229.5378	229.5778
1.6	215.1765	215.1965
1.7	202.5162	202.5562
1.8	191.241	191.281
1.9	1812051	181.2951
2	172.1412	172.1812

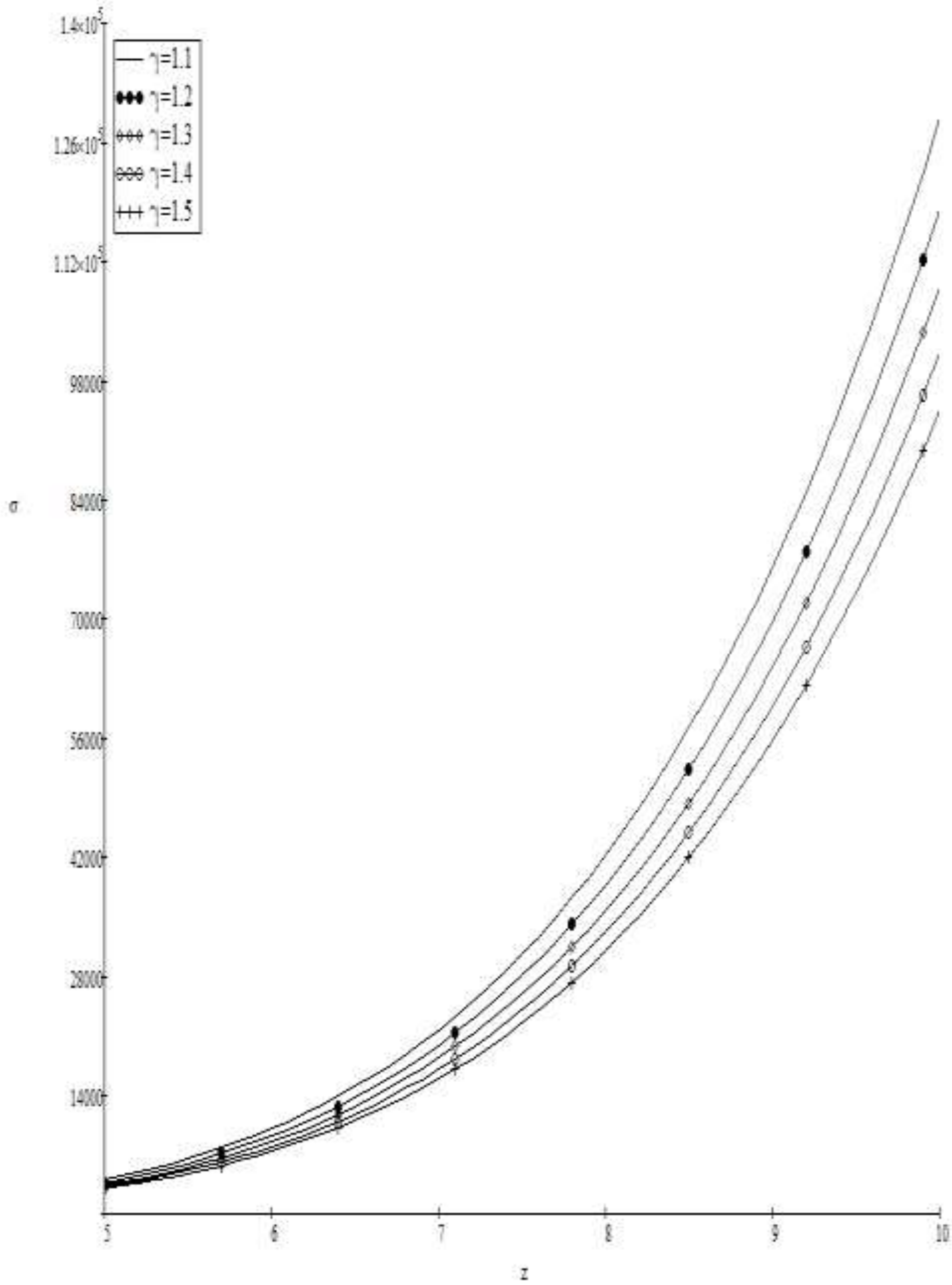
Table (3) give values of (σ) from eq (12) and emperically from [8] for z=3

γ	σ(from eq(12)$X10^{-33}cm^2$	σ emprically [8]$X10^{-33}cm^2$
1	1450.9056	1450.9756
1.1	1319.0144	1319.0349
1.2	1209.0368	1209.0786
1.3	1116.0576	1116.0980
1.4	1036.3596	1036.3876
1.5	967.2704	967.2894
1.6	906.752	906.762
1.7	853.4016	853.4034
1.8	802.888	802.898
1.9	763.5968	763.5348
2	725.4016	725.4876

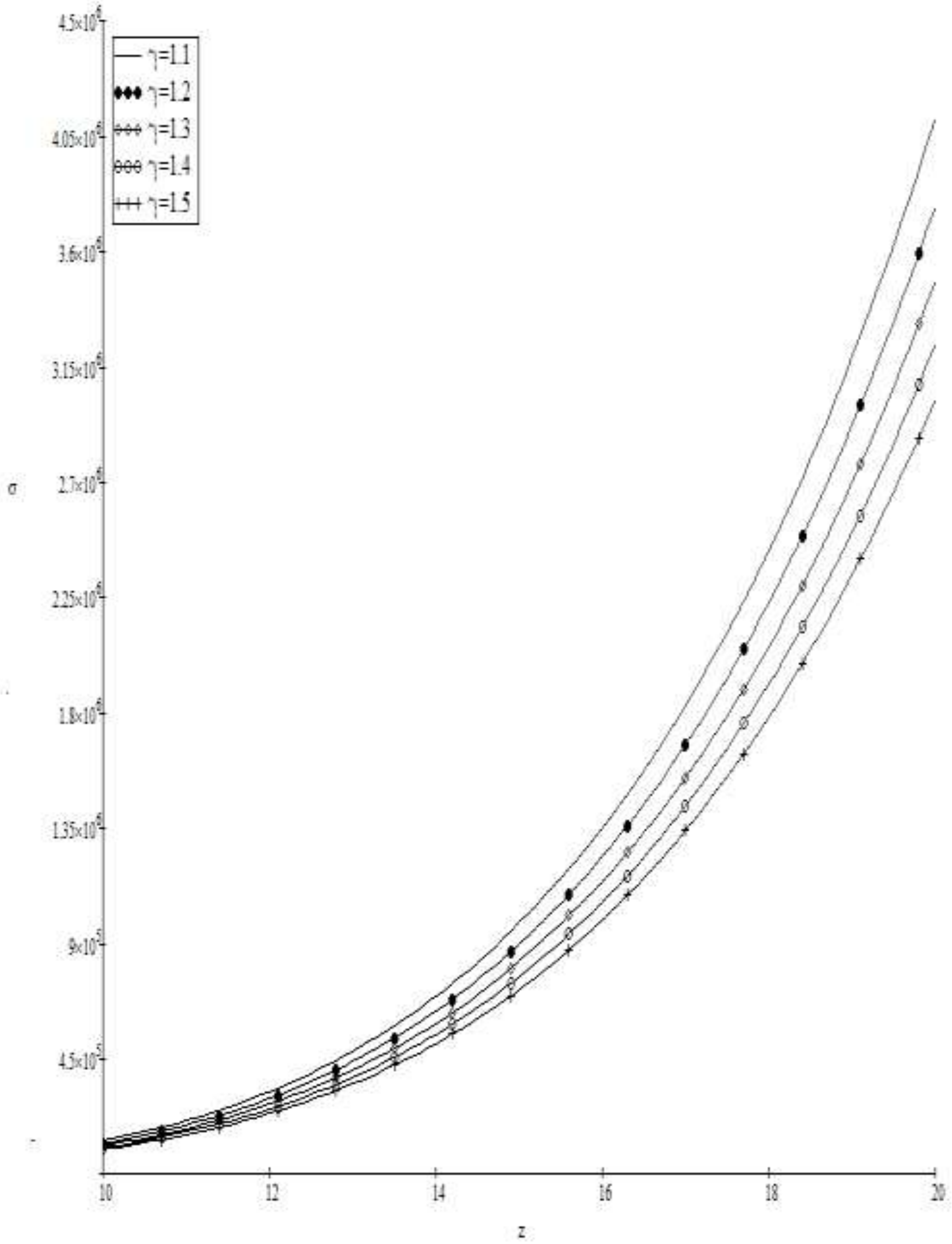
Table (4) give values of (σ) from eq (12) and emperically from [8] for z=4



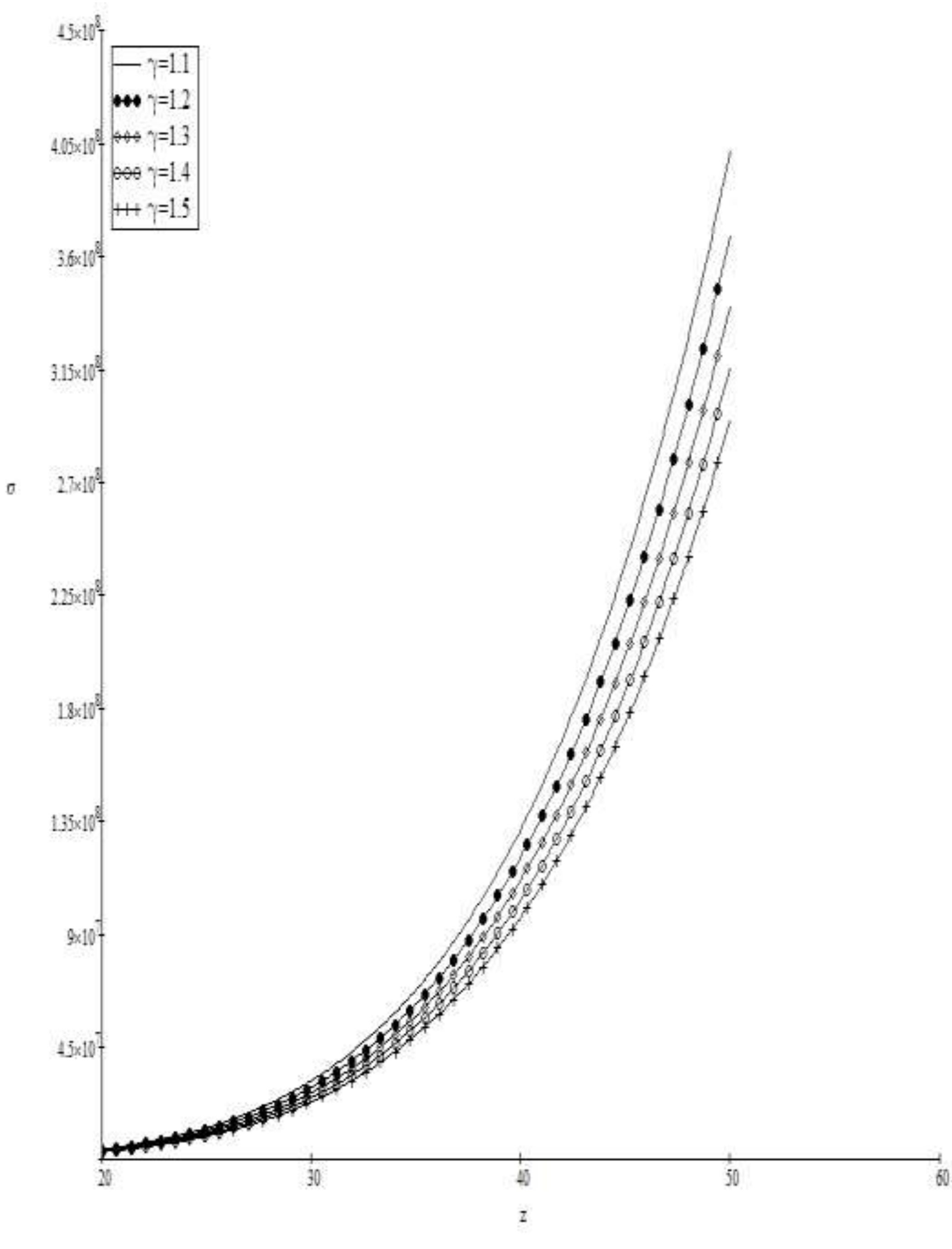
fig(1):arelation between (σ) and (z) when (z=1...5) for varous values of (γ)



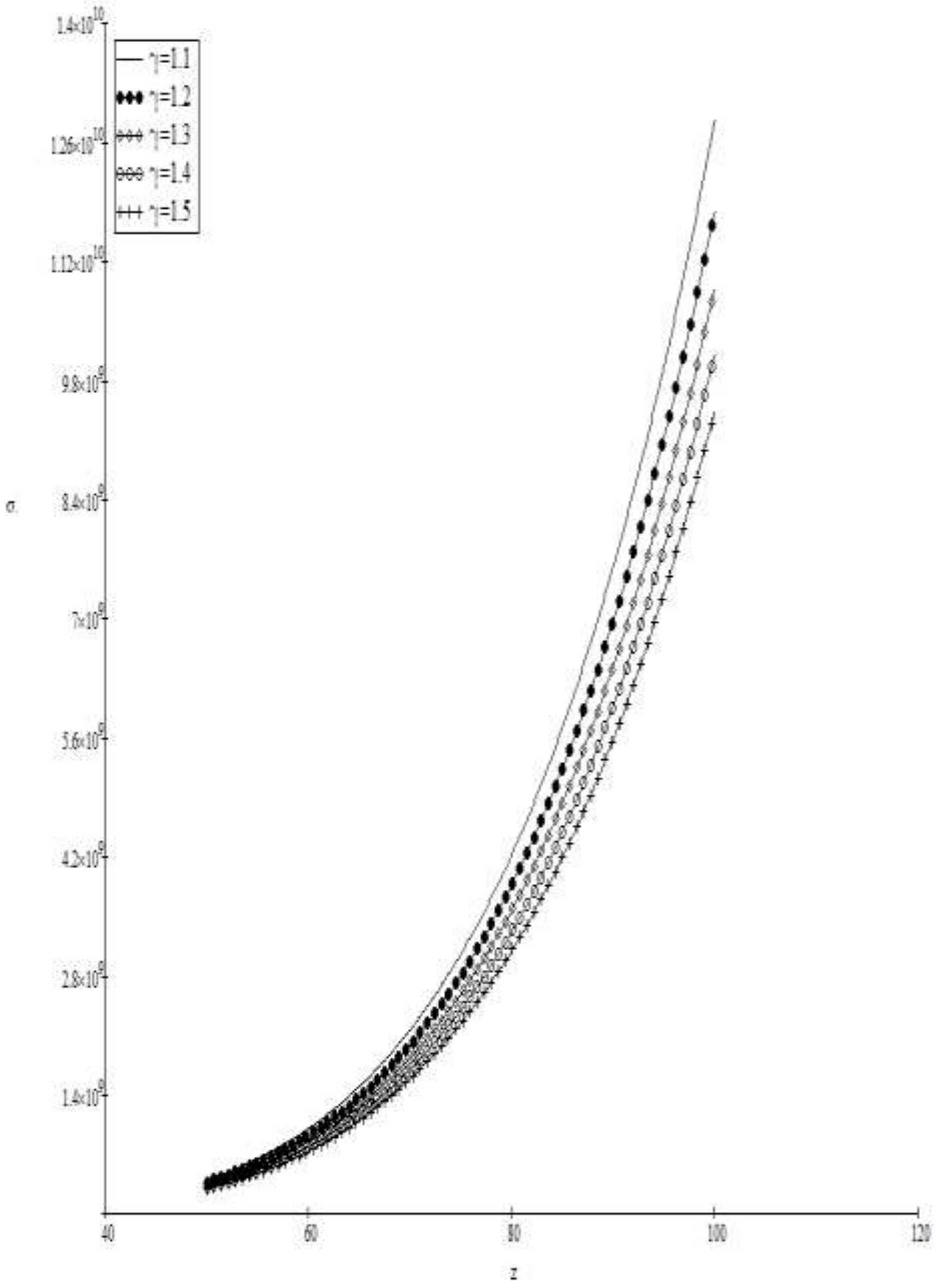
fig(2):relation between (σ) and (z) when (z=5...10) for varous values of (γ)



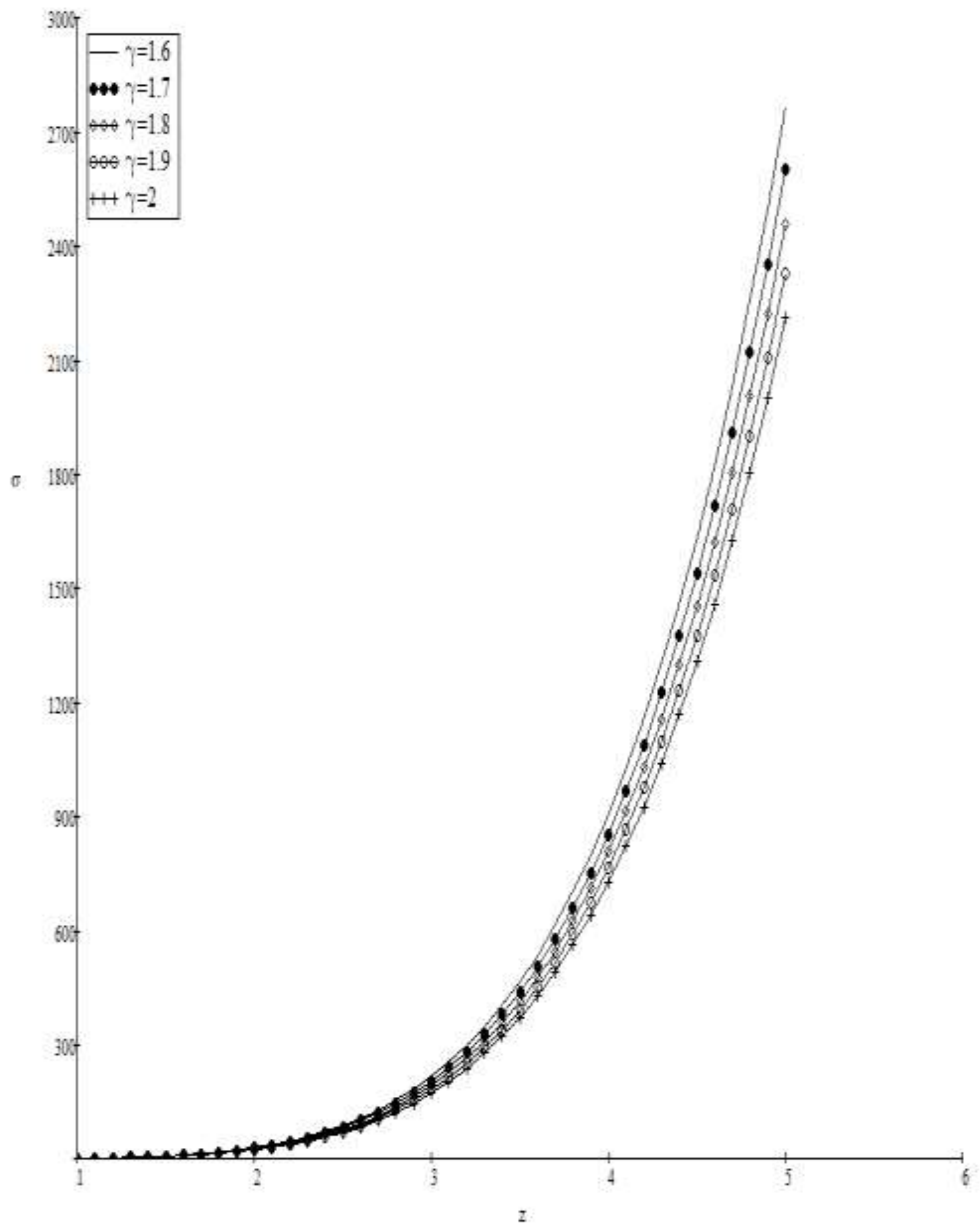
fig(3):arelation between (σ) and (z) when (z=10...20) for varous values of (γ)



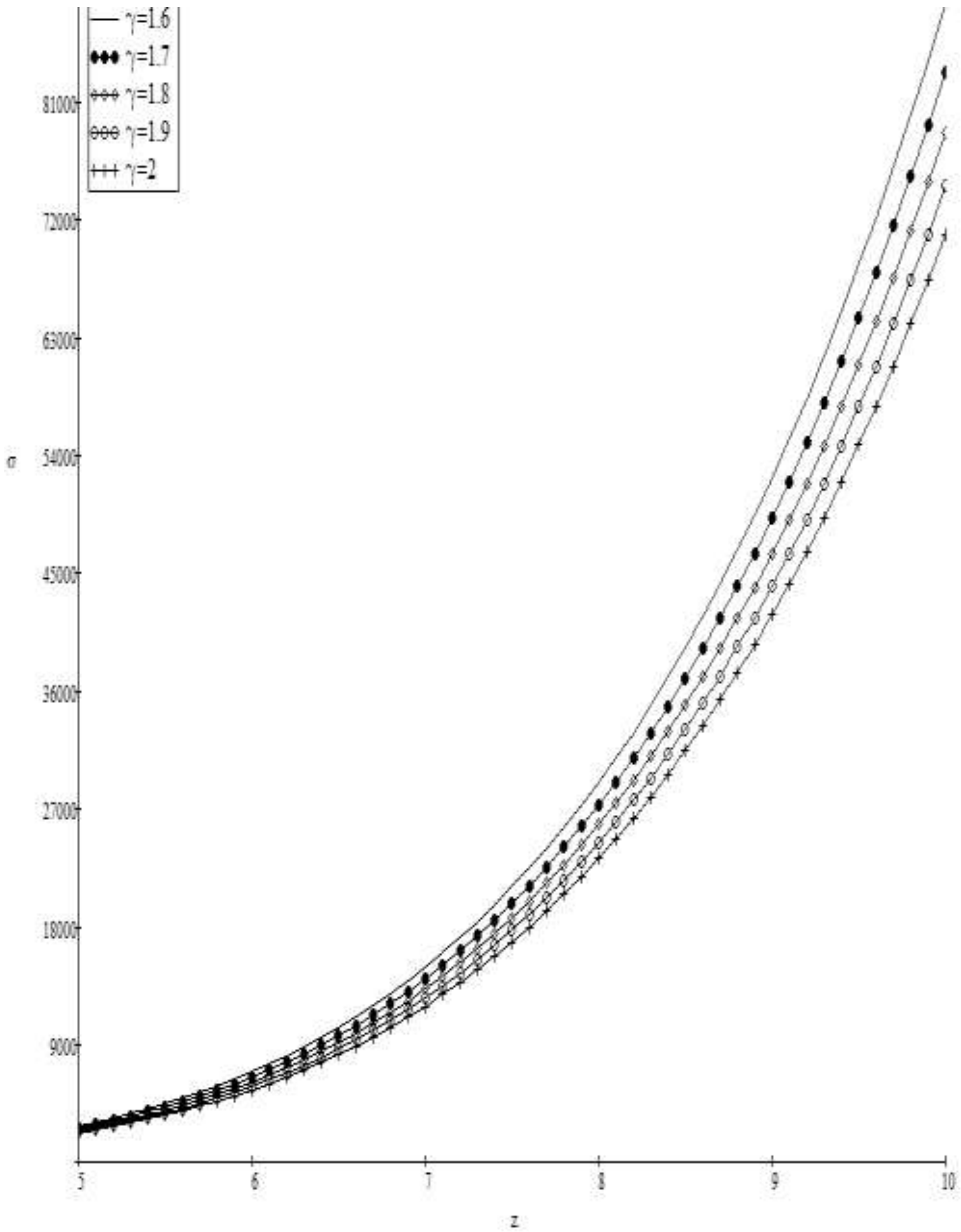
fig(4):arelation between (σ) and (z) when (z=20...50) for varous values of (γ)



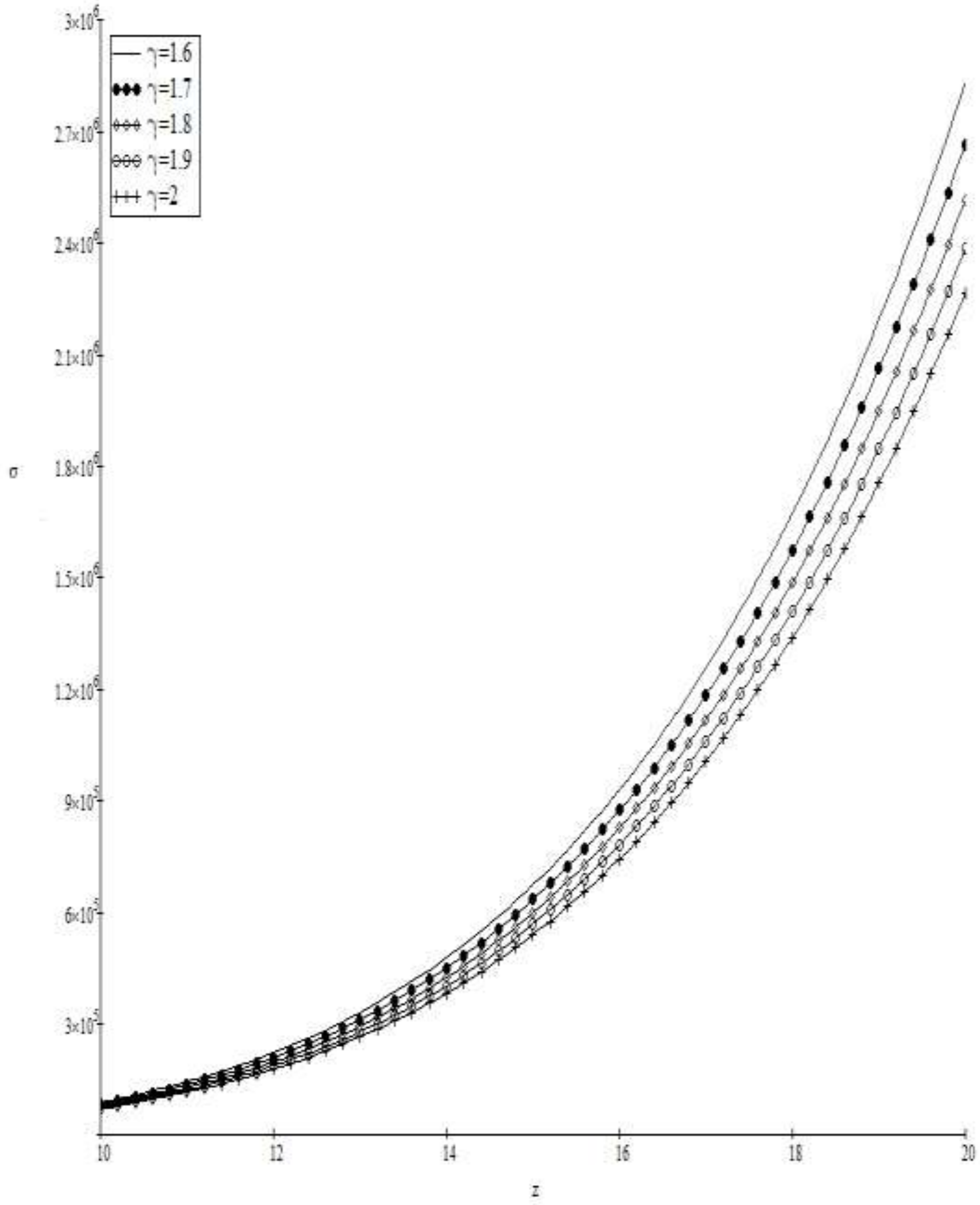
fig(5):arelation between (σ) and (z) when (z=50...100) for varous values of (γ)



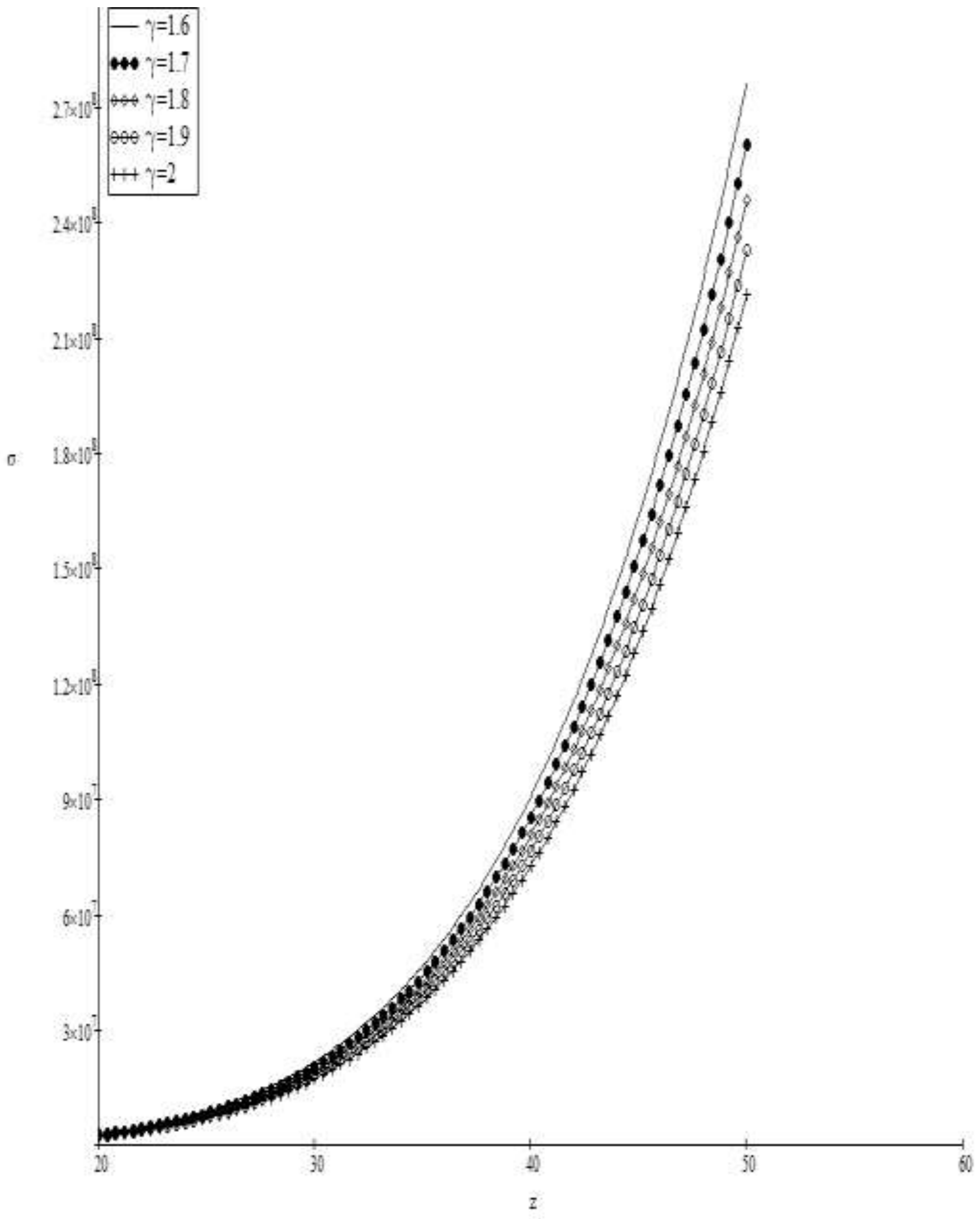
fig(6):arelation between (σ) and (z) when (z=1…5) for varous values of (γ)



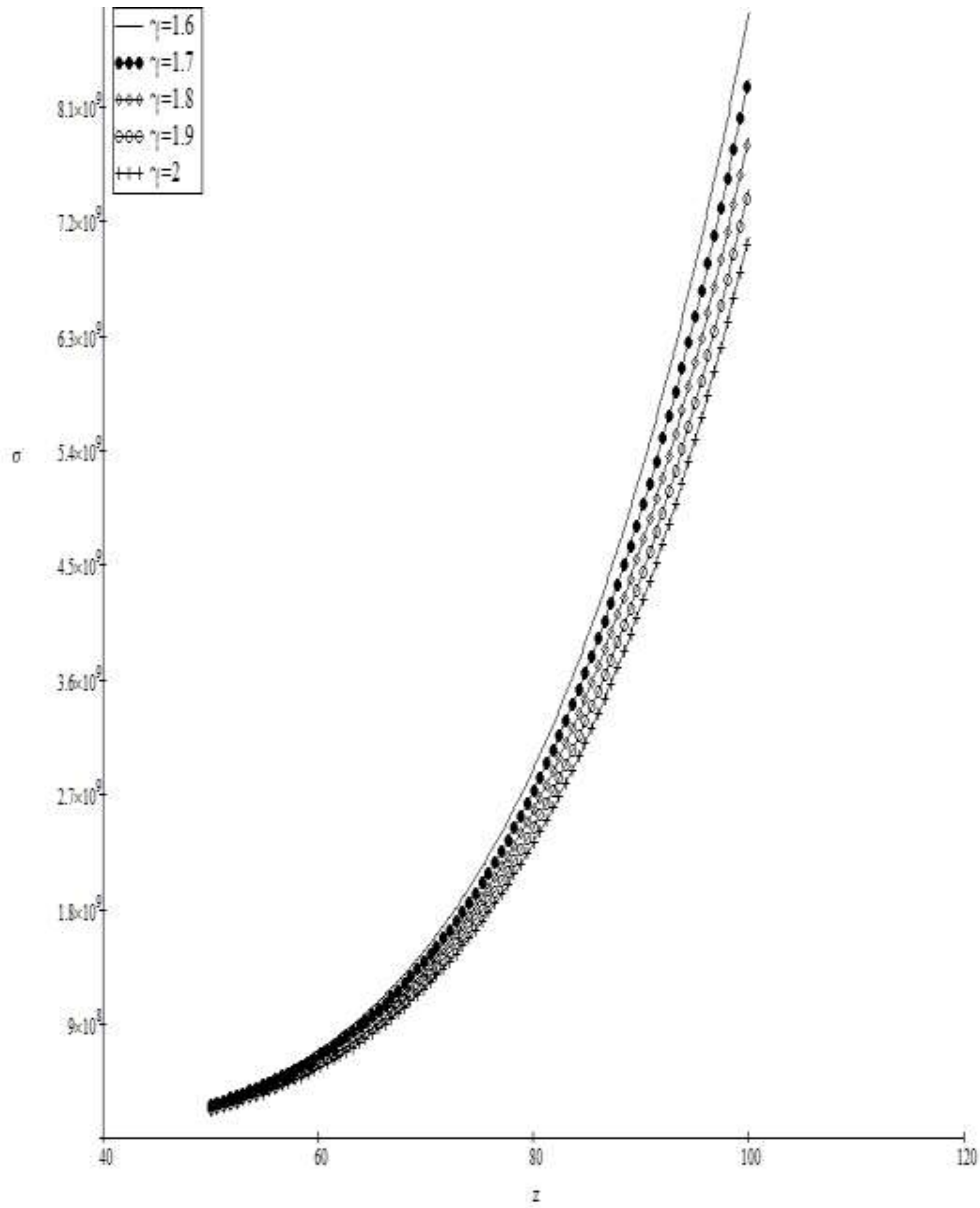
fig(7):arelation between (σ) and (z) when (z=5...10) for varous values of (γ)



fig(8):arelation between (σ) and (z) when (z=10...20) for varous values of (γ)



fig(9):arelation between (σ) and (z) when (z=20...50) for varous values of (γ)



fig(10):arelation between (σ) and (z) when (z=50...100) for varous values of (γ)

Conclusion;

1-We see in the case $I \ll \omega \ll mc^2$ i.e in the case of non relativistic limt eq(12) reduce to the well known relation for total cross section

$$\sigma = \frac{2^8\pi}{3}\alpha\, a_0^2 z^5 (\frac{I_0}{\hbar\omega})^{7/2}$$

and this is the thing which should be take place.

2-The empirical results in the tables have good concidense with that found from the equation (12).and this give confidence to the equation (12)

from (12) we see that when the velocity of electron become more i.e higher voltage for the emission electrons (σ) become more less and this is ordinary thing because less number of electrons can take heigh energy and this also true statistically.

From (12) we see larger velocity (v) for electrons make σ more less i.e there are less electrons can take this velocity and this agree with the statistical law's .

3-When the velocity of electrons become more less than the velocity of light we see that eq (12) give the classical relation for (σ)and this gives confidence to eq-(12).

we found that there is agood concidense of this values given by eq (12) with international values of (σ) given by [8],

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