

ARIMA

**

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ARIMA (2,1,4)₈

ARIMA(P,D,Q)_s

(2001-1971)

.AIC (K) MSE

: (Matlab)

Holt-Winters' Multiplicative Method, Winters' Three Parameters Exponential Smoothing Method, Holt-Winters' additive Method

**The Prediction of Seasonal ARIMA Model
by using Exponential Smoothing Methods with Application**

ABSTRACT

The reconciliation of one of the time series models with ARIMA (P, D, Q)_s seasonal to the average humidity in Mosul (1971-2001) has been carried out in this study.

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Prediction It is reached to the best Models in the proportional ARIMA (2, 1, 4)₈, because it has the less value for statistical standard AIC (k), MSE.

The current study, also arrived at to the reconciliation of seasonal time series models after performance of introduction for series by making Programs (MATLAB) for it. The method adopted is Winters' Three Parameters Exponential (Smoothing) method, method of Holt-Winters Multiplicative and Holt-Winters Additive.

The third method chosen is the best model of prediction for every method depending on the same statistic.

Finally, in this paper the best model is reconciliated for the introduction methods after performance, the difference for every method and the choice of the best prediction of every method depending on the same statistical methods.

:

(2005)

(Smoothing Data)

(2003)

:

Holt-Winters' Multiplicative :
 Method, Winters' Three- Parameters Exponential Smoothing
 Method, Holt-Winters' additive Method

:

Fourier

(1807)

(Cosine)

(Sine)

(Fourier Series)

(1906) (Schuster Stocks)

.(1922) (Beveridge)

(Makridaskis, S, et.al)

(1998)

Michael I.,Blake

. (Smoothing Fiter for Condensation)

(Celia F.,Balaji V.,Les S.,Asish Winters (2002) G.and Amar R.)

(Simon,2003) (2003)

Celia F.,Balaji V.,Les S.,Asish) (2004) (G.and Amar R.)

MATLAB

:
: _____

	T		t
	t	$x(t)$	$\{x(t), t \in T\}$
$\{x(t), -\infty \leq t \leq \infty\}$	"	"	
	$t = 0, \mp 1, \mp 2, \dots$		t $x(t)$
$\{x(t)\}$	$\{x(t), t = 0 \mp 1, \mp 2, \dots\}$	"	

Stationary Time Series

:___

$$\begin{aligned} & [t, t+h] \\ & [s, s+h] \end{aligned}$$

.Stationary

:

$$E(Y_t) = \mu \quad (1)$$

$$Var(Y_t) = \sigma^2 \quad (2)$$

$$Y_{t+k}, Y_t \quad (3)$$

$$\gamma_k = E[(Y_t - \mu)(Y_{t+k} - \mu)] \quad k$$

$$k = 1, 2, \dots, m \quad k$$

$$\chi^2_{(m-1)} = n \sum_{k=1}^m \rho_k^2 \quad \dots(1)$$

$$(n/2)$$

: m

Seasonal Time Series Models

:___

Seasonal

S

(4,8,12,...)

(1992).

Seasonal Auto Regressive

:
(1
Model

Yule

Walker AR(P)

1926

1931

: P

$$Y_t = C + \varphi_S Y_{t-S} + \varphi_{2S} Y_{t-2S} + \dots + \varphi_{PS} Y_{t-PS} + a_t \quad \dots (2)$$

S:

(2

Seasonal Moving Average Model

Stutzky (1937)

(q)

MA(q)

MA(q)

:

Q

$$Y_t = C + a_t - \theta_s a_{t-s} - \theta_{2s} a_{t-2s} - \dots - \theta_{Qs} a_{t-Qs} \quad \dots (3)$$

(3

Seasonal Auto Regressive Moving Average Model

1938

Wold

Auto-Regressive Mixed Moving Average Models (ARMA)

:

ARMA(p,q)

$$Y_t = C + \phi_s Y_{t-s} + \dots + \phi_{ps} Y_{t-ps} + a_t - \theta_s a_{t-s} - \dots - \theta_{qs} a_{t-qs} \quad \dots (4)$$

:

(4)

The Seasonal Integrated Auto Regressive Moving Average Model

(P,D,Q)_s

ARIMA

$$W_t = Y_t - Y_{t-s}$$

:

$$\phi_P (B^s) (1 - B)^D Y_t = \theta_Q (B^s) a_t \quad \dots (5)$$

:

$$\begin{aligned} \phi_P (B^s) &= 1 - \phi_s B^s - \phi_{2s} B^{2s} - \dots - \phi_{ps} B^{ps} \\ \theta_Q (B^s) &= 1 - \theta_s B^s - \theta_{2s} B^{2s} - \dots - \theta_{qs} B^{qs} \end{aligned}$$

. S

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...

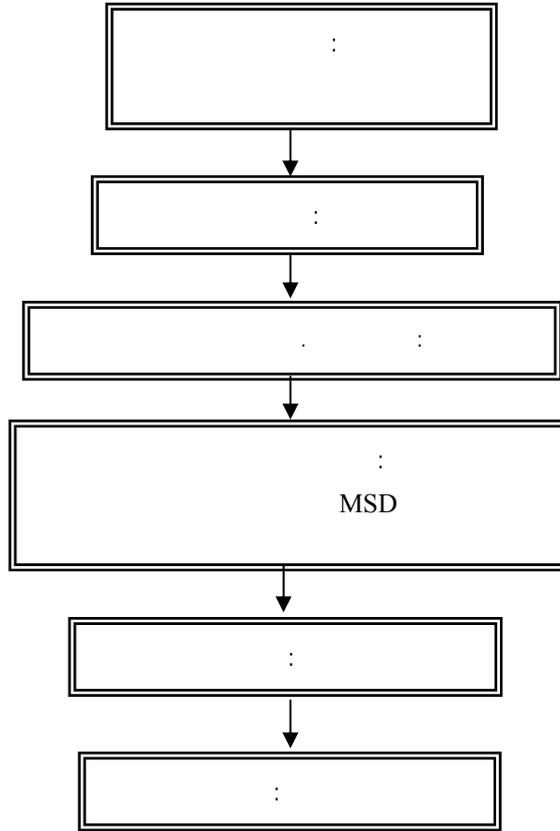
Pegels(1969)

(2002)

Hyndman

(Tylor,2003)

-:



:(1)

:

:_

Exponential Smoothing for Seasonal time Series

Winter's Three Parameter Exponential Smoothing Model

(-)
 : (-)
 $F_{t+M} = (l_t + b_t)S_{t-S+M}$... (6)
 Multiplicative Seasonal) (6)

.(Model

:

: l_t

: b_t

: M

: S_{t-S+M}

: F_{t+M}

:

Seasonal adjustment factor

:

$$l_t = \alpha(Y_t/S_{t-S}) + (1 - \alpha)(l_{t-1} + b_{t-1}) \quad \dots (7)$$

$$b_t = \beta(l_t - l_{t-1}) + (1 - \beta)b_{t-1} \quad \dots (8)$$

l_{t-1}, l_t
 Y_t () Period-to-Period

$$S_t = \gamma(Y_t/l_t) + (1 - \gamma)S_{t-S} \quad \dots (9)$$

$$F_{t+M} = (l_t + b_{t,M})S_{t-S+M} \quad \dots (10)$$

	: F_{t+M}	: l_t
. t		: Y_t
. $(0 < \alpha < 1)$	l_t	: α
. t		: l_t
. $(0 < \beta < 1)$	b_t	: β
. t		: b_t
. S ()		: S_{t-S}
(52- 12-)		: S
. $(0 < \gamma < 1)$ t		: γ
. t		: S_t
. F_{t+M}		: M

:

l_t

:

S_t b_t (Trend)

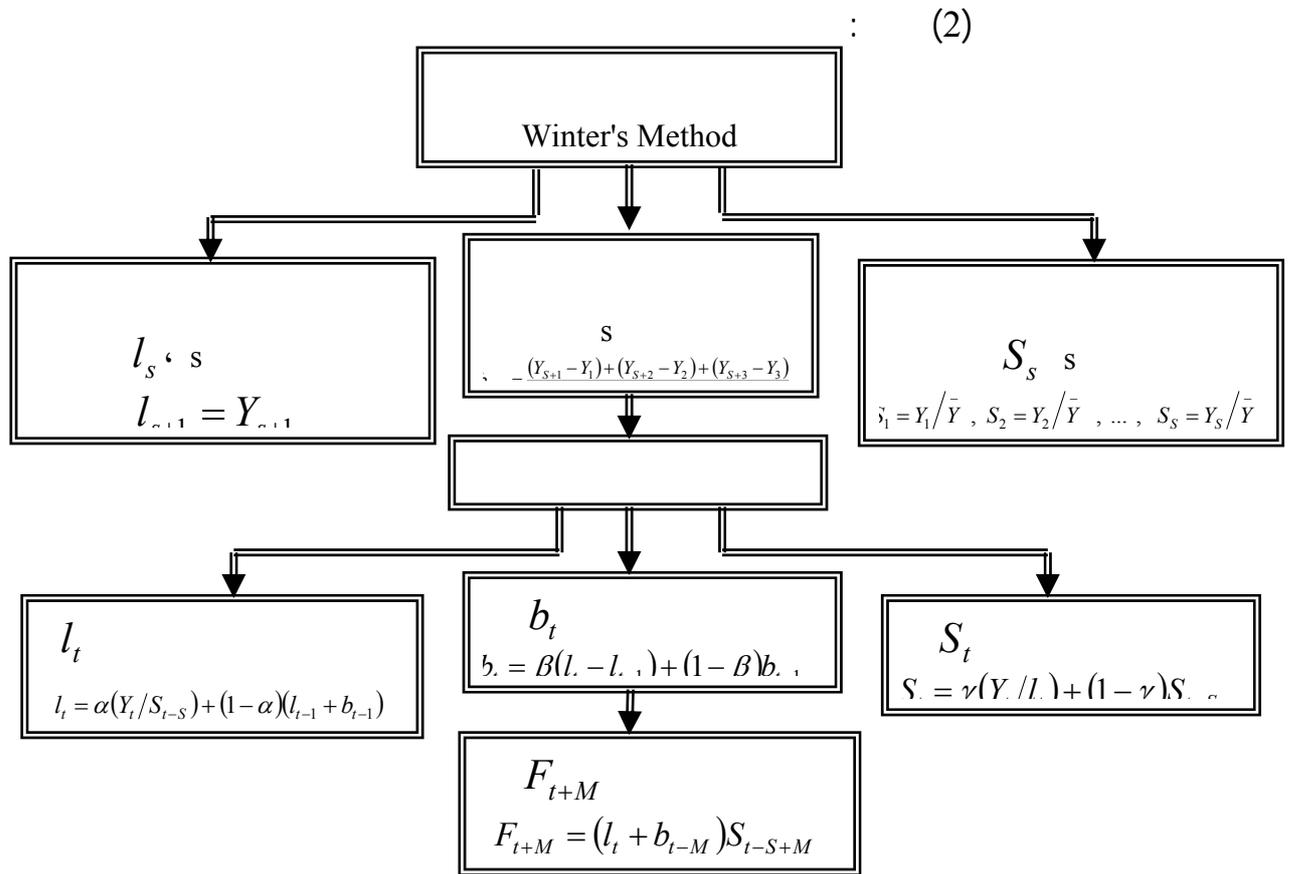
(Makridakis, et al., 1983)

$$l_{s+1} = Y_{s+1} \quad \dots (11)$$

$$S_1 = Y_1 / \bar{Y}, S_2 = Y_2 / \bar{Y}, \dots, S_S = Y_S / \bar{Y} \quad \dots (12)$$

$$\bar{Y} = \frac{\sum_{i=1}^S Y_i}{S}$$

$$b_{s+1} = \frac{(Y_{s+1} - Y_1) + (Y_{s+2} - Y_2) + (Y_{s+3} - Y_3)}{3(S)} \quad \dots (13)$$



(2)

Winters Three Parameter Exponential Smoothing Model

Holt- Winter's Multiplicative Seasonality Method

Holt's Linear

Winters(1960)

Holt

Holt-Winters

Holt's

Holt-

()

Winter's

(Tylor,2003) .Multiplicative()

Additive

Holt-Winter's Multiplicative

b_t

l_t

:

S_t

Seasonal adjustment
Factor

$$\frac{Y_t}{S_{t-s}} + (1-\alpha)(l_{t-1} + b_{t-1})$$

... (14)

S_{t-s}

S_{t-s}

Y_t

$t - s$

Y_t

$t - s$

S_{t-s}

s

l_t

S_t

Holt-Winter's Multiplicative

Holt-Winters Multiplicative

S_t b_t (Trend) l_t (Level)
(s period)

Makridakis,et) : S

(al.,1998

$$l_S = \frac{1}{S} (Y_1 + Y_2 + \dots + Y_S) \quad \dots (18)$$

2S)

(Period

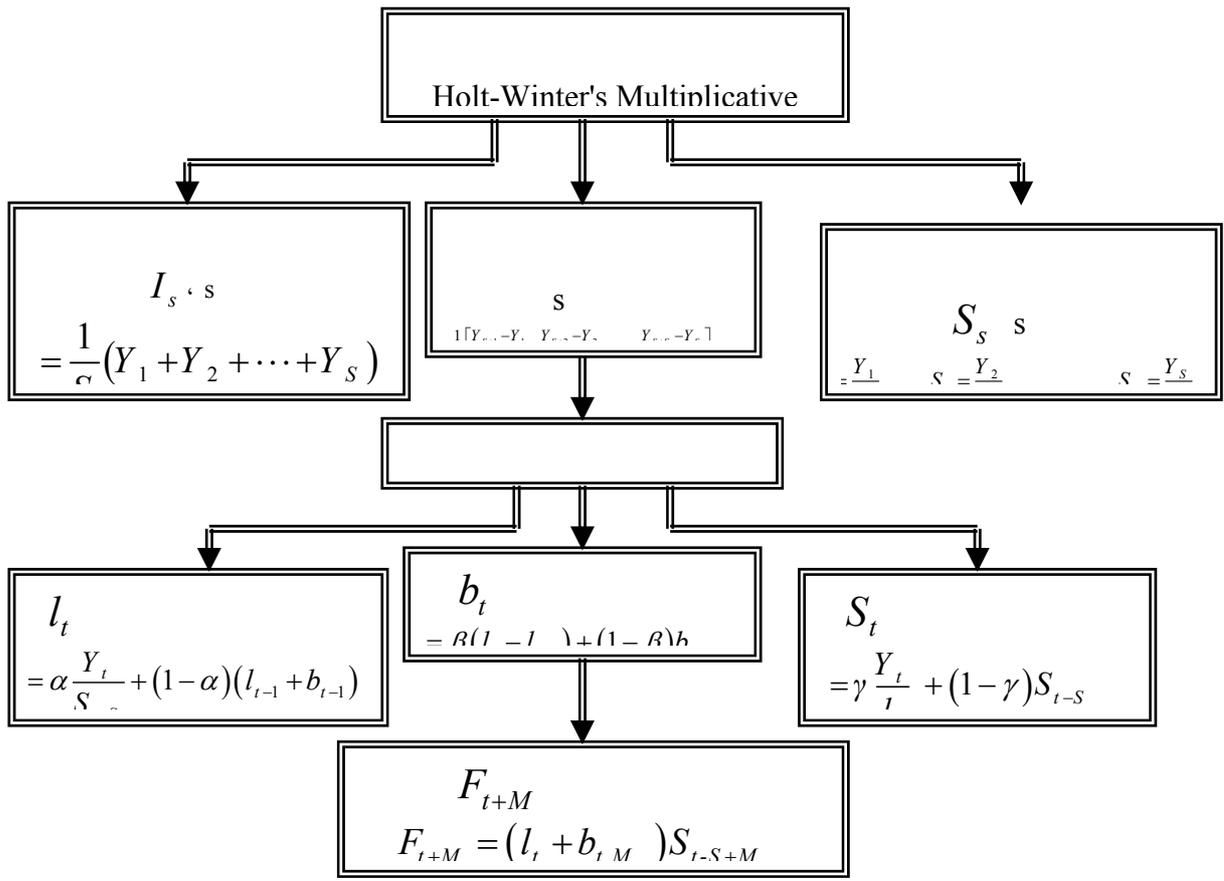
$$b_S = \frac{1}{S} \left[\frac{Y_{S+1} - Y_1}{S} + \frac{Y_{S+2} - Y_2}{S} + \dots + \frac{Y_{S+S} - Y_S}{S} \right] \quad \dots (19)$$

S b_S

$$S_1 = \frac{Y_1}{L_S}, \quad S_2 = \frac{Y_2}{L_S}, \quad \dots, \quad S_S = \frac{Y_S}{L_S} \quad \dots (20)$$

(A) Matlab

: (3)



: (3)

Holt-Winter's Multiplicative Method

Holt-Winter's Additive

Seasonality

Seasonal adjustment Factor $l_t = \alpha(Y_t - S_{t-S}) + (1 - \alpha)(l_{t-1} + b_{t-1})$... (21)

Trend $b_t = \beta(l_t - l_{t-1}) + (1 - \beta)b_{t-1}$... (22)

$$\text{Seasonal } S_t = \gamma(Y_t - l_t) + (1-\gamma)S_{t-S} \quad \dots(23)$$

$$\text{Forecast } F_{t+M} = l_t + b_{tM} + S_{t-S+M} \quad \dots(24)$$

(15)

(22)

Holt-Winter's additive

$$b_s \quad l_s$$

(Makridakis, et al.,1998) : (19) (18)

$$l_s = \frac{1}{S}(Y_1 + Y_2 + Y_3 + \dots + Y_S)$$

$$b_s = \frac{1}{S} \left[\frac{Y_{S+1} - Y_1}{S} + \frac{Y_{S+2} - Y_2}{S} + \dots + \frac{Y_{S+S} - Y_S}{S} \right]$$

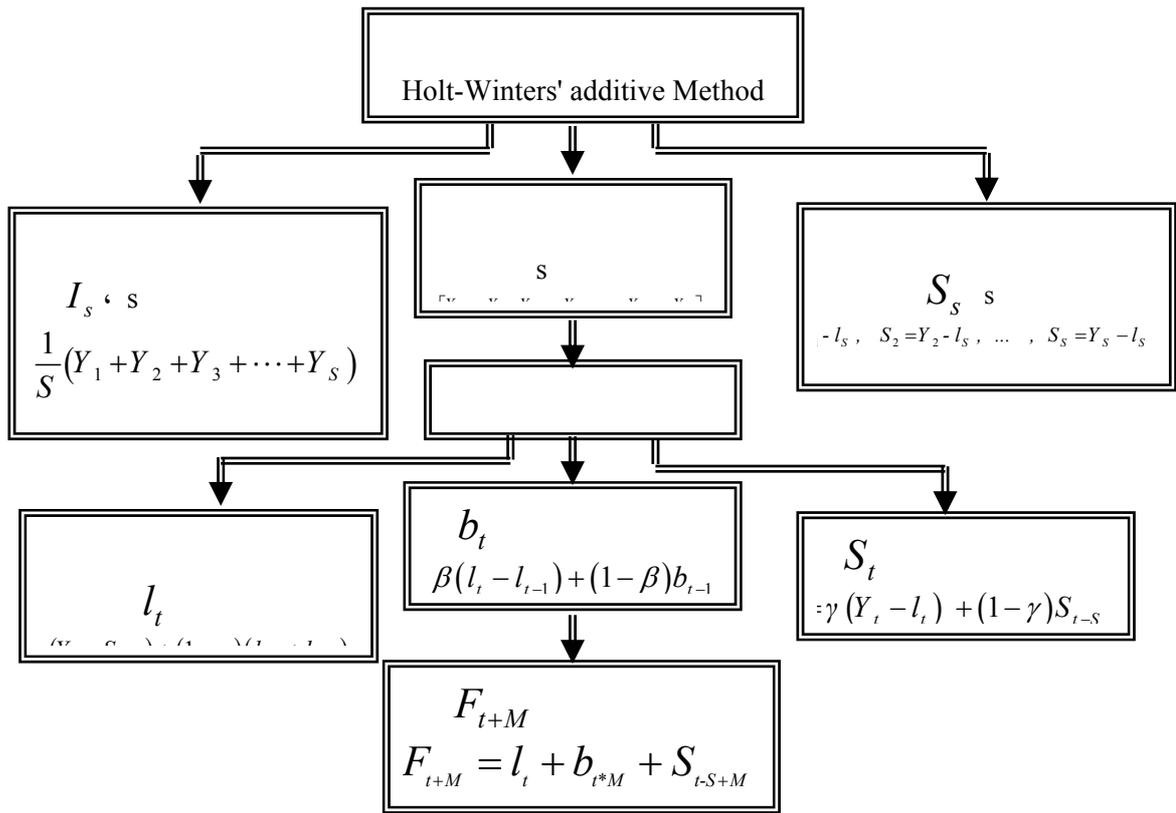
(Tylor,2003) :

$$S_1 = Y_1 - l_s, \quad S_2 = Y_2 - l_s, \quad \dots, \quad S_S = Y_S - l_s \quad \dots(25)$$

(A)

Matlab

.(4)



:(4)

Algorithm of Holt-Winters' additive

:
: .1

(Y)

(2001-1971)

.(5)

(5)

$$\chi^2 = 2439.336 \quad (10)$$

.(6)

(8)

: **SARIMA(P,D,Q)** .2

Box-Jenkins

: **Akaike's Information Criteria (AIC)** -

(k)

[(Akaike (1973,1974)]

Akaike's Information) (AIC)

: (Criteria

$$AIC(k) = n \ln \sigma_{\varepsilon}^2 + 2k \quad \dots (26)$$

n σ_{ε}^2 : (k)

AIC(k)

.(2003)

:(MSE) -

$$MSE = \frac{\sum_{t=1}^n (Y_t - \hat{Y}_t)^2}{n - (k + 1)} \dots (27)$$

: Y_t . : k . : n :
 (MSE) : \hat{Y}_t

.3

ARIMA(P,D,Q)_s

MSE AIC(K)

ARIMA(2,1,4)_s (1)

) MSE AIC(K)

: (28) (Minitab 13

$$Y = -0.51002 - 1.5350Y_{t-8} - 0.9510Y_{t-16} + a_t + 0.6350a_{t-8} - 0.5200a_{t-16} - 0.9463a_{t-24} - 0.1136a_{t-32} \dots (28)$$

ARIMA(P,D,Q)_s MSE AIC(K) :(1)

MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)
ARIMA(1,1,1)s	58.1	978.91	ARIMA(1,2,2)s	80.1	1057.98	ARIMA(3,1,3)s	57.2	983.17
ARIMA(1,1,2)s	58.2	981.33	ARIMA(1,2,3)s	63.4	1003.87	ARIMA(3,1,4)s	57.1	984.75
ARIMA(1,1,3)s	56.7	977.06	ARIMA(1,2,4)s	63.8	1007.38	ARIMA(4,1,1)s	58.0	984.50
ARIMA(1,1,4)s	58.2	985.33	ARIMA(1,2,5)s	64.4	1011.62	ARIMA(4,1,2)s	57.4	985.01
ARIMA(1,1,5)s	58.1	986.91	ARIMA(2,2,1)s	76.5	1046.94	ARIMA(4,1,3)s	56.0	980.08
ARIMA(2,1,1)s	58.3	981.74	ARIMA(2,2,2)s	72.2	1035.06	ARIMA(4,1,4)s	55.6	980.36
ARIMA(2,1,2)s	58.5	984.56	ARIMA(2,2,3)s	62.9	1003.97	ARIMA(4,1,5)s	56.5	986.21
ARIMA(2,1,3)s	56.9	979.91	ARIMA(2,2,4)s	59.6	993.03	ARIMA(5,1,1)s	58.8	989.97
ARIMA(2,1,4)s	55.7	976.79	ARIMA(2,2,5)s	56.9	983.91	ARIMA(5,1,2)s	62.2	1005.28
ARIMA(2,1,5)s	58.1	988.91	ARIMA(3,2,1)s	81.1	1062.96	ARIMA(5,1,3)s	58.1	990.91
ARIMA(3,1,1)s	58.4	984.15	ARIMA(3,2,2)s	70.6	1031.68	ARIMA(5,1,4)s	55.9	983.65
ARIMA(3,1,2)s	57.0	980.33	ARIMA(3,2,3)s	62.9	1005.97	ARIMA(1,2,1)s	85.8	1072.48
ARIMA(3,2,4)s	60.2	997.44	ARIMA(4,2,2)s	71.4	1036.39	ARIMA(4,2,5)s	60.2	1001.44
ARIMA(3,2,5)s	61.8	1005.73	ARIMA(4,2,3)s	77.0	1056.51	ARIMA(5,2,1)s	68.3	1025.73
ARIMA(4,2,1)s	109.6	1137.24	ARIMA(4,2,4)s	60.5	1000.63	ARIMA(5,2,2)s	67.4	1024.55
ARIMA(5,2,3)s	64.8	1017.11	ARIMA(5,2,4)s	56.4	985.79			

:

.4

(α, β, γ)

(MSD)

(1028)

(MSD)

(MSD)

(MSD)

(α, β, γ)

Mean Squared Deviation $MSD = \frac{\sum_{t=1}^n (Y_t - \hat{Y}_t)^2}{n}$

(\hat{Y}_t)

(Y_t)

(n)

.MSD

:(2)

MSD				
	γ	β	α	
61.7601	0.1	0.02	0.5	Winters Method
70.1447	0.2	0.01	0.2	Holt-Winters Multiplicative
65.2238	0.2	0.1	0.2	Holt-Winters Additive

:

() :__

Winter's Three Parameter Exponential Smoothing Model

$$\begin{aligned}
 (8) \quad & b_t & (7) \quad & (I_t) \\
 & (F_{t+M}) & (9) \quad & (S_t) \\
 & & & (M = 1) \quad (10)
 \end{aligned}$$

MSD

$$(7) \quad \alpha = 0.5, \beta = 0.02, \gamma = 0.1 \quad (22)$$

ARIMA(P,D,Q)_s 1-1

MSE AIC(K)

ARIMA(4,0,0)_s (3)

(MSE) (AIC(k))

(Minitab)

: (29)

$$Y_t = 0.6482 + 0.3598 Y_{t-8} + 0.2946 Y_{t-16} + 0.1123 Y_{t-24} + 0.2194 Y_{t-32} + a_t \quad \dots (29)$$

MSE AIC(k) : (3)

ARIMA(P,D,Q)_s

MODEL	MSE	AIC(k)
ARIMA(0,0,1) _s	126.7	1164.03
ARIMA(0,0,2) _s	87.8	1078.01
ARIMA(0,0,3) _s	76.2	1046.006
ARIMA(0,0,4) _s	67.8	1019.97
ARIMA(0,0,5) _s	60.6	995.03
ARIMA(1,0,0) _s	57.2	973.17
ARIMA(2,0,0) _s	45.8	921.82
ARIMA(3,0,0) _s	44.5	916.91
ARIMA(4,0,0) _s	43.1	911.24
ARIMA(5,0,0) _s	43.2	913.80

First Difference

2-1

Smoothing

(MSE) (AIC(k))

ARIMA(3,1,5)_s

$$Y_t = -0.55366 - 1.7224 Y_{t-8} - 0.6009 Y_{t-16} + 0.1315 Y_{t-24} + a_t + 0.157 a_{t-8} - 0.8006 a_{t-16} - 1.0088 a_{t-24} + 0.144 a_{t-32} + 0.0205 a_{t-40} \quad \dots (30)$$

-: (4)

$$ARIMA(P,D,Q)_s \quad (MSE) \quad (AIC(k)) \quad : (4)$$

.Winters First Difference

MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)
ARIMA(0,1,1)s	41.72	897.43	ARIMA(5,1,2)s	38.92	892.76	ARIMA(2,1,5)s	37.59	884.41
ARIMA(0,1,2)s	40.93	894.84	ARIMA(5,1,3)s	37.75	887.43	ARIMA(3,1,1)s	43.62	914.12
ARIMA(0,1,3)s	39.82	890.24	ARIMA(1,2,1)s	50.2	943.84	ARIMA(3,1,2)s	38.49	886.09
ARIMA(0,1,4)s	39.19	888.42	ARIMA(1,2,2)s	48.6	938.06	ARIMA(3,1,3)s	39.25	892.78
ARIMA(0,1,5)s	40.14	896.16	ARIMA(1,2,3)s	41.45	901.87	ARIMA(3,1,4)s	38.19	888.21
ARIMA(0,2,1)s	65.1	1004.22	ARIMA(1,2,4)s	40.77	899.90	ARIMA(3,1,5)s	37.17	883.72
ARIMA(0,2,2)s	48.7	936.56	ARIMA(1,2,5)s	41.32	905.12	ARIMA(4,1,1)s	43.16	913.57
ARIMA(0,2,3)s	47.7	933.58	ARIMA(2,2,1)s	93.3	1094.59	ARIMA(4,1,2)s	38.8	890.02
ARIMA(0,2,4)s	48.4	939.07	ARIMA(2,2,2)s	47.9	936.58	ARIMA(4,1,3)s	40.77	903.90
ARIMA(0,2,5)s	48.6	942.06	ARIMA(2,2,3)s	40.55	898.60	ARIMA(4,1,4)s	38.07	889.46
ARIMA(1,1,1)s	40.77	893.90	ARIMA(2,2,4)s	40.51	900.37	ARIMA(2,1,5)s	37.59	884.41
ARIMA(1,1,2)s	40.9	896.67	ARIMA(2,2,5)s	39.51	896.37	ARIMA(4,2,3)s	43.87	921.49
ARIMA(1,1,3)s	39.23	888.66	ARIMA(3,2,1)s	67.8	1019.97	ARIMA(4,2,4)s	40.07	901.75
ARIMA(1,1,4)s	39.53	892.49	ARIMA(3,2,2)s	45.04	923.81	ARIMA(4,2,5)s	42.76	919.34
ARIMA(1,1,5)s	38.03	885.21	ARIMA(3,2,3)s	40.03	897.51	ARIMA(5,2,1)s	45.91	930.40
ARIMA(2,1,1)s	46.3	926.43	ARIMA(3,2,4)s	43.92	921.76	ARIMA(5,2,2)s	42.82	915.68
ARIMA(2,1,2)s	38.9	886.63	ARIMA(3,2,5)s	37.99	888.95	ARIMA(5,2,3)s	42.87	917.96
ARIMA(2,1,3)s	38.4	885.53	ARIMA(4,2,1)s	45.59	926.72	ARIMA(5,2,4)s	40.03	903.51
ARIMA(2,1,4)s	39.35	893.39	ARIMA(4,2,2)s	43.87	919.49	ARIMA(5,1,1)s	40.05	897.63
ARIMA(4,1,5)s	37.71	889.18	ARIMA(4,2,3)s	43.87	921.49			

: - :_

Holt-Winters Multiplicative Seasonality

$$(15) \quad b_t \quad (14) \quad (I_t)$$

$$(16) \quad (S_t)$$

$$(S_s) \quad (b_s) \quad (I_s)$$

$$(20) \quad (19) \quad (18)$$

$$M = 1 \quad (17) \quad (F_{t+M})$$

$$(1) \quad B \quad (Matlab)$$

$$\gamma = 0.2, \beta = 0.01, \alpha = 0.2$$

(8) (- -)

.(22)

ARIMA(P,D,Q)_s

1-2

(AIC(k))

(5)

(MSE)

ARIMA(0,0,5)_s

: (Minitab 13)

$$Y_t = 56.645 - 0.6634a_{t-8} - 0.8878a_{t-16} - 0.5644a_{t-24} - 0.9652a_{t-32} - 0.1817a_{t-40} + a_t \quad \dots (31)$$

ARIMA(P,D,Q)_s AIC(k) : (5)

Holt-Winters Multiplicative

MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)
ARIMA(0,0,1)s	185	1254.88	ARIMA(1,0,4)s	106.6	1130.58	ARIMA(2,0,4)s	134.5	1188.37
ARIMA(0,0,2)s	86.2	1073.60	ARIMA(1,0,5)s	82	1069.61	ARIMA(2,0,5)s	94.8	1106.42
ARIMA(0,0,3)s	145.1	1200.58	ARIMA(2,0,0)s	124.7	1162.21	ARIMA(3,0,0)s	126.6	1167.84
ARIMA(0,0,4)s	66.1	1013.88	ARIMA(2,0,1)s	127.5	1169.54	ARIMA(3,0,1)s	125.5	1167.75
ARIMA(0,0,5)s	61.7	999.34	ARIMA(2,0,2)s	135.8	1186.68	ARIMA(3,0,4)s	147.2	1212.03
ARIMA(1,0,0)s	155.7	1213.50	ARIMA(4,0,3)s	137.3	1195.32	ARIMA(3,0,5)s	103.3	1129.03
ARIMA(1,0,1)s	152.2	1210.04	ARIMA(4,0,5)s	115.7	1158.24	ARIMA(4,0,0)s	134.7	1184.73
ARIMA(1,0,2)s	116	1146.86	ARIMA(5,0,0)s	114.2	1147.10	ARIMA(4,0,1)s	135.7	1188.50
ARIMA(1,0,3)s	106.3	1127.90	ARIMA(2,0,3)s	131.4	1180.77	ARIMA(4,0,2)s	126.6	1173.84

Difference

2-2

Holt-Winters Multiplicative

ARIMA(0,2,2)_s

(MSE)

(881.21)

AIC(k)

MSE AIC(k)

(3)

(38.67)

:

$$Y_t = -0.32990 - 1.4635a_{t-8} + 0.4952a_{t-16} + a_t \quad \dots (32)$$

ARIMA(P,D,Q)_s AIC(k) : (6)

Second Difference

MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)
ARIMA(0,1,1)s	114.1	1138.89	ARIMA(5,1,2)s	120.2	1163.39	ARIMA(2,1,2)s	117.7	1152.35
ARIMA(0,1,2)s	108.6	1129.04	ARIMA(5,1,4)s	119.7	1166.39	ARIMA(2,1,3)s	119.9	1158.79
ARIMA(0,1,3)s	108.8	1131.48	ARIMA(1,2,1)s	77.3	1047.44	ARIMA(2,1,4)s	162.8	1234.20
ARIMA(0,1,4)s	107.1	1129.70	ARIMA(1,2,2)s	71.5	1030.72	ARIMA(2,1,5)s	99.3	1117.55
ARIMA(0,1,5)s	56.7	979.06	ARIMA(1,2,3)s	70.5	1029.34	ARIMA(3,1,1)s	113.5	1143.63
ARIMA(0,2,1)s	51.1	946.10	ARIMA(1,2,4)s	59.4	990.23	ARIMA(3,1,2)s	174.5	1248.86
ARIMA(0,2,2)s	38.67	881.21	ARIMA(1,2,5)s	70.8	1034.36	ARIMA(3,1,3)s	116.8	1154.51
ARIMA(0,2,3)s	49.8	943.92	ARIMA(2,2,1)s	166.2	1233.16	ARIMA(3,1,4)s	86.7	1084.98
ARIMA(0,2,4)s	43.07	911.07	ARIMA(2,2,2)s	133.5	1182.58	ARIMA(3,1,5)s	84.7	1081.38
ARIMA(0,2,5)s	80.6	1063.47	ARIMA(2,2,3)s	61.7	999.34	ARIMA(4,1,1)s	122.2	1163.35
ARIMA(1,1,1)s	113.2	1138.99	ARIMA(2,2,4)s	48.9	945.54	ARIMA(4,1,2)s	126.7	1174.03
ARIMA(1,1,2)s	90.9	1088.34	ARIMA(2,2,5)s	54.8	974.88	ARIMA(4,1,3)s	124.9	1172.60
ARIMA(1,1,3)s	130.2	1176.57	ARIMA(3,2,1)s	121.7	1160.37	ARIMA(4,1,4)s	127	1178.60
ARIMA(1,1,4)s	136.4	1189.74	ARIMA(3,2,2)s	111.1	1140.50	ARIMA(2,1,2)s	117.7	1152.35
ARIMA(1,1,5)s	53.8	968.46	ARIMA(3,2,3)s	65.8	1016.78	ARIMA(2,1,3)s	119.9	1158.79
ARIMA(2,1,1)s	112.6	1139.72	ARIMA(3,2,4)s	52.8	965.96	ARIMA(2,1,4)s	162.8	1234.20
ARIMA(2,1,5)s	99.3	1117.55	ARIMA(4,2,2)s	110	1140.11	ARIMA(4,2,5)s	71.7	1043.39
ARIMA(3,2,5)s	81.5	1072.14	ARIMA(4,2,3)s	131	1184.04	ARIMA(5,2,1)s	118.6	1158.18
ARIMA(4,2,1)s	125.2	1169.17	ARIMA(4,2,4)s	164.4	1240.55	ARIMA(5,2,2)s	130.6	1183.31
ARIMA(5,2,3)s	78.2	1062.22	ARIMA(4,1,5)s	104.2	1133.11	ARIMA(5,1,1)s	122.4	1165.75
ARIMA(5,2,4)s	90.2	1098.48						

Holt-Winters Additive

- :__

Seasonality:

Holt-Winters Multiplicative

$$\begin{aligned}
 & (22) \quad (b_s) \quad (21) \quad (I_s) \\
 & \text{Holt-Winters} \quad (23) \quad (S_t) \\
 & (b_s) \quad (I_s) \quad \text{Additive} \\
 & (20) (19) (18) \quad (S_s) \\
 & \quad \quad \quad (F_{t+M}) \\
 & \quad \quad \quad M = 1 \quad (24)
 \end{aligned}$$

(10) $\gamma = 0.2, \beta = 0.1, \alpha = 0.2$
 (MATLAB) MSD

(2) A
 MSE AIC(k) 1-2

(7) ARIMA(P,D,Q)_s

: ARIMA(0,0,4)_s

$$Y_t = 57.535 - 0.5010 a_{t-8} - 0.8417 a_{t-16} - 0.430 a_{t-24} - 0.8574 a_{t-32} + a_t \quad \dots (33)$$

ARIMA(P,D,Q)_s MSE AIC(k) : (7)

Holt-Winters Additive

MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)
ARIMA(0,0,1)s	191.2	1262.79	ARIMA(1,0,4)s	120.1	1159.19	ARIMA(4,0,1)s	143.5	1201.984
ARIMA(0,0,2)s	93.9	1094.13	ARIMA(1,0,5)s	98.5	1113.61	ARIMA(4,0,2)s	144.5	2401.17
ARIMA(0,0,3)s	144	1198.75	ARIMA(2,0,0)s	134.4	1180.19	ARIMA(5,0,0)s	124.1	2326.12
ARIMA(0,0,4)s	77.3	1051.44	ARIMA(2,0,1)s	135.3	1183.79	ARIMA(5,0,2)s	115.4	2295.23
ARIMA(0,0,5)s	78	1055.61	ARIMA(2,0,2)s	143.4	1199.75	ARIMA(5,0,3)s	121.5	2321.95
ARIMA(1,0,0)s	161.9	1222.87	ARIMA(3,0,1)s	125	1166.79	ARIMA(5,0,4)s	142	2398.79
ARIMA(1,0,1)s	158.9	1220.38	ARIMA(3,0,3)s	130.7	1181.49	ARIMA(2,0,3)s	141	2387.4
ARIMA(1,0,2)s	136.1	1185.21	ARIMA(3,0,4)s	127.3	1177.17	ARIMA(2,0,4)s	158.8	2446.46
ARIMA(1,0,3)s	127.2	1170.98	ARIMA(4,0,0)s	139.8	1193.65	ARIMA(2,0,5)s	108.9	2267.4
						ARIMA(3,0,0)s	134.4	2360.39

2-2

Holt-Winter's Additive Smoothing
 (MSE) (AIC(k))

(8) ARIMA(0,2,1)_s

: (34)

$$Y_t = -0.51556 - 0.9695 a_{t-8} + a_t \quad \dots (34)$$

Holt- (MSE) (AIC(k)) : (8)
Winter's Additive

MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)	MODEL	MSE	AIC(k)
ARIMA(0,1,1)s	131.8	2347.01	ARIMA(1,2,1)s	84.8	2137.34	ARIMA(3,1,1)s	118.5	2301.95
ARIMA(0,1,2)s	123	2315.84	ARIMA(1,2,2)s	66.8	2024.81	ARIMA(3,1,2)s	103.9	2240.84
ARIMA(0,1,3)s	122.3	2315.10	ARIMA(1,2,3)s	85.1	2143.03	ARIMA(3,1,3)s	121	2315.97
ARIMA(0,1,4)s	120.7	2310.78	ARIMA(1,2,4)s	77.7	2101.37	ARIMA(3,1,5)s	88.3	2168.75
ARIMA(0,1,5)s	78.9	2108.72	ARIMA(1,2,5)s	88.9	2168.005	ARIMA(3,1,1)s	118.5	2301.95
ARIMA(0,2,1)s	58.9	1960.40	ARIMA(2,2,1)s	181.2	2503.80	ARIMA(3,1,2)s	103.9	2240.84
ARIMA(0,2,2)s	61.2	1980.79	ARIMA(2,2,2)s	138	2375.08	ARIMA(3,1,3)s	121	2315.97
ARIMA(0,2,3)s	60	1973.28	ARIMA(2,2,3)s	103	2236.66	ARIMA(3,1,5)s	88.3	2168.75
ARIMA(0,2,4)s	61.3	1985.57	ARIMA(2,2,4)s	78.3	2107.06	ARIMA(4,1,1)s	134.5	2364.75
ARIMA(0,2,5)s	63.6	2005.25	ARIMA(2,2,5)s	66.7	2032.09	ARIMA(4,1,2)s	126.4	2336.93
ARIMA(1,1,1)s	125.7	2326.27	ARIMA(3,2,1)s	122.4	2317.50	ARIMA(4,1,3)s	126.3	2338.55
ARIMA(1,1,2)s	96.8	2202.87	ARIMA(3,2,2)s	123.4	2323.40	ARIMA(4,1,4)s	139.5	2388.27
ARIMA(1,1,3)s	182.4	2508.97	ARIMA(3,2,3)s	90.4	2176.03	ARIMA(4,1,5)s	101.1	2235.73
ARIMA(1,1,4)s	154.8	2432.22	ARIMA(3,2,4)s	101	2231.25	ARIMA(4,2,1)s	119.8	2309.19
ARIMA(1,1,5)s	78	2105.22	ARIMA(3,2,5)s	76.9	2102.40	ARIMA(4,2,2)s	135.9	2371.72
ARIMA(2,1,1)s	125.9	2329.03	ARIMA(5,1,1)s	130.1	2350.78	ARIMA(4,2,3)s	130	2352.41
ARIMA(2,1,2)s	125.5	2329.50	ARIMA(5,1,2)s	100.5	2228.87	ARIMA(4,2,4)s	100.8	2232.30
ARIMA(2,1,3)s	127.9	2340.59	ARIMA(5,1,3)s	92.6	2191.57	ARIMA(4,2,5)s	64.4	2019.25
ARIMA(2,1,4)s	105.8	2251.54	ARIMA(5,1,2)s	100.5	2228.87	ARIMA(5,2,3)s	118.2	2308.74
ARIMA(2,1,5)s	105.6	2252.63	ARIMA(5,2,2)s	118.8	2309.17	ARIMA(5,2,4)s	145.6	2410.81
ARIMA(5,2,1)s	120.7	2314.78						

: ARIMA(P,D,Q) .5

ARIMA(P,D,Q)_s

Holt-)

(Winters Multiplicative, Holt-Winters Additive, Winters-Method

ARIMA(P,D,Q)

Difference

: AIC(k)

ARIMA(P,D,Q) : (9)

	Difference	MSE	AIC(k)	Difference	MSE	AIC(k)
Holt-Winters Multiplicative	ARIMA(0,0,5) _s	61.7	1990.69	ARIMA(0,2,2) _s	38.67	1760.43
Holt-Winters Additive	ARIMA(0,0,4) _s	77.3	2096.89	ARIMA(0,2,1) _s	58.9	1960.40
Winters Method	ARIMA(4,0,0) _s	43.1	1816.49	ARIMA(3,1,5) _s	37.17	1753.44
				ARIMA(2,1,4) _s	55.7	1943.59

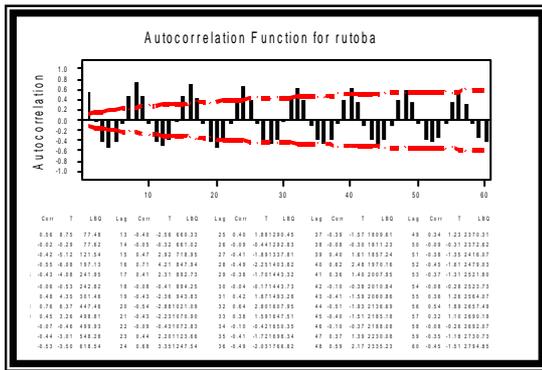
SARIMA(4,0,0)

(43.1) MSE (1816.49)

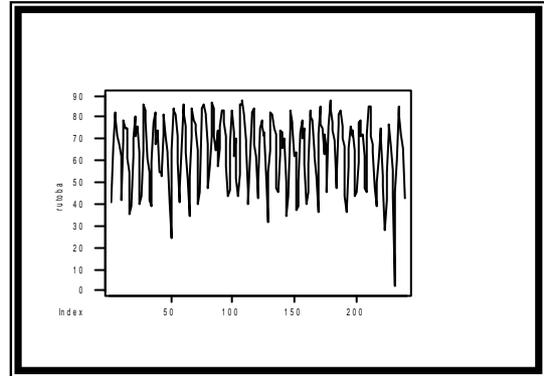
(9)

SARIMA(3,1,5)

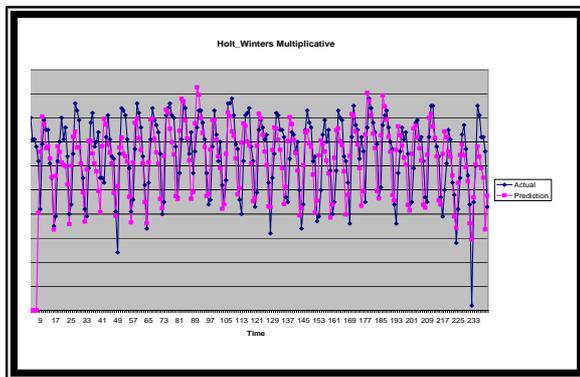
SARIMA(0,2,2)



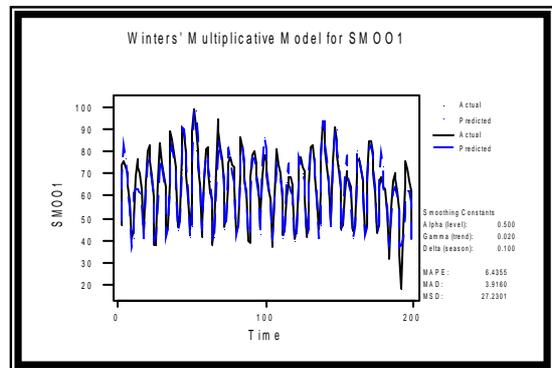
: (6)



: (5)



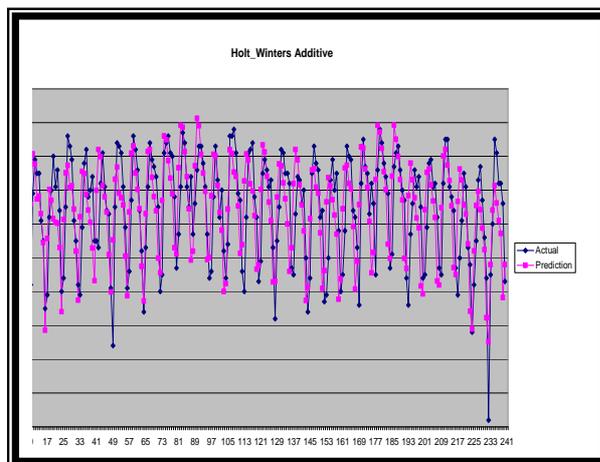
: (8)



: (7)

Holt-Winters
Multiplicative

Winter's
Three Parameter Exponential Smoothing
Model



: (9)

Holt-Winters
Additive

	:	.1
	:	.1
(8)	:	.2
	:	.3
	ARIMA(2,1,4) ₈	
	.(AIC(k)=1943.59) (MSE=55.7)	
Winters Three Parameters	:	.4
	Exponential Smoothing Model	
	ARIMA(4,0,0) ₈	-
	(MSE=43.1) (AIC(k)=1816.49)	
(3)	:	
	Winters-Method	-
	ARIMA(3,1,5) ₈	
()	(MSE=37.17) (AIC(k)=1753.44)	
	AIC(k)	
Holt-Winters Multiplicative Method	:	.5
Holt- Smoothing	:	-
Difference	Winters Multiplicative	

(MSE=61.7)	ARIMA(0,0,5) ₈	
	.(AIC(k)=1990.69)	
Difference	-	
ARIMA(0,2,2) ₈	Holt-Winters Multiplicative	
(AIC(k)=1760.43)	(MSE=38.67)	
	()	
	.	
Holt-Winters Additive Method		.6
		:
	ARIMA(0,0,4) ₈	-
	.(AIC(k)=2096.89) (MSE=77.3)	
Holt-Winters		-
	ARIMA(0,2,1) ₈	Additive
()	(AIC(k)=1960.40)	(MSE=58.9)
	.	
	.	
AIC(k),MSE		.7
	:	
	Winters Method	-
		.SAR(4)
Holt-Winters Multiplicative Method		-
	.SARIMA(0,2,2)	

: .2

.1

Winters Three .1

Holt-Winters .2 Parameter Exponential Smoothing Model

.Holt-Winters additive Method.3 Multiplicative

.2

Smooth Transition Exponential Smoothing

.3

MATLAB

.4

" : (2003)

.1

"(1989-1949)

.73

" (1992)

.2

" -

" (2005)

.3

"

4. Celia F., Balaji V. Les S. , Asish G.& Amar R., (2004) **"Afuzzy Foreca-sting Model for Women's Casual Sales"**, International Journal of Clothing Science and Technology 15(2), 107-125, (2004).
5. Makridaskis, S. Wheel Wright, S. and McGee, E. (1983) **"Forecasting: Methods and Applications"**, 2nd ed., John Wiley and Sons, New York, USA.

6. Makridaskis, S. Wheel Wright, S. and Hyndman, R. (1998) "**Forecasting: Methods and Applications**", 3rd ed., Jhon-Wiely and Sons, New York, USA.
7. Simon Shaw (2003) "**Exponential Smoothing Example**", s.c.shaw@maths.bath.ac.uk, 2003/04 semester II.
8. Taylor, J. W. (2003) "**Exponential Smoothing with a Damped Multiplicative Trend**", International Journal of Forecasting, Vol. 19, Pp. 715-725.

ملحق -A-

:(1)

```

%Holt -Winters multiplicative method
%Alpha,Beta and Gamma are smoothing constants
%S,b and L are initial values
%F(t+M) is prediction equation
%MSD1 is mean square division
clear
clc
kkk
s=8;
tall=240;
M=1;
S=zeros(s,1);
b=zeros(tall,1);
l=zeros(tall,1);
bs=0;
F=zeros(tall+M,1);
l(s)=sum(Y(1:s))/s;
    for i=1:s
        bs=bs+(Y(s+i)-Y(i))/s;
        S(i)=Y(i)/l(s);
    end
b(s)=bs/s;
F(s+1)=(l(s)+b(s))*S(M);
    Alpha1=0.2;
    Beta1=0.01;
    Gamma1=0.2;
    for t=s+1:tall
        l(t)=Alpha1*(Y(t)/S(t-s))+(1-Alpha1)*(l(t-1)+b(t-1));
        b(t)=Beta1*(l(t)-l(t-1))+(1-Beta1)*b(t-1);
        S(t)=Gamma1*(Y(t)/l(t))+(1-Gamma1)*S(t-s);
        F(t+M)=(l(t)+b(t)*M)*S(t-s+M);
    end
    MSD1=sum((Y(s+1:end)-F(s+1:end-M)).^2)/length(s+1:length(Y));
    disp([[1:length(F)]' F])

```

:(2)

```

%Holt-Winters Additive seasonal method
%Alpha, Beta and Gamma are smoothing constants
%S,b and L are initial values For combination
%F(t+M) is prediction equation
%MSD1 is mean square division
clear
clc
kkk
s=8;
tall=240;
M=1;
S=zeros(s,1);
b=zeros(tall,1);
l=zeros(tall,1);
bs=0;
F=zeros(tall+M,1);
l(s)=sum(Y(1:s))/s;
    for i=1:s
        bs=bs+(Y(s+i)-Y(i))/s;
        S(i)=Y(i)-L(s);
    end
b(s)=bs/s;
F(s+1)=l(s)+b(s)+S(M);
    Alpha1=0.2;
    Beta1=0.1;
    Gamma1=0.2;
    for t=s+1:tall
        l(t)=Alpha1*(Y(t)-S(t-s))+(1-Alpha1)*(l(t-1)+b(t-1));
        b(t)=Beta1*(l(t)-L(t-1))+(1-Beta1)*b(t-1);
        S(t)=Gamma1*(Y(t)-l(t))+(1-Gamma1)*S(t-s);
        F(t+M)=(l(t)+b(t)*M)+S(t-s+M);
    end
    MSD1=sum((Y(s+1:end)-F(s+1:end-M)).^2)/length(s+1:length(Y));
    disp([[1:length(F)]' F])

```