

## Decomposition of the Rotation Curves of Spiral Galaxies

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Received 29, April, 2012

Accepted 18, September, 2012

### Abstract:

Rotation curve of spiral galaxies manifests the distribution of surface mass density of spiral galaxies. In this paper the decomposition of the rotation curve of three spiral galaxies (NGC1365, NGC6946 and NGC5457) into a bulge, an exponential disk and Burkert dark halo respectively are presented. An addition contribution of local structure such as arms and bar can be included to suggest modified model. The results show that this modified model provides a good agreement with the observed rotation curve.

**Key words:** galaxies: disk - galaxies: bulge- galaxies: halo-galaxies: structure galaxies: rotation curve.

### Introduction:

Spiral galaxies are known to present an important mass discrepancy between their dynamical and visible masses. The commonly accepted hypothesis is to assume a more or less spherical halo of unseen matter in addition to the stars and gas. There were countless papers in recent years discussing the matter distribution in galaxies and, in particular, the contributions of both the stellar disc and the dark matter halo.

Bosma in 1981, Kent 1986 and Sofue in 1996 showed that the mass distribution has been obtained by decomposition of a rotation curve into several components such as a bulge, disk and dark halo [1, 2, 3].

S. Ninkovi´c in 1999 studied the rotation of a test spiral galaxy with two contributors the disc and the corona is considered. The disc is exponential, whereas the corona is the dark subsystem. For the latter several variants of mass distribution are considered [4].

It is well established that the luminosity profile of the spheroidal component in galaxies is represented

by the de Vaucouleurs ( $e^{-r^{1/4}}$  or  $r^{1/4}$  in 1953 and 1958) law with some modifications to  $r^n$  law including asymmetries [5, 6].

Fuchs B., Möllenhoff C., and Heidt J. in 2008 presented decompositions of the rotation curves of distant spiral galaxies into contributions due to their bulges, disks, and putative dark haloes. Galaxy models constrained in such a way show that the distant galaxies, which are much younger than nearby galaxies, are indeed also imbedded in dark haloes as expected from contemporary theories of the cosmogony of galaxies [7].

Yoshiaki S., Mareki H., and Toshihiro O. in 2008 presented a unified rotation curve by re-calculating the distances and velocities for a set of galactic constants  $R_0 = 8$  kpc and  $V_0 = 200$  km.s<sup>-1</sup>. Moreover Yoshiaki Sofue in 2008 construct a Galacto-Local Group rotation curve, and explain the behavior of some of the important features of rotation curve [8,9].

In the present work, the mass distributions in individual components of selected galaxies (NGC1365,

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NGC5457 and NGC6946) are determined to calculate decomposing rotation curves of these galaxies.

**Theory of galaxy dynamics**

The rotation curve of a spiral galaxy measures the variation of the rotation speed with radius in a galactic disk. This kinematic information is of crucial importance in understanding the structure and dynamics of spiral galaxies. In this section the theory of galaxy dynamics is described according to different regions such as bulge, disk and dark halo. Also a local structure such as bar and arms are described.

**Mass model**

Rotation curve is the fundamental tool to derive the mass distribution in the galaxy. The major goal of this study is to get detailed information of the mass distribution in individual components of selected galaxies (NGC1365, NGC5457 and NGC6946) such as bulge, disk and dark halo, which have smooth density distribution. Also it can be introduced some perturbations of the local structure such as bar and spiral arms which representing main differences between the observations and theoretical curves, each components of galaxy is described below.

**Bulge**

The inner region of the galaxy is assumed to be composed of two luminous components, which are a bulge and disk. The mass-to-luminosity ratio within each component is assumed to be constant, so that the mass density distribution has the same profile [8]. The bulge is assumed to have a spherically symmetric mass distribution, whose surface mass density obeys the de Vaucouleurs law. This profile for surface mass density can take as the following form [8]

$$\Sigma(r)_b = \Sigma_{bc} \exp[-k((\frac{r}{r_b})^{\frac{1}{4}} - 1)] \dots (1)$$

Where

k: constant = 7.6695.

r<sub>b</sub>: bulge radius.

Σ<sub>bc</sub>: bulge central mass density.

The total mass is calculated by

$$M_b(r) = 2\pi \int_0^{\infty} r \Sigma(r)_b dr \dots (2)$$

Consequently the circular velocity can be calculated by

$$V_b(r) = \sqrt{GM_b(r)/r} \dots (3)$$

**Exponential Disk**

The galactic disk is represented by an exponential disk and the surface mass density is expressed as the following [7]

$$\Sigma(r)_d = \Sigma_{dc} \exp(-\frac{r}{r_d}) \dots (4)$$

Where

Σ<sub>dc</sub>: disk central mass density.

The total mass of the exponential disk is given by

$$M_d = 2\pi \Sigma_{dc} r_d^2 \dots (5)$$

The rotation curve for a thin exponential disk is expressed by modified Bessel functions [10].

$$V_d^2(r) = \frac{1}{2} \frac{GM_d}{r_d} x^2 \left( I_0\left(\frac{x}{2}\right) K_0\left(\frac{x}{2}\right) - I_1\left(\frac{x}{2}\right) K_1\left(\frac{x}{2}\right) \right) \dots (6)$$

where x=r/r<sub>d</sub> and I<sub>0</sub>, K<sub>0</sub>, I<sub>1</sub>, K<sub>1</sub> are the modified Bessel functions computed at x/2.

**Dark Halo**

It was assumed that the dark halo mass distribution has Burkert models. The density profile is written as [11]

$$\rho_h(r) = \frac{\rho_h r_h^3}{(r+r_h)(r^2+r_h^2)} \dots (7)$$

where ρ<sub>he</sub> and r<sub>h</sub> represent central mass density and scale radius of the halo, respectively. This profile gives finite mass density at the center, but yields a flat rotation curve at large radius.

Consequently the circular velocity is given by [11]

$$V_h(r) = 6.4G\rho_{he}r_h^3/r\{\ln(1 + \frac{r}{r_h}) - \tan^{-1}(\frac{r}{r_h}) + 0.5 \ln[1 + (\frac{r}{r_h})^2]\} \dots(8)$$

G : universal gravity=6.67×10<sup>-11</sup>N.m<sup>2</sup>.kg<sup>-2</sup>.

**Bar**

A bar is an extreme case of a spiral arm with a pitch angle. It is an essential feature in galaxy evolution. About two-thirds of spiral galaxies are barred, one third being strongly barred (SB), the other third mildly barred (SAB) [12]. Since so many spiral galaxies have a bar structure, it is likely that it is a recurring phenomenon in spiral galaxy development. The bar is modeled as a Ferrers ellipsoid with a volume density of [13]

$$\rho(r) = \begin{cases} \rho_0(1-r^2)^n & \text{for } r < 1, \\ 0 & \text{elsewhere,} \end{cases} \dots(9)$$

where  $\rho_0$  is the central density of the bar,  $r^2 = y^2/a^2 + x^2/b^2$ , and x and y are the Cartesian coordinates and  $a > b$  the respective semi axes (ellipsoid:  $a > b$ ), with n an integer.

**New suggested model of Spiral arms**

In this assumption a contribution of local spiral arms will be added to disk, whose density distribution is expressed as

$$\rho(r, \theta) = \delta \Sigma_d \sin(\theta * s - \log(\frac{r}{aa}) * pr) \dots(10)$$

where s is the number of the arms and pr is the warp degree of the arms. Where  $\delta$  is the amplitude of the density wave and aa is a constant depending the position of an arm.

**Decomposition of the rotation curve**

A rotation curve of a system, which is composed of a spherical bulge, disk and a dark halo, can be adopted by

$$V^2(r) = V_b^2(r) + V_{disk+arms+bar}^2(r) + V_h^2(r) \dots (11)$$

Where  $V_b(r)$ ,  $V_{disk+ arms+ bar}(r)$  and  $V_h(r)$  are the circular velocity corresponding to individual components of the bulge, disk+ arms+ bar and dark halo, respectively.

**Results and Discussion:**

The sampled galaxies that the rotation curves calculated will be selected in different case according to the presence of a central bar and number of arms (NGC1365, NGC6946, NGC5457). The mass distribution has been obtained by decomposition of a rotation curve into several components such as a bulge, (disk+ bar+ arms) and halo. The function form of the bulge was so adopted that the surface mass density is represented by the de Vaucouleurs law as described in section (bulge). The exponential surface mass distribution in galaxy is obtained by Freeman as described in section (disk), and the halo by Burkert model in section (halo). The corresponding dynamical parameters for selected galaxies are listed in Table1.

**Table1: Dynamical parameters from the decomposition of the rotation curves.**

Galaxy (1)	Type (2)	Inclination( $^{\circ}$ ) (3)	$R_{out}$ (Kpc) (4)	$M_{gas}$ ( $10^{10}M_{\odot}$ ) (5)	$V_{rot}$ ( $Kms^{-2}$ ) (6)	Reference (7)
NGC1365	SBb	46	31	3.4	205.7	[14]
NGC5457	Sc	18	13.5	0.5	105	[15]
NGC6946	SABcd	30	17	2.7	207.75	[16]

**NOTE.** - Galaxy properties of the complete sample: Column (1) is the NGC galaxy number. Column (2) is the galaxy morphological type. Column (3) is inclination of each galaxy. Column (4) is the radius of galaxy. Column (5) is the total gas mass of galaxy. Column (6) is the observed rotation velocity of galaxy. Column (7) provides the primary original reference.

**NGC1365 galaxy**

NGC 1365 is one of the best known barred spiral galaxies with the straight bar and two very prominent outer spiral arms as shown in fig.(1).



**Fig.(1) shows NGC1365 image**

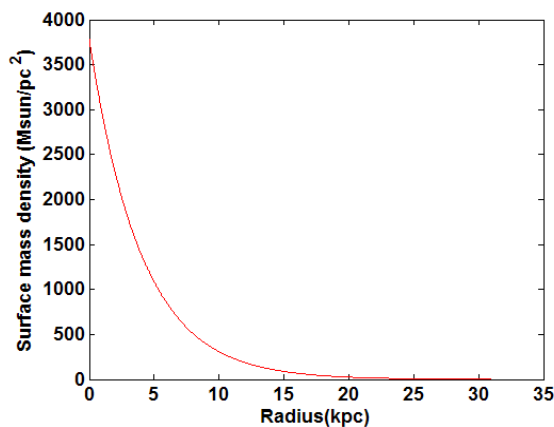
Table 2 lists the parameters for individual mass components of NGC1365 galaxy.

**Table 2.Parameters for Galactic mass components of NGC1365**

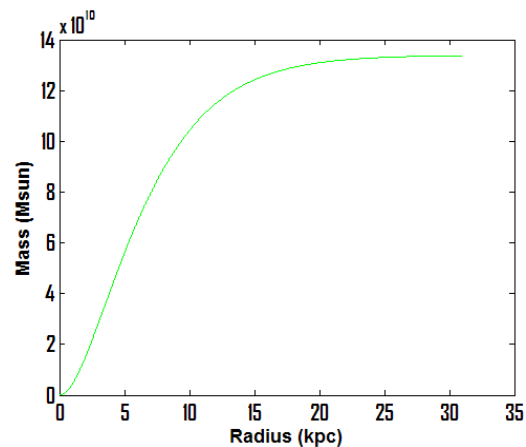
Component	Scale radius(kpc)	Central mass density( $M_{\odot}pc^{-2}$ )
Bulge	0.5	3.78e3 *
Disk	3.5	7.45e2 *
Halo	5.5	3.46e2 *

\* It was estimated from observed rotation curve.

Fig.(2)and fig.(3) show the obtained surface mass density and total mass for the exponential disk NGC1365 galaxy respectively.



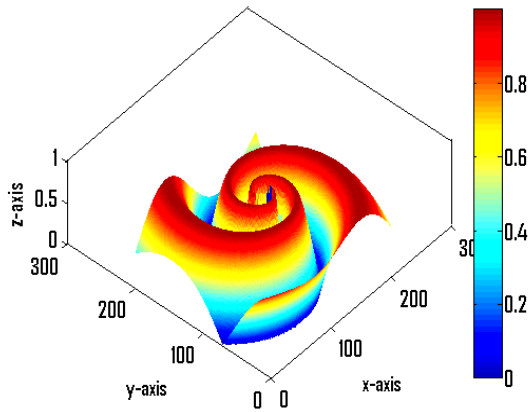
**Fig.(2 ) The surface mass density for the exponential disk of NGC1365 galaxy**



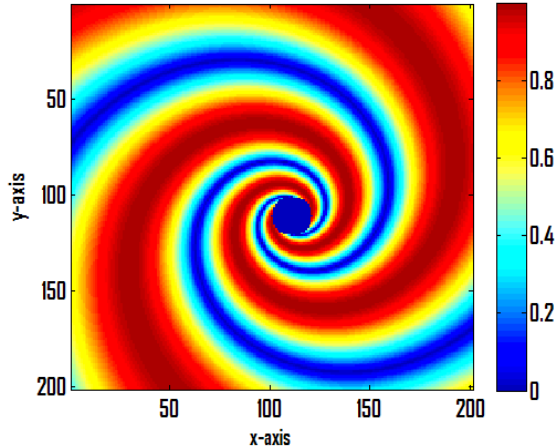
**Fig.(3 ) Total mass for the exponential disk of NGC1365 galaxy**

An addition contribution of local structure such as arms and bar can be included to suggest modified model, whose mass density distribution is described in section (bar) and section

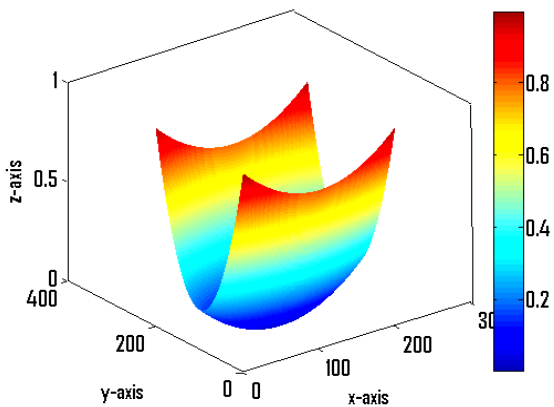
(new suggested model of spiral arms). Fig.(4) to fig.(7) show the surface mass density in 3-D and 2-D for the bar and arms in disk NGC1365 galaxy.



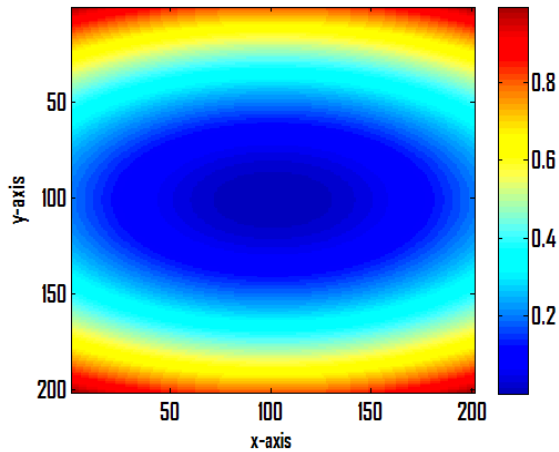
**Fig.(4) Density distribution of NGC1365 with two spiral arms in 3-D**



**Fig.(5) Image represents density distribution of NGC1365**



**Fig.(6) Density distribution of NGC1365 bar in 3-D**



**Fig.(7) Image represents density distribution of NGC1365 bar**

The expected number of spiral arms estimated using eq. (10) is shown in figures above. Almost all barred galaxies present two spiral arms, in this case the prediction is a two-armed spiral ( $s=1$ ) and ( $pr=3$ ). Fig.(6) and fig.(7) show the model of the ellipsoidal bar using eq.(9), in this model the following parameters are taken ( $a=1, b=2$ ).

The results of decomposition rotation curve (as describe in section 2) of NGC1365 galaxy include a bulge, exponential galactic disk and dark halo for a suggested model with two spiral arms and strong bar which is shown with the observations curve as in fig.(6).

NGC1365 fit well to the observation curve.

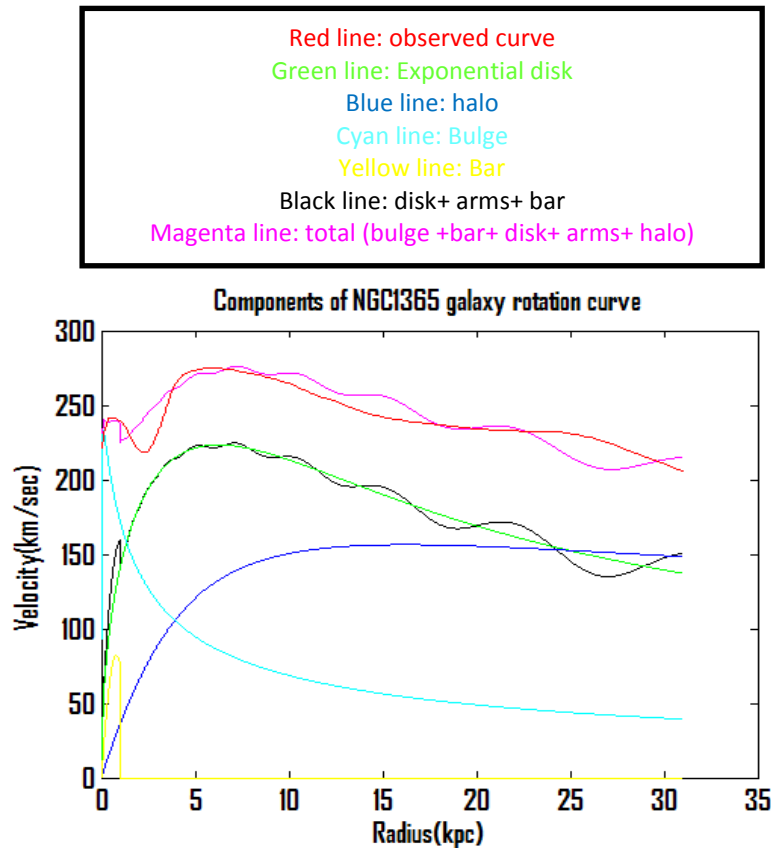


Fig.(8) The rotation curve of NGC1365 for new suggested model

In general from fig.(8), the following notes can be recorded; firstly the total curve rise rapidly in the first part reaching a peak. This increase corresponds to the bulge area of the galaxy at  $r=5$  kpc. After this part, the deep minimum at 2 kpc is the most prominent perturbations. This dip manifests the existence of a strong bar (bar radius in this galaxy  $< 2$  kpc). It can be seen that this additional part affects the rotation curve smoothness making it more complicated, more correlated with the observed curve in this region. After then the curve remain flat due to the contribution of the disk and halo. The prediction is a two-armed spiral, exactly as observed; this galaxy is called grand-design. This model also performs fits with dark halos. Finally As can be seen from the figures the model rotation curve of

**NGC6946 galaxy**

The spiral galaxy NGC6946 is classified as SAB (rs) cd, with an active nucleus. Its intermediate bar and ring feature is not clear as shown in fig.(9).



Fig.(9) shows NGC6946 image

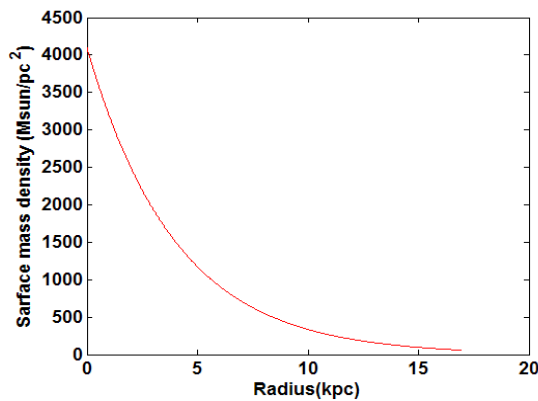


Table 3 lists the parameters for individual mass components of

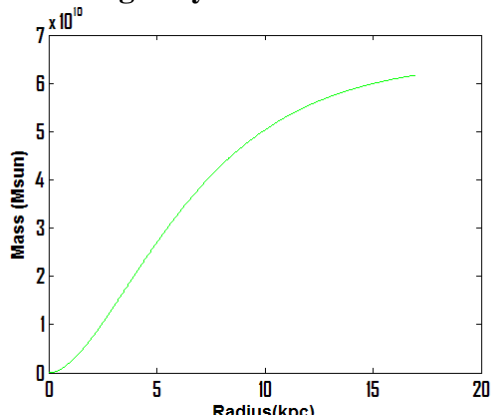
**Table 3: Parameters for Galactic mass components of NGC6946 galaxy**

Component	Scale radius(kpc)	Central mass density( $M_{\odot}pc^{-2}$ )
Bulge	0.5	4.1e3 *
Disk	3.5	4.77e2 *
Halo	5.5	2.86e2 *

Fig.(10) and fig.(11) show the obtained surface mass density and total mass for the exponential disk NGC6946 galaxy respectively.



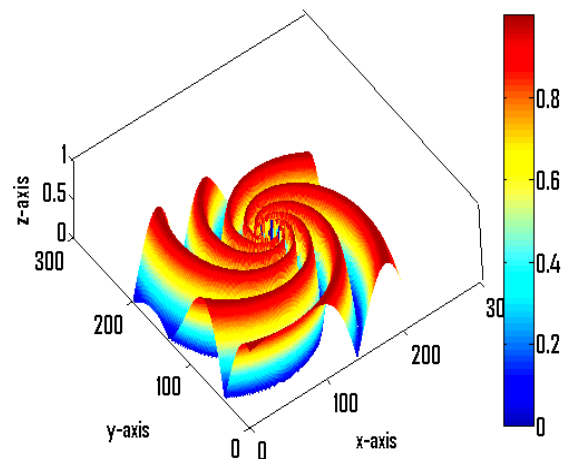
**Fig.(10 ) The surface mass density for the exponential disk of NGC6946 galaxy**



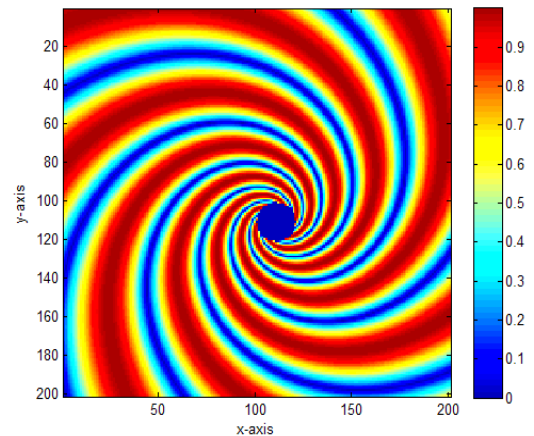
**Fig.(11 ) Total mass for the exponential disk of NGC6946 galaxy**

Fig.(12) to fig.(15) show the surface mass density in 3-D and 2-D for the bar and arms in disk NGC6946 galaxy.

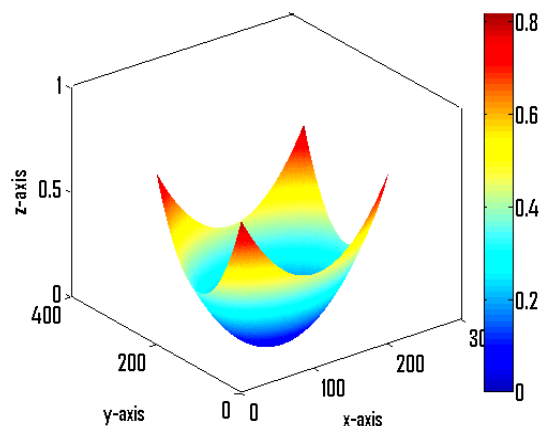
NGC6946 galaxy.



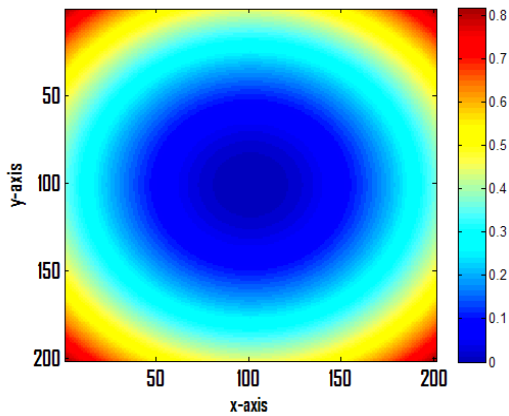
**Fig.(12) Density distribution of NGC6946 for spiral arms in 3-D**



**Fig.(13) Image represents density distribution of NGC6946**



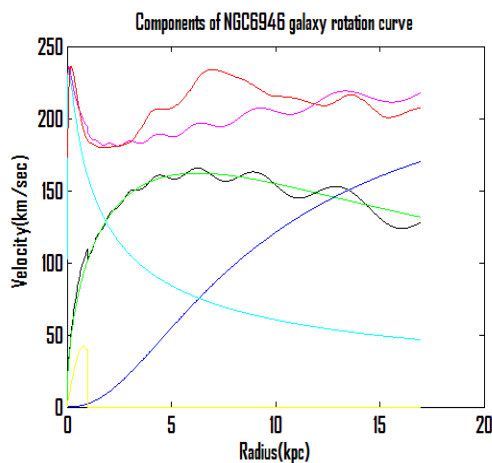
**Fig.(14 ) Density distribution of NGC6946 bar in 3-D**



**Fig.(15 ) Image represents density distribution of NGC6946 bar**

The expected number of spiral arms estimated is shown in fig.(13) . In this case the prediction is six-armed spiral ( $s=3$ ) and  $pr=6$ . Fig.(15) reveals density distribution lower for the center than for the outer part of the bar. The following parameters are taken to describe ellipsoid model of the bar ( $a=1.8, b=2$ ).

The results of decomposition rotation curve of NGC6946 galaxy include a bulge, exponential galactic disk and dark halo for a suggested model with six spiral arms and intermediate bar are shown with the observations curve in fig.(16).



**Fig.(16 ) The rotation curve of NGC6946 for new suggested model**

From the above figure, according to the proposed model, one can see the improved correlation between the decomposition curves with the observed curve. But it can be seen here at the dip ( $\approx 1.1$  kpc) an indication of a transition between bar and disk, in this region the intermediate bar becomes dominate. The fluctuations of the curve about the disk model are due to the presence of multi-spiral arms. Naturally the absolute correlation will be too hard to reach due to the nature of the unpredictable perturbation of the mass distribution. Finally, the part of the disk halo is conspiracy in which the disk and halo conspire to create a flat rotation curve. However, such transition seems difficult to be recognized on the theoretical rotation curves, even if it existed, because of the local structures in the disk of each galaxy.

**NGC5457 galaxy**

NGC5457 (M101) is a very large, relatively nearby, face-on spiral galaxy, also known as the Pinwheel galaxy as shown in fig.(17).



**Fig.(17) shows NGC5457 image**

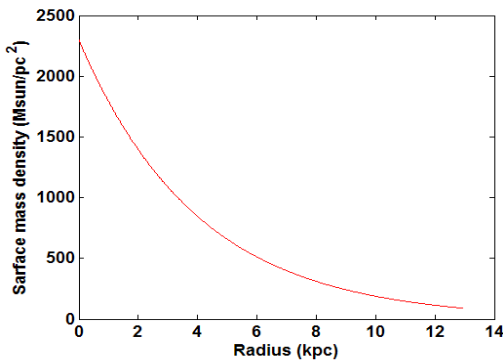
Table 4 lists the parameters for individual mass components of NGC5457 galaxy.



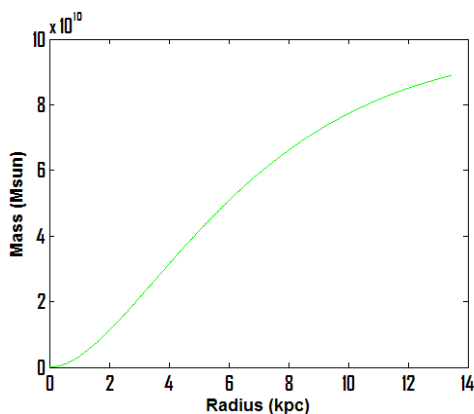
**Table 4:Parameters for Galactic mass components of NGC5457 galaxy**

Component	Scale radius(kpc)	Central mass density( $M_{\odot}pc^{-2}$ )
Bulge	0.5	2.3e3 *
Disk	3.5	4.77e2 *
Halo	5.5	3.46e2 *

Fig.(18) and fig.(19) show the obtained surface mass density and total mass for the exponential disk NGC5457 galaxy respectively.

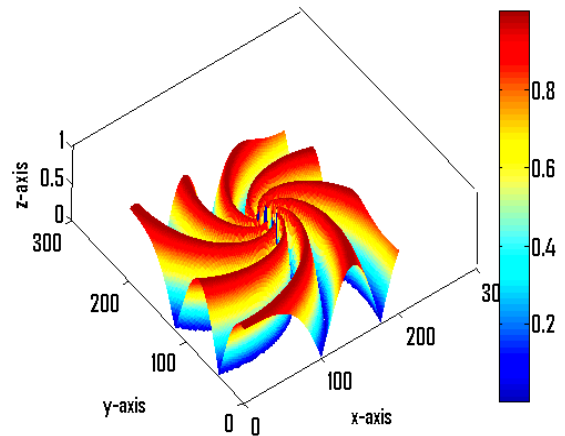


**Fig.(18 ) The surface mass density for the exponential disk of NGC5457 galaxy**

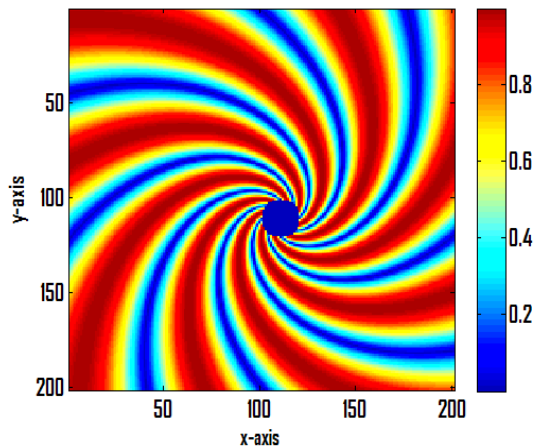


**Fig.(19) Total mass for the exponential disk of NGC5457 galaxy**

Fig.(20) and fig.(21) show the surface mass density in 3-D and 2-D for the arms in disk NGC5457 galaxy.



**Fig.(20 ) Density distribution of NGC5457 for spiral arms in 3-D**



**Fig.(21 ) Image represents density distribution of NGC5457**

The expected number of spiral arms estimated is shown in figures above. In this case the prediction is six-armed spiral ( $s=3$ ) and  $pr=4$ .

The results of decomposition rotation curve of NGC5457 galaxy include a bulge, exponential galactic disk and dark halo for a suggested model with six spiral arms which is shown with the observations curve as in fig.(22).

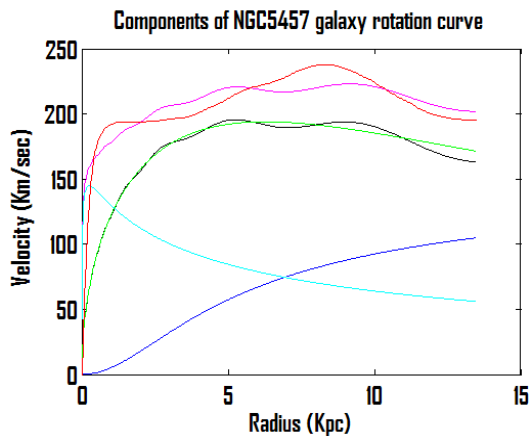


Fig.(22 ) The rotation curve of Pinwehell NGC5457 for new suggested model

From the above figure rotation curve of NGC5457 reveals the various regions of galaxy. The curve according to the profile of the bulge rise sharply in limited region then the ripple shape will be dominant then after. The ripple shape is proposed that describe galaxy multi-arms that change the smoothness of the rotation curve; then this region gives the output curve making it more correlated with the observation curve. Also it can be seen from above figure that the dip region is shown in NGC1365 disappear in this galaxy because NGC5457 is normal spiral (without bar); thus the main different between them are presence of a bar and number of arms.

### Conclusion:

In this paper a profile of the new spiral arms suggest to getting theoretical decomposition rotation curve more correlated and fitted with the observation curve for three spiral galaxies, which is different in the number of the arms. Also the bar region take into consideration for selected sample of galaxies, finally the results show that this modified model provides a good agreement with the observed rotation curve.

### References:

- 1- Bosma, A. 1981. The distribution and kinematics of neutral hydrogen in spiral galaxies of various morphological types, *Astronomical J.*86(1):1791-1846.
- 2- Kent, S. M. 1986. Dark matter in spiral galaxies. I - Galaxies with optical rotation curves, *Astronomical J.*91(1):1301- 1327.
- 3- Sofue, Y., 1996. The most completely sampled rotation curves of galaxies. *Astronomical J.*458(1):120-131.
- 4- Ninkovi, S.1999. The contribution of the dark matter to the rotation of spiral galaxies and its mass distribution, *Serbian Astronomical J.*160(1):1 – 4.
- 5- Ciotti, L.1991. Stellar systems following the  $R \exp 1/m$  luminosity law, *Astronomy & Astrophysics.*249(1):99-106.
- 6- Trujillo, I., Asensio A., Rubino J.A., and Geaham AA.w. 2002. Triaxial stellar systems following the  $r^{1/n}$  luminosity law: an analytical mass-density expression, gravitational torques and the bulge/disc interplay, *Monthly Notices of Royal Astronomical Soci.*333(1):510-516.
- 7- Fuchs B., Möllenhoff C., and Heidt J.2008. Decomposition of the rotation curves of distant field galaxies, *Astronomy & Astrophysics.*111(1):1-5.
- 8- Yoshiaki, S., Mareki H., and Toshihiro Omodaka.2008. Unified rotation curve of the galaxy-decomposition into deVaucouleurs bulge ,disk,dark halo, and the 9-kpc rotation dip , *Publications of Astronomical Society of Japan* .15(3):1–11.
- 9- Yoshiaki, S. 2008. Pseudo rotation curve connecting the galaxy,dark halo, and local group, *Publications*

- of Astronomical Society of Japan .15(1):1–10.
- 10- Paolp, S.,Irina A.Yegorova and NivDrory.2002. The disk mass of spiral galaxies. Monthly Notices of Royal Astronomical Society.133(1):1-6.
- 11- Salucci, P. and Burkert A.2000. Dark Matter Scaling Relations, Astronomical j.537(1):9-12.
- 12- Bournaud, F.and Combes F.2002 . Gas accretion on spiral galaxies:Bar formation and renewal, Astronomy & Astrophysics. 392(1):83–102.
- 13- Bárbara, P., Marco M. and Edmundo M.,2004. Models for the Gravitational Field of the Galactic Bar: An Application to Stellar Orbits in the Galactic Plane and Orbits of Some Globular Clusters. Astronomical j.609(1):144-165.
- 14- Burbidge, E. M. and Burbidge G. R.1960. Motions in Barred Spiral alaxies.The Nuclei of NGC 1097 and NGC 1365,Astronomical j.132(1):30-38.
- 15- Guelin, M. and Weliachew L.1970. A neutral Hydrogen study of the spiral galaxy NGC5457 , Astronomy & Astrophysics.7(1):141-149.
- 16- Sanders, R. H. 1996 . The published extended rotation curves of spiral galaxies:confrontation with modified dynamics ,Astronomical j.473(1):117-129.

## إعادة تركيب مركبات منحني دوران المجرات الحلزونية

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### الخلاصة:

أن منحني دوران المجرات الحلزونية إظهار لتوزيع كثافة الكتلة في المجرات الحلزونية. في هذا البحث تم إعادة تركيب مركبات منحني دوران ثلاث مجرات حلزونية (NGC1365, NGC6946, NGC5457) إلى الجزء المركزي للمجرة ( النواة ) ، القرص وهالة بركيرت المظلمة. كما تم إضافة أجزاء أخرى لاقتراح موديل مطور جديد لإعادة التركيب المحلي لمنحني الدوران مثل الأذرع والقضيب. أوضحت النتائج أن هذا الموديل المقترح المطور يعطي نتائج جيدة ومطابقة لمنحني دوران المجرات الحلزونية المرصودة.