Study the Effect of Solar Radiation Pressure at Several Satellite Orbits

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Abstract:

The effects of solar radiation pressure at several satellite (near Earth orbit satellite, low Earth orbit satellite, medium Earth orbit satellite and high Earth orbit satellite) have been investigated. Computer simulation of the equation of motion with perturbations using step-by-step integration (Cowell's method) designed by matlab a 7.4 where using Jacobian matrix method to increase the accuracy of result.

Key words:Solar radiation pressure,orbit of satellite, Cowell's method , Jacobian matrix

Introduction:

There are two kinds of perturbations of satellite which effect position and velocity of satellite then the lifetime of satellite, gravitational and non-gravitational. Gravitational perturbations include the spherical harmonics, Earth tide, ocean tide effect and effect of Sun and Moon attraction, second the non-gravitational perturbations include atmospheric drag force, solar radiation pressure, and magnetic forces etc. Rodolpho V. Moraes studied the joint effects of direct solar radiation pressure and atmospheric drag on the orbit of an artificial Earth satellite [1], D. Vokrouhlicky and his group studied the perturbation effect of the force due to direct solar radiation pressure on the dynamics of artificial satellites during penumbra [2], Some practical issues of interest such as the practical solar radiation pressure model formation flight of more than two satellites attitude control considerations sailing near earth space were performed by Zhong S. Wang [3].The rotational dynamics of a small solar system body subject to solar radiation torques were investigated by Daniel J. and Sepidehsadat [4]. In this paper, numerical simulation of the equation of motion of two body problem under the effects of solar radiation pressure at several satellite orbits (Near Earth Orbit, Low Earth Orbit, Medium Earth Orbit and High Earth Orbit) by using Matlab a7.4 of the Jacobian matrix was supplemented to improve the accuracy of the numerical solution.

Orbital Dynamic Model

Motion of a body or object in space is an integral part of the preliminary orbit determination process. The Stark problem represents the motion of a test particle, about a fixed Newtonian force center (i.e., the Kepler problem) subject in addition to a uniform force of constant magnitude and direction[5]. The equations of motion of two-body problem can be written in Cartesian coordinates as the form[6]

$$\frac{d^2\vec{r}}{dt^2} = -\frac{u}{r^3}\vec{r} \qquad \text{….. (1)}$$

Where r is the position vector of the satellite from Earth, and μ is the Earth gravitational constant (38601.2

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km^3/sec^2),
Eq.(1) can be written in Three direction as follows

$$\ddot{x} = -\frac{u}{r^3} x$$

$$\ddot{y} = -\frac{u}{r^3} y$$

$$\ddot{z} = -\frac{u}{r^3} z$$

… (2)

Solar Radiation Pressure

Solar radiation pressure is a force acting on the satellite's surface caused by the sunlight. The force acting directly on the satellite is proportional to the effective satellite surface area, to the reflectivity of the surface and to the solar flux; it is inversely proportional to the velocity of light, the acceleration result from solar pressure radiation is [7,8,9]

$$a_{SPR} = -\mu p_s C_R \frac{A}{m} \quad \text{.......(3)}$$

where
C_R =reflectivity coefficient
A= area / cross section of satellite
m= mass of satellite
μ=shadow function, equal 1 for complete sun light, 0 for umbra phase and 0 < μ < 1 for penumbra phase

P_S = is the luminosity of the sun which can be calculated from [10],[11],[12]

$$p_s = \frac{E}{c} \quad \text{……..(4)}$$

E= Solar radiant flux (nominal 1358 W/m²)
c =vacuum speed of light.

This method is a straightforward step-by-step integration equation of the two body equation of motion with perturbation. The equation of motion may be given[13]

$$\ddot{\vec{r}} + \frac{u}{r^3}\vec{r} = \vec{a}_p \quad \text{….(5)}$$

Which, for numerical integration, would be reduced to first-order differential equations

$$\dot{\vec{r}} = \vec{v} \quad \text{…. (6)}$$

$$\dot{\vec{v}} = -\frac{u}{r^3}\vec{r} + \vec{a}_p \quad \text{….. (7)}$$

here $\vec{a}_p$ is the vector sum of all the perturbing accelerations to be included in the integration. The equations of motion are first reduced into first-order differential equations as shown in the following matrix[14]

$$\frac{dy}{dt} = \begin{bmatrix} y(2) - \frac{u}{r} y(1) + a_{SPR} \\ y(4) - \frac{u}{r} y(3) + a_{SPR} \\ y(6) - \frac{u}{r} y(5) \end{bmatrix} \quad \text{… (8)}$$

Jacobian Matrix

The Jacobian matrix is the matrix of all first-order partial derivatives of a vector-valued function. That is, the Jacobian of a function describes the orientation of a tangent plan to the function at a given point. Let $x \in R^n$ and $y = f(x) \in R^n$ be a differentiable vector-valued function of x. The Jacobian matrix is defined by the following [15].

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} \frac{\partial f_1}{\partial x_2} \cdots\cdots \frac{\partial f_n}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} \frac{\partial f_2}{\partial x_2} \cdots\cdots \frac{\partial f_2}{\partial x_n} \end{pmatrix} = \left(\frac{df_i}{dx_j} \right)_{i=j=1,2,\ldots n} \qquad \ldots.(9)$$

Discussion of Simulation Results and Conclusions:

A computer simulation has been developed to the equation of orbital motion of two body problem with perturbation due the effects of solar radiation pressure using Matlab a7.4. The solar radiation pressure perturbation was calculated for 60 orbital cycles . The effect of solar radiation pressure on vector's position and the change of orbital velocity with time for Near Earth Orbit satellite (NEO) is shown in figures (1) and (2), the perturbation affect dos not appear in these type of satellites, this case is similar to the same effect done by Low Earth Orbit satellite (LEO) as shown in figures (3) and (4). Figures (5) and (6) show the position of vector and the change in orbital velocity with time when its experience perturbation resulted from solar radiation pressure for the Medium Earth Orbit satellite (MEO), in the beginning, the affection of perturbation would not appear clearly but after a period of several revolution the position vector, decreasing would be clear while the change of orbital velocity increased, and these effect increases with increasing of orbital cycles of the satellite. For the High Erath Orbit satellite (HEO) The effect of solar radiation pressure on position of vector and the change of orbital velocity with time are shown in figures (7) and (8), reveal that the position of vector decreased greatly compared with the same effect to the medium Earth orbit satellite, also that the orbital velocity increasing in clean-cut It can be concluded from these figures that ,the solar radiation pressure impacts at position and velocity of orbit satellite and subsequently causes reduction of the orbit position and steps-up the orbit velocity, then reduces the lifetime of satellite, and this perturbation affect on satellite that have altitude above 1000 km, and this kind of perturbation increased with increasing of altitude of satellite orbit.

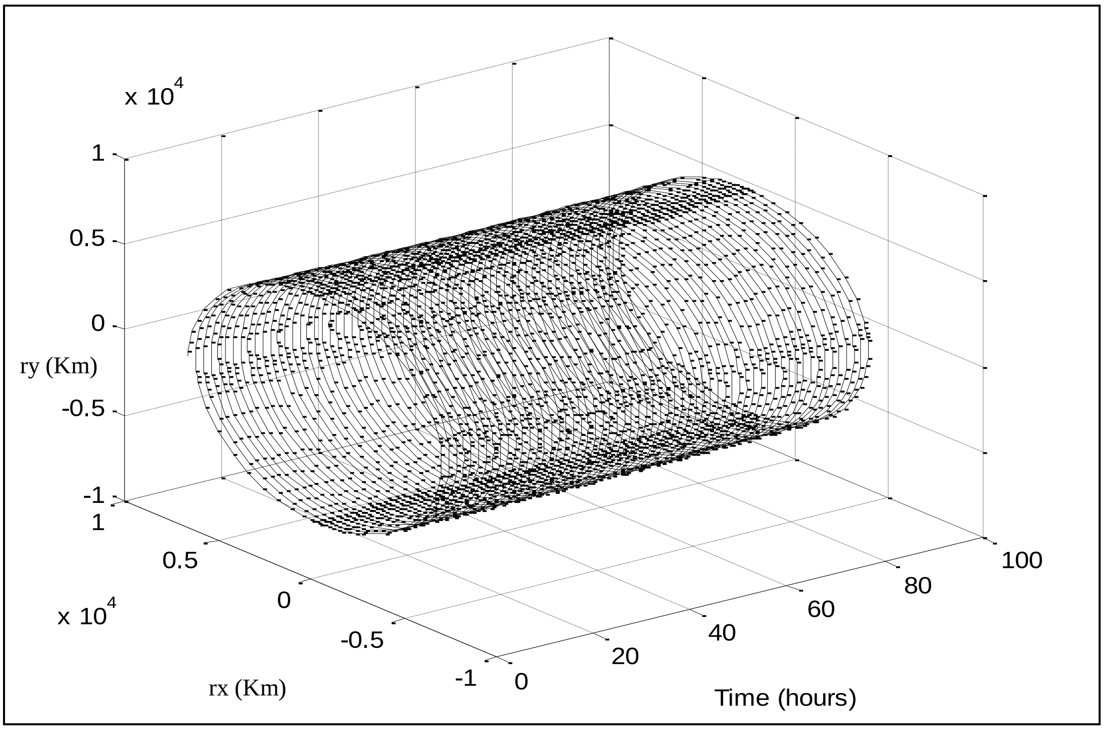

Fig.(1) Relationship between position of vector with time under solar radiation pressure effect

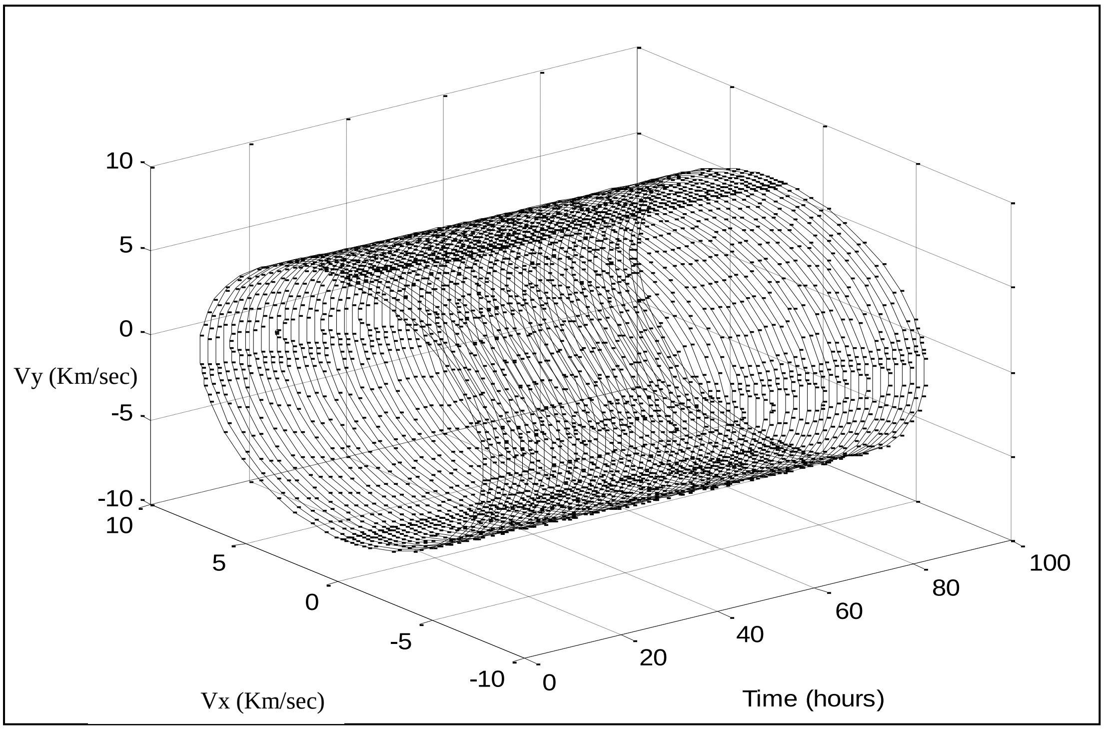

Fig.(2)The change of orbital velocity with time under the effect solar radiation pressure for NEO satellite

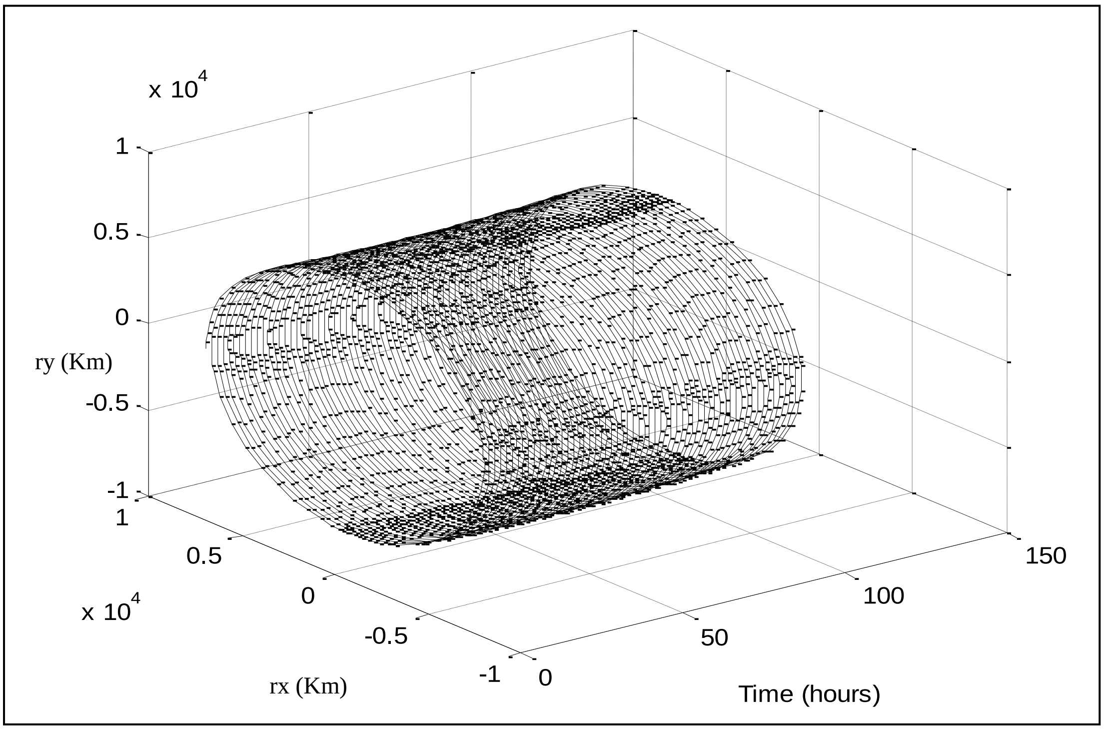

Fig.(3)) Relationship between position of vector with time under solar radiation pressure effect for LEO satellite

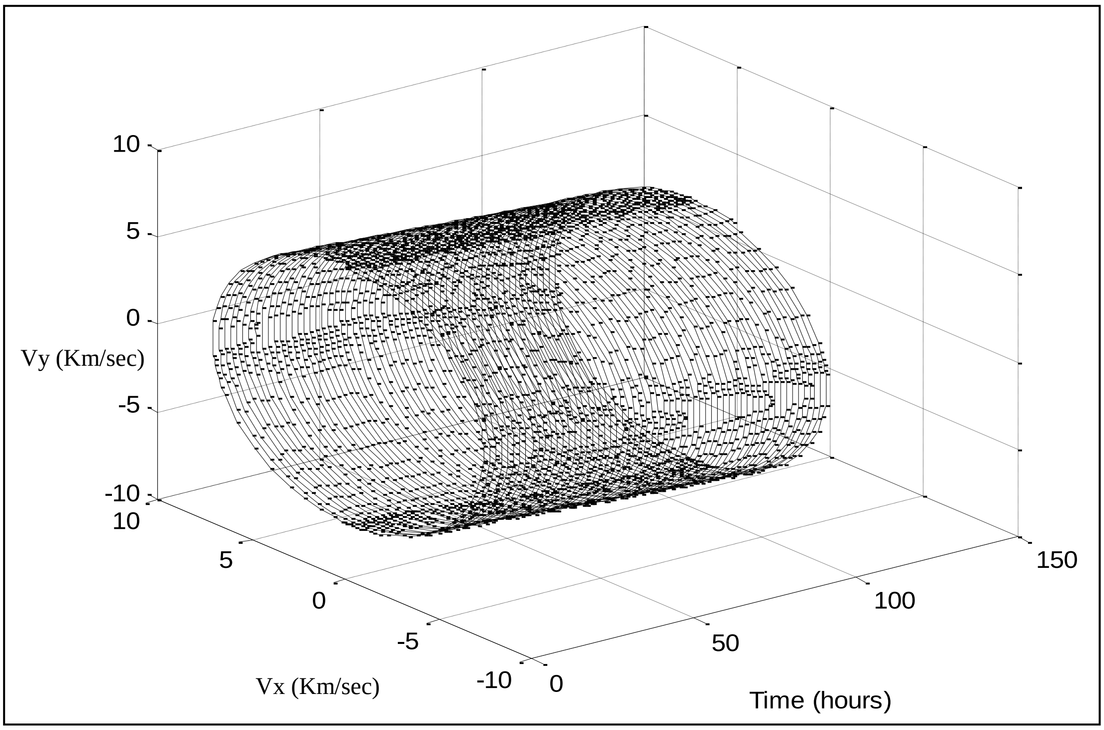

Fig.(4)The change of orbital velocity with time under the effect solar radiation pressure for LEO satellite

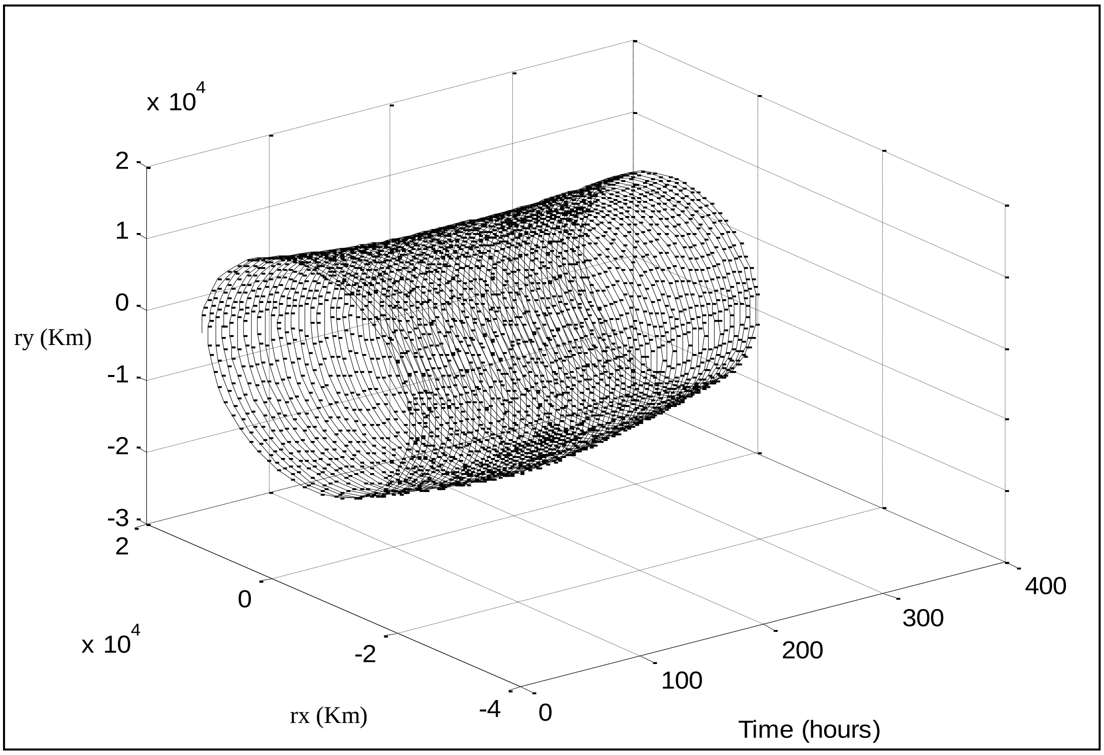

Fig.(5) Effect of solar radiation pressure at the position of vector with time to the MEO

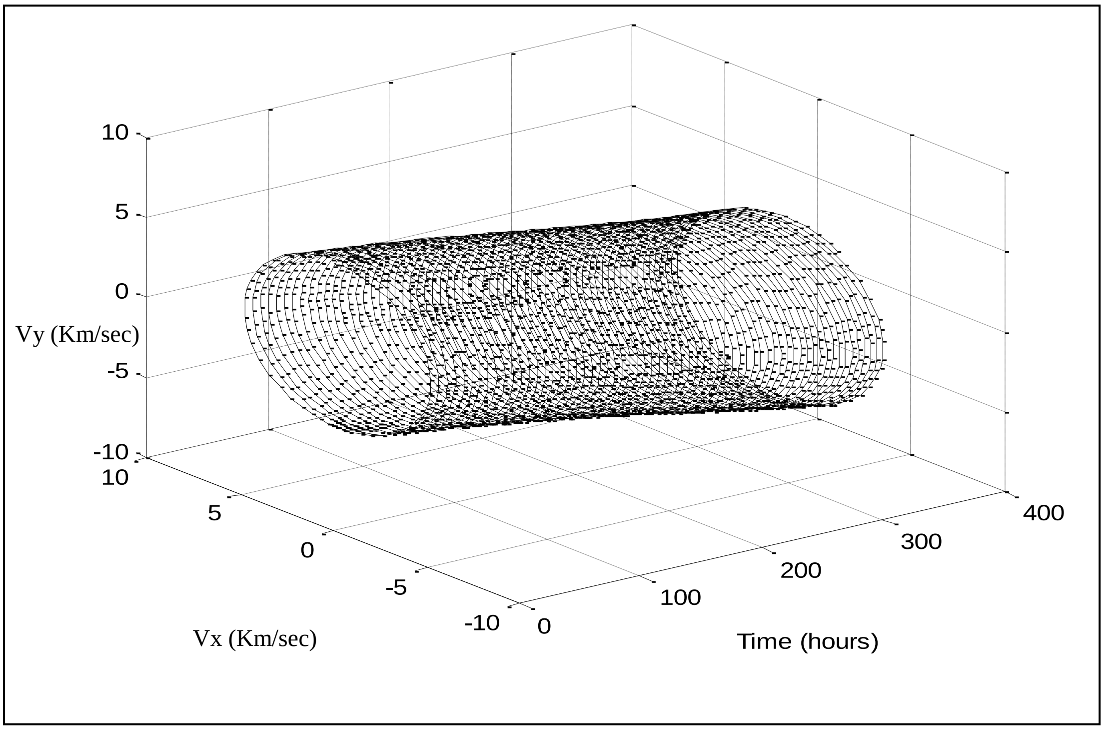

Fig.(6) Effect of solar radiation pressure at the change of orbital velocity with time to the MEO satellite

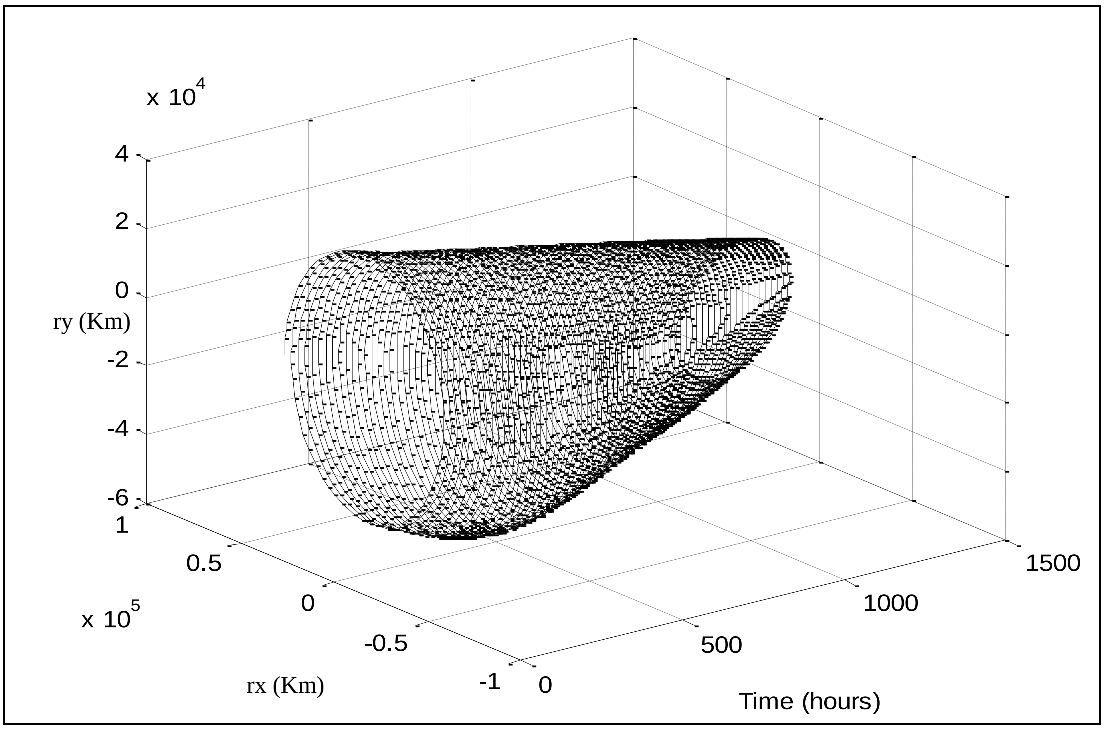

Fig.(7) Decreasing in the vector's position with time under effect of solar radiation pressure for HEO satellite

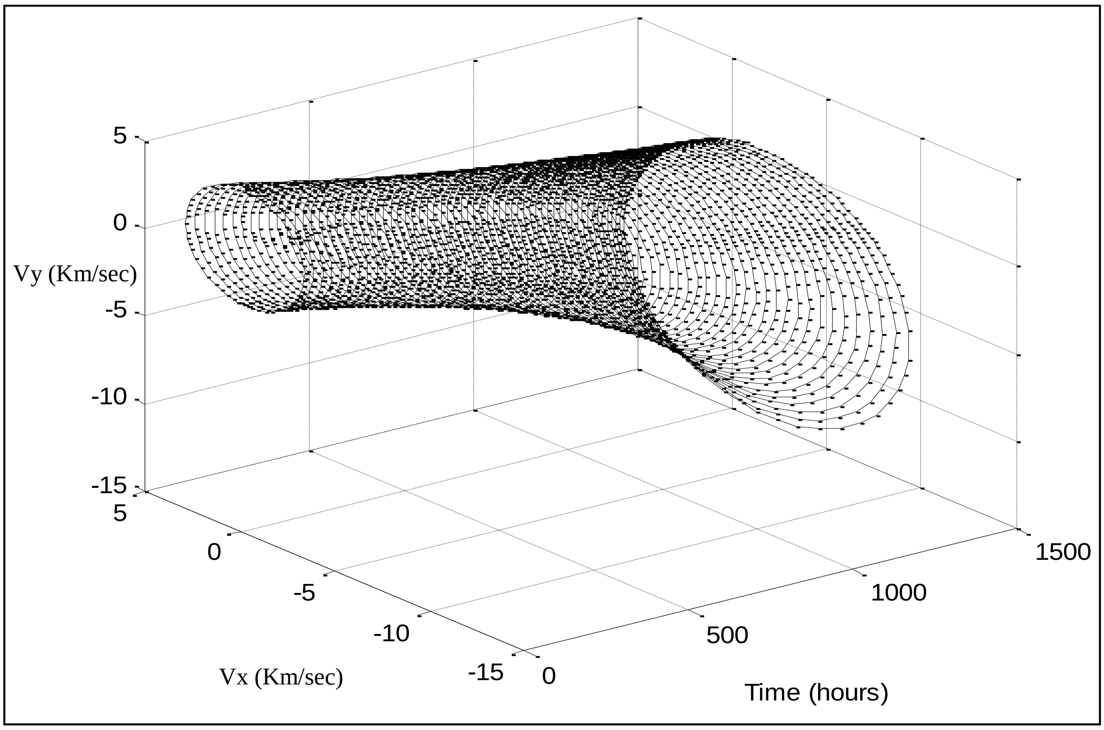

Fig.(8) Increasing in changeof orbital velocity with time under effect of solar radiation pressure for HEO satellite

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دراسة تأثير ضغط الإشعاع الشمسي على الأقمار الاصطناعية لمختلف المدارات

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الخلاصة:

تمت دراسة تأثير ضغط الإشعاع الشمسي على مجموعة من الأقمار الاصطناعية لمختلف المدارات (ذو المدار القريب من الأرض ، ذو المدار الواطئ ، ذو المدار المتوسط بالإضافة الى ذو المدار العالي) . حيث تم محاكاة معادلات الحركة مع الاضطرابات المدارية بالحاسوب باستخدام Matlab a 7.4 وتكاملها خطوة - خطوة (Cowell's method) حيث اعتمدت طريقة مصفوفة جاكوبيان لغرض زيادة دقة النتائج .