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Applications of Kalman and Extended Kalman Filtering to Target Tracking

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ABSTRACT

This study deals with the famous trackers named Kalman and extended Kalman filters. This is introduced by describing the state space representation approach to model the target system. A modification to the state prediction equation of Kalman and extended Kalman filters is given in order to offer an ability of multi-step ahead prediction of the target future position. The problem of missed measurements, with different percentages of missing, is studied and a method to estimate these missed measurements is then suggested. Some simulation experiments are performed and indicated that Kalman filtering techniques are promising when they deal with target tracking problem.

تطبيقات مرشح كالمن ومرشح كالمن الموسع في تعقب الهدف

الملخص

تتعامل هذه الدراسة مع المتعقبين المعروفين بمرشح كالمن ومرشح كالمن الموسع . ويتم التقديم لذلك بوصف معادلات تنبؤ فضاء الحالة لمرشحيه كالمن وكالمن الموسع وذلك لغرض إعطاء قابلية لتنبؤ متعدد الخطوات للموقع المستقبلي للهدف. وتدرس مسألة القراءات المفقودة وبنسب مختلفة من الفقدان وتقترح طريقة لتقدير هذه القراءات. وتنجز بعض التجارب بالمحاكاة ، حيث تظهر هذه التجارب ان أساليب مرشح كالمن تفيد كثيرا عندما نتعامل مع مسألة تعقب الهدف.

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1. Introduction

The Kalman filter is a set of mathematical equations that provides an efficient computational solution of the least squares method. This filter is very powerful in several aspects: it supports estimations of past, present and even future states, and it can do so even when the precise nature of the modeled system is unknown. It is the best linear estimator, which can produce an optimal estimate of the states of a linear dynamic system, subject to the disturbances having Gaussian distribution (Kalman [1960]).

The Kalman filter is the most famous filter in the time domain which can be used to compute the optimal estimate of the signal. Since 1960, when Kalman published his famous paper describing a recursive solution to discrete data linear filtering problem, due in large part to advances in digital computing, the Kalman filter has been the subject of extensive research and applications, particularly in the area of autonomous or assisted navigation.

Much of the early impetus for the development of Kalman filter theory and their applications come from problems in the aerospace sciences. Target Tracking is one of the important areas in which Kalman filter is widely used. The Kalman filter is considered a strong tracker, which serves in both non-maneuvering and maneuvering cases. To add, the most advantage of the Standard Kalman Filter (SKF) and Extended Kalman Filters (EKF) is the ability to deal with multi-dimensional problems, which means multi-dimension target and multi target tracking (Park et al.[1995]).

2. State Space Representation

It is well known that the classical approach for system representations is based on input and output variables only. However, according to a modern approach known as state space, systems can also be modeled by using some dependent variables, which depend on inputs, outputs or both, known as state variables. These variables contain complete historical information about the system, although they may not have any physical meaning and they may not be directly measurable.

In general, the state space approach is a very generalized mathematical representation, which is suggested by Kalman and Bucy in 1961. The state space model is expressed in terms of the inputs, the outputs and the state of the system. The state space model provides a description of the internal and external characteristics of the system, by incorporation of variables, which can relate to internal behavior of systems that cannot be accessed or measured, along with the measurable variables.

Usually, modeling by state space approach is done through mathematical model, which consists of two equations: the first is called the state (or system) equation, and the second is called the measurements (or observations) equation. If we denote the state vector by $\boldsymbol{s}_t$ and the measurement vector by $\boldsymbol{x}'_t$, a dynamic system (in discrete-time form) can be described by (Welch and Bishop [20001]):

$$s_{t+1} = h_t(s_t) + \eta_t \quad ; \quad t = 1,2,... \qquad (1)$$

$$f_t(x'_t, s_t) = 0 \quad ; \quad t = 1,2,... \qquad (2)$$

where $\boldsymbol{\eta}_t$ is the vector of random disturbance of the dynamic system and is usually modeled as a white noise with zero mean and covariance matrix $\boldsymbol{Q}_t$ i.e.:

$$E[\eta_t] = 0 \quad and \quad E[\eta_t \eta_t^T] = Q_t$$

In practice, the system noise covariance $\boldsymbol{Q}_t$ is usually determined on the basis of experience and intuition (i.e., it is guessed). The vector $\boldsymbol{x}'_t$ is called the measurement vector. Now, it is assumed that the measurement system is corrupted by additive white noise. That is the real observed measurement $\boldsymbol{x}_t$ which can be expressed as:

$$x_t = x'_t + \eta_t \qquad (3)$$

with

$$E[\eta_t \eta_k^T] = Q = \begin{cases} A_t & , \ for \quad t = k \\ 0 & , \ for \quad t \neq k \end{cases}$$

The measurement noise covariance $\boldsymbol{A}_t$ is either provided by some signal processing algorithm or guessed in the same manner as the system noise. In general, these noise levels are determined independently. It is assumed that there is no correlation between the noise process of the system and that of the observation, that is

$$E[\eta_t \eta_k^T] = 0 \quad , \qquad for \quad every \quad t \quad and \quad k$$

3. <u>The Standard Kalman Filter</u>

When $h_t(\boldsymbol{s}_t)$ in equation (1) is a linear function such that

$$s_{t+1} = H_t s_t + \eta_t \qquad (4)$$

and is able to write down explicitly a linear relationship

$$x_t = F_t s_t + \eta_t \qquad (5)$$

from $f_t(x'_t, s_t) = 0$, then the Standard Kalman Filter (SKF) is directly applicable.

The SKF addresses the general problem of estimating the state vector $\boldsymbol{s}_t$ of a discrete time controlled process governed by the linear stochastic difference equation (4) with measurement $\boldsymbol{x}_t$ in the equation (5). It estimates a process by using a feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. The SKF steps are summarized in the following Algorithm.

Algorithm (1): Standard Kalman Filter

Step 1: Initialization:

1.1: Initialize the covariance matrix of the states as:

$$P_{0|0} = A_{s_0}$$

1.2: Initialize the state vector as:

$$\hat{s}_{0|0} = E[s_0]$$

Step 2: The equations for the time update (Prediction) are:

2.1: Prediction of state vector as:

$$\hat{s}_{t|t-1} = H_{t-1}\hat{s}_{t-1} \qquad (6)$$

2.2: Prediction of the covariance matrix of the states as:

$$P_{t|t-1} = H_{t-1}P_{t-1}H_{t-1}^T + Q_{t-1} \qquad (7)$$

Step 3: The equations for measurement update (Correction) are:

3.1: Kalman gain matrix as:

$$K_t = P_{t|t-1}F_t^T\left(F_tP_{t|t-1}F_t^T + A_t\right)^{-1} \qquad (8)$$

3.2: Updating of the state estimation as:

$$\hat{s}_t = \hat{s}_{t|t-1} + K_t\left(x_t - F_t\hat{s}_{t|t-1}\right) \qquad (9)$$

3.3: Updating of the covariance matrix of states as:

$$P_t = \left(I - K_tF_t\right)P_{t|t-1} \qquad (10)$$

Notes:

1. At time *t*, the system model inherently in the filter structure generates $\hat{\mathbf{s}}_{t|t-1}$,the best prediction of the state, using the previous state estimate $\hat{\mathbf{s}}_{t-1}$. The previous state covariance matrix $\boldsymbol{P}_{t-1}$ is extrapolated to the predicted state covariance matrix $\boldsymbol{P}_{t|t-1}$. $\boldsymbol{P}_{t|t-1}$ is then used to compute the Kalman gain matrix $\boldsymbol{K}_t$ and to

update the covariance matrix $\boldsymbol{P}_t$. The system model generates also $F_t\ \hat{\mathbf{s}}_{t|t-1}$ which is the best prediction of what the measurement at time will be. The real measurement $\boldsymbol{x}_t$ is then read and the measurement residual is computed as:

$$r_t = x_t - F_t\hat{s}_{t|t-1} \qquad (11)$$

Finally, the residual $\boldsymbol{r}_t$ is weighted by the Kalman gain matrix $\boldsymbol{K}_t$ to generate a correction term and is added to $\hat{\mathbf{s}}_{t|t-1}$ to obtain the updated state $\hat{\mathbf{s}}_t$.

2. The SKF gives a linear, unbiased, and minimum error variance recursive algorithm to optimally estimate the unknown state of a linear dynamic system from noisy data taken at discrete real-time intervals.

3. Also the SKF yields at t an optimal estimate of $\boldsymbol{s}_t$, optimal in the sense that the spread of the estimate-error probability density is minimized. In other words, the estimate $\hat{\mathbf{s}}_t$ given by the SKF minimizes the following cost function (Al-Jabri,2000):

$$J_t(\hat{s}_t) = E\left[(\hat{s}_t - s_t)\ M\ (\hat{s}_t - s_t)^T\right] \qquad (12)$$

where $\boldsymbol{M}$ is an arbitrary, positive semidefinite matrix.

4. The optimal estimate $\hat{\mathbf{s}}_t$ of the state vector $\boldsymbol{s}_t$ is easily understood to be a least-squares estimate of $\boldsymbol{s}_t$ with the properties that: 1. The transformation that yields $\hat{\mathbf{s}}_t$ form is linear.

2. $\hat{\mathbf{s}}_t$ is unbiased in the sense that , $E[\ \hat{\mathbf{s}}_t\ ] = \boldsymbol{s}_t$.

3. It yields a minimum variance estimate with the inverse of covariance matrix of measurement as the optimal weight.

4. The Extended Kalman Filter

The EKF approach is to apply the SKF (for linear systems) to nonlinear systems with additive white noise by continually updating a linearization around the previous state estimate, starting with an initial guess. In other words, we only consider a linear Taylor approximation of the system function at the previous state estimate and that of the observation function at the corresponding predicted position. This approach gives a simple and efficient algorithm to handle a nonlinear model. However, convergence to a reasonable estimate may not be obtained if the initial guess is poor or if the disturbances are so large that the linearization is inadequate to describe the system.

Expanding $f_t(x_t^{'}, s_t)$ in the equation (2) into a two-dimensional Taylor series about ($\boldsymbol{x}_t$, $\hat{\mathbf{s}}_{t|t-1}$) yields:

$$f_t(x_t^{'}, s_t) = f_t(x_t, \hat{s}_{t|t-1}) + \frac{\partial f_t(x_t, \hat{s}_{t|t-1})}{\partial x_t^{'}}(x_t^{'} - x_t) + \frac{\partial f_t(x_t, \hat{s}_{t|t-1})}{\partial s_t}(s_t - \hat{s}_{t|t-1})$$
$$+ o((x_t^{'} - x_t)^2) + o((s_t - \hat{s}_{t|t-1})^2) + ... \qquad (13)$$

By ignoring the second order terms which are not significant, we get a linearized measurement equation:

$$y_t = M_t s_t + \xi_t \qquad (14)$$

where $\boldsymbol{y}_t$ is the new measurement vector, $\boldsymbol{\xi}_t$ is the noise vector of the new measurement, and $\boldsymbol{M}_t$ is the linearized transformation matrix. They are given by (Kanjilal ,1995), Welch and Bishop [2001]) :

$$M_t = \frac{\partial f_t(x_t, \hat{s}_{t|t-1})}{\partial s_t}, \qquad (15a)$$

$$y_t = -f_t(x_t, \hat{s}_{t|t-1}) + \frac{\partial f_t(x_t, \hat{s}_{t|t-1})}{\partial s_t}, \qquad (15b)$$

$$\xi_t = \frac{\partial f_t(x_t, \hat{s}_{t|t-1})}{\partial x_t^{'}}(x_t^{'} - x_t) = \frac{\partial f_t(x_t, \hat{s}_{t|t-1})}{\partial x_t^{'}}\eta_t. \qquad (15c)$$

We have:

The EKF equations are summarized in the following algorithm, where the derivative $\frac{\partial h_t}{\partial s_t}$ is computed at $\boldsymbol{s}_t = \hat{\mathbf{s}}_{t}$.

$$E[\xi_t] = 0, \quad and \quad E[\xi_t \xi_t^T] = \frac{\partial f_t(x_t, \hat{s}_{t|t-1})}{\partial x_t^{'}} A_t \frac{\partial f_t(x_t, \hat{s}_{t|t-1})^T}{\partial x_t^{'}} \cong A_t$$

Algorithm (2): Extended Kalman Filter

Step 1: Initialization:

1.1: Initialize the covariance matrix of the states as:

$$P_{0|0} = A_{s_0}$$

1.2: Initialize the state vector as:

$$\hat{s}_{0|0} = E[s_0]$$

Step 2: The equations for the time update (Prediction) are:

2.1: Prediction of state vector as:

$$\hat{s}_{t|t-1} = h_t(\hat{s}_{t-1}) \qquad (16)$$

2.2: Prediction of the covariance matrix of the states as:

$$P_{t|t-1} = \frac{\partial h_t}{\partial s_t} P_{t-1} \frac{\partial h_t}{\partial s_t}^T + Q_{t-1} \qquad (17)$$

Step 3: The equations for measurement update (Correction) are:

3.1: Kalman gain matrix as:

$$K_t = P_{t|t-1} M_t^T \left(M_t P_{t|t-1} M_t^T + A_t\right)^{-1} \qquad (18)$$

3.2: Updating of the state estimation as:

$$\hat{s}_t = \hat{s}_{t|t-1} + K_t\left(y_t - M_t \hat{s}_{t|t-1}\right) \qquad (19)$$

3.3: Updating of the covariance matrix of states as:

$$P_t = \left(I - K_t M_t\right) P_{t|t-1} \qquad (20)$$

5. Multi-step Prediction

This section concerns optimal prediction of the state vector $\hat{\mathbf{s}}_{t+p|t}$. The prediction of the output vector can be obtained from the predicted states following the measurement equation (5) (for SKF) as:

$$\hat{x}_{t+p|t} = F_t \hat{s}_{t+p|t} \qquad (21)$$

or equation (3.14) (for EKF) as:

$$\hat{y}_{t+p|t} = M_t \hat{s}_{t+p|t} \qquad (22)$$

The concept of prediction is inherent with the Kalman filtering, and it has already been shown that the Standard Kalman and Extended Klaman Algorithms generate one-step ahead prediction of the state vector. Multistep prediction follows as a natural extension. The objective is to produce p-step ahead estimate $\hat{\mathbf{s}}_{t+p|t}$, $p \geq 1$, given all data up to time t. The prediction will be optimal in a minimum variance sense, subject to the minimization of the cost function

$$J(\hat{s}_{t+p}) = E\left[(\hat{s}_{t+p} - s_{t+p})(\hat{s}_{t+p} - s_{t+p})^T\right] \qquad (23)$$

Rewriting equations (3,4):

$$s_{t+1} = Hs_t \quad ,$$

multistep prediction may be obtained as:

$$\begin{aligned} s_{t+2} &= Hs_{t+1} \\ &= HHs_t \\ &= H^2 s_t \quad , \end{aligned}$$

$$\begin{aligned} s_{t+3} &= Hs_{t+2} \\ &= HH^2 s_t \\ &= H^3 s_t \quad , \end{aligned}$$

Hence, in general we have

$$s_{t+p} = H^p s_t \quad p = 1,2,\ldots \qquad (24)$$

The optimal prediction can be expressed as (Kanjilal ,1995):

$$\hat{s}_{t+p|t} = H^p \hat{s}_{t|t} \qquad (25)$$

$$\hat{x}_{t+p|t} = F\hat{s}_{t+p|t} \qquad (26)$$

Also the covariance matrix corresponding to the optimal p-step prediction is given by:

$$P_{t+p} = F^p \quad P_t \quad (F^P)^T \qquad (27)$$

6. Simulation Experiments

Here, the performances of the SKF and EKF are tested. Firstly, the estimation of the current position of the maneuvered target is concerned. The simulation is done through 20 experiments of maneuvered target with white noise corrupting and other 20 experiments of maneuvered target with white noise corrupting. In each experiment, the RMSE is computed. In case of corrupting by white noise, Figure (1), in part a, shows estimated path of the tracked target using SKF, the X error and Y error in parts b and c, respectively. Figure (2) shows the same information but when using EKF. Figures(3) and (4) are similar to Figures (1) and (2) but in the case of EKF.

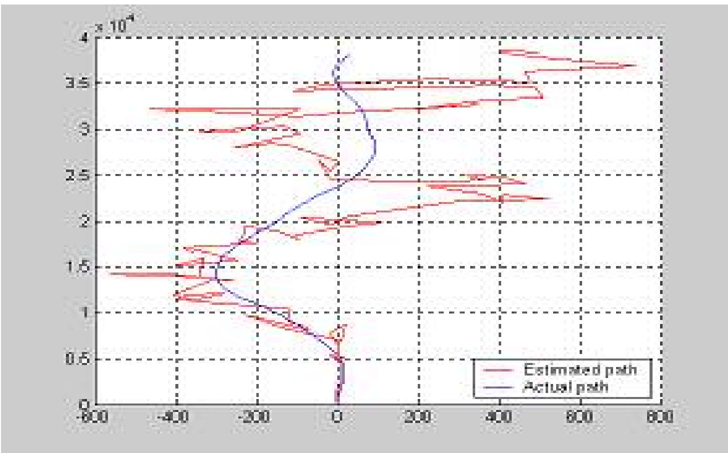

Figure (1a) : Estimated Path Using

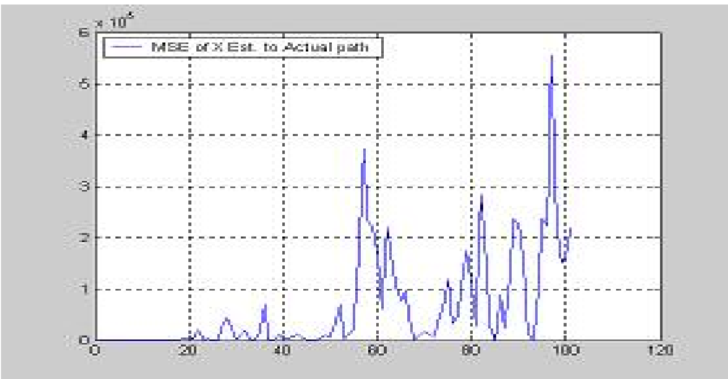

Figure (1b) : X- Coordinate

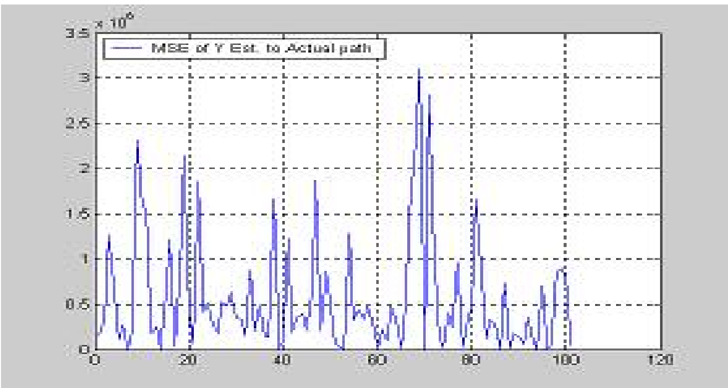

Figure (1c) : Y- Coordinate Error.

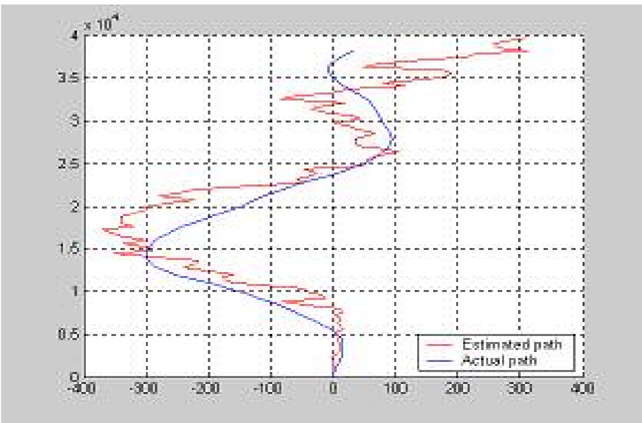

Figure (2a) : Estimated Path

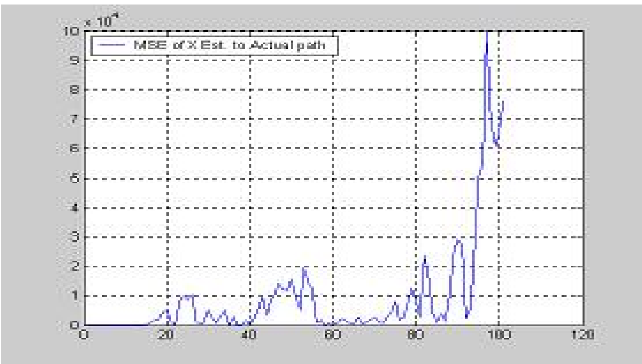

Figure (2b) : X- Coordinate

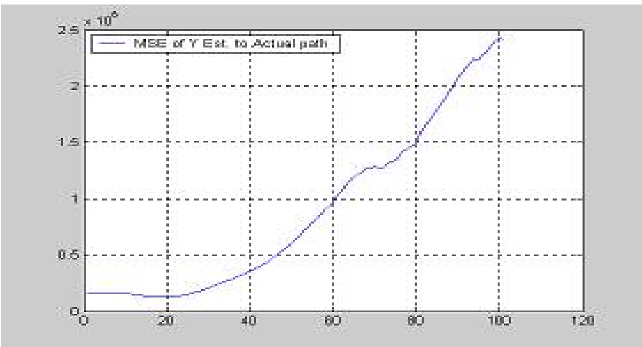

Figure (2c) : Y- Coordinate

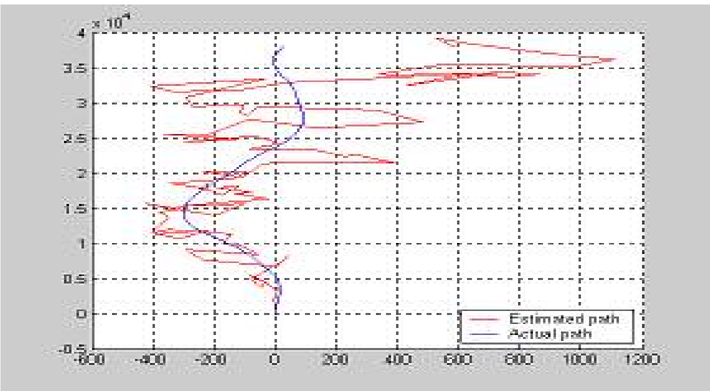

Figure (3a) : Estimated Path Using

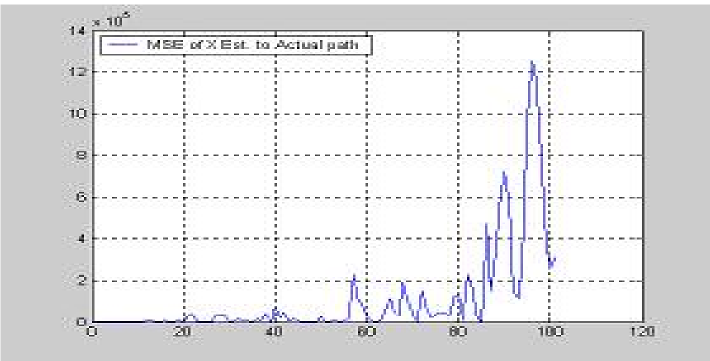

Figure (3b) : X- Coordinate

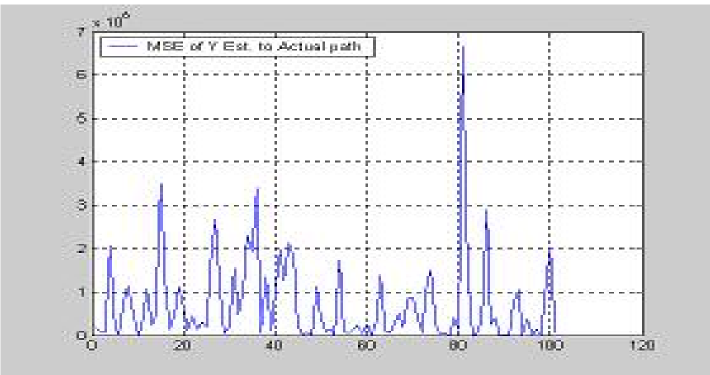

Figure (3c) : Y- Coordinate

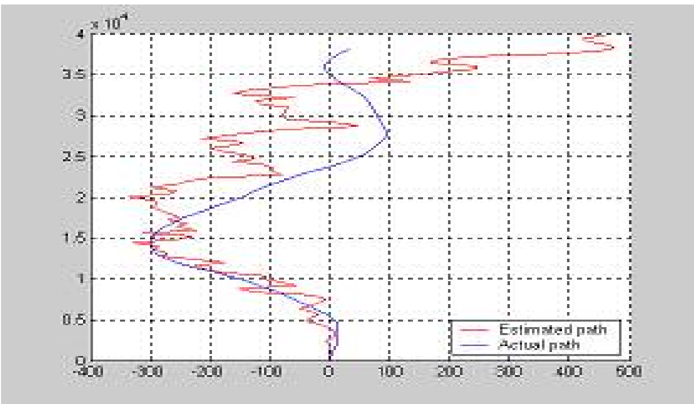

Figure (4a) : Estimated Path Using

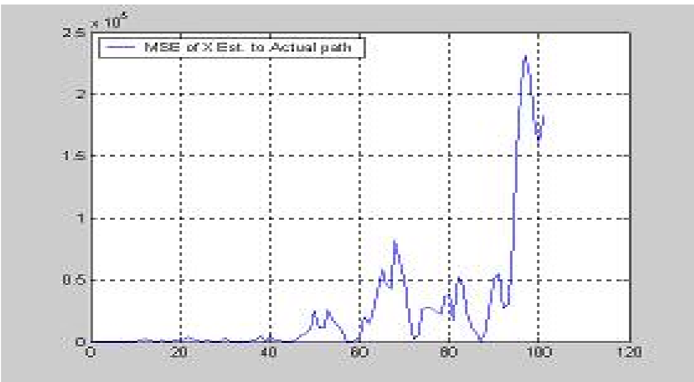

Figure (4b) : X- Coordinate

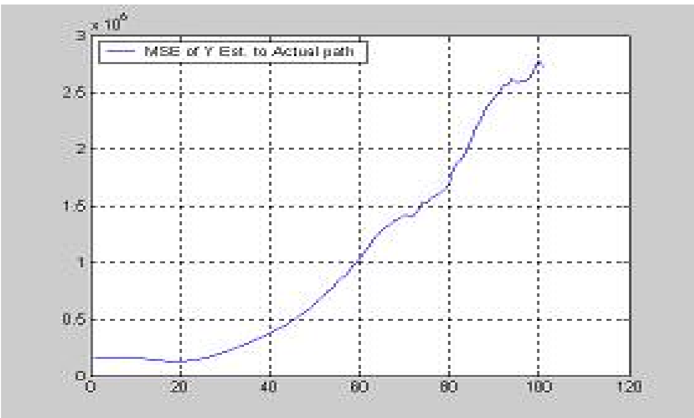

Figure (4c) : Y- Coordinate

Table (1): RMSE of tracking simulated data by SKF and EKF.

Experiment	Coordinates	White Noise Case		Colored Noise Case	
		SKF	EKF	SKF	EKF
1	X	11.3463	4.7218	17.4312	8.7599
	Y	0.0891	0.0683	0.0795	0.0697
2	X	4.5433	4.5730	3.4390	4.4427
	Y	0.0903	0.0853	0.0807	0.0862
3	X	5.7577	1.2668	4.2614	2.4526
	Y	0.0881	0.0432	0.0785	0.0443
4	X	3.1475	1.3232	16.5509	9.2747
	Y	0.0897	0.0830	0.0801	0.0843
5	X	2.5695	1.3505	1.6803	2.6450
	Y	0.0871	0.0581	0.0774	0.0570

On the other side, the experiments of testing Kalman and Extended Kalman filters efficiency in multi-step prediction field include:

1st. The prediction of the first step ahead prediction.
2nd. The prediction of the second step ahead prediction.
3rd. The prediction of the third step ahead prediction.
4th. The prediction of the fifth step ahead prediction.
5th. The prediction of the tenth step ahead prediction.

There are 20 experiments that are executed in each case for testing purpose with considering the type of noise corruption. Tables (2) and (3) show the *RMSE* of X and Y coordinates respectively in case of white noise corruption and using Kalman Filter. Tables (4) and (5) show the same but by using Extended Kalman Filter. Tables 6) and (7) show the *RMSE* of X and Y coordinates respectively in case of colored noise corruption and using Kalman Filter. Tables (8) and (9) show the same but by using Extended Kalman Filter.

Table (2) : RMSE of SKF – white noise X-Coordinate.

Experiment	I	II	III	IV	V
1	24.7863	28.5600	30.3644	34.6816	44.9959
2	24.9299	29.1427	31.4426	38.1890	54.7147
3	26.2601	29.9873	31.9295	36.6011	47.6176
4	28.5263	32.8307	35.5028	42.0494	56.9208
5	28.5227	33.0733	35.7161	42.5007	56.7857
6	28.3444	32.9589	35.1650	40.1801	52.1824
7	28.4157	32.8572	34.9846	4.00754	51.6365
8	26.6288	30.8724	32.8074	37.2006	47.5703
9	27.5625	31.2420	33.8981	41.4813	58.4479
10	26.9283	32.7414	34.6003	39.8615	55.2606
11	29.2511	34.0379	36.8389	44.1533	60.8013
12	24.9002	28.4265	30.1024	33.9268	42.0693
13	25.3926	29.2938	31.4911	37.4243	51.7256
14	25.7761	30.2541	31.7265	34.7637	43.1043
15	29.3561	33.5446	36.0110	42.0797	55.5055
16	27.4133	31.6726	34.1724	40.5395	54.2546
17	24.6605	28.1390	29.7991	33.3900	42.7501
18	26.9503	30.7603	32.8802	38.1084	50.2891
19	27.5004	31.9077	33.8233	38.0610	48.3779
20	26.2926	30.9667	32.8985	38.3313	53.6845s

Table (3) : RMSE of SKF – white noise Y-Coordinate.

Experiment	I	II	III	IV	V
1	74.5357	79.0162	99.6725	200.6943	396.1428
2	74.6282	79.2313	99.9831	201.2338	396.5478
3	74.6548	79.7351	101.1304	203.7786	400.5661
4	74.6180	80.3628	104.1806	215.8569	429.7168
5	74.5964	78.9832	99.6013	200.6046	395.6795
6	74.4210	80.4522	104.4600	216.2184	430.0691
7	74.5217	81.0781	107.0338	225.3556	448.7399
8	74.5608	79.7937	102.3614	209.7110	415.2183
9	74.6395	79.5355	101.1421	205.3378	405.8918
10	74.3138	79.4757	100.1175	199.1358	389.9342
11	74.3978	79.6081	102.2200	210.0292	417.9211
12	74.5367	78.5134	97.8928	194.7231	383.8231
13	74.6976	79.2301	100.2098	202.4789	399.8306
14	74.4214	79.0118	99.8665	201.4861	398.4807
15	S74.4524	80.4641	104.3555	215.4620	426.8662
16	74.4549	78.2272	97.0394	191.7622	376.5992
17	74.6555	79.0734	99.5607	200.2039	395.5398
18	74.6772	80.3387	103.6453	213.1354	421.9922
19	74.5026	80.7652	106.0101	222.1001	442.2814
20	74.6231	80.0514	102.7082	209.9242	415.2079

Table (4) : RMSE of EKF – white noise X-Coordinate.

Experiment	I	II	III	IV	V
1	9.6364	10.2122	10.6581	12.4995	18.5790
2	27.1180	29.6397	31.8673	38.8973	48.8668
3	21.0432	22.0725	23.2777	27.2068	33.6110
4	16.7398	16.9183	17.8539	20.7953	26.3107
5	18.3067	19.6224	20.6797	23.8240	28.2030
6	15.8667	17.0434	17.7179	19.7387	26.1101
7	9.0969	9.8345	10.2269	11.6480	13.9039
8	13.9766	14.5205	15.5115	18.8093	24.1618
9	39.4156	40.7822	43.4195	52.0766	69.2784
10	28.1634	25.4192	28.5258	38.9587	59.1702
11	30.6778	32.9665	34.7916	40.4763	52.6273
12	13.5800	14.3601	14.4434	14.5967	16.4378
13	30.8111	33.6124	35.9856	43.6075	55.7292
14	20.9053	20.2719	21.8785	27.5162	40.6672
15	33.2583	35.8384	37.6051	43.1570	55.2296
16	30.0246	32.3677	33.8184	38.1530	47.5539
17	21.2741	22.8260	24.3747	29.4881	37.9800
18	23.8733	25.7772	27.5220	33.1536	41.9353
19	17.0223	17.5977	18.9014	23.3939	32.1484
20	42.5122	43.9806	47.2851	57.8857	75.5651

Table (5) : RMSE of EKF – white noise Y-Coordinate.

Experiment	I	II	III	IV	V
1	42.9064	61.8278	93.6594	120.5764	322.4355
2	44.7893	68.2836	101.1163	11.3246	311.2834
3	52.3513	80.8995	96.6376	114.9097	293.7123
4	48.6236	76.0868	112.5665	234.7780	453.5914
5	47.4411	68.7042	100.5588	115.3532	315.2723
6	119.7911	155.7720	194.9059	321.1858	547.5791
7	119.7911	155.7720	194.9059	321.1858	547.5791
8	13.6735	27.4044	53.2653	176.2592	390.4042
9	18.1845	29.8250	66.7514	132.7194	340.1024
10	50.7183	76.8864	96.5150	109.1271	286.5120
11	27.1348	45.7390	82.2638	204.3630	420.8200
12	82.7433	105.5766	106.3377	136.2517	288.1373
13	25.6568	44.4479	79.1850	124.5422	330.3243
14	37.5181	47.0143	60.2885	154.1810	360.5416
15	29.5691	52.5155	89.3884	211.6629	428.2476
16	105.7074	115.4435	140.2238	169.3044	263.0721
17	39.9936	64.2022	97.7815	11.6835	313.3356
18	30.5425	36.8032	61.1619	180.4157	394.0607
19	37.5486	88.7525	159.5045	283.1175	505.5137
20	16.7540	30.4790	52.7615	173.7813	386.4308

Table (6) : RMSE of SKF – colored noise X-Coordinate.

Experiment	I	II	III	IV	V
1	33.1171	37.8144	40.3330	44.9660	52.4134
2	31.8524	37.6460	40.3124	45.9214	58.2061
3	35.6793	41.6703	44.8432	52.0225	52.0225
4	36.6381	42.2189	45.5342	51.7706	62.0065
5	35.8536	42.0391	45.3907	52.7301	65.3228
6	37.2856	43.0199	46.1638	52.2872	61.7849
7	37.1018	42.9072	45.7719	51.116	60.0795
8	35.0651	40.9007	43.6404	48.8067	57.8629
9	34.2112	38.7614	41.7728	48.2885	61.3033
10	33.6818	42.0005	44.5170	50.4882	63.6778
11	36.3846	41.6805	44.9955	51.7948	63.3459
12	33.1861	38.0592	40.6717	45.9515	54.2024
13	33.6190	39.4250	42.3376	48.5107	60.7004
14	34.8970	41.2417	43.9186	49.5640	59.8282
15	38.0453	43.3724	46.7617	54.0729	66.3634
16	34.2988	39.3721	42.5291	49.3739	60.9117
17	33.4841	38.8099	41.5704	46.8369	56.5179
18	35.6790	41.4647	44.4649	50.5883	62.0759
19	37.2591	43.3279	46.2645	51.7853	60.8208
20	33.8862	40.9239	43.5654	49.3516	62.8040

Table. (7) : RMSE of SKF – colored noise Y-Coordinate.

Experiment	I	II	III	IV	V
1	84.2446	83.7878	99.3604	192.9785	384.1331
2	83.8954	84.3509	99.5231	193.3022	384.3310
3	84.1522	84.1681	100.4609	195.6755	387.8947
4	84.3084	84.4590	102.9105	207.3653	415.2239
5	83.6744	84.1561	99.2744	192.8705	383.4370
6	84.0600	84.5060	103.1885	207.8301	417.4858
7	84.1517	85.0213	105.7233	216.9484	436.1375
8	84.1500	84.1602	101.5745	201.6371	402.7591
9	83.9746	84.3379	100.2984	196.9469	393.2452
10	83.8880	84.4251	100.3291	192.2687	378.0567
11	83.9571	84.1073	101.2676	201.8004	405.5456
12	83.4438	84.1889	97.8292	187.3352	372.1320
13	83.8904	84.3513	99.7674	194.6050	387.7217
14	83.8096	84.0285	99.7000	194.2748	386.9267
15	84.0128	84.4482	103.0654	206.9392	414.0375
16	83.3011	84.1205	97.1832	184.4509	364.7716
17	83.7554	84.2837	99.1288	192.4077	383.5426
18	84.3134	84.4733	102.5048	204.6153	409.2634
19	84.1129	84.5705	104.4338	213.2796	429.2392
20	84.1563	84.2828	101.7694	201.7532	402.6381

Table (8) : RMSE of EKF – colored noise X-Coordinate.

Experiment	I	II	III	IV	V
1	16.0294	16.5185	16.6959	16.6570	18.9664
2	24.4590	27.4151	29.9056	38.1286	51.1048
3	22.4262	24.0870	25.8035	31.2682	40.9816
4	16.9909	18.5465	19.8550	23.7746	30.8546
5	21.3871	22.2592	22.7503	23.2802	26.1342
6	10.7468	11.3468	11.6439	12.5337	15.6111
7	10.7468	11.3468	11.6439	12.5337	15.6111
8	16.2811	17.1296	18.3416	22.3370	30.1471
9	35.9853	36.8850	39.2439	46.9690	63.2018
10	29.5573	27.1881	30.5602	41.8789	64.1942
11	31.8531	33.5853	34.9381	38.5447	47.1585
12	16.6982	17.3790	17.4199	18.6807	19.5035
13	30.0927	33.4742	36.1845	45.1088	60.1783
14	27.0216	27.1065	28.8432	34.5421	47.2996
15	34.1479	36.3409	37.8337	41.9823	51.5041
16	31.3079	33.2376	34.3652	37.0521	434646
17	22.9804	25.1246	27.0300	33.3078	44.4525
18	23.1033	25.6311	27.7524	34.7451	46.7065
19	21.7204	22.7972	24.2764	29.0965	38.9150
20	42.4575	44.3298	48.0245	60.1479	81.1716

Table (9) : RMSE of EKF – colored noise Y-Coordinate.

Experiment	I	II	III	IV	V
1	47.5225	67.3310	98.7858	118.7594	319.7514
2	48.5743	72.4371	105.0599	109.8487	309.1290
3	55.3096	84.0960	95.0233	118.0164	219.5349
4	44.2918	71.4662	108.2566	231.0824	450.6212
5	51.5862	73.2451	104.8274	114.1571	313.1929
6	46.2421	73.6362	110.3159	232.8483	452.1314
7	115.1772	151.2930	190.6112	317.2812	544.2901
8	11.2888	30.7837	50.2022	173.1390	387.4944
9	16.2042	33.1609	70.3986	129.6329	337.7442
10	54.5637	80.8302	94.7534	112.8565	283.3921
11	27.2312	42.0406	78.2408	200.5338	417.5442
12	87.8749	106.1365	111.5728	141.3061	286.3477
13	28.5394	48.4484	82.7930	122.5852	328.1294
14	64.6381	41.3839	47.6507	151.8982	375.7436
15	24.9489	46.9968	84.5964	207.7881	425.2329
16	108.1526	121.2959	146.1265	175.0919	261.6161
17	43.8632	68.3524	101.6527	110.1403	311.1200
18	26.5329	36.9360	57.2839	177.1971	391.3726
19	84.1700	117.2888	155.1492	279.0195	501.9521
20	15.4143	33.1528	50.2163	170.9027	383.5070

Some cases of missing measurements are considered as:

1st. There is 1 missed measurement each 20 measurements.
2nd. There is 1 missed measurement each 10 measurements.
3rd. There is 1 missed measurement each 5 measurements.

There are 20 experiments that are executed in each case for testing purpose with considering the type of noise corruption. Tables (10) and (11) show the *RMSE* of X and Y coordinates respectively in case of white noise corruption and using Kalman Filter. Tables (12) and (13) show the same but by using Extended Kalman Filter. Tables (14) and (15) show the *RMSE* of X and Y coordinates respectively in case of colored noise corruption and using Kalman Filter. Tables (16) and (17) show the same but by using Extended Kalman Filter.

Table (10) : RMSE of SKF – white noise – Missing – X-Coordinate.

Experiment	I	II	III	IV	V
1	26.3913	27.4392	23.6180	22.6875	22.7658
2	26.9425	27.6615	25.2149	23.6536	21.9877
3	27.1433	28.4999	25.8245	27.3149	27.3580
4	30.5188	31.4993	28.1357	27.3331	25.8917
5	30.0995	31.4169	28.8052	28.6428	28.1583
6	29.8240	31.1205	27.1759	26.7659	27.3427
7	30.0273	31.2313	27.6385	26.5989	25.0897
8	28.1388	29.3027	25.8651	25.3660	24.7192
9	29.1872	30.4177	28.3157	27.2088	24.8491
10	28.1012	29.6956	26.8187	26.16230	29.2209
11	31.0512	32.3968	29.0751	27.6946	27.7766
12	25.9678	27.3022	24.2273	23.6171	22.5262
13	27.2081	28.0054	25.1849	25.0394	23.9743
14	26.7468	28.3149	24.6757	24.6383	26.3296
15	30.4305	31.9805	28.9474	29.4643	28.6133
16	28.7714	30.3826	27.4845	26.7029	27.0230
17	26.0150	27.0375	23.7773	24.0600	23.3676
18	28.5467	29.6705	26.6618	26.7414	25.7221
19	29.0338	30.2700	26.1801	25.9956	26.5331
20	27.9787	29.1427	26.3090	25.7003	26.5013

Table (11) : RMSE of SKF – white noise – Missing – Y-Coordinate.

Experiment	I	II	III	IV	V
1	80.1939	90.5721	96.6992	87.6751	105.7382
2	80.3247	90.6999	96.8041	88.1451	105.6267
3	80.3638	90.7540	96.8118	87.7241	106.0156
4	80.2294	90.5978	96.7324	87.9700	105.5657
5	80.2683	90.6490	96.6990	87.6589	105.9508
6	80.1184	90.4727	96.5535	87.2459	105.9078
7	80.1758	90.5265	96.6242	87.5871	105.8386
8	80.2663	90.6018	96.6823	87.6363	105.7939
9	80.2462	90.6570	96.7675	87.8524	105.9436
10	80.1858	90.4454	96.3627	86.9152	106.0154
11	80.0383	904112	96.4952	87.2408	106.8788
12	80.2307	90.6042	96.6986	87.5414	105.9458
13	80.3158	90.6777	96.8227	88.2469	105.4428
14	80.1673	90.4786	96.5364	87.2829	105.8998
15	80.1424	90.5383	96.6222	87.1615	106.2502
16	80.1429	90.5285	96.5933	87.3385	105.9837
17	80.2910	90.6623	96.7720	87.9897	105.6210
18	80.3062	90.6875	96.8057	88.1361	105.5230
19	80.1791	90.5091	96.6102	87.6390	105.6342
20	80.3557	90.7176	96.7586	87.8251	105.8948

Table (12) : RMSE of EKF – white noise – Missing – X-Coordinate.

Experiment	I	II	III	IV	V
1	10.2642	10.8276	8.3637	12.3574	18.9751
2	27.6891	27.8092	31.0843	34.7369	34.8040
3	20.9511	20.7330	22.4403	25.3826	26.9057
4	16.1089	16.2913	18.8866	21.5112	15.1529
5	18.1358	17.9506	18.7127	18.5713	17.1769
6	16.6538	17.8727	15.4663	15.1960	27.4222
7	8.9041	9.0687	8.7283	8.5746	12.6534
8	14.8387	14.9721	16.1626	20.6117	28.2855
9	40.0629	42.1170	47.5659	52.5921	55.7671
10	31.5394	34.7828	38.9347	49.9034	82.9616
11	31.3721	33.0678	33.2399	31.9693	34.6391
12	13.0096	13.6839	12.6033	10.7706	11.9135
13	31.7796	32.0075	35.9232	41.9341	43.3328
14	23.3884	25.3026	26.5226	34.5347	57.3548
15	33.6938	35.5133	37.7260	38.9413	40.5369
16	30.3286	31.7499	31.3458	29.2883	30.7833
17	22.0571	22.1660	24.9680	30.09894	34.3353
18	24.3709	24.3562	27.6239	32.7189	32.7718
19	18.8618	19.7954	21.2457	28.4504	41.7092
20	44.9058	46.6055	51.2818	59.9466	75.9739

Table (13) : RMSE of EKF – white noise – Missing – Y-Coordinate.

Experiment	I	II	III	IV	V
1	94.1231	94.3147	97.0237	100.3705	101.4174
2	101.8305	102.3341	104.7025	106.7123	108.5408
3	115.7573	116.3359	118.5577	121.1168	123.3288
4	49.0306	49.7532	50.1470	51.3020	53.8813
5	101.4111	101.8559	104.2200	106.9839	108.8181
6	51.7987	52.8745	52.8121	52.0149	55.1177
7	120.9454	122.6307	124.2878	126.7921	132.6327
8	27.4303	27.1548	28.0150	28.8755	28.0887
9	67.5527	68.1556	69.2556	70.8832	74.3118
10	110.0720	110.4211	113.9661	120.9629	126.4050
11	27.1981	27.2674	27.2564	26.6669	26.5755
12	137.3659	138.0182	141.1915	145.4923	148.3128
13	79.6143	79.9104	81.9727	83.1074	84.2127
14	60.6148	60.3316	63.4875	67.9162	68.7128
15	29.5470	30.0574	30.0075	29.2651	30.3599
16	170.8396	171.8229	175.7823	182.1283	186.6777
17	98.4042	98.7817	101.3698	103.5951	104.9922
18	36.7954	36.8316	36.7656	368306	37.0464
19	89.7483	91.3225	91.5794	92.4788	97.5344
20	30.4724	30.2716	31.0755	32.5494	32.1550

Table (14) : RMSE of SKF – colored noise – Missing – X-Coordinate.

Experiment	I	II	III	IV	V
1	26.3913	27.4392	23.6180	22.6875	22.7658
2	26.9425	27.6615	25.2149	23.6536	21.9877
3	27.1433	28.4999	25.8245	27.3149	27.3580
4	30.5188	31.4993	28.1357	27.3331	25.8917
5	30.2294	31.4169	28.8052	28.6428	28.1583
6	29.8240	31.1205	271759	26.7659	27.3427
7	30.0273	31.2313	27.6385	26.5989	25.0897
8	28.1388	29.3027	25.8651	25.3660	24.7192
9	29.1872	30.4177	28.3157	27.2088	24.8491
10	28.1012	29.6956	26.8187	26.1623	29.2209
11	31.0512	32.3968	29.0751	27.6946	27.7766
12	25.9678	27.3022	24.2273	23.6171	22.5262
13	80.3158	90.6777	96.8227	88.2469	105.4428
14	26.7468	28.3149	24.6757	24.6383	26.3296
15	30.4305	31.9805	28.9474	29.4643	28.6133
16	28.7714	30.3826	27.4845	26.7029	27.0230
17	26.0150	27.0375	23.7773	24.0600	23.3676
18	80.3062	90.6875	96.8057	88.1361	105.5230
19	29.0338	30.2700	26.1801	25.9956	26.5331
20	27.9787	29.1427	26.3090	25.7003	26.5013

Table (15) : RMSE of SKF – colored noise – Missing – Y-Coordinate.

Experiment	I	II	III	IV	V
1	80.1939	90.5721	96.6992	87.6751	105.7382
2	80.3247	90.6999	96.8041	88.1451	105.6267
3	80.3638	90.7540	96.8118	87.7241	106.0156
4	80.2294	90.5978	96.7324	87.9700	105.5657
5	80.2683	90.6490	96.6990	87.6589	105.5908
6	80.1184	90.4727	96.5535	87.2459	105.9078
7	80.1758	90.5265	96.6242	87.5871	105.8386
8	80.2663	90.6018	96.6823	87.6363	105.7939
9	80.2462	90.6570	96.7675	87.8524	105.9436
10	80.1858	90.4454	96.3627	86.9152	106.0514
11	80.0383	90.4112	96.4952	87.2408	106.8788
12	80.2307	90.6042	96.6986	87.5414	105.9458
13	80.3158	90.6777	96.8227	88.2469	105.4428
14	80.1673	90.4786	96.5364	87.2829	105.8998
15	80.1424	90.5383	96.6222	87.1615	106.2502
16	80.1429	90.5285	96.5933	87.3385	105.9837
17	80.2910	90.6623	96.7720	87.9897	105.6210
18	80.3062	90.6875	96.8057	88.1361	105.5230
19	80.1791	90.5091	96.6102	87.6390	105.6342
20	80.3557	90.7176	96.7586	87.8251	105.8948

Table (16) : RMSE of EKF – colored noise – Missing – X-Coordinate.

Experiment	I	II	III	IV	V
1	17.5555	17.0853	15.1682	25.1725	18.7450
2	23.2755	25.0183	28.2248	42.3951	32.3957
3	20.7454	21.4786	21.5848	35.6663	24.5986
4	13.2669	12.6900	11.5606	20.3679	15.1669
5	15.5869	16.2587	15.2953	25.3991	15.0817
6	23.2399	23.2737	21.5765	24.3315	28.2286
7	10.3509	10.6264	8.9475	20.2662	11.5381
8	16.1819	17.3597	18.4506	33.9319	26.2295
9	36.7142	37.9250	41.1294	40.2345	57.4816
10	32.6549	36.8820	43.6861	61.8471	81.6525
11	33.9424	34.2107	33.6371	24.8865	37.1514
12	16.8149	16.3926	13.4388	16.1258	13.3160
13	29.3629	31.0142	34.4194	51.2749	40.7973
14	29.3378	31.4057	34.1309	48.9573	56.2781
15	35.3181	35.7188	35.8975	29.8839	42.9762
16	32.8367	32.9971	31.6582	23.4160	33.0944
17	22.3675	23.6233	25.6746	42.8836	31.8492
18	21.8657	23.2326	25.4874	41.6817	30.1751
19	22.9401	24.5623	26.7656	43.3710	40.0014
20	43.7449	47.1790	53.2122	72.1453	73.7570

Table (17) : RMSE of EKF – colored noise – Missing – Y-Coordinate.

Experiment	I	II	III	IV	V
1	98.3307	98.1842	100.3397	102.9562	102.4901
2	105.0658	105.2800	107.5834	106.8200	109.9244
3	118.3511	118.8608	121.2736	122.1843	124.3364
4	45.2303	46.3338	46.7856	50.7546	52.4638
5	105.0536	105.2700	107.7428	103.7084	110.0512
6	47.5372	48.8840	49.2010	48.2097	54.2917
7	117.0389	118.9903	120.7891	124.5776	131.5142
8	30.1164	29.6423	30.3851	30.5520	28.8831
9	71.2582	71.4501	73.0659	71.2380	75.7915
10	113.0608	113.4158	116.7263	126.7124	126.5281
11	27.0836	27.1005	27.0639	26.8555	26.4875
12	141.6565	142.0393	145.0581	148.3069	149.4470
13	82.3828	82.4623	84.3530	82.6889	85.5938
14	63.5856	63.4026	65.9719	72.4932	69.1658
15	25.3077	26.0638	26.0154	26.3209	29.3455
16	176.0158	176.5501	180.2943	186.1934	187.7271
17	101.3483	101.5714	103.9380	104.2823	106.2040
18	36.83337	36.7417	36.6084	36.6565	36.8875
19	86.3055	88.0381	88.8500	89.8823	96.6401
20	32.6420	32.2283	32.7390	34.2691	32.7041

7. Discussions and Conclusions

It is clear from tables and figures that SKF and EKF obtain good estimations of target positions in both white and colored noisy corruption. However, EKF results are more accurate, this means that the consideration of nonlinear nature affects positively on the accuracy, since the maneuvering has nonlinear behavior. In addition, it is clear that the estimated trajectory yielded using EKF is smoother than the one yielded using SKF.

The obtained results for predictions show the high ability of both SKF and EKF in multi-step ahead prediction. It is clear that EKF has higher efficiency than SKF as the number of steps of prediction increases. When the missed measurements appear, it is important to estimate their values before continuing the tracking process. This situation affects the result accuracy negatively. Although, the results show that when using SKF or EKF, the accuracy decreases reasonably. This means that SKF and EKF are stable against missed measurements but EKF has higher ability of estimating the missed data.

The main conclusions, which can be drawn are that both SKF and EKF are good trackers. EKF is more efficient, especially with higher maneuvering, EKF is better than SKF in cases of multi-step ahead prediction and tracking with missed measurements.

References

AL-Jabri, A. M. [2000], A Study of Some Filtering Approaches of Time Series with Applications,Unpublished Ph. D. Thesis, College of Computer Sciences and Mathematics, University of Mousl, Mousl, Iraq.

Kalman, R. E. [1960], A New Approach to Linear Filtering and Prediction Problems, Trans. ASME. J. Basic Eng., Series D 82, PP. 35-45.

Kanjilal, P. P. [1995], Adaptive Prediction and Predictive Control., Peter Peregrins Ltd. London, U K.

Park, Y. H.; Seo, J. H. and Lee G. J. [1995], Tracking Using the Variable-Dimension Filter with Input Estimation., IEEE Transaction on Aerospace and Electronic Systems, Vol. 31, No. 1, PP.399-408.

Welch, G. and Bishop, G. [2001], An Introduction to the Kalman Filter, Report NC2799-3175, Dept. of Computer Science, University of North Carolina, USA.