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Single Index Model with Ordinal Data

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Abstract : Modeling and forecasting ordinal outcomes has become a core study for many statisticians because of the many forms of data encountered in real life, that have such a format. Several authors have proposed different approaches in modeling this type of data either in classical approaches (Mc Cullagh, 1980) or from a Bayesian perspective (Albert and Chib, 1993) (Cowles et al., 1996). One commonly adopted method of modeling ordinal data is that the observed ordinal scores have a correspondence with the latent variable through a set of cut points. Sometimes there are some difficulties in estimation. One of these categories is koto. (Albert and Chib, 1993) proposed an ordinal model in a Bayesian framework that fuzzy-collaborates prior on the cut-point parameters. The approach is used to estimate these parameters through their posterior distribution. We then compare our results after prediction with ordinal logistic regression to see which methods through which we obtain the best estimates.
Keywords: ordinal data, prediction, ordinal results, logistic model.

Introduction: Ordinal scores arise naturally in fields as diverse as environment, economics, finance, and social studies. For example, in social studies in surveying Regarding the level of agreement on a particular question, the results can be recorded as follows:
: 1 for "strongly disagree", 2 for "disagree", 3 for "no opinion", 4 for "agree", 5 for
'Strongly Agree'. The results in this example have an ordinal meaning but no cardinal meaning We usually refer to the words (strongly disagree, disagree, disagree, no opinion, agree, strongly agree) as labels and numbers (1, 2, 3, 4, 5) as values.
There is a large body of work on estimation of ordinal data (see, for example, Albert and Chip 1993; Chen and Dai 2000; Albert and Chip 2001; Kutas, Mueller, and Quintana 2005; Jeliazkov, Greaves, and Kutzbach 2008). Apart from the average regression, there is also...
Studies on(QR) for ordinal data (see Hong and He 2010; Zhou 2010; Hong and Zhou 2013). Compared with mean regression, QR models belong to a strong model family (Conker 2005).where as there is No distributional assumption is imposed on the error term. However, the conditional quantity of the error term is zero.
Asingle index for ordinal data model can be determined based on a continuous hafent variable y_i^* as follows:

$$y_i^{*}= g\,(x_i'\beta) + \varepsilon_i \;,\; i=1,2,\ldots\ldots,n \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1)$$

Where y_i^* is the unobserved variable reletes for the observed discrete response variable y_i, x_i is a covariates vector with $k\times 1$ dimenstion ,β is $k\times 1$ index parameter vector ,$g(.)$is the unknownnon parameteric function and ε_i is the error term follows normal distribution with $N(0,\sigma^2)$. Assume that the categories or outcome are, J

$$Y_i=j \;,\; \delta_{j-1} < y_i^* \le \delta_j \;\forall\, i = 1,2,\ldots\ldots n, j = 1,\ldots\ldots J$$

Wher $\delta_j, j = 1,2,\ldots\ldots J$ are cut-point ,where its satisfy the coordinates $-\infty = \delta_0 < \delta_1 < \cdots < \delta_{j-1} < \delta_j = \infty$
In this study, Gaussian process set as aprior distribution for the non parametric function with zero mean and variance-covariance matrix $E(.,.)$
Therefore ,we can show as follows.
$g(.)\sim(0, E(.,.))$

$$E(\beta' x_i, \beta' x_j)=\Upsilon \exp\;\{-(x_i'\beta - x_j'\beta)^{\,2}/h\}\ldots\ldots\ldots\ldots 2)$$

Where Υ and h are unknown hyparameters δ_0 that the prior distribution et al (2011)and Gramcy and lain (2012) can be shown as:

$$\prod(h/\beta,\Upsilon) = \det\,[En]^{-\frac{1}{2}} \exp\,[-\frac{gn' E^{-1} gn}{2}]$$

Many researchers assum that $\|\beta\|=1$ for identifiable. Where as Gramcy and lain(2012) mention that ,this codition is unnecessary when Gaussion process used as aprior distribution .
They indicated $(\beta/\sqrt{h})$ is identifiable without the constriaant $\|\beta\|=1$, we will replaced $\beta/\sqrt{h}$ by the parameters vector β which is identifiable then variance covariance matrix is:

$$E(\beta' x_i, \beta' x_j)=\Upsilon \exp\;\{-(x_i'\beta - x_j'\beta)\}$$

Invars Gamma set as aprior to the hyperparameter Υ, $\Upsilon\sim IG(a,b)$where a,b Laplace distribution put as prior distribution for the coefficient vector β_j, j=1,2,…..p

$$\prod(\beta/\lambda,\tau) = \frac{\lambda}{2\tau}\ \exp\{-\frac{\lambda|\beta|}{\tau}\} \ldots\ldots\ldots\ldots\ldots\ldots 3)$$

Where$(\lambda > 0)$ is apenalty parameter follow Andrews and Mallows(1974).Laplace distribution can represent as ascale mixture of normal and exponential density for any $\theta \geq 0$,then

$$\frac{\theta}{2}\ e^{-\theta|z|} = \int_0^{\infty} \frac{1}{\sqrt{2\pi s}} \exp\{-\frac{z^2}{2s}\} \frac{\theta^2}{2} \exp\ \{-\frac{\theta^2}{2}\ s\} ds \ldots\ldots 4)$$

So, Based onthis equality ,Let $\theta = \frac{\lambda}{\tau}$,then

$$\pi(\beta/\theta) = \prod_{k=1}^{p} \frac{\theta}{2}\ \exp\{-\theta|\beta|\} \ldots\ldots\ldots\ldots\ldots\ldots 5)$$

$$= \prod_{k=1}^{p} \frac{1}{\sqrt{2\pi\, s_j}} \exp\{-\frac{\beta_j^2}{2s_j}\} . \frac{\theta^2}{2} \exp\left\{-\frac{\theta^2}{2} s\right\} ds \ldots\ldots\ldots 6)$$

On the other side ,for the unobserved dependent variable$y_i^*$the error termdistributed as normal with mean zero and varianc σ^2

Then the cumulative distribution function for the j category of observed respose y_i is given as:

$P(y_i \leq j/y_i^*, \delta j) = p_r(y_i^* \leq \delta_j/\beta)$

$= p_r(gx_i'\beta) + \varepsilon_i \leq \delta_j)$

$= p_r(\varepsilon_i \leq \delta_j - g(x_i'\beta))$

$= p_r(\frac{\varepsilon_i}{\sigma_\epsilon} \leq \frac{\delta j - g(x_i'\beta)}{\sigma_\epsilon})$

$= \emptyset\left(\frac{\delta_j - g(x_i'\beta)}{\sigma\ \epsilon}\right)$

Based on $\emptyset$, we have

$P_r(\delta_{\ j-1} < \ y_i^* \leq \delta_j/\beta)$

$= \emptyset\left(\frac{\delta_j - (g(x_i'\beta)}{\sigma_\epsilon}\right) - \emptyset(\frac{\delta_{\ j-1} - g(x_i'\beta)}{\sigma_\epsilon})$

As same as the metivn by sorenser et al.(1995) Montesinos-Lopez et al.(2015) and Alhamzawi and Ali (2018),an order statistics from $U(\delta_{\ min}, \delta_{max})$ distribution for the J-1unknown cut-point .

$P(\delta) = (J-1)!\ (\frac{1}{\delta min - \delta \max})^{\ J-1} I(\delta_0)$

Where $\delta = (\delta_1, \delta_2, \ldots \ldots, \delta_J)$ and $Z=\{(\delta_{min}, \delta_{1,\ldots\ldots}, \delta_{max})/\delta_{min} <, \ldots\ldots < \delta_{max})\}$

Then the hierarchical Bayesian for single index model when the response variable is ordinal can be shown as:

y_i^*/β, g ,ɤ$\sim N(g(x_i'\beta), \sigma^2)$

Yi=j if $(\delta_{j-1} < y_i^* \leq \delta_j)$

g/β,ɤ$\sim N\ (0, E)$

$\beta/\theta, S \sim \int_0^{\infty} \frac{1}{\sqrt{2\pi\, s_j}} \exp\left\{-\frac{\beta_j^2}{2\, s_j}\right\} . \frac{\theta^2}{2} \exp\left\{-\frac{\theta^2}{2} s\right\} ds$

ɤ$\sim IG(a, b)$

$\sigma^2 \sim IG(c, d)$

$\theta \sim IG(a, b)$

2.Posterior Distribution

We can write the full condition posterior distribution as follws:

P(β,g,ɤ,$\delta, \theta, \sigma^2/y_i^*) \propto$ p($y_i^*/\beta, \delta, g$)p(g/ɤ,β)

P(β,s/θ) p(ɤ) p(θ) p(σ^2) p(δ)

$\propto [\prod_{i=1}^{n} \prod_{j=1}^{J} I\{\delta_{j-1} < y_i^* < \delta j\} \frac{1}{\sqrt{\sigma^2}} \{\exp\{-\frac{(y_i^* - g(x_i'\beta)\ ^2}{2\sigma^2}\}$

$\propto |E|^{-\frac{1}{2}} \exp\ \{-\frac{g\ E^{-1} g}{2}\} \prod_{k=1}^{p} \frac{1}{\sqrt{2\pi\, s_k}} \exp\left\{-\frac{\beta_k^2}{2\, s_k}\right\} \frac{\theta^2}{2} \left\{\frac{\theta^2}{2}\ s_k\right\}$

$\propto [\frac{1}{ɤ}]^{\ a+1} \exp\{-\frac{b}{ɤ}\} \times [\frac{1}{\theta}]^{\ c+1} \exp\{-\frac{d}{\theta}\} . \left\{\frac{1}{\sigma^2}\right\}^{\ a_{1+1}} . \exp\{-\frac{b1}{\sigma^2}\}$

$\propto (J-1)!\ [\frac{1}{\delta_{max} - \delta_{min}}]^{\ J-1} I(\delta \in R)$

Where I{.} is anindicator function .In the for we will summarizd the MCMC algorithm:

1-Truncated normal distribution $TN(\delta_j, \delta_{j-1})(g(x_i'\beta, \sigma^2)$ will be u sed to sample the latent variable y_i^*.

2-The full conditional posterior for the cut points δ will compute as follows

$P(\delta_j/y_i) \propto p(y_i/\delta_j).p(\delta_j) \propto \prod_{i=1}^{n} \sum_{j=1}^{J} I(y_{i=}j)\, I(\delta_{j-1} < y_i^* \leq \delta_j)\, I(\delta \epsilon R)$

Therefore the full conditional of δ_j is a uniform and it will be draw from
$P(\delta_j/y_i)=\frac{1}{\min((y_i^*|y1=j+1)-max(y_i^*|y=j)}\, I(\delta \epsilon R)$

3-To sample g we will use:
$\prod(g/\beta, y^*, \theta, \Upsilon, \delta) \propto p(y^*/\beta, g, \sigma^2)^{\times} \prod(g/\beta, \Upsilon)$
$\propto |D|^{-\frac{1}{2}} \exp\left\{-\frac{(y_i^*-gn)' D^{-1}(y_i^*-gn)}{2}\right\} \times |E|^{-\frac{1}{2}} \exp\{-\frac{gn' E^{-1} gn}{2}\}$
Then gn will by sampling form normal distribution N(A, B) where
$A=E(E+D)^{-1} y^*$
$B=E(E+D)^{-1}(D)$
4-sampling the coefficient vector β from the following posterior

$$\prod(\beta/g, y^*, \theta, \Upsilon, \delta, \sigma^2) \propto p(y^*/g, \beta)\, p(g/\beta, \Upsilon) \pi(\beta/\delta, \theta)$$
$$\propto |D+E|^{-\frac{1}{2}} \exp\{-\frac{y_i^{*'}(D+E)^{-1} y^*}{2}\}. \prod_{k=1}^{p} \exp \frac{\beta_k^2}{2s_k}$$
Metropolis Algorithm will be used to sampl β.

5-sampling Υ
$\pi(\Upsilon/g, \beta, y^*, \theta, \delta, \sigma^2) \propto \pi(y^*/g, \beta, \sigma^2). \pi(g/\beta, \Upsilon). \pi(\Upsilon)$
$$\propto [|D+E|^{-\frac{1}{2}} \exp\left\{-\frac{y^*(D+E)^{-1} y^*}{2}\right\} \times (\frac{1}{\Upsilon})^{a+1} \exp\{-\frac{b}{\Upsilon}\}$$
Metropolis Agorithm will use to sample Υ.

6-samplig θ
$\pi(\theta/g_n, \beta, \sigma^2, \delta, \Upsilon) \propto \pi(s_{k/\theta}). \pi(\theta)$
$\propto \prod_{k=1}^{p} \frac{\theta}{2} \exp\{-\frac{\theta^2}{2} s_k\}. \theta^{a1+1} \exp\{-b_{1,\theta}\}$
The conditional posterior for θ is $Ga(a_1+2p, b_1+\frac{\theta}{2})$

7-samplings_k
$\pi(s_k/y_i^*, g_n, \beta, \theta, \delta, \sigma^2) \propto \pi(\beta/s_k). \pi(s_k/\theta)$
$$\propto (2\pi s_k)^{-\frac{1}{2}} \exp\ \{-\frac{\beta^2}{2s_k}\} \exp\{-\frac{\theta^2}{2} s_k\}$$
Then the conditipnal posterior is Generalizwd inverse Gaussian (GIG)
$(s_k)^{-\frac{1}{2}} \exp\{-\frac{1}{2}\theta^2 s_k + \beta_k^2 s_k^{-1}\}$

8-sampling σ^2
$\pi(\sigma^2/g_n, \beta, \Upsilon, \theta, \delta) \propto \pi(y^*/g_n, \beta) \pi(\frac{1}{\sigma^2})$
$\propto \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\{\sum \frac{(y_i^*-g_n)^2}{2\sigma^2}\}\ .(\frac{1}{\sigma^2})^{c+1} \exp\{-\frac{d}{\sigma^2}\}$
$$\propto (\frac{1}{\sigma^2})^{\frac{n}{2}+c+1} \exp\{-\frac{1}{\sigma^2}\}[\frac{1}{2}\sum (y_i^* - g_n)^2 + d)\}$$
Invers Gamma is the posterior distribution for σ^2

3. Simulation of Ordinal Single Index Model .

In this part of chapter three we conducted two simulation example to check the accuraty of our proposed method Bayesian ordinal SIM and compar results with the method (BOSI) of Bayesian Ordinal Quantil regression (BOQR) with quarfole level (0.5).

3.1-Example One

The dataset in this example are generated from the following regression form:

$$y^* = \sin(4z) + 0.1\varepsilon, \quad y_i = \begin{cases} 1 & if \quad -\infty < y_i^* \le 0 \\ 2 & if \quad 0 < y_i^* \le 0.500 \\ 3 & if \quad 0.500 < y_i^* \le 1 \\ 4 & if \quad 1 < y_i^* \le \infty \end{cases}$$

where $z = \boldsymbol{x}'\boldsymbol{\beta}$, $\boldsymbol{x}$ is the design matrix with dimension 5 columns, five independent variables and sample size *n=25,50,100 and 150* $x_i \sim Unif[0,1]$, $(i = 1,2,..,5)$, $\boldsymbol{\beta} = (1,2,0,0,0)/\sqrt{5}$, the quantile coefficient β_τ will be estimated for the five different quantile levels $\tau = 0.05, 0.20, 0.50, 0.70$ and 0.95, and the error term will be considered with a mixed distribution $\varepsilon \sim 0.90\,\mathrm{N}(0,1) + 0.10 Cauchy(0,1)$ (Kuruwita, 2015). Table 3-1 shows a brief summary of the parameter estimates for simulation example one.

Table 3-1 the parameter estimates of simulation example one ordinal SIM

Sample Size	Methods	β_1	β_2	β_3	β_4	β_5
N=25	BOSI	0.41041	0.33152	0.61258	0.60940	0.24068
	BOQR	0.11910	1.43240	-1.57336	-0.69546	0.34936
N=50	BOSI	0.36830	0.39799	0.37641	0.44651	0.51080
	BOQR	-0.30758	0.69392	0.06304	-0.41690	1.44962
N=100	BOSI	0.12966	0.13797	0.51014	0.36915	0.38501
	BOQR	0.70080	-0.24725	0.25290	0.14005	0.08985
N=150	BOSI	0.35272	0.50031	0.37536	0.61735	0.53729
	BOQR	-0.85395	-0.38165	1.17395	0.91055	0.18315

The simulation results in Table $(3-1)$ including the parameter estimates of the ordinal single index model (SIM)over sample sizes $(25,50,100,150)$ It can be observed that the proposed method $(BOSI)$performs better than other methods $(BOQR)$ especially when the sample size getting bigger, where the true value of $\beta_1 = 0.44$. As the values for estimated bias for BOSI and BOQR in methods in ordinal.

In Table (3-2) Estimated Bias for simulation example one using different methods.

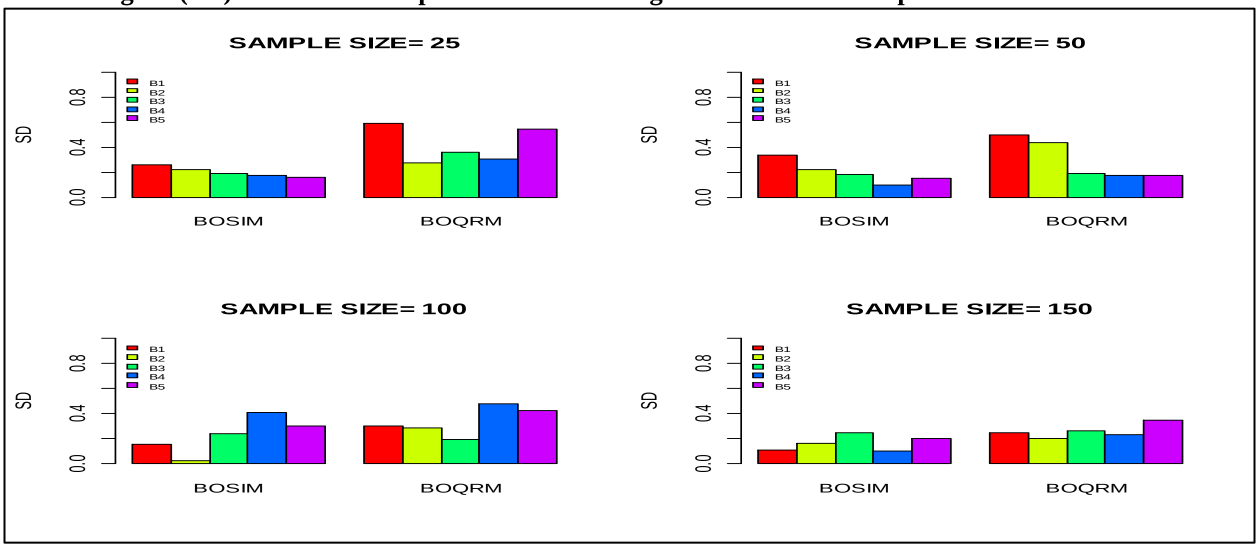
Table 3-2 Estimated Bias for simulation example one using different methods

Sample Size	Methods	Bias (β_1)	Bias (β_2)	Bias (β_3)	Bias (β_4)	Bias (β_5)
N=25	BOSI	0.03681	0.56291	0.61258	0.60940	0.24068
	BOQR	0.32811	0.53797	1.57336	0.69546	0.34936
N=50	BOSI	0.07892	0.49644	0.37641	0.44651	0.51080
	BOQR	0.75479	0.20051	0.06304	0.41690	1.44962
N=100	BOSI	0.31756	0.75645	0.51014	0.36915	0.38501
	BOQR	0.25359	1.14168	0.25290	0.14005	0.08985

N=150	BOSI	0.09450	0.39412	0.37536	0.61735	0.53729
	BOQR	1.30116	1.27608	1.17395	0.91055	0.18315

table $(3-2)$ show that the obtained bias values from our proposed method (BOSI) is much smaller at different sample sizes than the competing method $(BOQR)$ for all the five parameter estimates . We can see that as the sample size become more larger, the proposed method $(BOSI)$ yields lower bias values, which indicated that $(BOSI)$ method peforms well .
Moreover, we draw the valnes of the standard errors for the estimated values of the parameters estimates $(\beta_{1-}\beta_5)$ along with BOSI and BOQR methods.

Figure (3-1) the SD value of parameter estimating for simulation example one in Ordinal SIM



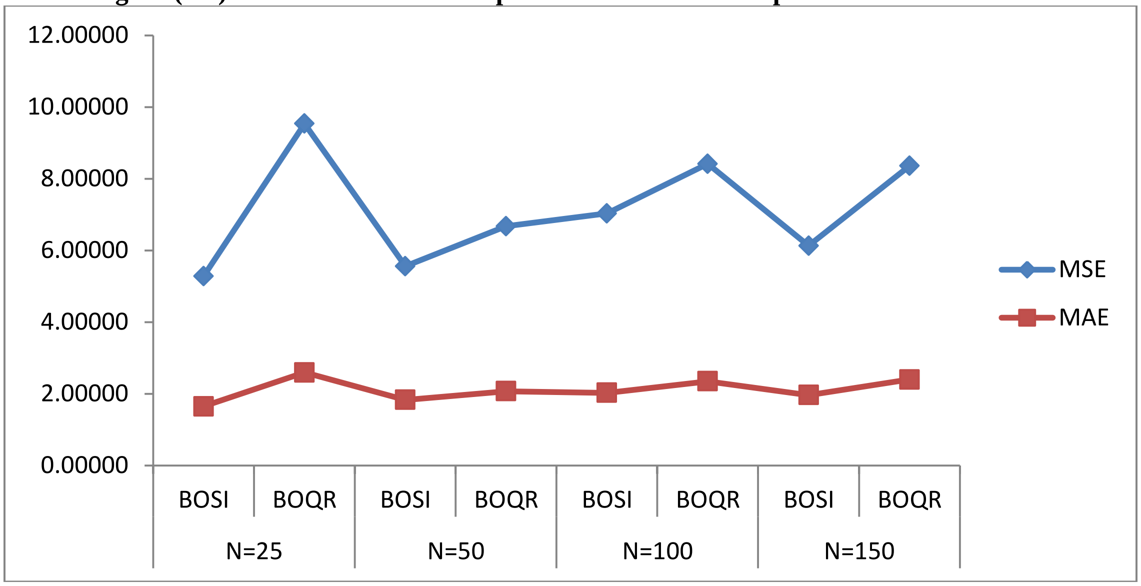
It is ving clear from figures $(3-1)$ That the value of SD for the proposed method $(BOSI)$ Are less than the values of SD that obtaimal from $(BOSI)$ Method , and that indicates the good from $(BOSI)$ Perfrom of the propted method.

Table 3-3 MSE and MAD of simulation example one

sample size	Methods	MSE	MAE
N=25	BOSI	5.27339	1.64769
	BOQR	9.53420	2.59238
N=50	BOSI	5.55599	1.82967
	BOQR	6.67400	2.07213
N=100	BOSI	7.03031	2.02433
	BOQR	8.41190	2.34842
N=150	BOSI	6.12743	1.96520
	BOQR	8.35379	2.39524

The results of MSE and MAD that listed in table $(3-3)$ shows that the proposed method $(BOSI)$ performs better than $(BOQR)$ methods ocer all different sample size . However , we observed that $(BOSI)$ method tends to be the best meted in terms of the values of MSE and MAD Criteria.

Figure (3-2) MSE and MAD values polts of simulation example one for Ordinal SIM



In Figure (3-2) the values of MSE and MAD Criteria are plotted against the methods $(BOSI, BOQR, BOSI)$It can be observed that the values of MAE criterion with different methods are better than the performance of MSE criterion .The closer value to zero the better perform. But in estimed the resulte of the popcered method $(BOSI)$ *are befte in them of MAE and MSE ontrm*

3.3-Example Two :

In this example, the samples size we considers *n=25,50,100* and 150 observations are generated from the regression model:

$$y^* = \sin\left(\frac{\pi(z-A)}{B-A}\right) + 0.5\varepsilon \ , \quad y_i = \begin{cases} 1 & if \quad -\infty < y_i^* \le -2 \\ 2 & if \quad -2 < y_i^* \le 0 \\ 3 & if \quad 0 < y_i^* \le 2 \\ 4 & if \quad 2 < y_i^* \le 4 \\ 5 & if \quad 4 < y_i^* \le \infty \end{cases}$$

where $= \boldsymbol{x}'\boldsymbol{\beta}$, $\boldsymbol{x}$ is the design matrix $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5)'$, $\boldsymbol{\beta}$ is the coefficient vector $\boldsymbol{\beta} = (1,1,0,0,1)/\sqrt{3}$, and the $x_i \ (i = 1,2,3,4,5)$ are i.i.d in a uniform distribution $[0,1]^5$. A and B are constants that can be shown to be $(\frac{\sqrt{3}}{2} + \frac{1.645}{\sqrt{12}}, \frac{\sqrt{3}}{2} - \frac{1.645}{\sqrt{12}})$ respectively. ε is the error term, and we consider density distributions of the error term to evaluate the robustness of our proposed approach (Benoit et al., 2013).

$$\varepsilon \sim N(0,1),$$

In each level 10,000 iterations are run in the MCMC algorithm with 2,000 burn-in Table (3-4) shows abrief summing of the parameter estimates for simulation example two

Table 3-4 the parameter estimates of simulation example two

Sample Size	Methods	β_1	β_2	β_3	β_4	β_5
N=25	BOSI	0.33446	0.62063	0.20104	0.41123	0.67259
	BOQR	1.68870	1.36485	0.13475	3.14425	0.38655
N=50	BOSI	0.49593	0.08846	0.50133	0.64144	0.31773
	BOQR	1.07100	1.94110	-1.04940	0.81170	3.01440
N=100	BOSI	0.44483	0.40846	0.38780	0.44186	0.45855
	BOQR	0.77743	0.80620	1.96608	1.02379	-0.03009

N=150	BOSI	0.55623	0.32565	0.42486	0.49516	0.40414
	BOQR	1.49374	1.16375	0.92632	0.64043	0.92769

The simulation results in Table (3-4) including the parameter estimation of the ordinal single index model (SIM) with four sample sizes $(25,50,100,150)$ It can be seen that the proposed method $(BOSI)$ performs better than other methods $(BOQR)$ especially when the sample size getting larger, where the true value of $\beta_1 = 0.58$ with in sample size $(n = 150)$ the $(\hat{\beta_1} = 0{,}55)$ parameter estimates getting closer to the true value as sample size getting bigger . As the sample size become more larger, the parameter estimates are close to true values with the proposed method .In Table 3-4 the value of estimated bias for different sample size with different estimation method of Ordinal SIM .Further more,we calculate the estimated values of the quality criteria ,MSE and MAD.

Table (3-5) shows the values of the MSE and MAD Criteria.

Table 3-5 Estimated Bias for simulation example one using Ordinal SIM

Sample Size	Methods	Bias (β_1)	Bias (β_2)	Bias (β_3)	Bias (β_4)	Bias (β_5)
N=25	BOSI	0.24289	0.04327	0.20104	0.41123	0.09524
	BOQR	1.11135	0.78750	0.13475	3.14425	0.19080
N=50	BOSI	0.08142	0.48889	0.50133	0.64144	0.25963
	BOQR	0.49365	1.36375	1.04940	0.81170	2.43705
N=100	BOSI	0.13252	0.16890	0.38780	0.44186	0.11880
	BOQR	0.20008	0.22885	1.96608	1.02379	0.60744
N=150	BOSI	0.02112	0.25170	0.42486	0.49516	0.17321
	BOQR	0.91639	0.58640	0.92632	0.64043	0.35034

from table $(3-5)$ that the obtained bias values from our proposed method $(BOSI)$ is much smaller at different sample sizes than the competing method$(BOQR)$ for all parameter estimates .Also we can say that the prosed method $(BOSI)$ performs better than other method .cosquently, as the sample size become more larger, the proposed method $(BOSI)$ gives lower bias values, which suggesting a good performance of$(BOSI)$ method.

Next ,we draw the SD values for the estimated parameters $(\beta_{1-}\beta_5)$

With BOSI and BOQR methods.

Figure (3-3) MSE and MAD values plots of simulation example two

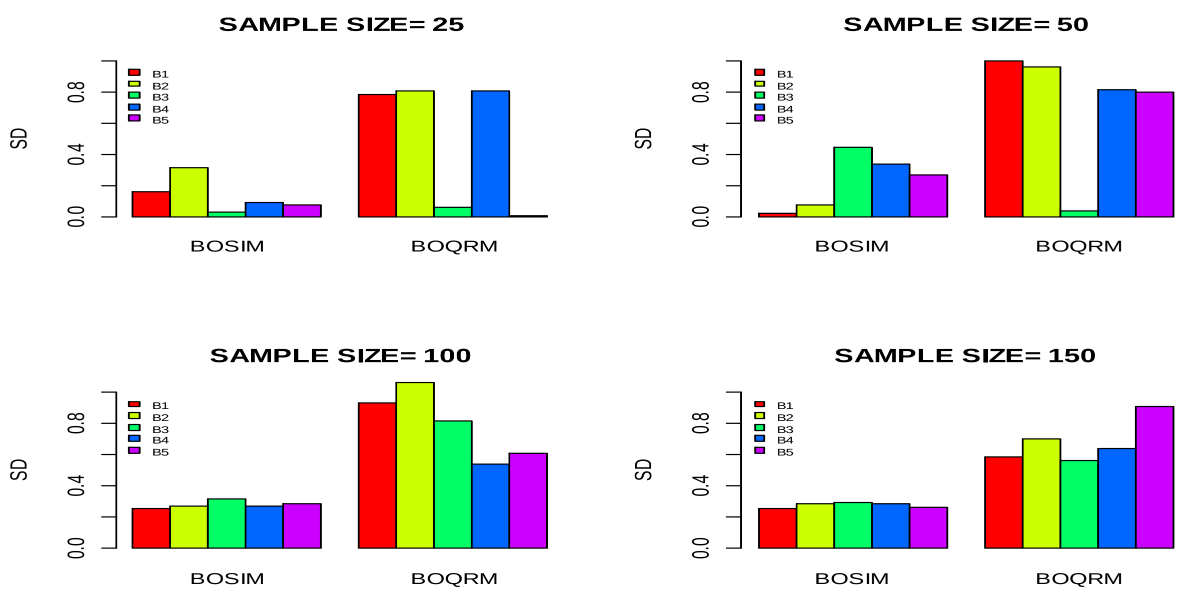


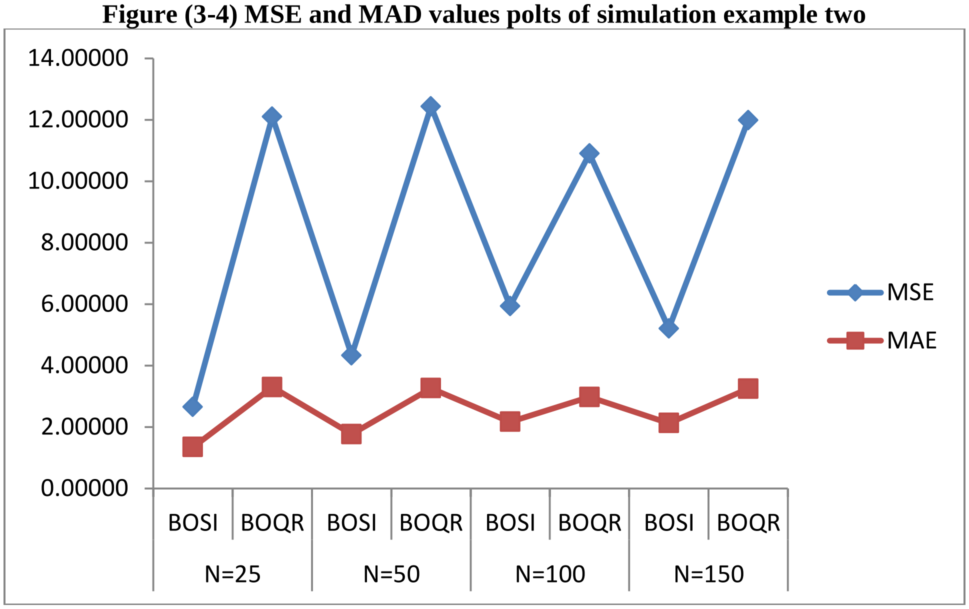
Figure (3-3) we can see that the SD result from proposed method (BOSI) are less then the values of SD criterim that obtainal from (BOQR) method ,this indicates the well performing of proposed method (BOSI) in terins of standard prror for the parameters estimates.
Next table $(3-6)$ Show the values of MSE and MAE measures under different sample sizes.

Table 3-6 MSE and MAD of simulation example two for Ordinal SIM

sample size	Methods	MSE	MAE
N=25	BOSI	2.64807	1.34439
	BOQR	12.09748	3.29586
N=50	BOSI	4.32750	1.76307
	BOQR	12.43013	3.25683
N=100	BOSI	5.93040	2.16584
	BOQR	10.89384	2.97045
N=150	BOSI	5.20379	2.12142
	BOQR	11.98434	3.24380

The results of MSE and MAD that obtaned in table (3-6) indicates that the proposed method (BOSI) generally performs better than $(BOQR)$ method overall different sample size . However , we observed that (BOSI) method tends to be have better than the other method in terms of the values of MSE and MAD Criteria. To summary the values of MSE and MAE we draw the following figures.

Figure (3-4) MSE and MAD values polts of simulation example two



In Figure (3-4) the values of MSE and MAD Criteria are plotted against the methods (BOSI) and(BOQR) .It can be observed that the values of MAE criterion with different methods are better than the performance of MSE criterion .The closer value to zero the better perform.

4. Conclusions

1. the Ordinal SIM, the proposed method (BOSI) performe better in terms of MSE and MAE values.

5. Recommendations

1.we recommerd the usage of (BBSI) in Binary Single Index Model and (BOSI) method when dealing with Ordinal SIM because the flexibility of these model.

2.we also recommend and suggest the of Bayesian semi parametric logistic regression ueder the same hierarichal prior distributions.

4. Apply the proposed method in medical research labs to support the medical staff with the necessary information that help patients to recover as soon as possible

6-Sources

1.Antoniadis, A., Gr_egorie, G., and McKeague, I. (2004). \Bayesian Estimation of Single-Index Models." Statistica Sinica, 14, 1147{1164.

2.Bastos, L. and O'Hagan, A. (2009). \Diagnostics for Gaussian Process Emulators." Tech-nometrics, 51, 4, 425{438.

3.Brillinger, D. (1977). \The identi_cation of a particular nonlinear time series system."

3.Biometrika, 64, 509{515.20 | (1982). \A generalized linear model with \Gaussian" regressor variables." In A Festscrift

for Erich L. Lehman, eds. P. Bickel, K. Doksum, and J. Hodges, 97{114. New York:

Wadsworth.

4.Broderick, T. and Gramacy, R. B. (2010). \Classi_cation and Categorical Inputs with Treed Gaussian Process Models." Journal of Classi_cation. To appear.

5.Carvalho, C., Johannes, M., Lopes, H., and Polson, N. (2008). \Particle Learning and Smoothing." Discussion Paper 2008-32, Duke University Dept. of Statistical Science.

6.Chipman, H., George, E., and McCulloch, R. (2002). \Bayesian Treed Models." Machine Learning, 48, 303{324.

7.Choi, T., Shi, J., and Wang, B. (2011). \A Gaussian process regression approach to a single-index model." Journal of Nonparametric Statistics, 23, 21{36.

8.Craig, P. (2008). \A new reconstruction of multivariate normal orthant probabilities." Jour- nal of the Royal Statistical Society: Series B, 70, 227{243.Friedman, J. and Stuetzle, W. (1981). \Projection Pursuit Regression." Journal of the American Statistical Association, 76, 817{823.

9.Gramacy, R. and Lee, H. (2010). \Optimization under unknown constraints." In Proceedings of the ninth Valencia International Meetings on Bayesian Statistics, eds. J. Bernardo, S. Bayarri, J. Berger, A. Dawid, D. Heckerman, A. Smith, and M. West. Oxford University

10.Gramacy, R. and Polson, N. (2010). \Particle learning of Gaussian process models for sequential design and optimization." Tech. Rep. arXiv:0909.5262, University of Cambridge.