A note on an R–module with $(m, \ n)$-pure intersection property

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Abstract

Let R be a ring. Given two positive integers m and n, an R module V is said to be (m,n)-presented, if there is an exact sequence of R -modules $0 \to K \to R^m \to V \to 0$ with K is n-generated. A submodule N of a right R -module M is said to be (m,n)-pure in M, if for every (m,n)-Presented left R -module V the canonical map $N \otimes_R V \to M \otimes_R V$ is a monomorphism. An R -module M has the (m,n)-pure intersection property if the intersection of any two (m,n)-pure submodules is again (m,n)-pure. In this paper we give some characterizations, theorems and properties of modules with the (m,n)-pure intersection property.

Key words:- (m,n)-pure submodule, (m,n)-flat module, module with (m,n)-pure intersection property.

Introduction

Throughout, this paper, R is an associative ring with non-zero identity, and all modules are unitary right R-modules. A submodule N of an R-module M is called pure submodule, if for every finitly generated ideal I of R $MI \cap N = NI$ [1]. Following [2], an R-module M has the PIP, if the intersection of any two pure submodules is again pure. For an abelian group G, we write $G^{m\times n}$ for the set of all formal $m\times n$ matrices with entries in G and write G^n (resp. G_n) for $G^{1\times n}$ (resp. $G^{n\times 1}$). For two position integer m,n. A submodule N of an R-module M is (m,n) –pure in M if and only if $MI \cap N^m = NI$, for all n–generated submodule I of $_R R^m$ [3]. An R-module M is $(m, \ n)$-flat, if $1_M \otimes L_I \ : \ M \otimes_R I \to M \otimes_R R^m$ is monomorphism for all n-generated submodule I of $_R R^m$ [4]. In this paper, for two fixed positive integers m and n, we introduce the concept of an R-module M has $(m, \ n)$-PIP. We prove that if M is an R-module such that for any two $(m, \ n)$-pure submodules A and B of M, $A+B$ is $(m, \ n)$- flat R-module, then M has the $(m, \ n)$-PIP.

Propeties of module which has $(m, \ n)$-PIP

Definition 2.1:- An R-module M has the $(m, \ n)$-pure intersection property (briefly $(m, \ n)$-PIP) if the intersection of any two (m,n)-pure submodules is again (m,n)-pure. An R-module M has the $(m,*)$-PIP (resp. $(*,n)$) if for all positive integer n (resp. m) M has the $(m, \ n)$-PIP.

It is clear that if M has the $(1,1)$ -PIP, then M has the PIP. The converse is not true.

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Proposition 2.2:-
(1) If an R-module M has the (m, n)-PIP, then every (m, n)-pure submodule of M has the (m, n)- PIP .
(2) Let N be (m, n)-pure submodule of an R-module M . M has the (m, n)-PIP, if and only if $\frac{M}{N}$ has the (m, n)-PIP .

Proof:- (1) trivial
(2) $\Rightarrow$) Let $\frac{A}{N}$ and $\frac{B}{N}$ be (m, n)-pure submodules of $\frac{M}{N}$.We want to show that $\frac{A}{N} \cap \frac{B}{N} = \frac{A \cap B}{N}$ is (m, n)-pure. Now, A and B are (m, n)-pure in M [3,Proposition(1.9-4)]. And since M has the (m, n)-PIP, then $A \cap B$ is (m, n)-pure. Then $\frac{A \cap B}{N} = \frac{A}{N} \cap \frac{B}{N}$ is (m, n)-pure in $\frac{M}{N}$ [4,Proposition(1.9-3)].
$\Leftarrow$) Let E and F are (m, n)- pure submodules in M .Then $\frac{E}{N}$ and $\frac{F}{N}$ is (m, n)-pure submodules of $\frac{M}{N}$. [3,Proposition(1.9-3)]. Since $\frac{M}{N}$ has the (m, n)-PIP, then $\frac{E}{N} \cap \frac{F}{N} = \frac{E \cap F}{N}$ is (m, n)-pure in $\frac{M}{N}$ .hence $A \cap B$ is (m, n)-pure in M [4,Proposition (1.9-4)]. Thus M has the (m, n)-PIP.

Remarks 2.3:-
(1) Let A and B be R-modules, it is clear that $(A \cap B)^m = A^m \cap B^m$
(2) Every summand submodule N of an R-module M is (m, n)-pure submodule.

In [2, CH 2, theorem 3.1] an R-module M has the PIP, if and only if $(A \cap B)I = AI \cap BI$ for every finitely generated ideal I of R and for every pure submodules A and B of M .

Theorem 2.4:- Let M be an R-module. Then M has the (m, n)-PIP, if and only if $(A \cap B)I = AI \cap BI$ for every $n-$generated submodule I of ${}_R R^m$ and for every (m, n)- pure submodules A and B of M .

Proof:- Suppose that M has the (m, n)-PIP and let each of A and B is (m, n)-pure. Let I be n-generated submodule of ${}_R R^m$, then $(A \cap B)^m \cap MI = AI \cap BI$ ${}_R R^m$, [3,theorem (1.5)]. It is clear that $(A \cap B)I \subseteq AI \cap BI$ .But $AI \cap BI \subseteq (MI \cap A^m) \cap B^m = MI \cap (A^m \cap B^m) = MI \cap (A \cap B)^m = (A \cap B)I$. Conversely, let A and B be (m, n)-pure submodules in M and let I be n-generated submodule of ${}_R R^m$ .Then $MI \cap (A \cap B)^m = (MI \cap A^m) \cap B^m = AI \cap B^m$.
Similarly, $MI \cap (A \cap B)^m = BI \cap A^m$, because A and B are (m, n)-pure in M . Thus, $MI \cap (A \cap B)^m = AI \cap BI = (A \cap B)I$. Therefore M has the (m, n)-PIP.

Corollary 2.5:- Let M be an R-module, then M has the $(1, *)$-PIP, if and only if $(A \cap B)I = AI \cap BI$ for every finitely generated ideal I of R and for every for every $(1, *)$-pure submodules A and B of M .

Proof:- It follows by [3, corollary 1.6]

In [2, CH 2, theorem 3.3], an R-module M has the PIP, if and only if for every pure submodules A and B of

M and for every R- homomorphism $f:(A\cap B)\to M$ such that $A\cap \operatorname{Im} f=0$ and $A+\operatorname{Im} f$ is pure in M, $\ker f$ is pure in M.

Theorem 2.6 :- Let M be an R-module, then M has the $(m,\ n)$-PIP, if and only if for every $(m,\ n)$-pure submodules A and B of M and for every R-homomorphism $f:(A\cap B)\to M$ such that $A\cap \operatorname{Im} f=0$ and $A+\operatorname{Im} f$ is $(m,\ n)$-pure in M, $\ker f$ is $(m,\ n)$-pure in M.

Proof:- Assume that M has the $(m,\ n)$-PIP. Let A and B be $(m,\ n)$-pure submodules of M and $f:(A\cap B)\to M$ be an R- homomorphism such that $A\cap \operatorname{Im} f=0$ and $A+\operatorname{Im} f$ is $(m,\ n)$-pure in M. Let $T=\{x+f(x)/x\in A\cap B\}$,
It is clear that T is a submodule of M. To show that T is $(m,\ n)$-pure in M. Let I be n- generated submodule of ${}_R R^m$, $I=\mathrm{Rb}_1+\mathrm{Rb}_2+\ldots+\mathrm{Rb}_n$, $b_j=(\alpha_{1j}\ldots,\alpha_{mj})\in R^m$ and $y\in MI\cap T^m$, $m_j\in M$, $b_j\in R^m$, $\forall$ $\mathrm{j}=1,\ldots,\mathrm{n}$. Hence $y=\sum_{j=1}^{n} m_j b_j=(u_1\ldots,u_m)$, $\mathrm{u}_1\ldots,\mathrm{u}_\mathrm{m}\in T$, $u_i=x_i+f(x_i)$, $\mathrm{i}=1,\ldots,\mathrm{m}$. For some $x_i\in A\cap B$. Sine $y=\sum_{j=1}^{n} m_j b_j=$ $(\mathrm{x}_1+\mathrm{f}(\mathrm{x}_1),\ldots.\mathrm{x}_\mathrm{m}+\mathrm{f}(\mathrm{x}_\mathrm{m}))=(\mathrm{x}_1\ldots\mathrm{x}_\mathrm{m})+(\mathrm{f}(\mathrm{x}_1),\ldots,\mathrm{f}(\mathrm{x}_\mathrm{m}))\in(A\cap B)^m+(\operatorname{Im} f)^m$ $\subseteq A+(\operatorname{Im} f)^m=(A+\operatorname{Im} f)^m$ and $A+\operatorname{Im} f$ is $(m,\ n)$-pure in M. Thus $y=\sum_{j=1}^{n} m_j b_j\in MI\cap(A+\operatorname{Im} f)^m=(A+\operatorname{Im} f)I$ [4,theorem(1.5)]. Therefore $\sum_{j=1}^{n} m_j b_j=$ $\sum_{j=1}^{n}(a_j+c_j)b_j$, $a_j\in A$, $c_j\in \operatorname{Im} f$.

Thus $y=\sum_{j=1}^{n} m_j b_j=\sum_{1}^{n} a_j b_j+\sum_{1}^{n} c_j b_j$, hence $(\mathrm{x}_1,\ldots,\mathrm{x}_\mathrm{m})-\sum_{j=1}^{n} a_j b_j=\sum_{j=1}^{n} c_j b_j$ - $(f(x_1),\ldots,f(x_m))\in(A\cap \operatorname{Im} f)^m$. Since $A\cap \operatorname{Im} f=0$, then $(\mathrm{x}_1,\ldots,\mathrm{x}_\mathrm{n})=\sum_{j=1}^{n} a_j b_j$ $\in AI\cap(A\cap B)^m$. But $A\cap B$ is $(m,\ n)$-pure in M, hence $A\cap B$ is $(m,\ n)$-pure in A, thus $AI\cap(A\cap B)^m=(A\cap B)I$. Then $(\mathrm{x}_1,\ldots,\mathrm{x}_\mathrm{m})\in(A\cap B)I$. Let $(\mathrm{x}_1,\ldots,\mathrm{x}_\mathrm{m})=\sum_{j=1}^{n} w_j b_j$, $w_j\in A\cap B$, then $(\mathrm{f}(\mathrm{x}_1),\ldots,\mathrm{f}(\mathrm{x}_\mathrm{m}))=\sum_{j=1}^{n} f(w_j)b_j$. Now, $y=(x_1,\ldots,x_m)+(\mathrm{f}(\mathrm{x}_1),\ldots,\mathrm{f}(\mathrm{x}_\mathrm{m}))=\sum_{j=1}^{n} w_j b_j+\sum_{j=1}^{n} f(w_j)b_j=\sum_{j=1}^{n}(w_j+f(w_j))b_j\in TI$.
Therefore T is $(m,\ n)$-pure in M. Next we show that $\ker f=(A\cap B)\cap T$. Let $x\in\ker f$, then $x\in A\cap B$ and $f(x)=0$. Hence $x\in T$. Now, let $x\in(A\cap B)\cap T$,
then $x=y+f(y)$, $y\in A\cap B$. $x-y=f(y)\in A\cap \operatorname{Im} f=0$. Therefore $f(x)=f(y)=0$ and $x\in\ker f$. Since M has the $(m,\ n)$-PIP, $(A\cap B)\cap T=\ker f$ is $(m,\ n)$-pure in M. Conversly, let A and B be $(m,\ n)$-pure submodules of M. Define $f:(A\cap B)\to M$ by $f(x)=0$ $\forall$ $x\in A\cap B$. It is clear that $A\cap \operatorname{Im} f=0$ and $A+\operatorname{Im} f=A$ is $(m,\ n)$-pure in M. Then $\ker f=A\cap B$ is $(m,\ n)$-pure in M. Then M has the $(m,\ n)$-PIP.

By the same argument one can prove the following:

Theorem 2.7:- Let M be an R-module. Then M has the $(m,\ n)$-PIP if and only if for every $(m,\ n)$-pure submodules A and B of M and for ever R-homomorphism $f:(A\cap B)\to C$ where C is a submodule of M such that $A\cap C$ =0 and $A+C$ is $(m,\ n)$-pure, in M , ker f is $(m,\ n)$-pure in M .

Proof:- it is clear

Corollary 2.8:- Let M be an R-module with the $(m,\ n)$-PIP. Then for every decomposition $M=A\oplus B$ and for every R- homomorphism f : $A\to B$, ker f is $(m,\ n)$-pure in M .

Proof:- Since $(A\cap B)$=0 and $A+B$= M is $(m,\ n)$-pure, in M and A= $A\cap M$ .Then by (theorem 2.9) , ker f is $(m,\ n)$-pure in M .

Corollary 2.9:- Let M be an R-module with the $(m,\ n)$-PIP. let A and B be $(m,\ n)$-pure submodules in M such that $(A\cap B)$=0 and $A+B$ is $(m,\ n)$-pure, in M .Then for each R-homomorphism $f:A\to B$, ker f is $(m,\ n)$-pure in M .

Remark 2.10:- Let A be $(m,\ n)$-pure submodule of an R-module, M then there exists $(m,\ n)$-pure $\overline{A}$ in M such that $\overline{A}$ is maximal with respect property $A+\overline{A}$ is $(m,\ n)$-pure in M and $A\cap\overline{A}=0$.

Proof :- Let F ={ $B:B$ is $(m,\ n)$-pure in M such that $(A\cap B)$=0 and $A+B$ is $(m,\ n)$-pure in M }.It is clear that $(0)\in F$ and hence $F\neq\phi$. Let $\{C_\alpha\}_{\alpha\in I}$ be a chain in F .It is clear that $\bigcup_{\alpha\in I} C_\alpha$ is a submodule of M and since $C_\alpha\cap A$=0 $\forall\ \alpha\in I$ .Then $(\bigcup_{\alpha\in I} C_\alpha)\cap A=0$ .To show that $\bigcup_{\alpha\in I} C_\alpha$ is $(m,\ n)$-pure in M .Let I = $Rb_1+Rb_2+.........+Rb_n$ be $n-$generated submodule of ${}_RR^m$ and, $\sum_{i=1}^{n} m_ib_i\in$ $MI\cap(\bigcup_{\alpha\in I} C_\alpha)^m$ .Then $\sum_{i=1}^{n} m_ib_i\in$ $MI\cap(C_{\alpha o})^m$ for some $\alpha_o\in I$ .Thus, $\sum_{i=1}^{n} m_ib_i\in C_{\alpha o}I\subseteq(\bigcup_{\alpha\in I} C_\alpha)I$ .Therefore, $MI\cap(\bigcup_{\alpha\in I} C_\alpha)^m=(\bigcup_{\alpha\in I} C_\alpha)I$ .To show that $A+\bigcup_{\alpha\in I} C_\alpha$ is $(m,\ n)$-pure in M .Let $\sum_{i=1}^{n} m_ib_i\in$ $MI\cap(A+\bigcup_{\alpha\in I} C_\alpha)^m$,then $\sum_{i=1}^{n} m_ib_i\in MI\cap(A+C_{\alpha o})^m$ for some $\alpha_o\in I$,and hence $\sum_{i=1}^{n} m_ib_i\in(A+C_{\alpha o})I$.

Thus $\sum_{i=1}^{n} m_ib_i\in(A+\bigcup_{\alpha\in I} C_\alpha)I$. By zeron 's lemma F has a maximal element say $\overline{A}=\bigcup_{\alpha\in I} C_\alpha$.

In [2, theorem 3.8], let M be an R-module such that for every pure submodules A and B of M either $A\subseteq B\oplus\overline{B}$ or $B\subseteq A\oplus\overline{A}$,then M has the PIP if and only if for every R-homomorphism $f:A\cap(B\oplus\overline{B})\to\overline{A}$, ker f is pure in M .

Theorem 2.11:- Let M be an R-module such that for every $(m,\ n)$-pure submodules A and B of M either $A\subseteq B\oplus\overline{B}$ or $B\subseteq A\oplus\overline{A}$,then M has the $(m,\ n)$-PIP if and only if for every

R-homomorphism $f: A\bigcap(B\oplus \overline{B}) \to \overline{A}$, $\ker f$ is $(m, \ n)$-pure in M .

Proof:- Suppose that M has the $(m, \ n)$-PIP and A and B are $(m, \ n)$-pure submodules of M . Let $f: A\bigcap(B\oplus \overline{B}) \to \overline{A}$, be an R-homomorphism, then by (theorem 2.9), $\ker f$ is $(m, \ n)$-pure in M .The converse ,let A and B be $(m, \ n)$-pure submodules of M .Assume that $A \subseteq B\oplus \overline{B}$. $\pi_1 : A\oplus \overline{A} \to A$ and $\pi_{2:}$ $B\oplus \overline{B} \to \overline{B}$ be the natural projections. Put h = $\boldsymbol{\pi_2\pi_1}|_{B\cap(A\oplus B)}$.

We show that $\ker h = (B\bigcap A)\oplus(B\bigcap \overline{A})$.

Let $x\in \ker h$, $x\in B\bigcap(A\oplus \overline{A})$ and then $x = a+\overline{a}$, $a\in A, \overline{a}\in \overline{A}$. Now, $\pi_2 o \pi_1(a+\overline{a}) = \pi_2(a)$=0 .So $a\in B$, then . Thus $\overline{a}\in B$, $x\in (B\bigcap A)\oplus(B\bigcap \overline{A})$. Now, let $x\in (B\bigcap A)\oplus(B\bigcap \overline{A})$, then x $= a+\overline{a}$, $a\in B\bigcap A, \overline{a}\in B\bigcap \overline{A}$.

Thus, $\pi_2 o \pi_1(a+\overline{a}) = \pi_2(a)$=0 . Therefore $\ker h = (B\bigcap A)\oplus(B\bigcap \overline{A})$ is $(m, \ n)$-pure in M . $B\bigcap A$ is $(m, \ n)$-pure in $\ker h$ (Remark 2.12), then $B\bigcap A$ is $(m, \ n)$-pure in M [3,prop.1.9]. That is M has the $(m, \ n)$-PIP.

In [3, proposition 3.10], for any two pure submodule A and B of an R-module M , if A+B is flat, then M has the PIP.

Proposition 2.12:- Let M be an R-module such that for any two $(m, \ n)$-pure submodules A and B of M , $A+B$ is $(m, \ n)$- flat R-module, then M has the $(m, \ n)$-PIP.

Proof:- Let A and B be $(m, \ n)$-pure submodules of M . Consider the following short exact sequence

$$0 \to A\bigcap B \xrightarrow{i_1} A \xrightarrow{f_1} \frac{A}{A\bigcap B} \to 0$$

$$0 \to B \xrightarrow{i_{21}} A+B \xrightarrow{f_2} \frac{A+B}{B} \to 0$$

Where i_1, i_2 are the inclusion maps and f_1, f_2 are the natural epimorphism by the second isomorphism theorem, $\frac{A}{A\bigcap B} \cong \frac{A+B}{B}$ .Since $A+B$ is $(m, \ n)$- flat R-module, and B is $(m, \ n)$-pure submodule of M ,then [4, theorem 3.6] $\frac{A+B}{B}$ is $(m, \ n)$- flat and B is $(m, \ n)$-pure in $A+B$. Thus $\frac{A}{A\bigcap B}$ is $(m, \ n)$-flat and hence $A\bigcap B$ is $(m, \ n)$-pure in A .But A is $(m, \ n)$-pure in M , so $A\bigcap B$ is $(m, \ n)$-pure in M , thus M has the $(m, \ n)$-PIP.

Lemma 2.13 :- Let $M = \oplus_{i\in I} M_i$ where M_i is a submodule of M $\forall$ $i\in I$ and let W_i be a submodule of M_i ,for each $i, j \in I$. Then $\oplus W_i$ is $(m, \ n)$-pure in M if and only if W_i is $(m, \ n)$-pure in M_i $\forall$ i .

Proof :- Assume $\oplus_{i\in I} W_i$ is $(m, \ n)$-pure in M .since W_i is a summand of $\oplus W_i$, then W_i is $(m, \ n)$-pure in $\oplus_{i\in I} W_i$. So W_i is $(m, \ n)$-pure in M [3,proposition1.9]. Since W_i is a submodule of M_i ,then W_i is $(m, \ n)$-pure in M_i $\forall$ i, [3,proposition 1.9]. The converse ,let J be $n-$generated submodule of $_R R^m$ and $x \in MJ\bigcap(\oplus_{j\in I} W_i)^m$. $x = \sum_{j=1}^{n} m_j b_j$, $m_j \in \oplus_{i\in I} M_i$. Then $m_j = \sum_{i\in I} m_{ij}$, $m_{ij} \in M_i$

for $i \in I$. Thus $x \in \sum_{j=1}^{n}\sum_{i\in I} m_{ij}bj = \sum_{i\in I}\sum_{j=1}^{n} m_{ij}b_j$. Since $\sum_{j=1}^{n} m_{ij}b_j \in M_i$, $M = \oplus_{i\in I} M_i$. The element x can be written uniquely as $\sum_{i\in I}\sum_{j=1}^{n} m_{ij}b_j$ .But $x \in \oplus_{i\in I} W_i$. Thus $\sum_{j=1}^{n} m_{ij}b_j \in W_i \quad \forall\, i$ and hence $\sum_{j=1}^{n} m_{ij}b_j \in M_i J \cap (W_i)^m = W_i J$ (W_i is $(m,\ n)$-pure in M_i), $\sum_{j=1}^{n} m_{ij}b_j = \sum_{j=i}^{n} w_{ij}b_j \quad w_{ij} \in W_i$ for each j. Thus $x = \sum_{i\in I}\sum_{j=1}^{n} w_{ij}b_j \in (\oplus_{i\in I} W)_i J$.

Proposition 2.14 :- Let $M = \oplus_{i\in I} M_i$ be an R-module where each M_i is a submodule of M, If M has the $(m,\ n)$-PIP, then each M_i has the $(m,\ n)$-PIP. The converse is true if each $(m,\ n)$-pure submodule of M is fully invariant.

Proof:- Suppose that M has the $(m,\ n)$-PIP. Since M_i is a summand of M, then M_i is $(m,\ n)$- pure in M [Remark 2.3], and hence M_i has the $(m,\ n)$- PIP .To prove the converse, let S be $(m,\ n)$- pure submodule of M and $\pi_i : M \to M_i$ be the natural projection on M_i, for each $i \in I$. Let $x \in S$,then $x = \sum_{i\in I} m_i$, $m_i \in M_i$, $\pi_i(x) = m_i$. Since S is $(m,\ n)$-pure in M, then S is fully invariant (By assumption),and hence $\pi_i(S) \subseteq S \cap M_i$ .Thus $\pi_i(x) = m_i \in S \cap M_i$. i.e $x \in \oplus_{i\in I}(S \cap M_i)$. Therefore $S \subseteq \oplus_{i\in I}(S \cap M_i)$. But $\oplus_{i\in I}(S \cap M_i) \subseteq S$, so $S = \oplus_{i\in I}(S \cap M_i)$. Now suppose S and T are $(m,\ n)$-pure submodules of M, then $S \cap T = (\oplus_{i\in I}(S \cap M_i)) \cap (\oplus_{i\in I}(T \cap M_i)) = \oplus((S \cap M_i) \cap (T \cap M_i))$. Since $S = \oplus_{i\in I}(S \cap M_i)$, then $S \cap M_i$ is $(m,\ n)$ -pure in S [Remark 2.3]. But S is $(m,\ n)$-pure in M, so $S \cap M_i$ is $(m,\ n)$-pure in M. Then $S \cap M_i$ is $(m,\ n)$-pure in M_i. By lemma 2.13, $\oplus((S \cap M_i) \cap (T \cap M_i))$ is $(m,\ n)$-pure in $\oplus_{i\in I} M_i = M$.

Proposition 2.15 :- Let M and N be R-module with the $(m,\ n)$- PIP, such that $r_R(M) + r_R(N) = R$, then $M \oplus N$ has the $(m,\ n)$-PIP .

Proof:-Let C and D be $(m,\ n)$-pure submodules of $M \oplus N$ .Since $r_R(M) + r_R(N) = R$, then by the same way of the proof of ([5],proposition 4.2,CH.1), $C = A \oplus B$ and $D = A_1 \oplus B_1$ where A and A_1 are submodules of M , B and B_1 are submodules of N .Since M and N has the $(m,\ n)$-PIP ,then $A \cap A_1$ is $(m,\ n)$-pure in M and $B \cap B_1$ is $(m,\ n)$-pure in N .Thus by lemma 2.13 $(A \cap A_1) \oplus (B \cap B_1)$, is $(m,\ n)$-pure in $M \oplus N$. But $(A \cap A_1) \oplus (B \cap B_1)$ = $(A \cap A_1) \cap (B \cap B_1)$ = $C \cap D$. So $C \cap D$ is $(m,\ n)$-pure in $M \oplus N$ and that is $M \oplus N$ has $(m,\ n)$-PIP .

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ملاحظات حول المقاسات التي تمتلك خاصية التقاطع النقي من النمط-(m,n)

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الكلمات المفتاحيه: مقاس جزئي نقي من النمط(m,n) ، مقاس مسطح من النمط(m,n) ، مقاس يمتلك خاصية التقاطع النقي من النمط(m,n) .

الخلاصة:

لتكن R حلقة فأن المقاس يسمى رئيسي من النمط(m,n) لكل عددين صحيحين موجبين m,n اذا وجدت متتابعة مضبوطة بالشكل $0 \to K \to R^m \to V \to 0$ بحيث K هو مقاس متولد ب n من العناصر. المقاس الجزئي N من المقاس الأيمن M يسمى نقي من النمط(m,n) في M. إذا كان كل مقاس رئيسي من النمط(m,n) وليكن V بحيث إن التشاكل $N \otimes_R V \to M \otimes_R V$ متباين .أما المقاس M فأنه يسمى مقاس يمتلك خاصية التقاطع النقي من النمط(m,n) اذا كان تقاطع كل مقاسين نقيين من النمط(m,n) يكون مقاس نقي من النمط(m,n).الهدف من هذا البحث هو اعطاء بعض الخواص والنظريات والخصائص للمقاسات بأستخدام خاصية التقاطع النقي من النمط (m,n) للمقاسات .