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Biased estimators in Poisson regression model in the presence of multicollinearity: A subject review

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Abstract: *The presence of high correlation among predictors in regression mode has undesirable effects on the regression estimating. In the literature, there are several available biased methods to overcome this issue. The Poisson regression model (PRM) is a special model from the generalized linear models. The PRM is a well-known model in research application when the response variable under the study is count data. Numerous biased estimators for overcoming the multicollinearity in Poisson regression model have been proposed in the literature using different theories. An overview of recent biased methods for PRM is provided. A comparison among these biased estimators allows us to gain an insight into their performance. Simulation and real data application results show that the Liu-type estimator is comparable to other estimators.*

Keywords: *Multicollinearity, biased estimator, Poisson regression model, Monte Carlo simulation.*

1. Introduction

Poisson regression model is widely applied for studying several real data problems, such as in mortality studies where the aim is to investigate the number of deaths and in health insurance where the target is to explain the number of claims made by the individual [1,2,3]. In dealing with the Poisson regression model, it is assumed that there is no correlation among the explanatory variables [4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24]. However, this assumption often not holds, which leads to the problem of multicollinearity. In the presence of multicollinearity, when estimating the regression coefficients for Poisson regression model using the maximum likelihood (ML) method, the estimated coefficients are usually become unstable with a high variance, and therefore low statistical significance [25]. Numerous remedial methods have been proposed to overcome the problem of multicollinearity [26,27,28,29,30]. The ridge regression method [31] has been consistently demonstrated to be an attractive and alternative to the ML estimation method.

Ridge regression is a biased method that shrinks all regression coefficients toward zero to reduce the large variance [32]. This done by adding a positive amount to the diagonal of $\mathbf{X}^T\mathbf{X}$. As a result, the ridge estimator is biased but it guaranties a smaller mean squared error than the ML estimator.

In linear regression, the ridge estimator is defined as

$$\hat{\boldsymbol{\beta}}_{Ridge} = (\mathbf{X}^T\mathbf{X} + k\,\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y}, \tag{1}$$

where $\mathbf{y}$ is an $n\times 1$ vector of observations of the response variable, $\mathbf{X} = (\mathbf{x}_1,...,\mathbf{x}_p)$ is an $n\times p$ known design matrix of explanatory variables, $\boldsymbol{\beta} = (\beta_1,...,\beta_p)$ is a $p\times 1$ vector of unknown regression coefficients, $\mathbf{I}$ is the identity matrix with dimension $p\times p$, and $k \geq 0$ represents the ridge parameter (shrinkage parameter). The ridge parameter, k, controls the shrinkage of $\boldsymbol{\beta}$ toward zero. The OLS estimator can be considered as a special estimator from Eq. (1) with $k = 0$. For larger value of k, the $\hat{\boldsymbol{\beta}}_{Ridge}$ estimator yields greater shrinkage approaching zero [31,33].

2. Poisson regression model

Count data are often arise in epidemiology, social, and economic studies. This type of data consists of positive integer values. Poisson distribution is a well-known distribution that fit to such type of data. Poisson regression model is used to model the relationship between the counts as response variable and potentially explanatory variables [9,34,35,36,37,38,39,40].

Let y_i be the response variable and follows a Poisson distribution with mean ϕ_i, then the probability density function is defined as

$$f\,(y_i) = \frac{e^{-\phi_i}\phi_i^{\,y_i}}{y_i\,!}, \quad y_i = 0,1,\ldots;\; i = 1,2,\ldots,n. \tag{2}$$

In a Poisson regression model, $\ln(\phi_i) = \mathbf{x}_i^T\boldsymbol{\beta}$ is expressed as a linear combination of explanatory variables $\mathbf{x}_i = (x_{i1},...,x_{ip})^T$. The $\ln(\phi_i)$ is called as canonical link function which making the relationship between explanatory variables, and response variable linear. The most common method of estimating the coefficients of Poisson regression model is to use the maximum likelihood method. Given the assumption that the observations are independent, the log-likelihood function is defined as

$$\ell(\boldsymbol{\beta}) = \sum_{i=1}^{n}\left\{y_i\,\mathbf{x}_i^T\boldsymbol{\beta} - \exp(\mathbf{x}_i^T\boldsymbol{\beta}) - \ln y_i\,!\right\}. \tag{3}$$

The ML estimator is then obtained by computing the first derivative of the Eq. (3) and setting it equal to zero, as

$$\frac{\partial\ell(\boldsymbol{\beta})}{\partial\boldsymbol{\beta}} = \sum_{i=1}^{n}\left[\,y_i - \exp(\mathbf{x}_i^T\boldsymbol{\beta})\,\right]\mathbf{x}_i = 0. \tag{4}$$

Because Eq. (4) is nonlinear in $\boldsymbol{\beta}$, the iteratively weighted least squares (IWLS) algorithm can be used to obtain the ML estimators of the Poisson regression parameters (PR) as

$$\hat{\boldsymbol{\beta}}_{ML} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X})^{-1}\mathbf{X}^T\hat{\mathbf{W}}\hat{\mathbf{v}}, \tag{5}$$

where $\hat{\mathbf{W}} = \mathrm{diag}(\hat{\phi}_i)$ and $\hat{\mathbf{v}}$ is a vector where i^{th} element equals to $\hat{v}_i = \ln(\hat{\phi}_i) + ((y_i - \hat{\phi}_i)/\hat{\phi}_i)$. The ML estimator is asymptotically normally distributed with a covariance matrix that corresponds to the inverse of the Hessian matrix

$$\mathrm{cov}(\hat{\boldsymbol{\beta}}_{ML}) = \left[-E\left(\frac{\partial^2\ell(\boldsymbol{\beta})}{\partial\beta_i\,\partial\beta_k}\right)\right]^{-1} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X})^{-1}. \tag{6}$$

The mean squared error (MSE) of Eq. (5) can be obtained as

$$\begin{aligned}\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{ML}) &= E\,(\hat{\boldsymbol{\beta}}_{ML} - \hat{\boldsymbol{\beta}})^T(\hat{\boldsymbol{\beta}}_{ML} - \hat{\boldsymbol{\beta}}) \\ &= tr[(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X})^{-1}] \\ &= \sum_{j=1}^{p}\frac{1}{\lambda_j},\end{aligned} \tag{7}$$

where λ_j is the eigenvalue of the $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}$ matrix.

3. Ridge estimator

In the presence of multicollinearity, the matrix $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}$ becomes ill-conditioned leading to high variance and instability of the ML estimator of the Poisson regression parameters. As a remedy, Månsson and Shukur [41] proposed the Poisson ridge estimator (PRE) as

$$\begin{aligned}\hat{\boldsymbol{\beta}}_{PRE} &= (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X} + k\mathbf{I})^{-1}\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}\hat{\boldsymbol{\beta}}_{ML} \\ &= (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X} + k\mathbf{I})^{-1}\mathbf{X}^T\hat{\mathbf{W}}\hat{\mathbf{v}},\end{aligned} \tag{8}$$

where $k \ge 0$. The ML estimator can be considered as a special estimator from Eq. (8) with $k = 0$. Regardless of k value, the MSE of the $\hat{\boldsymbol{\beta}}_{PRE}$ is smaller than that of $\hat{\boldsymbol{\beta}}_{ML}$ [25] because the MSE of $\hat{\boldsymbol{\beta}}_{PRE}$ is equal to

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{PRE}) = \sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j + k)^2} + k^2\sum_{j=1}^{p}\frac{\alpha_j}{(\lambda_j + k)^2}, \tag{9}$$

where α_j is defined as the j^{th} element of $\gamma\hat{\boldsymbol{\beta}}_{ML}$ and γ is the eigenvector of the $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}$ matrix. Comparing with the MSE of Eq. (5), $\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{PRE})$ is always small for $k>0$.

4. Liu estimator

Another popular biased estimator which is known as Liu estimator has been adopted in Poisson regression model. The Poisson Liu estimator (PLE) is defined as

$$\hat{\boldsymbol{\beta}}_{PLE} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+\mathbf{I})^{-1}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{ML}, \tag{10}$$

where $0<d<1$. Regardless of d value, the MSE of the $\hat{\boldsymbol{\beta}}_{PLE}$ is smaller than that of $\hat{\boldsymbol{\beta}}_{ML}$ [25] because the MSE of $\hat{\boldsymbol{\beta}}_{PLE}$ is equal to

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{PLE}) = \sum_{j=1}^{p}\frac{(\lambda_j+d)^2}{\lambda_j(\lambda_j+1)^2}+(d-1)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j+1)^2}. \tag{11}$$

5. Liu-type estimator

Alternative to Liu estimator, the Liu-type estimator was proposed by Liu [42] to overcome the problem of severe multicollinearity. The Poisson Liu-type estimator (PLTE) is defined as

$$\hat{\boldsymbol{\beta}}_{PLTE} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I})^{-1}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}-d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{ML}, \tag{12}$$

where $-\infty<d<\infty$ and $k\geq 0$. In Eq. (12), the parameter k can be used totally to control the conditioning of $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I}$. After the reduction of $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I}$ is reach a desirable level, then the expected bias that is generated can be corrected with the so-called bias correction parameter, d [43,44,45,46,47].

Liu [42] proved that, in terms of MSE, the Liu-type estimator has superior properties over ridge estimator. The MSE of $\hat{\boldsymbol{\beta}}_{PLTE}$ is defined as

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{PLTE}) = \sum_{j=1}^{p}\frac{(\lambda_j-d)^2}{\lambda_j(\lambda_j+k)^2}+(d+k)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j+k)^2}. \tag{13}$$

6. Two-parameter estimator

Following Asar and Genç [48] and Huang and Yang [49] the two-parameter estimator in linear regression model is defined as:

$$\hat{\boldsymbol{\beta}}_{TPE} = (\mathbf{X}^T\mathbf{X}+k\,\mathbf{I})^{-1}(\mathbf{X}^T\mathbf{X}+k\,d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{OLS}, \tag{14}$$

where $0<d<1$ and $k\geq 0$. For PRM, the two-parameter estimator (PTPE) is defined as:

$$\hat{\boldsymbol{\beta}}_{PTPE} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I})^{-1}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{ML}. \tag{15}$$

It is obviously noted that the $\hat{\boldsymbol{\beta}}_{PTPE}$ is a combination of two different estimators PRE and PLE. Furthermore, if $k=1$, Eq. (15) will be the $\hat{\boldsymbol{\beta}}_{PLE}$ while if $k=0$, Eq. (15) will be the $\hat{\boldsymbol{\beta}}_{ML}$. Besides, when $d=0$, then Eq. (15) will equal $\hat{\boldsymbol{\beta}}_{PRE}$.

In terms of MSE, the two-parameter estimator has superior properties over ML estimator. The MSE of $\hat{\boldsymbol{\beta}}_{PTPE}$ is defined as

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{PTPE}) = \sum_{j=1}^{p+1}\left[\frac{(\lambda_j+kd)^2}{\lambda_j(\lambda_j+k)^2}+k^2(d-1)^2\frac{\alpha_j^2}{(\lambda_j+k)^2}\right]. \tag{16}$$

7. Jackknifed Ridge estimator

In generalized ridge estimator, the Jackknifing approach was used [50,51,52]. Batah *et al.* [53] proposed a modified Jackknifed ridge regression estimator in linear regression model. Related to Poisson regression model, Türkan and Özel [54] proposed a modified Jackknifed Poisson ridge estimator depending on the study of Singh *et al.* [52].

Let $\mathbf{M}=(m_1,\mathrm{m}_2,...,m_p)$ and $\mathbf{\Lambda}=\mathrm{diag}(\lambda_1,\lambda_2,...,\lambda_p)$, respectively, be the matrices of eigenvectors and eigenvalues of the $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}$ matrix, such that $\mathbf{M}^T\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}\mathbf{M}=\mathbf{S}^T\hat{\mathbf{W}}\mathbf{S}=\mathbf{\Lambda}$, where $\mathbf{S}=\mathbf{X}\mathbf{M}$. Consequently, the Poisson regression estimator of Eq. (5), $\hat{\boldsymbol{\beta}}_{ML}$, can be written as

$$\begin{aligned}\hat{\boldsymbol{\gamma}}_{ML} &= \mathbf{\Lambda}^{-1}\mathbf{S}^T\hat{\mathbf{W}}\hat{\mathbf{v}}\\ \hat{\boldsymbol{\beta}}_{ML} &= \mathbf{M}\,\hat{\boldsymbol{\gamma}}_{ML}.\end{aligned} \tag{17}$$

Accordingly, the Poisson ridge estimator, $\hat{\boldsymbol{\beta}}_{PRE}$, is rewritten as

$$\begin{aligned}\hat{\boldsymbol{\gamma}}_{PRE} &= (\mathbf{\Lambda}+\mathbf{K})^{-1}\mathbf{S}^T\hat{\mathbf{W}}\mathbf{v}\\ &= (\mathbf{I}-\mathbf{K}\mathbf{D}^{-1})\hat{\boldsymbol{\gamma}}_{ML},\end{aligned} \tag{18}$$

where $\mathbf{D}=\mathbf{\Lambda}+\mathbf{K}$ and $\mathbf{K}=\mathrm{diag}(k_1,k_2,...,k_p);\ k_i\geq 0, i=1,2,...,p$. Equation (18) represents the generalized ridge Poisson regression estimator (GRPR) [51,53,55].

Following the study of Batah *et al.* [53], let the Jackknife estimator (JE), in Poisson regression, is defined as

$$\hat{\boldsymbol{\gamma}}_{JE} = (\mathbf{I}-\mathbf{K}^2\mathbf{D}^{-2})\hat{\boldsymbol{\gamma}}_{ML}, \tag{19}$$

and the modified Jackknife estimator (MJE) of Batah *et al.* [53], in Poisson regression model, is defined as

$$\hat{\boldsymbol{\gamma}}_{MJE} = (\mathbf{I}-\mathbf{K}\mathbf{D}^{-1})(\mathbf{I}-\mathbf{K}^2\mathbf{D}^{-2})\hat{\boldsymbol{\gamma}}_{ML}. \tag{20}$$

The MSE of $\hat{\boldsymbol{\gamma}}_{MJE}$ is defined as:

$$\mathrm{MSE}(\hat{\boldsymbol{\gamma}}_{MJE}) = (\mathbf{I}-k^2\mathbf{D}^{-2})\mathbf{\Lambda}^{-1}(\mathbf{I}-k^2\mathbf{D}^{-2})^T + k^4\mathbf{D}^{-2}\boldsymbol{\tau}\boldsymbol{\tau}^T\mathbf{D}^{-2}. \tag{21}$$

8. Simulation study

In this section, a Monte Carlo simulation experiment is used to examine the performance of the used estimators. The response variable of n observations is generated from PRM by

$$\ln(\phi_i) = \mathbf{x}_i^T\boldsymbol{\beta} \tag{22}$$

where $\boldsymbol{\beta}=(\beta_0,\beta_1,...,\beta_p)$ with $\sum_{j=1}^{p}\beta_j^2=1$ and $\beta_1=\beta_2=...=\beta_p$. The explanatory variables $\mathbf{x}_i^T=(x_{i1},x_{i2},...,x_{in})$ have been generated from the following formula

$$x_{ij} = (1-\rho^2)^{1/2}w_{ij} + \rho w_{ip},\ i=1,2,...,n,\quad j=1,2,...,p, \tag{23}$$

where ρ represents the correlation between the explanatory variables and w_{ij} 's are independent standard normal pseudo-random numbers. Three representative values of the sample size are considered: 30, 50 and 100. In addition, the number of the explanatory variables is considered as $p=4$ and $p=8$ because increasing the number of explanatory variables can lead to increase the MSE. Further, because we are interested in the effect of multicollinearity, in which the degrees of correlation considered more important, three values of the pairwise correlation are considered with $\rho=\{0.90,0.95,0.99\}$. For a combination of these different values of n,p, and ρ the generated data is repeated 500 times and the averaged mean squared errors (MSE) is calculated as

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}) = \frac{1}{500}\sum_{i=1}^{500}(\hat{\boldsymbol{\beta}}-\boldsymbol{\beta})^T(\hat{\boldsymbol{\beta}}-\boldsymbol{\beta}), \tag{24}$$

where $\hat{\boldsymbol{\beta}}$ is the estimated coefficients for the used estimator.

The estimated MSE of Eq. (25) for ML, PRE, PLE, PLTE, PTPE, and MJE, for all the combination of n, p, and ρ, is summarized in Table1. Two observations can be made. First, in terms of ρ values, there is increasing in the MSE values when the correlation degree increases regardless the value of n, p. However, PLTE performs better than the others estimators. Second, regarding the value of p, it is easily seen that there is increasing in the MSE values when the p increasing from four variables to eight variables. Although this increasing can affected the quality of an estimator, PLTE is achieved the lowest MSE comparing with the other used estimators. To summary, all the considered values of n, ρ, p, PLTE is superior to ML, PRE, PLE, PTPE, and MJE indicating that this estimator is more efficient.

Table 1: MSE values of simulation study for the ML, PRE, PLE, PLTE, PTPE, and MJE estimators.

		ρ	ML	PRE	PLE	PLTE	PTPE	MJE
$p=4$	$n=30$	0.90	4.7044	4.5914	4.4634	3.3974	4.1914	4.4984
		0.95	5.3324	5.2194	5.0914	4.0254	4.8194	5.1264
		0.99	5.7304	5.6174	5.4894	4.4234	5.2174	5.5244
	$n=50$	0.90	3.0754	2.9624	2.8344	1.7684	2.5624	2.8694
		0.95	4.1504	4.0374	3.9094	2.8434	3.6374	3.9444
		0.99	4.3424	4.2294	4.1014	3.0354	3.8294	4.1364
	$n=100$	0.90	2.9184	2.8054	2.6774	1.6114	2.4054	2.7124
		0.95	3.1284	3.0154	2.8874	1.8214	2.6154	2.9224
		0.99	3.8834	3.7704	3.6424	2.5764	3.3704	3.6774
$p=8$	$n=30$	0.90	4.8094	4.6964	4.5684	3.5024	4.2964	4.6034
		0.95	5.4284	5.3154	5.1874	4.1214	4.9154	5.2224
		0.99	5.8434	5.7304	5.6024	4.5364	5.3304	5.6374
	$n=50$	0.90	3.3444	3.2314	3.1034	2.0374	2.8314	3.1384
		0.95	4.4874	4.3744	4.2464	3.1804	3.9744	4.2814
		0.99	4.8124	4.6994	4.5714	3.5054	4.2994	4.6064
	$n=100$	0.90	3.2544	3.1414	3.0134	1.9474	2.7414	3.0484
		0.95	3.5294	3.4164	3.2884	2.2224	3.0164	3.3234
		0.99	4.0874	3.9744	3.8464	2.7804	3.5744	3.8814

9. Real application

To investigate the usefulness of reviewed biased estimators, an application related to the football English league (https://www.efl.com), season 2016-2017 is employed. This data contains twenty teams, where the response variable represents the number of won matches. The six considerable predictors included the number of yellow cards (x_1), the number of red cards (x_2), the total number of substitutions (x_3), the number of matches with 2.5 goals on average (x_4), the number of matches that ended with goals (x_5), and the ratio of the goal scores to the number of matches (x_6).

First, the deviance test [56] is used to check whether the Poisson regression model is fit well to this data or not. The result of the residual deviance test is equal to 8.373 with 14 degrees of freedom and the p-value is 0.869. It is indicated form this result that the Poisson regression model fits very well to this data.

Second, to check whether there are relationships between the explanatory variables or not, Figure 1 displays the correlation matrix among the six explanatory variables. It is obviously seen that there are correlations greater than 0.86 between x_1 and x_6, x_1 and x_4, x_2 and x_4, and, x_4 and x_6.

Third, to test the existence of multicollinearity, the eigenvalues of the matrix $\mathbf{X}^T \hat{\mathbf{W}}\mathbf{X}$ are obtained as 993.758, 142.907, 75.560, 38.999, 21.424, and 1.016. The determined condition number $\text{CN} = \sqrt{\lambda_{\max} / \lambda_{\min}}$ of the data is 31.274 indicating that the multicollinearity issue is exist.

The estimated Poisson regression coefficients, standard errors which are computed by using bootstrap with 500 replications, and MSE values for the ML, PRE, PLE, PLTE, PTPE, and MJE estimators are listed in Table 2. According to Table 2, it is clearly seen that the PLTE estimator shrinkages the value of the estimated coefficients efficiently. Additionally, in terms of the calculated standard errors, the PLTE and PTPE show substantial decreasing comparing with ML.

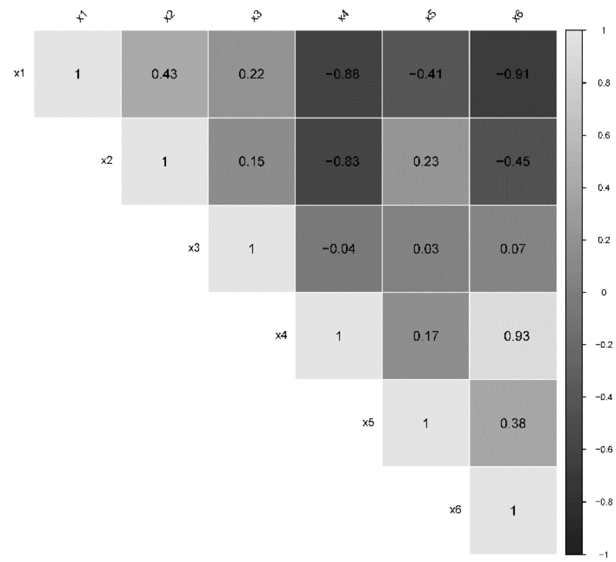

Figure 1: The correlation matrix among the six predictors.

Table 2: The estimated coefficients and MSE values for the ML, PRE, PLE, PLTE, PTPE, and MJE estimators. The number in parenthesis is the standard error.

	ML	PRE	PLE	PLTE	PTPE	MJE
$\hat{\beta}_1$	-1.219	0.057	-0.016	-0.052	-0.015	-1.223
	(0.151)	(0.022)	(0.007)	(0.018)	(0.011)	(0.127)
$\hat{\beta}_2$	0.441	-0.035	-0.004	-0.032	-0.004	0.440
	(0.151)	(0.013)	(0.001)	(0.011)	(0.002)	(0.139)
$\hat{\beta}_3$	0.575	0.007	-0.016	0.006	-0.012	0.576
	(0.175)	(0.004)	(0.008)	(0.004)	(0.008)	(0.108)
$\hat{\beta}_4$	-3.476	0.066	0.034	0.063	0.014	-3.447
	(0.313)	(0.004)	(0.008)	(0.004)	(0.007)	(0.231)
$\hat{\beta}_5$	-2.432	0.017	-0.008	0.0162	-0.007	-2.419
	(0.160)	(0.010)	(0.004)	(0.011)	(0.003)	(0.133)
$\hat{\beta}_6$	5.121	0.073	-0.004	0.066	-0.003	5.084
	(0.387)	(0.009)	(0.003)	(0.008)	(0.001)	(0.249)
MSE	3.681	1.184	1.065	0.977	1.032	1.152

10. Conclusions

In this paper, we presented a thorough review of literature regarding the biased estimators in Poisson regression model when the multicollinearity is existing. According to the simulation and real data application results, the Liu-type estimator has better performance than ML, PRE, PLE, PTPE, and MJE, in terms of MSE. In conclusion, the use of the Liu-type estimator is recommended when multicollinearity is present in the Poisson regression model.

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