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Artin's characters table of the group $(Q_{2m}\times C_2)$ when $m=2^h$, $h \in Z^+$

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Abstract

The main purpose of this paper is to find the general form of Artin's characters table of the group $(Q_{2m} \times C_2)$ When $m=2^h$,$h \in Z^+$ where Q_{2m} is the Quaternion group of order 4m and C_2 is the Cyclic group of order 2 this table depends on Artin's characters table of a quaternion group of order 4m when $m=2^h$, $h\in Z^+$. which is denoted by $Ar(Q_{2^{h+1}}\times C_2)$.

المستخلص

الهدف الرئيسي لهذا البحث هو ايجاد الصيغة العامة لجدول شواخص ارتن للزمرة $(Q_{2m} \times C_2)$ عندما $m=2^h$ $h \in$ Z^+,حيث ان Q_{2m} هي الزمرة الرباعية العمومية ذات الرتبة 4m و C_2 هي الزمرة الدائرية ذات الرتبة 2 ,وقد وجدنا أن هذا الجدول يعتمد على جدول شواخص آرتن للزمرة الرباعية العمومية ذات الرتبة 4m عندما $m=2^h$, $h \in Z^+$. الذي يعبر عنه بـ $Ar(Q_{2^{h+1}}\times C_2)$

Introduction

For a finite group G, let R(G) denote the group generated by z- valued characters of the group G .Inside this group, we have a subgroup generated by Artin's characters (the characters induced from the principal characters of cyclic subgroups) of G which will be denoted by T(G). the factor group R(G)/T(G) which is denoted by AC(G) is called Artin's cokernal of G characters and it is a finite abelain group of the exponent A(G) which is called Artin's exponent. Let x and y be two elements of G, x and y are called Γ-conjugate if the cyclic subgroups which they generate, are Γ-conjugate in G. this is defined at an equivalent relation on G, its classes are called Γ- classes of G.
The square matrix whose rows correspond to Artin's characters and columns correspond to the Γ- classes of G is called Artin's characters table . this matrix is very important to find the cyclic decomposition of the factor group AC(G) and Artin's exponent A(G).
In 1967 T.Y. lam [9] studied A(G) extensively for many groups.In 1970 K.Yamauchi[6] studied 2- part A(G).In 1976 G.David [3] studied A(G) of arbitrary characters of the cyclic subgroups.
In 1996 K.K Nwabuez [5] studied A(G) of p-groups. In 2009 S.J.Mahmood [8] studied the general from of Artin's characters table $Ar(Q_{2m})$ when m is an even number.
The aim of this paper is to find the general from of the Artin's characters table of the group $(Q_{2m}\times C_2)$ when $m=2^h$,$h \in Z^+$.

1.Preliminaries

This section introduce some important definitions and basic concepts of the Artin's characters tables, the Artin's characters table of C_{p^s} , the Artin's characters table of the Quaternion group Q_{2m} when m is an even number, the Artin's characters table of the Quaternion group Q_{2m} when $m=2^h$,$h\in Z^+$ and the Group $(Q_{2m}\times C_2)$.

1.1 Definition: [7]
Two elements of G are said to be **Γ- conjugate** if the cyclic subgroups they generate are conjugate in G , this defines an equivalence relation on G. It is classes are called Γ- classes.

1.2 Example:
Consider a cyclic group C_4=< x > such that:
1 is Γ- conjugate 1
Then the Γ- class [1] = {1}
< x > = < x^3>
Then x and x^3 are Γ- conjugate , and [x] = { x , x^3}
There is another Γ- class [x^2] = {x^2}
So that there are three Γ- classes of C_4 :[1] , [x] and [x^2]

In general for C_{p^s} *where p is any prime number, so that are s+1 distinct Γ- classes Which are* [1], [x], [x^p], … ,[$x^{p^{s-1}}$] .

1.3 Definition: [5]
Let H be a subgroup of G and let ϕ be a class function on H, ***the induced class function on G***, is given by :

$$\phi'(g) = \frac{1}{|H|}\sum_{x\in G}\phi^{\circ}(xgx^{-1})$$

where ϕ° is defined by:

$$\phi^{\circ}(h) = \begin{cases}\phi(h) & if \quad h\in H\\ 0 & if \quad h\notin H\end{cases}$$

1.4 Proposition: [3]
Let H be a subgroup of G and ϕ be a character of H, then ϕ' is a character of G and it is called ***induced character*** on G

1.5 Example:
Take H=C_4 as acyclic subgroup of Q_4 the character φ on C_4 is defined as follows : $\phi(1) = 1, \phi(x) = \omega, \phi(x^2) = \omega^2, \phi(x^3) = \omega^3$

Where $\omega = e^{2\pi i/4}$

$$\phi'(1) = \frac{1}{|H|}\sum_{r\in Q_4}\phi^{\circ}(r.1.r^{-1}) = \frac{1}{|H|}\sum_{r\in Q_4}\phi(1)$$

$$= \frac{1}{4}(1+1+1+1+1+1+1+1) = \frac{1}{4}.8 = 2$$

$$\phi'(x) = \frac{1}{|H|}\sum_{r\in Q_4}\phi^{\circ}(r.x.r^{-1}) = \frac{1}{|H|}\sum_{r\in Q_4}\phi(x)$$

$$= \frac{1}{|H|}[\phi(x)+\phi(x)+\phi(x)+\phi(x)+\phi(x^3)+\phi(x^3)+\phi(x^3)+\phi(x^3)] = (1/4).4(\phi(x)+\phi(x^3)) = \omega+\omega^3$$

$$\phi'(x^2) = \frac{1}{|H|}\sum_{r\in Q_4}\phi^{\circ}(r.x^2.r^{-1}) = \frac{1}{|H|}\sum_{r\in Q_4}\phi(x^2)$$

$$= \frac{1}{|H|}[\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)+\phi(x^2)] = (1/4).8\phi(x^2) = 2\omega^2$$

$$\phi'(x^3) = \frac{1}{|H|}\sum_{r\in Q_4}\phi^{\circ}(r.x^3.r^{-1}) = \frac{1}{|H|}\sum_{r\in Q_4}\phi(x^3)$$

$= \frac{1}{|H|}[\phi(x^3)+\phi(x^3)+\phi(x^3)+\phi(x^3)+\phi(x)+\phi(x)+\phi(x)+\phi(x)] = (1/4).4(\phi(x^3)+\phi(x)) = \omega^3+\omega$

Since $y, xy, x^2y, x^3y \notin C_4$ then $\phi'(y)=\phi'(xy)=\phi'(x^2y)=\phi'(x^3y)=0$

Hence ϕ' is induced characters of Q_4.

1.6 Theorem:[4]

Let H be a cyclic subgroup of G and $h_1, h_2, \dots, h_m$ are chosen representative for m-conjugate classes, then :

1- $\varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i \in H \cap CL(g)$

2- $\varphi'(g) = 0$ if $H \cap CL(g) = \phi$

1.7 Example:

To find the Artin's character of C_4 ,there are three cyclic subgroups of C_4 , which are {1}, $<x>$ and $<x^2>$, there are three Γ-classes which are $[1]=\{1\},[x^2]=\{x^2\}$ and $[x]=\{x,x^3\}$

So we have three distinct Artin's characters, then by using theorem (1.6)

$$\varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) \quad \text{if} \quad h_i \in H \cap CL(g)$$

$$\varphi'(g) = 0 \quad \text{if} \quad H \cap CL(g) = \phi .$$

(i) if $H = \{1\}$ and $G = C_4$

since $H \cap CL(1) = \{1\}$, then

$\varphi_1'(1) = \frac{2^2}{1} . \varphi(1) = 2^2.1 = 2^2$

since $H \cap CL(x) = \phi$, then $\varphi_1'(x) = 0$

since $H \cap CL(x^2) = \phi$, then $\varphi_1'(x^2) = 0$

(ii) if $H = <x^2> = \{1,x^2\}$

$\varphi_2'(1) = \frac{2^2}{2} . \varphi(1) = 2.1 = 2$, since $H \cap CL(1) = \{1\}$

$\varphi_2'(x^2) = \frac{2^2}{2} . \varphi(1) = 2.1 = 2$, since $H \cap CL(x^2) = \{x^2\}$

since $H \cap CL(x) = \phi$, then $\varphi_1'(x) = 0$

(iii) if $H = <x> = \{1,x,x^2,x^3\}$

$\varphi_3'(1) = \frac{2^2}{2^2} . \varphi(1) = 1.1 = 1$, since $H \cap CL(1) = \{1\}$

$\varphi_3'(x^2) = \frac{2^2}{2^2} . \varphi(1) = 1.1 = 1$, since $H \cap CL(x^2) = \{x^2\}$

$\varphi_3'(x) = \frac{2^2}{2^2} . \varphi(1) = 1.1 = 1$, since $H \cap CL(x) = \{x\}$

Then we get three Artin's characters φ_1', φ_2' and φ_3'.

1.8 Definition:[9]

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called ***Artin's characters of G.***

In theorem (1.6) , if φ is the principal character , then $\varphi(h_i) = \varphi(1) = 1$, where $h_i \in H$

1.9 Proposition:[2]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G.

Furthermore , Artin's characters are constant on each Γ-classes.

1.10 Definition: [1]

Artin's characters of finite group G can be displayed in table ***called Artin's characters table of G*** which is denoted by Ar (G).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest row contain the values of Artin's characters.

1.11Example:

In the Artin's character table of C_4 there are three Γ- classes, [1], [x^2] and [x] then, from proposition (1.9) they obtain three distinct Artin's characters

And From example (1.7) we obtain the values of Artin's characters, then the table of it as follows:

Ar(C_4)=

Γ- classes	[1]	[x^2]	[x]
$\lvert CL_\alpha \rvert$	1	1	1
$\lvert C_{C_3}(CL_\alpha) \rvert$	2^3	2^3	2^3
φ'_1	2^2	0	0
φ'_2	2	2	0
φ'_3	1	1	1

Table (1)

1.12 Theorem:[1]

The general form of Artin's character table of C_{p^s} when p is a prime number and s is an integer number is given by:

Ar(C_{p^s})=

Γ-classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	…	$[x^p]$	$[x]$
$\lvert CL_\alpha \rvert$	1	1	1	1	…	1	1
$\lvert C_{p^s}(CL_\alpha) \rvert$	P^s	p^s	P^s	p^s	…	p^s	P^s
φ'_1	p^s	0	0	0	…	0	0
φ'_2	P^{s-1}	P^{s-1}	0	0	…	0	0
φ'_3	P^{s-2}	P^{s-2}	P^{s-2}	0	…	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮
φ'_s	P	P	P	P	…	P	0
φ'_{s+1}	1	1	1	1	…	1	1

Table (2)

1.13 Example:

Consider the cyclic group C_{128} ,To find the Artin's character table we use theorem (1.12) as follows : The group $C_{128} = C_{2^7}$ then $Ar(C_{2^7})$=

Γ- classes	[1]	$[x^{2^6}]$	$[x^{2^5}]$	$[x^{2^4}]$	$[x^{2^3}]$	$[x^{2^2}]$	$[x^2]$	[x]
$\lvert CL_\alpha \rvert$	1	1	1	1	1	1	1	1
$\lvert C_{C_{2^7}}(CL_\alpha)\rvert$	2^7	2^7	2^7	2^7	2^7	2^7	2^7	2^7
φ'_1	2^7	0	0	0	0	0	0	0
φ'_2	2^6	2^6	0	0	0	0	0	0
φ'_3	2^5	2^5	2^5	0	0	0	0	0
φ'_4	2^4	2^4	2^4	2^4	0	0	0	0
φ'_5	2^3	2^3	2^3	2^3	2^3	0	0	0
φ'_6	2^2	2^2	2^2	2^2	2^2	2^2	0	0
φ'_7	2	2	2	2	2	2	2	0
φ'_8	1	1	1	1	1	1	1	1

Table (3)

1.14 Theorem: [8]

The Artin's characters table of the Quaternion group Q_{2m} when m is an even number is given as follows :

Ar (Q_{2m}) =

Γ- classes	Γ- classes of C_{2m}						
	[1]	$[x^m]$				[y]	[xy]
$\lvert CL_\alpha \rvert$	1	1	2	2 …	2	m	m
$\lvert C_{Q_{2m}}(CL_\alpha)\rvert$	4m	4m	2m	2m …	2m	4	4
Φ_1	2Ar(C_{2m})					0	0
Φ_2						0	0
⋮						⋮	⋮
Φ_l						0	0
Φ_{l+1}	m	m	0	0 …	0	2	0
Φ_{l+2}	m	m	0	0 …	0	0	2

Table(4)

where l is the number of Γ- classes of C_{2m} and Φ_j ; $1 \le j \le l+2$ are the Artin characters of the Quaternion group Q_{2m}.

Let $m=2^h$, $h \in Z^+$ then $Ar(Q_{2m})=Ar(Q_{2^{h+1}})$ and it is given by:

$Ar(Q_2^{h+1})$=

Γ- classes	Γ- classes of C_{2m}							
	[1]	$[x^{2^h}]$					$[y]$	$[xy]$
$\lvert CL_\alpha \rvert$	1	1	2	2	…	2	2^h	2^h
$\lvert C_{Q_{2^{h+1}}}(CL_\alpha) \rvert$	2^{h+2}	2^{h+2}	2^{h+1}	2^{h+1}	…	2^{h+1}	4	4
Φ_1	**$2Ar(C_2^{h+1})$**						0	0
Φ_2							0	0
⋮							⋮	⋮
Φ_l							0	0
Φ_{l+1}	2^h	2^h	0	0	…	0	2	0
Φ_{l+2}	2^h	2^h	0	0	…	0	0	2

Table (5)

1.15 Example:

To construct $Ar(Q_{128})$ by using theorem (1.14) we get the following table :

$Ar(Q_{128})=Ar(Q_{2^7})$=

Γ- classes	[1]	$[x^{2^6}]$	$[x^{2^5}]$	$[x^{2^4}]$	$[x^{2^3}]$	$[x^{2^2}]$	$[x^2]$	$[x]$	[y]	[xy]
$\lvert CL_\alpha \rvert$	1	1	2	2	2	2	2	2	64	64
$\lvert C_{Q_{2^7}}(CL_\alpha) \rvert$	256	256	128	128	128	128	128	128	4	4
Φ_1	28	0	0	0	0	0	0	0	0	0
Φ_2	27	27	0	0	0	0	0	0	0	0
Φ_3	26	26	26	0	0	0	0	0	0	0
Φ_4	25	25	25	25	0	0	0	0	0	0
Φ_5	24	24	24	24	24	0	0	0	0	0
Φ_6	23	23	23	23	23	23	0	0	0	0
Φ_7	22	22	22	22	22	22	22	0	0	0
Φ_8	2	2	2	2	2	2	2	2	0	0
Φ_9	26	26	0	0	0	0	0	0	2	0
Φ_{10}	26	26	0	0	0	0	0	0	0	2

Table (6)

1.16The Group ($Q_{2m} \times C_2$) [10]

The direct product group ($Q_{2m} \times C_2$) where Q_{2m} is Quaternion group of order 4m with tow generators x and y is denoted by

$$Q_{2m} =\{x^k y^j : x^{2m} = y^4=1, yx^m y^{-1}=x^{-m}, 0\leq k\leq 2m-1, j=0,1\}$$

and C_2 is acyclic group of order 2 consisting of elements {I,z} .the generalized the group ($Q_{2m} \times C_2$) is denoted by

$$(Q_{2m} \times C_2) = \{(q,c): q\in Q_{2m}, c \in C_2\} \text{ and } |Q_{2m}\times C_2|=|Q_{2m}|.|C_2|=4m.2=8m$$

2. The main results

In this section is to find the general form of Artin's characters table of the group ($Q_{2m} \times C_2$) When $m=2^h$,$h \in Z^+$

2.1 Proposition:

The general from of the Artin's characters table of the group ($Q_{2^{h+1}} \times C_2$) when $m=2^h$,$h \in Z^+$ is give as follows:

$Ar(Q_{2^{h+1}} \times C_2)=$

<table>
<tr><th></th><th colspan="6">Γ- classes of $(Q_{2^{h+1}}) \times \{I\}$</th><th colspan="6">Γ- classes of $(Q_{2^{h+1}}) \times \{z\}$</th></tr>
<tr><td>Γ- classes</td><td>[1,I]</td><td>$[x^{2^h}, I]$</td><td>…</td><td>[x,I]</td><td>[y,I]</td><td>[xy,I]</td><td>[1,z]</td><td>$[x^{2^h}, z]$</td><td>…</td><td>[x,z]</td><td>[y,z]</td><td>[xy,z]</td></tr>
<tr><td>$|CL_\alpha|$</td><td>1</td><td>1</td><td>…</td><td>2</td><td>2^h</td><td>2^h</td><td>1</td><td>1</td><td>…</td><td>2</td><td>2^h</td><td>2^h</td></tr>
<tr><td>$|C_{Q_{2^{h+1}} \times C_2}(CL_\alpha)|$</td><td>$2^{h+3}$</td><td>$2^{h+3}$</td><td>…</td><td>$2^{h+2}$</td><td>8</td><td>8</td><td>2^{h+3}</td><td>2^{h+3}</td><td>…</td><td>2^{h+2}</td><td>8</td><td>8</td></tr>
<tr><td>$\Phi_{(1,1)}$</td><td colspan="6" rowspan="6">$2Ar(Q_{2^{h+1}})$</td><td colspan="6" rowspan="6">0</td></tr>
<tr><td>$\Phi_{(2,1)}$</td></tr>
<tr><td>⋮</td></tr>
<tr><td>$\Phi_{(l,1)}$</td></tr>
<tr><td>$\Phi_{(l+1,1)}$</td></tr>
<tr><td>$\Phi_{(l+2,1)}$</td></tr>
<tr><td>$\Phi_{(1,2)}$</td><td colspan="6" rowspan="6">$Ar(Q_{2^{h+1}})$</td><td colspan="6" rowspan="6">$Ar(Q_{2^{h+1}})$</td></tr>
<tr><td>$\Phi_{(2,2)}$</td></tr>
<tr><td>⋮</td></tr>
<tr><td>$\Phi_{(l,2)}$</td></tr>
<tr><td>$\Phi_{(l+1,2)}$</td></tr>
<tr><td>$\Phi_{(l+2,2)}$</td></tr>
</table>

Table (7)

Proof :

Let $g \in (Q_{2^{h+1}} \times C_2)$;g=(q,I) or g=(q,z),$q \in Q_{2^{h+1}}$,I,z $\in C_2$

Case (I):

If H is a cyclic subgroup of ($Q_{2^{h+1}} \times \{I\}$),then:

1-H=$\langle (x,I) \rangle$ 2- H=$\langle (y,I) \rangle$ 3- H=$\langle (xy,I) \rangle$

And φ the principal character of H, Φ_j Artin characters of $Q_{2^{h+1}}$ $1 \le j \le l+2$ then by using theorem (1.6)

1- $\Phi_j(g) = \dfrac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \varphi(h_i)$ if $h_i \in H \cap CL(g)$

2- $\Phi_j(g) = 0$ if $H \cap CL(g) = \phi$

1- IF H=$\langle (x,I) \rangle$

(i) If g=(1,I)

$$\Phi_{(j,1)}((1,I)) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{2^{h+3}}{|C_H(I,1)|}.1 = \frac{2.2^{h+2}}{|C_H(I,1)|}.1 = \frac{2|C_{Q_{2^{h+1}}}(1)|}{|C_{\langle x \rangle}(1)|}.\varphi(1) = 2.\Phi_j(1)$$

since $H \cap CL(1,I)=\{(1,I)\}$

(ii) if g=($(x^{2^h}, I), g \in H$

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{2^{h+3}}{|C_H(g)|}.1 = \frac{2.2^{h+2}}{|C_H(g)|}.1 = \frac{2|C_{Q_{2^{h+1}}}(x^{2^h})|}{|C_{\langle x \rangle}(x^{2^h})|}.\varphi(g) = 2.\Phi_j(x^{2^h})$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g≠ $(x^{2^h}, I), g \in H$

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{2^{h+2}}{|C_H(g)|}.(1+1) =$$

$$\frac{2.2^{h+1}}{|C_H(g)|}.(1+1) = \frac{2|C_{Q_{2^{h+1}}}(q)|}{|C_{\langle x \rangle}(q)|}.(\varphi(g) + \varphi(g^{-1})) = 2.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_2^{h+1} and q≠ x^{2^h}

(iv) if g∉H

$\Phi_{(j,1)}(g) = 2.0 = 2.\Phi_j(q)$ SinceH∩CL(g)=ϕ

2- IF H=$\langle (y, I) \rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{8.2^h}{4}.1 = 2.2^h = 2.\Phi_{l+1}(1)$$

(ii) If g= (x^{2^h}, I)=(y^2,I) and g∈ H

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{8.2^h}{4}.1 = 2.2^h = 2.\Phi_{l+1}(x^{2^h})$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g≠ (x^{2^h}, I) and g∈ H,i.e.{g=(y,I) or g=(y^3,I)}

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{8}{4}.(1+1) = 2.2 = 2.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,1)}(g) = 0$ since H∩CL(g)=∅

3-IF H=$\langle (xy, I) \rangle$ ={(1,I),(xy,I),($(xy)^2$,I)=(y^2,I),($(xy)^3$,I)=(xy^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{8.2^h}{4}.1 = 2.2^h = 2.\Phi_{l+2}(1)$$

(ii) If g= (x^{2^h}, I)=($(xy)^2$,I)=(y^2,I) and g∈ H

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot \varphi(g) = \frac{8.2^h}{4}.1 = 2.2^h = 2.\Phi_{l+2}(x^{2^h})$$

Since H∩CL(g)={g} , φ(g)=1

(iii) If g≠ (x^{2^h}, I) and g∈ H,i.e.{g=(xy,I) or g=($(xy)^3$,I)}

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2^{h+1}} \times C_2}(g)|}{|C_H(g)|} \cdot (\varphi(g) + \varphi(g^{-1})) = \frac{8}{4}.(1+1) = 2.2 = 2.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+2,1)}(g) = 0$ since H∩CL(g)=ϕ

Case (II):

If H is a cyclic subgroup of ($Q_{2^{h+1}}\times$ {z}),then:

1-H=$\langle(x,z)\rangle$ 2- H=$\langle(y,z)\rangle$ 3- H=$\langle(xy,z)\rangle$

And φ the principal character of H, Φ_j Artin characters of $Q_{2^{h+1}}$ $1\le j\le l+2$,

then by using theorem (1.6)

1- $\Phi_j(g)=\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i\in H\cap CL(g)$

2- $\Phi_j(g)=0$ if $H\cap CL(g)=\phi$

1- IF H=$\langle(x,z)\rangle$

(i) If g=(1,I) or g=(1,z)

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(1,I)|}\cdot\varphi(g)=\frac{2^{h+3}}{|C_H(1,I)|}.1=\frac{2.2^{h+2}}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{2|C_{Q_{2^{h+1}}}(1)|}{2|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={(1,I),(1,z)}

(ii) if g= $(1,I)\,or\;\; g=(x^{2^h},I)\,or\;\; g=(x^{2^h},z)\,or\;\; g=(1,z),g\in H$

if g=(1,I) or g=(1,z)

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=$$

$$\frac{2^{h+3}}{|C_H(g)|}.1=\frac{2.2^{h+2}}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{2|C_{Q_{2^{h+1}}}(1)|}{2|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$ since H ∩CL(g)={g}, φ (g)=1

if g=($(x^{2^h},I)\,or\;\; g=(x^{2^h},z),g\in H$

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{2^{h+3}}{|C_H(g)|}.1=\frac{2.2^{h+2}}{|C_H(g)|}.1=\frac{2|C_{Q_{2^{h+1}}}(x^{2^h})|}{2|C_{\langle x\rangle}(x^{2^h})|}.\varphi(x^{2^h})=\Phi_j(x^{2^h})$$

since H ∩CL(g)={g}, φ (g)=1

(iii) if {g≠$(x^{2^h},I)\,or\;\; g\ne(x^{2^h},z)$ }, $g\in H$

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{2^{h+2}}{|C_H(g)|}(1+1)=$$

$$\frac{2.2^{h+1}}{|C_H(g)|}.(1+1)=\frac{2|C_{Q_{2^{h+1}}}(q)|}{2|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,z),q∈ $Q_{2^{h+1}}$ and q≠x^{2^h}

(iv) if g∉H

$\Phi_{(j,2)}(g)=0$ SinceH∩CL(g)=ϕ

2- IF H=$\langle(y,I)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z)}

(i) If g=(1,I) or g=(1,z) H∩CL(g)={(1,I),(1,z)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{8.2^h}{8}.1=2^h\;=.\Phi_{l+1}(1)$$

(ii) If g= (x^{2^h}, I)=(y^2,I),(y^2,z) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{8.2^h}{8}.1=2^h=\Phi_{l+1}(x^{2^h})$$

Since H∩CL(g)={g} , φ(g)=1

(iii) If g≠ (x^{2^h}, I) and g∈ H,i.e.{g=(y,I),(y,z) or g=(y^3,I),(y^3,z)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8}{8}.(1+1)=2=\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g $^{-1}$)=1

Otherwise

$$\Phi_{(l+1,2)}(g)=0 \qquad \text{since } H\cap CL(g)=\phi$$

3.IF H=$\langle(xy,I)\rangle$ ={(1,I),(xy,I),((xy)^2,I)=(y^2,I),((xy)^3,I)=(xy^3,I),(1,z),(xy,z),((xy)^2,z), ((xy)^3,z)}

(i) If g=(1,I) or g=(1,z) H∩CL(g)={g}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{8.2^h}{8}.1=2^h=\Phi_{l+2}(1)$$

(ii) If g= (x^{2^h}, I)=((xy)^2,I)=(y^2,I) ,((xy)^2 ,z) and g∈ H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{8.2^h}{8}.1=2^h=\Phi_{l+2}(x^{2^h})$$

Since H∩CL(g)={g} , φ(g)=1

(iii) If g≠ (x^{2^h}, I) and g∈ H ,i.e.g={(xy,I),((xy)^3,I),(xy,z),((xy)^3,z)}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_2}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8}{8}.(1+1)=2=\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g $^{-1}$)=1

Otherwise

$$\Phi_{(l+2,2)}(g)=0 \qquad \text{since } H\cap CL(g)=\phi$$

2.2 Example:

To construct Ar($Q_{128}\times C_2$) by using proposition (2.1) we have

1. $\Phi_{(j,1)}(x^i,I)=2\Phi_j(x^i)$; $x^i \in Q_{2m}$ when $m=2^h$,h∈Z^+,$\Phi_{(j,1)}(y,I)=2\Phi_j(y)$, $\Phi_{(j,1)}(xy,I)=2\Phi_j(xy)$ and $\Phi_{(j,1)}(g)=0$ otherwise ;g∈($Q_{2m}\times C_2$) when $m=2^h$,h∈Z^+.
2. $\Phi_{(j,2)}(x^i,I)=\Phi_j(x^i)$; $x^i \in Q_{2m}$ when $m=2^h$,h∈Z^+,$\Phi_{(j,2)}(y,I)=\Phi_j(y)$, $\Phi_{(j,1)}(xy,I)=\Phi_j(xy)$ and $\Phi_{(j,2)}(x^i,z)=\Phi_j(x^i)$; $x^i \in Q_{2m}$ when $m=2^h$,h∈Z^+,$\Phi_{(j,2)}(y,z)=\Phi_j(y)$, $\Phi_{(j,1)}(xy,z)=\Phi_j(xy)$

Then $Ar(Q_2^7 \times C_2)$=

Γ-classes	[1,I]	$[x^{64},I]$	$[x^{32},I]$	$[x^{16},I]$	$[x^8,I]$	$[x^4,I]$	$[x^2,I]$	[x,I]	[y,I]	[xy,I]	[1,z]	$[x^{64},z]$	$[x^{32},z]$	$[x^{16},z]$	$[x^8,z]$	$[x^4,z]$	$[x^2,z]$	[x,z]	[y,z]	[xy,z]
$\|CL_\alpha\|$	1	1	2	2	2	2	2	2	64	64	1	1	2	2	2	2	2	2	64	64
$\|C_{Q_2^7}(CL_\alpha)\|$	512	512	256	256	256	256	256	256	8	8	512	512	256	256	256	256	256	256	8	8
$\Phi_{(1,1)}$	512	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	256	256	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	128	128	128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	64	64	64	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	32	32	32	32	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	16	16	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	8	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	128	128	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(10,1)}$	128	128	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	256	0	0	0	0	0	0	0	0	0	256	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	128	128	0	0	0	0	0	0	0	0	128	128	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	64	64	64	0	0	0	0	0	0	0	64	64	64	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	32	32	32	32	0	0	0	0	0	0	32	32	32	32	0	0	0	0	0	0
$\Phi_{(5,2)}$	16	16	16	16	16	0	0	0	0	0	16	16	16	16	16	0	0	0	0	0
$\Phi_{(6,2)}$	8	8	8	8	8	8	0	0	0	0	8	8	8	8	8	8	0	0	0	0
$\Phi_{(7,2)}$	4	4	4	4	4	4	4	0	0	0	4	4	4	4	4	4	4	0	0	0
$\Phi_{(8,2)}$	2	2	2	2	2	2	2	2	0	0	2	2	2	2	2	2	2	2	0	0
$\Phi_{(9,2)}$	64	64	0	0	0	0	0	0	2	0	64	64	0	0	0	0	0	0	2	0
$\Phi_{(10,2)}$	64	64	0	0	0	0	0	0	0	2	64	64	0	0	0	0	0	0	0	2

Table (8)

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