Some Generalizations of Near (Seminear) Rings

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Abstract:

In this paper we study smarandache near (seminear) ring, anti-smarandache seminear ring, seminear (near) ring homomorphisim. We obtain some interesting results and many examples about them. We also define and study semiequiprime rings and semiequiprime ideals.

Key words: near (seminear) ring, smaranche near (seminear ring), anti-smarandache seminear ring, seminear (near) ring homomorphisim, semiequiprime ring, equisemiprime ideals.

Introduction:

Many authors studied right (left) near rings and right (left) seminear rings ([1], [2], [3], [4]). [5] introduced the notions of smarandache near (seminear) rings, and anti-smarandache seminear rings. Also he introduced the notion of near (seminear) ring homomorphism.

In section one, we study smarandache near (seminear) rings, and anti-smarandache seminear rings. Also, we gvie many relationships between them, and supported these by several examples. Beside this we give an answer of problem (1) which is given in [5].

In section two, we introduce the notion of semiequiprime near ring (ideal) as a generalization of equiprime near ring (ideal) which are introduced and studied in [6]. We study and give some properties related with these concepts.

Finally all near (seminear) rings in section one of this paper are right near (seminear) rings and all of them (in section two) are left near (seminear) rings.

1- Smarandache Near (Seminear) Rings and Anti-smarandache Seminear Rings

In this section we give many properties about smarandache near (seminear) rings and anti-smarandache seminear ring. We give an answer for problem (1) which is given [5].

Finally, we remark that all near (seminear) rings in this section are right near rings.

First, we recall some definitions which will be used later.

1.1 Definition, [1], [7]:

An algebraic system $(N,+,\cdot)$ is called a right (left) near ring, if it satisfies the following three conditions:

(i) $(N,+)$ is a group (not necessarily abelian).

(ii) $(N,\cdot)$ is a semigroup.

(iii) $(n_1 + n_2)\cdot n_3 = n_1\cdot n_3 + n_2\cdot n_3$ (or $n_3 \cdot (n_1 + n_2) = n_3\cdot n_1 + n_3\cdot n_2$) for all n_1, n_2, $n_3 \in N$ (right (left) distributive law).

1.2 Definition, [1]:

An algebraic system $(N,+,\cdot)$ is called a right (left) seminear ring, if it satisfies the following three conditions:

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(i) (N,+) is a semigroup (not necessarily abelian).
(ii) (N,·) is a semigroup.
(iii) $(n_1 + n_2)\cdot n_3 = n_1\cdot n_3 + n_2\cdot n_3$ (or $n_3 \cdot (n_1 + n_2) = n_3\cdot n_1 + n_3\cdot n_2$) for all $n_1, n_2, n_3 \in N$ (right (left) distributive law).

1.3 Definition, [8]:

An near ring (N,+,·) is called a near field, if it satisfies the following three conditions:

(i) (N,+) is a group (not necessarily abelian).
(ii) (N\{0},·) is a group.
(iii) $(n_1 + n_2)\cdot n_3 = n_1\cdot n_3 + n_2\cdot n_3$ (or $n_3 \cdot (n_1 + n_2) = n_3\cdot n_1 + n_3\cdot n_2$) for all $n_1, n_2, n_3 \in N$ (right (left) distributive law).

1.4 Definition, [5]:

N is said to be a smarandache near ring if (N,+,·) is a near ring and has a proper subset A such that (A,+,·) is a near field.

Now, we have some remarks and examples.

1.5 Remarks and Examples:

1. Every near ring is seminear ring. But the converse is not true in general for example:
$(Z^+,+,\cdot)$ is a seminear ring, but it is not a near ring since $(Z^+,+)$ is not a group, where +, · are the usual addition and multiplication on Z^+, where Z^+ is the set of positive integers.

2. Let X be a nonempty set. Then $(P(X),\cup,\cap)$ is a seminear ring, but it is not a near ring, where P(X) is the power set of X.

3. A near ring may not be a ring, for example:
$(Z,+,\circ)$, where $a \circ b = a$ for all a, $b \in Z$ and + is the usual addition on Z, is a near ring, but it is not a ring.

4. Let $N = \{\bar{1},\bar{8}\} \subseteq Z_9$. Then $(N,\cdot_9,\circ)$ is a near field, where $a \circ b = a$ for all a, $b \in N$. Since $(N,\cdot_9)$ is a group and $(N-\{\bar{1}\},\circ)$ is a group and the condition (iii) of definition (1.3) holds.

5. Let X be a nonempty set, which has more than one element. Then $(P(X),\Delta,\cap)$ is a smarandache near ring, where P(X) is the power set of X and $\Delta = (A\cup B) - (A\cap B)$ for all $A, B \in P(X)$. Since there exists $N = \{\varnothing, X\} \subsetneq P(X)$ and $(N,\Delta,\cap)$ is a near field.

6. A ring $(Z_6,+_6,\cdot_6)$ is a near ring and let $A = \{\bar{0},\bar{3}\} \subset Z_6$. Then $(A,+_6,\cdot_6)$ is a near field. Therefore $(Z_6,+_6,\cdot_6)$ is a smarandache near ring.

7. Not every ring is a smarandache near ring, for example:
Consider the ring $(Z_8,+_8,\cdot_8)$. Then for all $\{\bar{0}\} \neq A \subset Z_8$ such that $(A,+_8)$ is a group, then $A = \{\bar{0},\bar{2},\bar{4}\}$ or $A = \{\bar{0},\bar{4}\}$. But $(\{\bar{2},\bar{4}\},\cdot_8)$ is not a group, also $(\{\bar{4}\},\cdot_8)$ is not a group. Hence $(A,+_8,\cdot_8)$ is not a near field. Thus $(Z_8,+_8,\cdot_8)$ is not a smarandache near ring.

8. A smarandache near ring need not be a ring, for example:
$(Z_6,+_6,\circ)$, where $a \circ b = a$ for all a, $b \in Z_6$ is a near ring, but it is not a ring.
Take $A = \{\bar{0},\bar{3}\} \subset Z_6$, then $(\{\bar{3}\},\circ)$ is a group. Also $(A,+_6)$ is a group and the binary operation $\circ$ is right distributive over the binary operation $+_6$. Thus $(A,+_6,\circ)$ is a near field and $(Z_6,+_6,\circ)$ is a smarandache near ring.

9. Let $N = \{0,x,y,z\}$ with addition and multiplication tables defined as follows

+	0	*x*	*y*	*z*
0	0	*x*	*y*	*z*
x	*x*	0	*z*	*y*
y	*y*	*z*	0	*x*
z	*z*	*y*	*x*	0

·	0	*x*	*y*	*z*
0	0	0	0	0
x	0	*x*	0	*x*
y	0	0	0	0
z	0	*z*	0	*x*

Then (N,+,·) is a near ring. There exists B = {0,x} ⊂ N such that (B,+,·) is a near field, since (B,+) is a group and (B\{0},·) is a group and · distributive over +. Thus (N,+,·) is a smarandache near ring.

Recall the following definition.

1.6 Definition, [5]:

Let (N,+,·) be a seminear ring. Then (N,+,·) is said to be a smarandache seminear ring if there exists a proper subset A of N such that (A,+,·) is a near ring, where +, · are the same operations on N.

1.7 Remarks and Examples:

1. A seminear ring may not be smarandache seminear ring, for example:
(Z^+,+,·), where +, · are the usual addition and multiplication on Z^+. Then it is a seminear ring, but there is no a proper subset A of Z^+ such that (A,+,·) is a near ring. Thus (Z^+,+,·) is not a smarandache near ring.

2. A smarandache seminear ring may not be near ring, for example: (Z_{18},$+_{18}$,∘), where $a∘b=a$ for all a, b∈ Z_{18}. It is clear that it is not a near ring. But it is a smarandache seminear ring, since there exists A = $\{\bar{1}\} \subsetneq Z_{18}$ such that (A,$+_{18}$,∘) is a near ring.

3. For all $n \in Z^+$, consider (Z_n,$\cdot_n$,∘), where $a∘b = a$ for all a, b∈ Z_n. Then (Z_n,$\cdot_n$,∘) is a smarandache seminear ring, since (Z_n,$\cdot_n$,∘) is a seminear ring and there exists A = $\{\bar{1}\} \subsetneq Z_n$ such that (A,$\cdot_n$,∘) is a near ring.

4. As a generalization of example (3), we have the following:
If (N,+,·) is a seminear ring such that N has identity e with respect +, then (N,+,·) is a smarandache seminear ring, since ({e},+,·) is a near ring.

5. Let N = {0}. Then (N,+,·) is a near ring, so it is seminear ring, where +, · are the usual addition and multiplication on N. But it is not smarandache seminear ring, since N = {0} has no proper subset A such that A is a near ring.

We introduce the following:

1.8 Definition:

Let (N,+,·) be a seminear ring. (N,+,·) is called strongly smarandache seminear ring if there exists A $\subsetneq$ N, A ≠ singleton element such that (A,+,·) is a near ring.

Clearly, every strongly smarandache seminear ring is smarandache seminear ring.

1.9 Proposition:

For all $n > 2$, consider (Z_n,$\cdot_n$,○), where $a○b = a$ for all a, b∈ Z_n. Then (Z_n,$\cdot_n$,○) is strongly smarandache seminear ring, and hence smarandache seminear ring.

proof: Let A = $\{x: x \in Z_n, x \cdot_n x = \bar{1}\}$. It is clear that A ⊂ Z_n and (A,$\cdot_n$) is a group. Then (A,$\cdot_n$,○) is a near ring. Therefore (Z_n,$\cdot_n$,○) is a strongly smarandache seminear ring.

By using (Rem. And Ex. 1.7 (2)) and prop. 1.9, we get directly the following result which appeared in [5].

1.10 Corollary:

Let $n = p^k$, p is prime, $k \in Z^+$, $k \geq 1$. Then (Z_n,$\cdot_n$,∘) is a smarandache seminear ring.

Now, we can compute the set A = $\{x: x \in Z_n, x \cdot_n x = \bar{1}\}$ for special cases for n

If $n = 2^2$, then A = $\{\bar{1},\bar{3}\}$

If $n = 2^3$, then A = $\{\bar{1},\bar{3},\bar{5},\bar{7}\}$

If $n = 3^2$, then A = $\{\bar{1},\bar{8}\}$

If $n = 5$, then A = $\{\bar{1},\bar{4}\}$

If $n = 7$, then A = $\{\bar{1},\bar{6}\}$

W.B.Vasantha Kandasamy in [5], gave the following problem: if $Z_n = \{\bar{0}, \bar{1}, ..., \overline{n-1}\}$, $n = p_1^{\alpha_1} ... p_t^{\alpha_t}$ where p_1, …, p_t are distanct primes, $t > 1$. Consider $(Z_n, \cdot_n, \circ)$, where $a \circ b = a$ for all a, $b \in Z_n$. Let A = $\{1, q_1, …, q_r\}$ where q_1, q_2,…, q_r are all odd primes different from p_1, …, p_t and q_1, …, $q_r \in$ Z. Then $(A, \cdot_n, \circ)$ is a near ring.

By proposition 1.9, $(A, \cdot_n, \circ)$ is a near ring, where A = $\{x: x \in Z_n, x \cdot_n x = \bar{1}\}$. However in this case, A = $\{1, q_1, …, q_r\}$ where q_1, …, q_r are odd primes different from p_1, …, p_t. For examples:

Take n = 6 = $2^1 \cdot 3^1$, the set A = $\{x: x \in Z_6, x \cdot_6 x = \bar{1}\} = \{\bar{1}, \bar{5}\}$

5 is prime and 5 ≠ 2, 5 ≠ 3.

If n = 12 = $2^2 \cdot 3^1$, the set A = $\{x: x \in Z_{12}, x \cdot_{12} x = \bar{1}\} = \{\bar{1}, \bar{5}, \bar{7}, \overline{11}\}$

5, 7, 11 are primes and 5, 7, 11 are different from 2, 3.

If n = 24 = $2^3 \cdot 3^1$, the set A = $\{x: x \in Z_{24}, x \cdot_{24} x = \bar{1}\} = \{\bar{1}, \bar{5}, \bar{7}, \overline{11}, \overline{13}, \overline{17}, \overline{19}, \overline{23}\}$

5, 7, 11, 13, 17, 19, 23 are odd primes different from 2, 3.

Now, we turn our attention to direct sum of smarandache seminear rings. We have the following result:

1.11 Theorem:

Let $(N_1, +, \cdot)$ and $(N_2, +', \cdot')$ be two smarandache seminear rings. Then N = $(N_1 \times N_2, \oplus, □)$ is a smarandache near (seminear) ring, where

$(a,b) \oplus (c,d) = (a + c, b +' d)$ for all $(a,b), (c,d) \in N_1 \times N_2$

(a,b) □ $(c,d) = (a \cdot c, b \cdot' d)$ for all $(a,b), (c,d) \in N_1 \times N_2$

proof: If $(N_1, +, \cdot)$, $(N_2, +', \cdot')$ are smarandache seminear rings. Since $(N_1, +, \cdot)$, $(N_2, +', \cdot')$ are seminear rings. $(N_1, +)$, $(N_2, +')$, $(N_1, \cdot)$, $(N_2, \cdot')$ are semigroups. Hence each $(N, \oplus)$ and (N,□) is semigroup. Also □ is right distributive over ⊕ (it is easy to check). Moreover since each of N_1, N_2 are smarandache seminear ring. There exists $A_1 \subset N_1$, $B_1 \subset N_2$ such that $(A_1, +, \cdot)$, $(B_1, +', \cdot')$ are near rings. This implies C = $A_1 \times B_1$ with ⊕,□ is a near ring and $C \underset{\neq}{\subset} N$. Thus (N,⊕,□) is a smarandache seminear ring.

1.12 Definition, [5]:

Let (N,⊕,□) be a near ring. Then (N,⊕, □) is called anti-smarandache seminear ring if there exists $A \underset{\neq}{\subset} N$, such that (A,⊕,□) is a seminear ring.

In fact this definition implies that any near ring is anti-smarandache seminear ring since there exists A = {e}, where e is the identity of (N,⊕), such that (A,⊕,□) is a seminear ring.

Hence, we call a near ring (N,⊕,□) an anti-smarandache seminear ring if there exists $A \underset{\neq}{\subset} N$, A ≠ {e}, where e is the identity of (N,⊕), such that (A,⊕,□) is a seminear ring.

1.13 Remarks and Examples:

1. $(Z, +, \cdot)$ is a near ring and $Z^+ \underset{\neq}{\subset} Z$ such that $(Z^+, +, \cdot)$ is a seminear ring. Thus $(Z, +, \cdot)$ is an anti-smarandache seminear ring, [5].

2. Every smarandache near ring is an anti-smarandache seminear ring.

proof: Let $(N, +, \cdot)$ be smarandache near ring. Then $(N, +, \cdot)$ is near ring and there exists A ⊂ N such that $(A, +, \cdot)$ is a near field, which implies $(A, +, \cdot)$ is seminear ring. Hence $(N, +, \cdot)$ is an anti-smarandache seminear ring.

3. Not $(Z_3 - \{\bar{0}\}, \cdot_3, \circ)$ is anti-smarandache seminear ring, since A = $\{\bar{1}, \bar{2}\} \subset Z_3$ and A ≠ $\{\bar{1}\}$, is a seminear ring, where $a \circ b = a$ for all a, $b \in Z_3$.

4. Every ring which has non-trivial subring is anti-smarandache seminear ring, for example: $(Z_4,+_4,\cdot_4)$ is an anti-smarandache seminear ring.

5. $(Z_2,+_2,\cdot_2)$ and $(Z_5,+_5,\cdot_5)$ have only trivial subring. Hence they are not anti-smarandache seminear ring.

Next, we have the following:

1.14 Theorem:

Let $(N_1,+,\cdot)$ and $(N_2,+',\cdot')$ be two anti-smarandache seminear rings and let $N = N_1\times N_2$. Then $(N,\oplus,\odot)$ is anti-smarandache near ring, where $\oplus,\odot$ defined by

$(a,b) \oplus (c,d) = (a + c,b +' d)$, $(a,b) \odot (c,d) = (a \cdot c,b \cdot' d)$ for all (a,b), (c,d) in N.

for all $(a,b), (c,d) \in N_1\times N_2$

proof: It is easy, so is omitted.

The converse of theorem (1.14) is not true in general as the following example shows.

1.15 Example:

Consider $(Z_6,+_6,\cdot_6)$, $N_1 = <\bar{2}> \underset{\neq}{\subset} Z_6$ and $N_1 = <\bar{3}> \underset{\neq}{\subset} Z_6$. Then $(N_1,+_6,\cdot_6)$ and $(N_2,+_6,\cdot_6)$ are seminear rings. But $N_2 = \{\bar{0},\bar{3}\}$ has no non-trivial proper subset A such that $(A,+_6,\cdot_6)$ is a near ring.

Similarly, N_1 has no non-trivial proper subset B such that $(B,+_6,\cdot_6)$ is a near ring. Thus $(Z_6,+_6,\cdot_6)$ is an anti-smarandache seminear ring. But each of N_1 and N_2 are not anti-smarandache seminear ring.

Recall the following definition in [5].

1.16 Definition, [5]:

A mapping h between two near rings (seminear rings) is called near ring homomorphisim (seminear ring homomorphism) if h is homomorphism.

Next, we have the following results.

1.17 Proposition:

Let $h:(N,+,\cdot) \longrightarrow (N',+',\cdot')$ be a near ring homomorphism such that ker $h \subseteq A$, where A is any seminear ring. Then

1. If N is anti-smarandache seminear ring, then N' is anti-smarandache seminear ring.

2. ker h is a near subring of N.

proof:

1. Since N is an anti-smarandache seminear ring. There exists $A \underset{\neq}{\subset} N$, $A \neq \{e\}$ such that $(A,+,\cdot)$ is a seminear ring. Then $(h(A),\oplus',\odot')$ is a seminear ring (since h is homomorphism). Moreover $A \subset N$, implies $h(A) \underset{\neq}{\subset} N'$.

Suppose $h(A) = N'$. Since $A \neq N$, then there exists $x \notin A$, $x \in N$, implies $h(x)\in N' = h(A)$. Thus there exists $a \in A$ such that $h(x) = h(a)$. Then $x - a \in$ ker $h \subseteq A$, implies $x \in A$ which is a contradiction. Therefore $h(A) \underset{\neq}{\subset} N'$. Hence N' is an anti-smarandache seminear ring.

2. It is clear.

1.18 Proposition:

Let $h:(N,+,\cdot) \longrightarrow (N',+',\cdot')$ be a seminear ring homomorphism such that ker $h \subseteq A$, where A is a near subring of N. If N is a smarandache seminear ring, then N' is a smarandache seminear ring.

proof: We have N is smarandache seminear ring, then there exists $A \underset{\neq}{\subset} N$ such that $(A,+,\cdot)$ is a near ring. It is easy to see that $(h(A,+,\cdot)$ is a near ring. Since ker $h \subseteq A$ (by assumption), we get $h(A) \neq N'$. Thus N' is a smarandache seminear ring.

1.19 Proposition:

Let $h:(N,+,\cdot) \longrightarrow (N',+',\cdot')$ be a near ring homomorphism such that h is one to one. If (N,+,·) is a smarandache near ring, then (N',+',·') is a smarandache near ring.

proof: (N,+,·) is a smarandache near ring, then there exists $A \subsetneq N$ such that (A,+,·) is a near field. We want to prove that $h(A) \neq N'$. Suppose $h(A) = N'$. Then there exists $x \in N$ and $x \notin A$ (since $A \neq N$). Now, $h(x) \in h(A)=N'$, implies $h(x)=h(a)$ for some $a \in A$. Then $x - a \in \ker h = \{0\}$, implies $x = a \in A$. This is a contradiction. Therefore $h(A) \neq N'$ and hence N' is a smarandache near ring.

2- Semiequiprime Near Rings

Equiprime near rings, which generalize prime rings were defined by G.L.Booth together N.J.Gronewald and S.Veldsman, see [6] where a ring R is called prime ring if (0) is prime ideal. That is for each a, $b \in R$, $aRb = 0$, implies $a = 0$ or $b = 0$. In this section we extend this concept to semiequiprime near rings, also semiequiprime ideal is defined. Many properties of these concepts are given.

Note that every near ring in this section means left near ring.

First we list the following definitions which are needed later.

2.1 Definition, [6]:

An ideal of a near ring N is a normal subgroup I of N such that $I N \subseteq I$, $N I \subseteq I$.

2.2 Definition, [6]:

Let (N,+,·) be a near ring. N is called equiprime ring if a, x, $y \in N$, $anx = any$ for all $n \in N$ implies that $a = 0$ or $x = y$.

2.3 Definition, [6]:

An ideal I of a near ring N is called equiprime ideal if N / I is an equiprime ring.

Now, we introduce the following definition.

2.4 Definition:

Let (N,+,·) be a near ring. N is called semiequiprime if whenever a, x, $y \in N$, $a^2nx = a^2ny$ for all $n \in N$, implies that $a^2 = 0$ or $x = y$.

2.5 Definition:

An ideal I of a near ring N is called semiequiprime if N / I is an semiequiprime ring.

Next, we have the following remarks.

2.6 Remark:

Every equiprime ring is a semiequiprime ring, but the converse is not true. For example:

Let $(Z_4,+_4,\cdot_4)$ be a ring. It is easly to show that this ring is semiequiprime but it is not equiprime.

2.7 Remark:

Let (N,+,·) be a near ring. Then the following statements are equivalent:

1. (0) is a semiequiprime ideal.
2. N/(0) is a semiequiprime ring.
3. N is a semiequiprime ring.

Now, we give the following proposition.

2.8 Proposition:

Let I be an ideal of a near ring N. Then the following are equivalent:

a. I is a semiequiprime ideal of N.

b. **(i)** $0N \subseteq I$, where 0 is the additive identity of a ring N.
(ii) whenever a, x, $y \in N$, $a^2nx - a^2ny \in I$ for all $n \in N$, implies $a \in I$ or $x - y \in I$.

c. **(i)** $0N \subseteq I$, where 0 is the additive identity of a ring N.
(ii) whenever x, $y \in N$, $a^2Ny \subseteq I$, implies either $x \in I$ or $y \in I$.
(iii) A is an invariant subgroup of I, $A \not\subseteq I$, $a^2x - a^2y \in I$ for all $a \in A$, implies $x - y \in I$.

proof: It is abvious.

Next, we have the following.

2.9 Proposition:

Let N be a semiequiprime near ring and I be a prime ideal of N. Then $\frac{N}{I}$ is a semiequiprime near ring.

proof: Let (a + I), (x + I), (y + I) $\in \frac{N}{I}$ and let I $\neq n$ + I $\in \frac{N}{I}$. Suppose that $(a + I)^2(n + I)(x + I) = (a + I)^2(n + I)(y + I)$. Then $a^2nx + I = a^2ny + I$, implies $a^2nx - a^2ny \in I$. We must prove that either $a^2 + I = I$ or $x + I = y + I$.
Suppose $a^2 + I \neq I$, then $a^2 \notin I$.
Now, $a^2 (nx - ny) \in I$. Since I is a prime ideal, then either $a^2 \in I$ or $nx - ny \in I$. But $a^2 \notin I$ contradict the hypothesis), so $nx - ny \in I$, which implies $n(x - y) \in I$. Since I is a prime ideal and $n \notin I$ (by hypothesis). Then $x - y \in I$, therefore $x + I = y + I$. This completes the proof.

Now, we deduce the following corollary.

2.10 Corollary:

Let N be a semiequiprime near ring and I,J be two ideals of N such that I $\subseteq$ J and J is a prime ideal of N. Then J / I is semiequiprime in N / I

proof: We have N is semiequiprime near ring and J is prime. From proposition (2.9) we get $\frac{N}{J}$ is semiequiprime. Now, $N/I\big/J/I \cong N/J$. Thus $N/I\big/J/I$ is semiequiprime ring, which implies $\frac{J}{I}$ is semiequiprime ideal (by definition (2.5)).

However, we give the following result.

2.11 Proposition:

Let I and J be two ideals of a near ring N such that I $\subseteq$ J and $\frac{N}{J}$ is semiequiprime near ring, then J / I is a semiequiprime ideal.

proof: We have N / J is semiequiprime near ring, then by (2.5) J is semiequiprime ideal of N. Since $N/I\big/J/I \cong N/J$, implies $N/I\big/J/I$ is semiequiprime ring. Thus by (2.5) J / I is semiequiprime ideal.

From proposition (2.11), we get the following corollary.

2.12 Corollary:

Let I, J be two ideals of a near ring N. If N / [I:J] is semiequiprime near ring. Then $\frac{[I:J]}{I}$ is semiequiprime ideal in N.

proof: Since I $\subseteq$ [I:J]. The result follows from proposition (2.11) and hypothesis.

References:

1. Pilz G. 1977. Near Rings, North-Holland Publ, pp.350.

2. Booth L.G.2005. Primeness in Near Rings of Continuous Function 2, J. of Alg. and Geom., 46(1):207-214.

3. Kandasamy Vasantha W.B. 1996. Idempotent and Semi-idempotent in Near Ring, J. of Sich. Univ., 33(1):330-332.

4. Abujabal, H.A.S., Obaid, M.A. and Khan, M.A. 2000. On Structure and Commutativity of Near Rings, Proyecciones, Antofagasta, 19(2):113-124.

5. Kandasamy Vasantha W.B. 2009. Smarandache Near Rings and Their Generalizations, from Net.

6. Booth G.L., Gronewold N.J. and Veldsman S. 1990. A Kurosh-Amitsure Prime Radical for Near Rings, Comm. Algebra, 18 (9):3111-3122.

7. Wendth G. 2009. On Zero Divisors in Near Rings, J. of Int. of Alg., 3(1):21-32.

8. Abbasi S.J. and Ambreen Zehra Rizvi. 2008. Study of Prime Ideals in Near Rings, J. of Eng. and Sci., 2(1):7-15.

بعض الاعمامات للحلقات التقريبية (شبه التقريبية)

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الخلاصة:

في هذا البحث درسنا الحلقات تقريباً (شبه الحلقات تقريباً) من النمط سمرقند، وشبه الحلقات تقريباً من النمط سمرقند المضادة والتشاكل الحلقي وشبه الحلقي تقريباً. لقد حصلنا على بعض النتائج الشيقة والعديد من الامثلة. كذلك قدمنا ودرسنا مفهوم الحلقات شبه الأولية المعتدلة والمثاليات شبه الأولية المعتدلة.